10.018 Modelling Space and Systems
1D Project - Part 1

Glacier Decline and Climate Change

F07 Group 9

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SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

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Question 1: *What is the approximate value of H(50,- 40)in an appropriate unit? No explanation is needed.*

H(-50,40) = 1050 m

Question 2: *Interpret in your own words what H(x,50) means.*

The heights or thickness of the glacier along y = 50 km due to varying x values.

Question 3: *A geologist friend of yours is interested to study the mathematical equation in the form of H(-50, y) = 1100 m by using the contour plot in Fig.1. Explain the meaning of his/her research and provide the approximated solution(s) through the study.*

Her research is to find out all the distances in the y direction where the height of the glacier is 1100 m, along x=-50 km. Approximated solutions are (-50,48), (-50,26) and (-50,62.5)

Question 4: *Sketch a graph of the H values against the x- distance along the line y = -x for* $x \in [-70, 70]$ *.(i.e. sketch the graph of H(x,-x) against x.) Your graph should clearly include the approximated values and location of all critical points, boundary values and intercepts.*

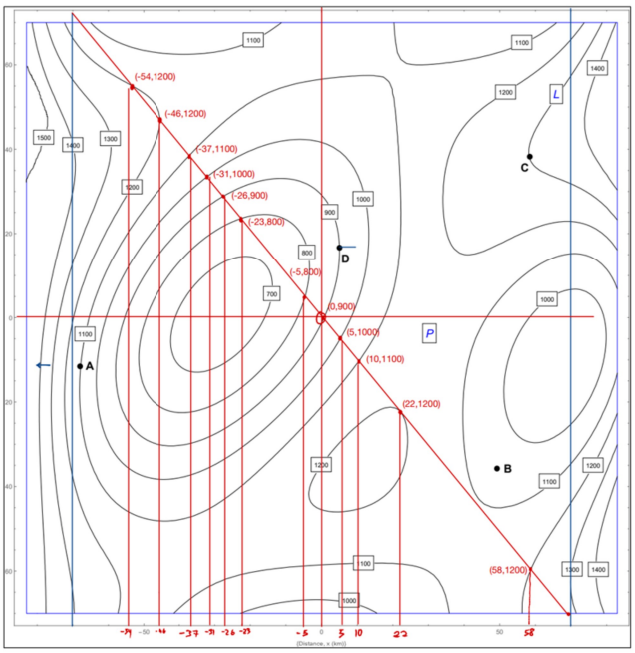

Figure 1.0 Intersection between H(x,y) and y =-x for $x \in [-70, 70]$.

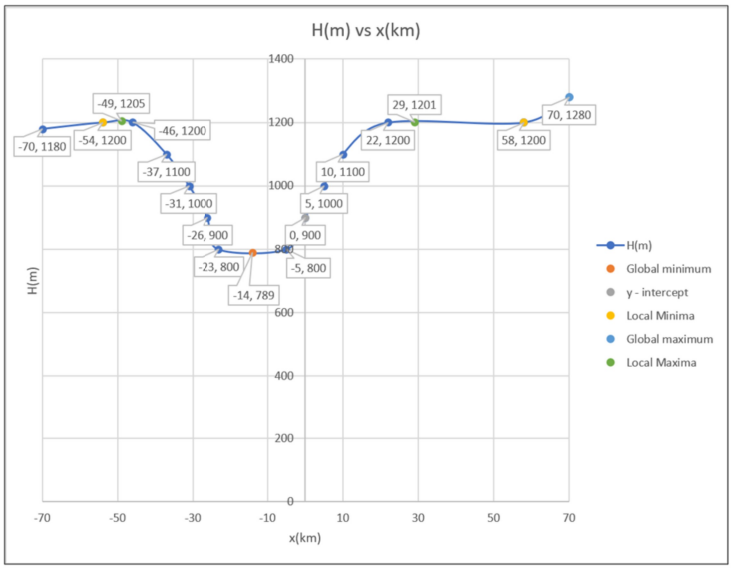

Figure 1.1 Graph of the H values against the x-distance along the line y =-x for $x \in [-70, 70]$.

Question 5: *Consider the level curve L in Fig.1. What is the value H(x, y) for this level set? Explain your answer clearly*

1300 m, by symmetry of the model above and below y = 0 m as well as the average of the nearest two level set curves, 1200 m and 1400 m.

