10.018 Modelling Space and Systems – 1D Project

Due Date: 28 Feb 2023, 6.00 PM (Week 6)

Glacier Decline and Climate Change

Disclaimer: This is a work of fiction. Names, characters, organizations and events are the product of the author's imagination.

Right after their Freshmore year, a group of XUTD students conducted a research study on the issue of global warming and climate change that could potentially affect human population and ecosystems in the coming century if the problems continued to worsen. As part of their study, they focus on the glacier change in terms of the net glacier loss, measured through the volume of a glacier over time for a particular region of about twenty-three thousand km^2. Climate change and the loss of glaciers will lead to severe flooding, water shortage for millions of people, and sea level rise destroying coastal communities and habitats. Hence, climate change has a tremendous impact on the ability of future generations to meet their sustainability needs.

Let $H(x,y) : R^2 \to R$ be a scalar function describing the height or thickness of the famous Glacier "Avatar" at a coordinate (x,y). The units for x and y are given in kilometer (km). The unit for $H(x,y)$ is given in meter (m). Fig.1 shows the level curves of $H(x,y)$ for the Glacier Avatar at a specific time, t_0. ALL level sets (in interval of 100 m) for 700 m, 800 m, 900 m, 1000 m, 1100 m, 1200 m, 1300 m, 1400 m and 1500 m are shown explicitly. The blue lines in Fig.1 are the boundaries of the rectangular closed and bounded region $\mathscr{G}$ defined by $-83 \text{ km} \leq x \leq 83 \text{ km}$ and $-70 \text{ km} \leq y \leq 70 \text{ km}$, respectively. Noted that Fig. 1 is purely a toy model.

The 1D project consists of two parts. For Part I (50 marks), you need to analyse the contour plot and level curves of the height/thickness of the Glacier "Avatar". In Part II (25 marks), you need to perform math modelling. The 1D project shall contribute 25% towards your final total aggregate.

Instructions[1] (A bonus of 2 marks is set aside for following the instructions closely)

- You will be working in teams for this project. The teams will be the default team in your cohort.
- Type or write your solutions with all workings in report format.
- Please state the **unit(s)** clearly for all your answers.
- Include **Name, Student ID** and **Project Contributions** of each team member in your reports (for BOTH for Part I and Part II).
- For Part II, you need to prepare a short 10 minutes video on your math modelling. Upload the video to YouTube (set to private) and place the URL in the report so that we can assess it easily. Refer to Part II for more details.
- Prepare your Part I report in a SINGLE pdf file named FXX_teamYY_Part1.pdf where XX is your cohort number and YY is your team number.

[1] In total, you should submit TWO separate hard-copy reports (Part I + Part II) and TWO separate soft-copy reports (Part I + Part II)! You are also required to prepare a short 10-minute video on Part II (explain your math modelling). Upload the video to YouTube and submit the URL to us.

- Prepare your Part II report in a SINGLE pdf file named FXX_teamYY_Part2.pdf where XX is your cohort number and YY is your team number.
- Submit HARDCOPY of BOTH reports to an appointed class rep.
- Submit BOTH reports (softcopy) by EMAIL to BOTH of your instructors. Their email address can be found in eDimension.

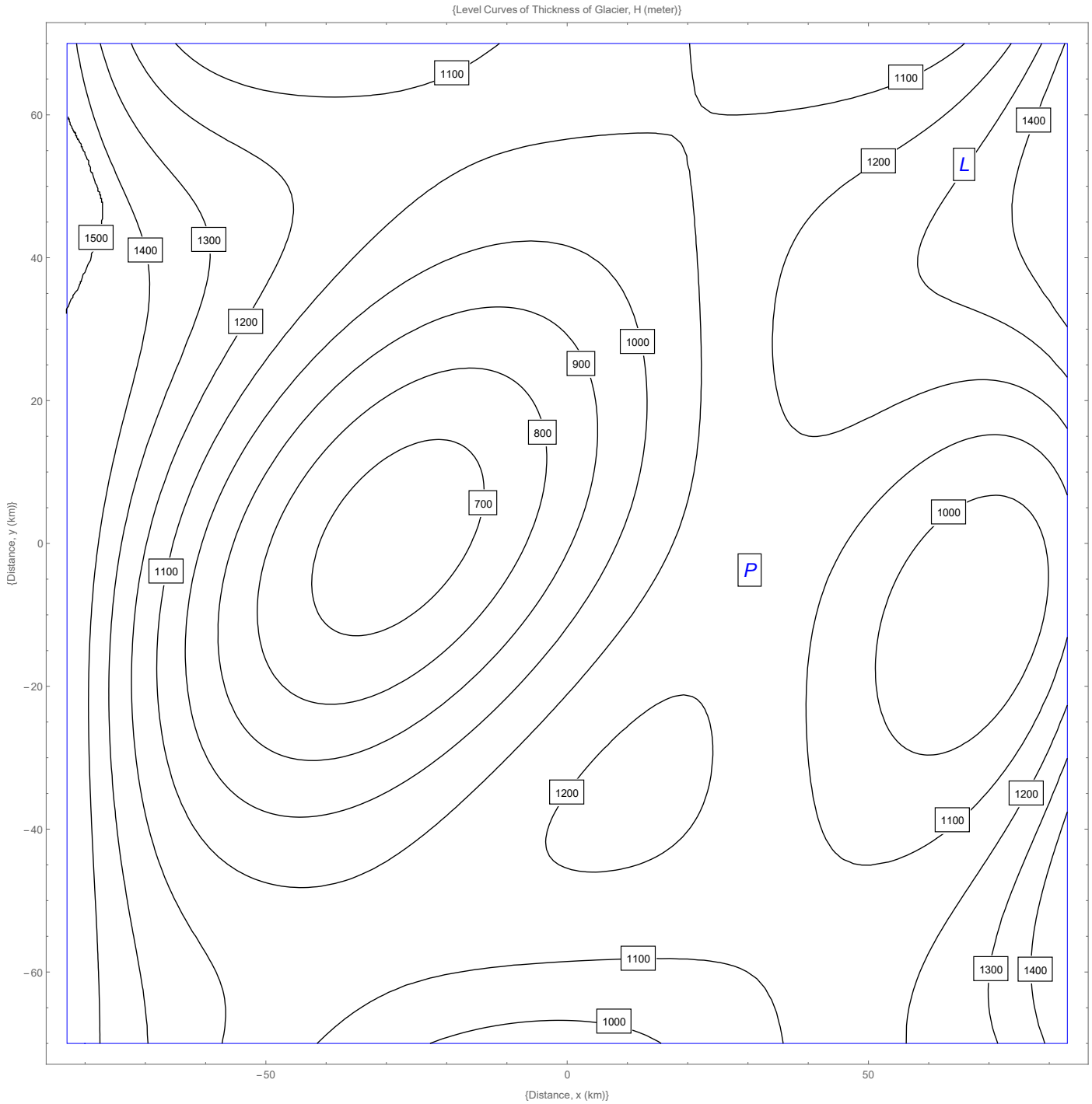

Figure 1: Contour plot and level curves of the height/thickness of the Glacier "Avatar", $H(x,y)$ (in m).

Part I: Multi-variable Calculus

1. What is the approximate value of $H(50,-40)$ in an appropriate unit? No explanation is needed. **[2 marks]**

2. Interpret in your own words what $H(x,50)$ means. **[2 marks]**

3. A geologist friend of yours is interested to study the mathematical equation in the form of $H(-50,y) = 1100$ m by using the contour plot in Fig.1. Explain the meaning of his/her research and provide the approximated solution(s) through the study. **[4 marks]**

4. Sketch a graph of the H values against the x-distance along the line $y = -x$ for $x \in [-70,70]$. (i.e. sketch the graph of $H(x,-x)$ against x.) Your graph should clearly include the approximated values and location of all critical points, boundary values and intercepts. **[7 marks]**

5. Consider the level curve L in Fig.1. What is the value $H(x,y)$ for this level set? Explain your answer clearly. **[2 marks]**