Question 6: *Consider the coordinate point (-50,0) in Fig.1. What is the sign of the partial derivative of H(x, y) w.r.t x and y, i.e. the sign of Hx and Hy? Explain your answer clearly*

The sign of $H_x(-50, 0)$ is negative as the function decreases from 800 m to 700 m along positive ∂x. The sign of $H_y(-50, 0)$ is positive as the function increases from 800 m to $900m$ along positive ∂y

Question 7: *Using Fig.1, approximate the directional derivative of H at the origin in the direction of* $\hat{u} = u_1\hat{i} + u_2\hat{j} = \frac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$. *Hint: graphical method can be useful*

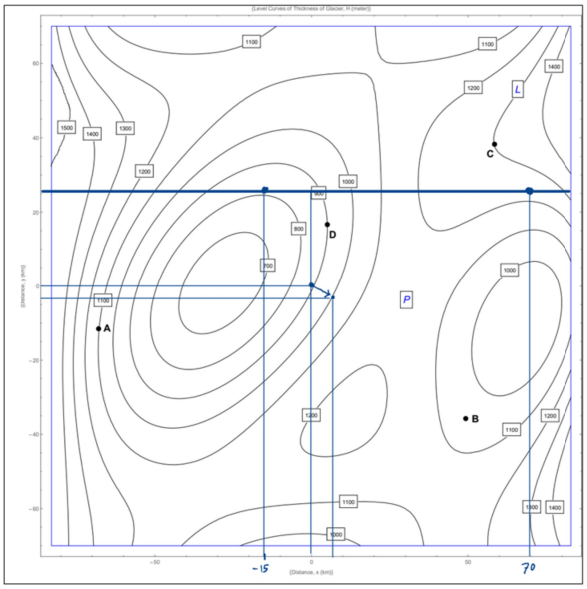

Figure 1.2 Method to finding the cartesian coordinates of the gradient vector.

From Figure 1.2, the gradient vector points to an increase by 7 in the x direction and decrease by 8 in the y direction as the level set increases from 900 m to 1000 m. Therefore,

$$\nabla H(x,y) = 7\hat{\imath} - 8\hat{\jmath}$$

$$\vec{u} = -\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$$

$$D\vec{u}H(x,y) = \left(7 * \frac{-1}{\sqrt{2}}\right)\hat{\imath} * \hat{\imath} + \left(-8 * \frac{1}{\sqrt{2}}\right)\hat{\jmath} * \hat{\jmath}$$

$$= \frac{-15}{\sqrt{2}}$$

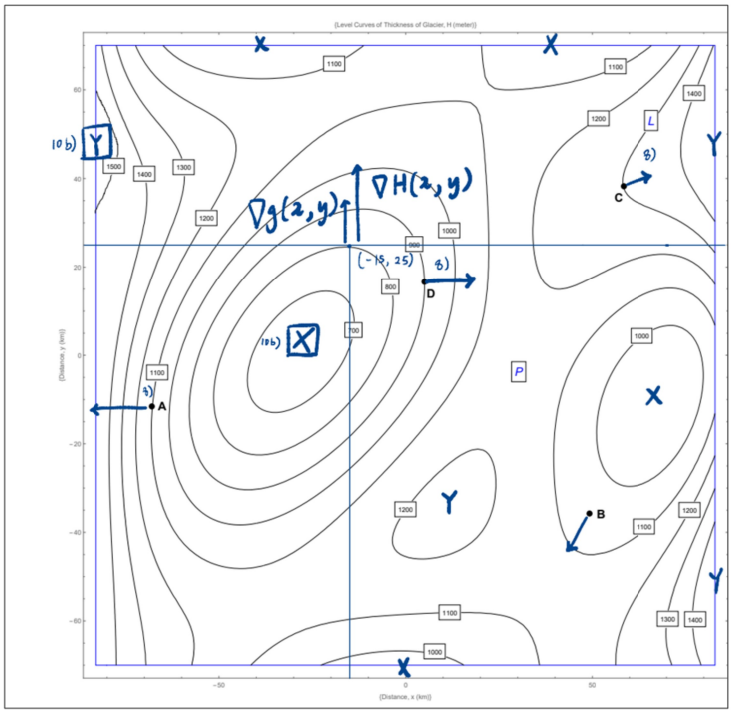

Figure 1.3 Method to finding the cartesian coordinates of the gradient vector.