6. Consider the coordinate point $(-50,0)$ in Fig.1. What is the sign of the partial derivative of $H(x,y)$ w.r.t x and y, i.e. the sign of H_x and H_y? Explain your answer clearly. **[2 marks]**

7. Using Fig.1, approximate the directional derivative of H at the origin in the direction of $\hat{u} = u_1\hat{i} + u_2\hat{j} = \frac{1}{\sqrt{2}}\left(-\hat{i} + \hat{j}\right)$. Hint: graphical method can be useful. **[3 marks]**

8. On Fig.8, sketch the direction of gradient vector $\nabla H(x,y)$ for the points A, B, C and D. Note: Leave blank (and provide a reason) for point(s) that are inconclusive from the given information. **[4 marks]**.

9. Refer to your answer in question 8, rank the magnitude (or norm) of $\nabla H(x,y)$ at points A, B, C and D in descending order, i.e. starting from the largest. **[2 marks]**

10. Finding extrema and checking the criticality of a point

    (a) Indicate on the same Fig.8, the approximate location of local minima of $H(x,y)$ with an "X" and the approximate location of local maxima of $H(x,y)$ with an "Y". Hint: There are many local extrema. You will be awarded full marks if you can indicate more than 8 correct local extrema. **[4 marks]**

    (b) Does a global extrema for $H(x,y)$ exist? If yes, box the "X" and "Y" that represent the global minima/maxima in the same diagram as in part (a). **[2 marks]**

    (c) Based on the contour plot, is the point P indicated on Fig.8 a critical point?

    - If YES, provide the nature of the point with a supporting argument.
    - If NO, give detailed reasoning.

    Explain your answer clearly. **[3 marks]**

11. Consider the constraint $y = 25$.

    (a) What is the expression for $g(x,y)$, where $g(x,y) = 0$ produces the above constraint? **[1 mark]**

    (b) There is a point along the constraint $y = 25$ where the condition for Lagrange multiplier theorem is satisfied. Write down the approximate coordinates of this point. **[2 marks]**

(c) Sketch on the same Fig.8 as in question 8, the $\nabla g(x,y)$ and $\nabla H(x,y)$ at the above point in question b. **[2 marks]**

(d) Is the point you have indicated a maximum or a minimum? Explain your answer. **[2 marks]**

12. To study the effect of changes in glacier thickness/height in relation to climate changes, environmentalists plot $H(x,y,t)$ over time. As a simplified approach, one can measure $H(x,y,t)$ starting from the origin O, varies y-coordinate over time and keep $x = 0$ throughout. One obtained the following,

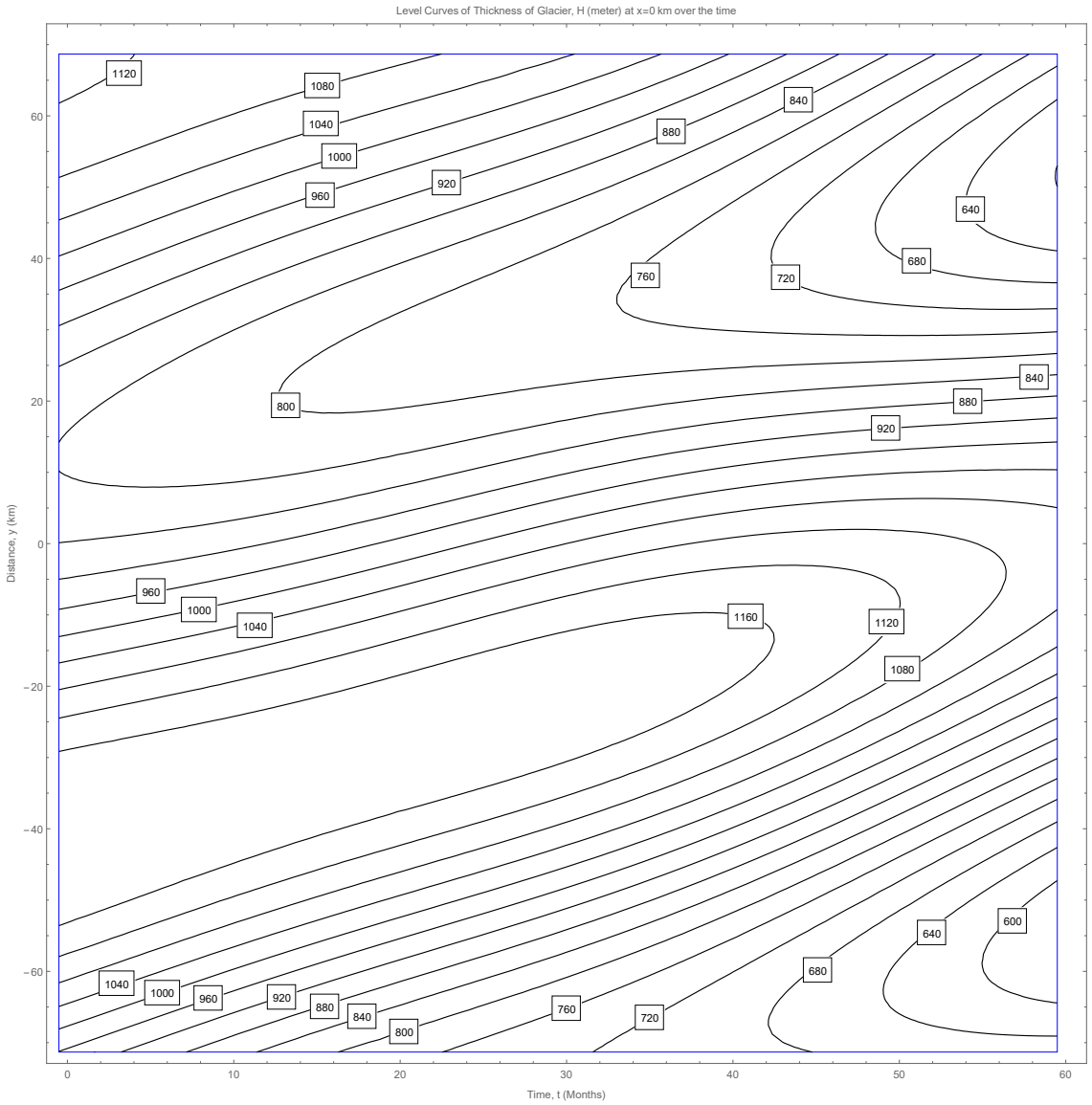

Figure 6: Level Curves of the thickness of the Glacier "Avatar", $H(0,y,t)$ (in m) over time (in months).

(a) Interpret in your own words what $H(0,30,t)$ means. Numerically, $H(0,30,t)$ is bounded, i.e. $\alpha > H(0,30,t) > \beta$. Based on the level curves, determine the best value of both α and β. **[2 marks]**

(b) Through observations, environmentalists came up with a phenomenological proposal stating that the index of climate change is proportional to the absolute rate of change of the glacier thickness/height locally, i.e $\left|\frac{\partial H(0,y,t)}{\partial t}\right|$. Indicate in Fig.9 the possible range of y-values with the worst climate change index with an index "R". Explain your reasoning. **[2 marks]**

(c) To carry out the geological research, the environmentalists decide to set up a research facility along $x = 0$ km over the period of 5 years. Based on the level curves in Fig.6, can you suggest a suitable location? Indicate in Fig.9 the range of y-values for the suitable location with an index "S". Briefly explain your reasoning. **[2 marks]**

Part II: Math Modelling

What is the best sustainable effort to fight climate change?

In your group, devise a concise restatement of the above question and perform the required math modelling by following the steps in our cohort activities.