Question 8: *On Fig.8, sketch the direction of gradient vector $\nabla H(x, y)$ for the points A, B, C and D. Note:Leave blank (and provide a reason) for point(s) that are inconclusive from the given information*

Refer to Figure 1.3

Question 9: *Here is my second question*

$A > D > C > B$

Question 10: *Finding extrema and checking the criticality of a point (a) Indicate on the same Fig.8, the approximate location of local minima of $H(x, y)$ with an "X" and the approximate location of local maxima of $H(x, y)$ with an "Y". Hint: There are many local extrema. You will be awarded full marks if you can indicate more than 8 correct local extrema. b) Does a global extrema for $H(x, y)$ exist? If yes, box the " X " and " Y " that represent the global minima/maxima in the same diagram as in part (a). (c) Based on the contour plot, is the point P indicated on Fig. 8 a critical point? - If YES, provide the nature of the point with a supporting argument. - If NO, give detailed reasoning. Explain your answer clearly.*

a.) Refer to Figure 1.3 for solution

b) Yes, as the region provided for the function of H(x,y) is bounded and closed, by the extreme value theorem, global extrema exist within this region.

c) Yes. Moving in the x direction, P is a local maximum point, while moving in the y direction P is a local minimum point, hence P is a saddle point.

Question 11: *Consider the constraint $y = 25$. (a) What is the expression for $g(x, y)$, where $g(x, y) = 0$ produces the above constraint? (b) There is a point along the constraint $y = 25$ where the condition for Lagrange multiplier theorem is satisfied. Write down the approximate coordinates of this point.*

a) y-25=0
b) (-15,25)
c) Refer to Figure 1.3 for solution.
d) The point indicated is a minimum point, as the height of the contour tangent to the constraint represented by line y=25 is smaller than the rest of the contours intersecting y=25.

Question 12: *To study the effect of changes in glacier thickness/height in relation to climate changes, environmentalists plot H(x, y,t) over time. As a simplified approach, one can measure H(x, y,t) starting from the origin O, varies y-coordinate over time and keep $x = 0$ throughout. One obtained the following.Figure 6: Level Curves of the thickness of the Glacier "Avatar", $H(0, y, t)$ (in m) over time (in months). (a) Interpret in your own words what $H(0, 30, t)$ means. Numerically, $H(0, 30, t)$ is bounded, i.e. $\alpha > H(0, 30, t) > \beta$. Based on the level curves, determine the best value of both α and β. [2 marks] (b) Through observations, environmentalists came up with a phenomenological proposal stating that the index of climate change is proportional to the absolute rate of change of the glacier thickness/height locally, i.e $\left|\frac{\partial H(0,y,t)}{\partial t}\right|$. Indicate in Fig.9 the possible range of y-values with the worst climate change index with an index "R". Explain your reasoning. [2 marks]*

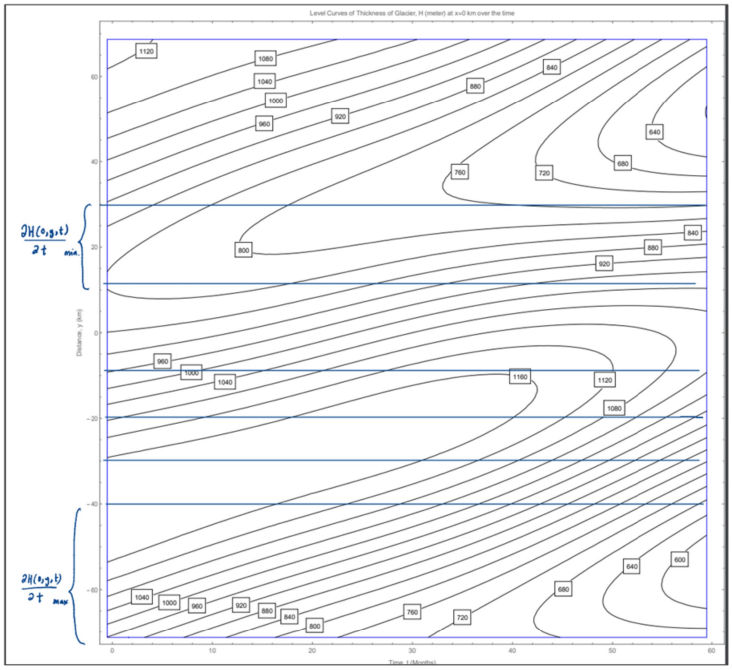

Figure 1.4 Solution to 12 on the second contour map provided.

a) H(0,30,t) is the line representing the height of the glacier at x=0 km and y=30 km against time t months. a = 750 b = 920

b) $-70 < y < -40$ The contour line density in this region of the graph of H(x,y) vs t is the greatest, so as t increases, each point along x = 0 for $-70 < y < -40$, the rate of change of H(x,y) would be the greatest.

c) $10 < y < 30$ Points along x=0 within this region intersect the least number of contour curves, so there would be the least amount of fluctuation in H(x,y) and would pose the least amount of danger to the scientist.