Format of the submission:

- One (1) page executive summary
- At most four (4) pages report which should include
  - clear problem definition;
  - list of assumptions and variables used;
  - problem solution;
  - analysis and model assessment;
  - strength and weakness;
  - references.
- A 10-minute video presentation. Upload the video to the YouTube (set to private) and provide us with the URL in your report.
- In summary, 1-page executive summary + 4-page report + YouTube video URL to be submitted. **Please state all units clearly**

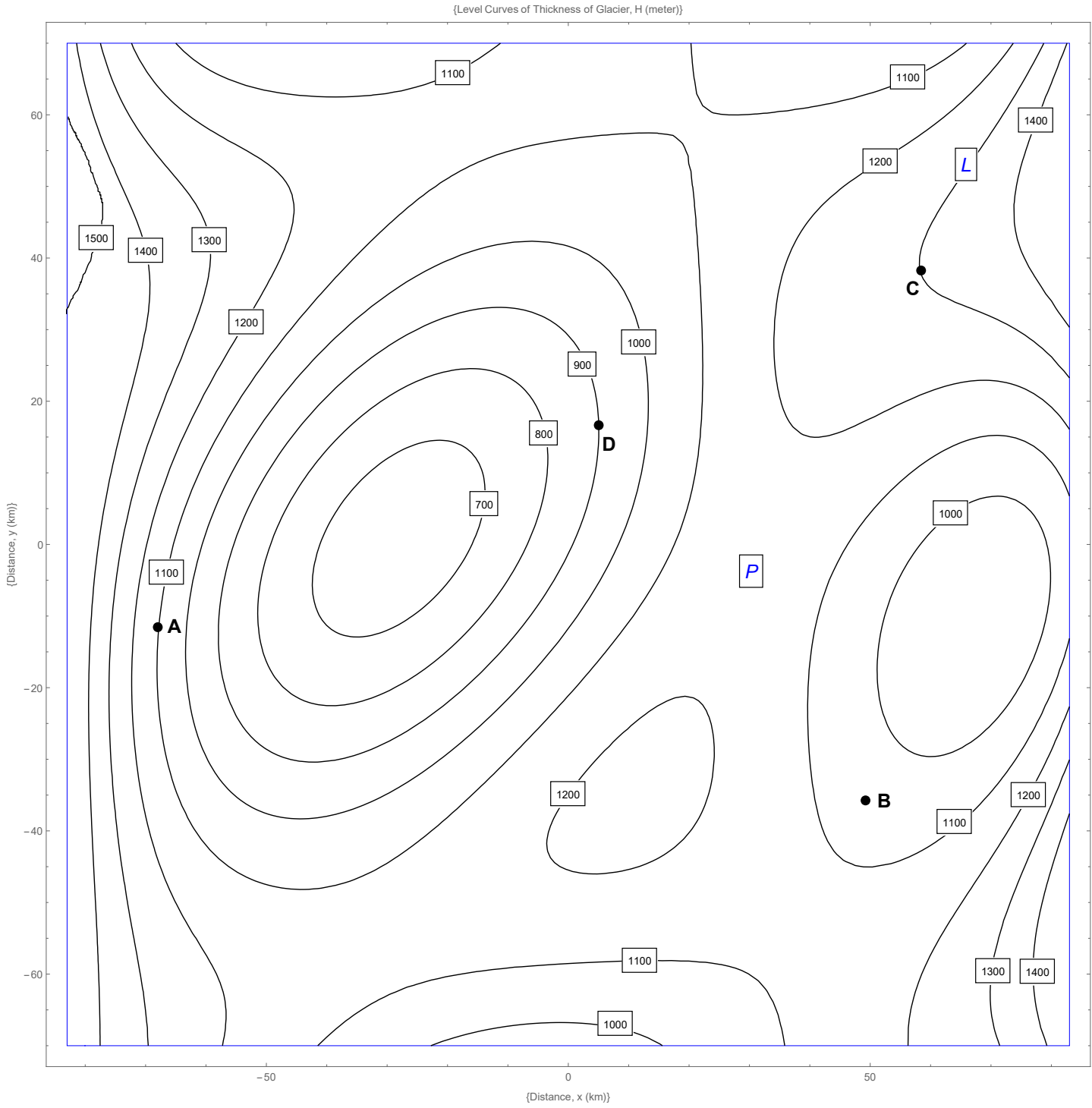

Figure 8: Answers for question 8, 10 and 11 are to be sketched or indicated on this figure. Attach this figure with your submission.

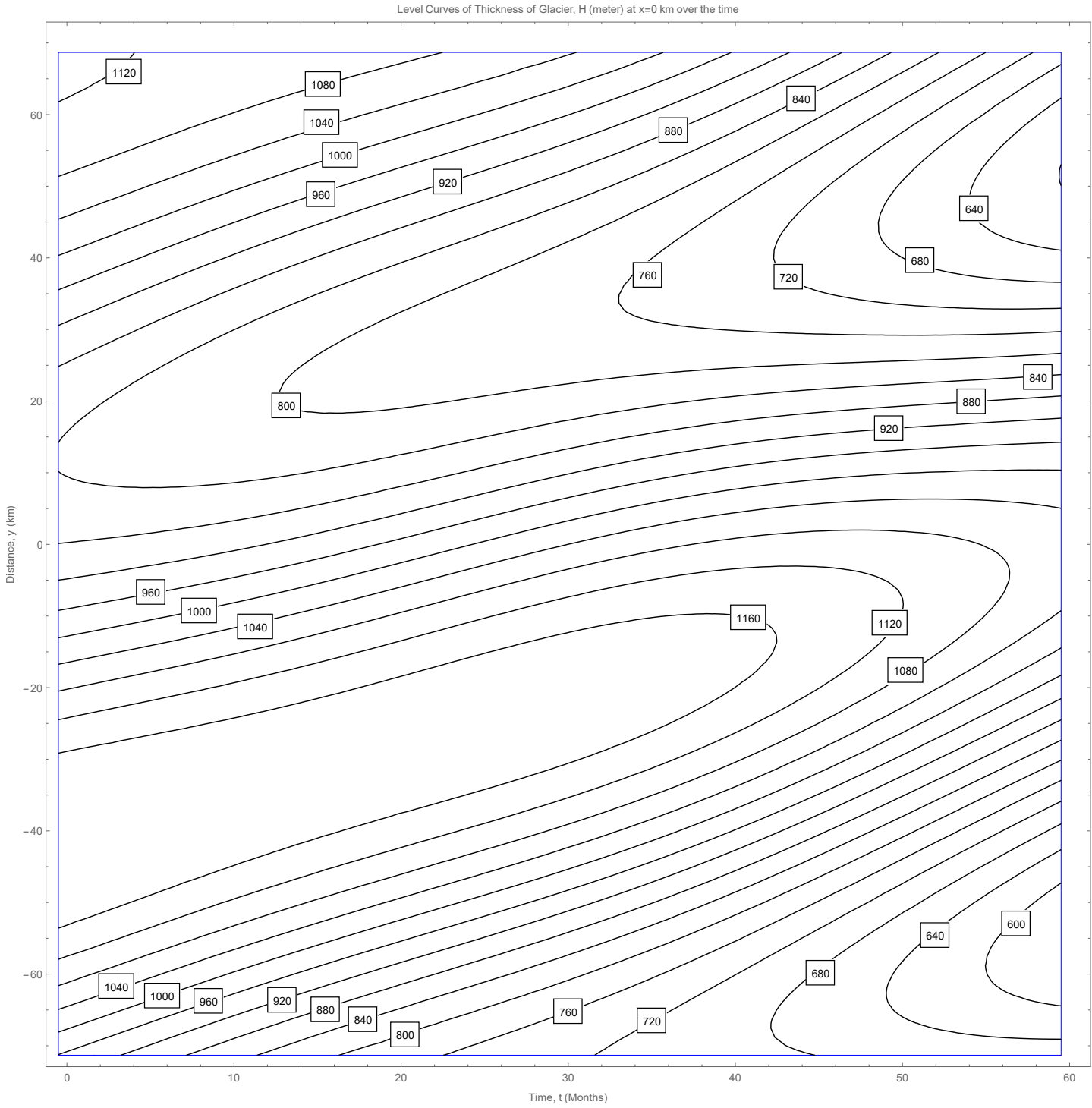

Figure 9: Answers for question 12 are to be indicated on this figure. Attach this figure with your submission.