# 10.018 Modelling Space and Systems
# 1D Project - Part 2

# Comparison of Alternative Vehicle Power Type Ratios on Greenhouse Gas Emissions Using 3-Dimensional Linear Programming


## F07 Group 9

| Name | Student ID | Project Contribution |
|---|---|---|
| Cyan Koh Shi-An | 1007230 | Collected, aggregated and extrapolated data from a range of sources to be processed using the model, defined the model variables, provided troubleshooting support throughout the coding process, and compiled the executive summary. |
| Jervis Lu Shi Tian | 1007204 | Assisted the team in the writing of the report, recorded, edited, and uploaded the 10-minute video, and reached out to the instructors for consultation. |
| Nathan Ansel | 1007492 | Developed the foundations and mathematical equations for the math model, programmed the Python code for the linear program, and compiled the executive report and summary alongside the tables and graphs. |
| Sean Hung Xiang Hui | 1007171 | Assisted the team in the writing of the report, specifically ensuring all references were documented properly, both in the references section and the in-text citations within the document. |
| Zia Mohammad Saif | 1007107 | Thought of a strength and a weakness. |



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## EXECUTIVE SUMMARY


### Overview

The Singapore Green Plan 2030 targets nationwide long-term net-zero emissions by 2050. The Land Transport Authority (LTA) of Singapore aims to reduce emissions from land transport by 80%. One way it has tried to achieve this aim is through the introduction of electric vehicles (EVs) and providing incentives for the private transport sector to adopt them. While EVs may have significantly lower carbon emission rates compared to their internal combustion engine (ICE) counterparts, they may produce considerably higher emissions during their production phases. With advancements in technology, EVs and hybrid vehicles have the potential to become even more efficient and sustainable. As such, the adoption of electric vehicles is an issue that Singapore must address. Our problem statement is summarised as follows:

*How might we develop a mathematical model to predict the right combination of EVs, ICE, and hybrid cars to minimise the total carbon emissions of private transportation in Singapore?*

### Methodology

Data was aggregated from a variety of sources to get an estimate of the $CO_2$ equivalent produced by ICE, electric and hybrid cars over their lifespan at a specific year. We then extrapolated current trends to estimate the total emissions over a 30-year period. We introduced scenarios in which the maximum proportion of cars of the different types was varied, representing alternate policy changes.

We then passed this data into our model, given below, where *t* is the year, *i* is the car type, *x* is the projected number of cars, and *LCA* is the projected $CO_2$ equivalent. We then attempt to minimise the output to get the best-case scenario that fits the scenarios defined earlier.

$$\sum_{t=2023}^{T=2053} CO_2 = \sum_{t=2023}^{T=2053} \sum_{i=0}^{n=3} x_{t,i} \cdot LCA_{t,i}$$

### Results

From our data aggregation step, we noted that the emissions (defined by $CO_2$ equivalent from the LCA of the vehicle) of EVs were increasing as new models were released, while that of hybrid vehicles was decreasing. Emissions of ICE vehicles would remain stationary. Despite this trend, purely in terms of $CO_2$ production, our model predicted that switching to EVs would be best, followed by hybrid vehicles, with continuing with the current ICE-dominated population being the worst outcome.

### Strengths & Limitations

The model adapts easily to additional types of cars, scenarios and changes in car efficiency over time. However, it relies on sparse data from all over the world, which may not be applicable to Singapore's context. Current trends in emissions may also not hold in the long term. The model does not account for future changes in Singapore's energy sources too, which may cause EVs to seem less ideal than they might otherwise be. With more complete data and more time, the model can be adapted to become much more accurate.

# I. INTRODUCTION

In its effort to battle the effects of climate change, the Singapore government has introduced The Singapore Green Plan 2030 which targets nationwide long-term net-zero emissions by 2050. As part of this multi-ministerial effort, the Land Transport Authority of Singapore aims to reduce emissions from land transport by 80% (Land Transport Authority (LTA), 2022). One way it has tried to achieve this aim is through the introduction of electric vehicles (EVs) and providing incentives for the private transport sector to adopt them.

However, the road to EV adoption is not without its bumps. While EVs may have significantly lower carbon emission rates compared to their combustion engine counterparts (i.e. internal combustion engine (ICE) vehicles or hybrid vehicles), they may produce considerably higher emissions during their production phases as Lithium-ion batteries are emission-intensive to produce (MIT Climate, n.d.). Furthermore, the initial cost of purchasing EVs is much more expensive, which might impede demand for these cars (Anthony, 2022).

Despite these challenges, however, EVs undoubtedly offer a glimmer of hope for a greener future. With advancements in technology, they have the potential to become even more efficient and sustainable. As such, the adoption of electric vehicles is a pressing issue that Singapore must address. Our problem statement is summarised as follows:

*How might we develop a mathematical model to predict the right combination of EVs, ICE, and hybrid cars to minimise the total carbon emissions of private transportation in Singapore?*

# II. ASSUMPTIONS, VARIABLES, & DATA

### A. Assumptions

The following assumptions are made due to a lack of available data or to simplify our model:

1. Electric vehicles are charged using 2% renewable and 98% non-renewable sources. As cleaner sources of energy become available, this may make EVs less polluting
2. The total number of cars in Singapore remains constant at 650,000,
3. A car's lifespan in Singapore is 10 years and no car is used for a longer or shorter period,
4. Car age is evenly distributed across the different types of cars,
5. The vehicle and fuel LCA[1] applied (Green NCAP, 2022) is easily adapted to represent that of cars and fuel in Singapore,
6. The LCA rate of a car is linear in relation to the change in time and can be predicted with sufficient accuracy using linear regression analysis. This means that a car type's LCA rate either decreases or increases constantly over time.
7. The small sample size of car models used in our calculation for LCA is representative of the current market in SG.
8. The car models used in the LCA calculation are representative of the technology and hence LCA at the time of the release of its first generation.

[1] See part B. Variables for explanation on the term 'LCA'

### B. Variables

Our model was built upon several variables:

**Bounds**

- ***T***: The upper bound on the year (i.e. 2053).
- ***n***: The number of types of cars.

**Independent Variables**

- ***x***: The number of a certain type of car.
- ***LCA***: The $CO_2$ equivalent of the cars over their lifespan, measured in tons $CO_2$ eq (i.e. the total $CO_2$ produced throughout the production, usage, and end-of-life phases of the car).[2]
- ***D***: Demand for cars. Every year, new cars have to be produced in order to replace old cars as we assume that 11-year-old or older cars are to be discarded. The demand variable represents the number of cars to be produced in a certain year such that the overall number of cars stays at 650,000. Thus, the demand for cars in a year is the number of 10-year-old cars in the previous year.

**Dependent Variable**

- $\boldsymbol{CO_2}$: The total $CO_2$ equivalent generated by the cars, measured in tons $CO_2$ eq.

Our variables are represented by a 2D matrix of indices $t$ and $i$, where $t$ is the year in question (also the production year of the car) and $i$ is the type of car. Theoretically, our model can be applied to more types of cars, given that sufficient data is provided. However, due to a lack of data accessibility, we will only consider three types of cars in this study: Internal Combustion Engine (ICE, $i=1$), Electric ($i=2$) and Hybrid ($i=3$). A sample of the matrix can be seen below, where $x_{2025,1}$ represents the number of ICE cars in the year 2025.

| YEAR | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| 2025 | $x_{2025,1}$ | $x_{2025,2}$ | $x_{2025,3}$ |

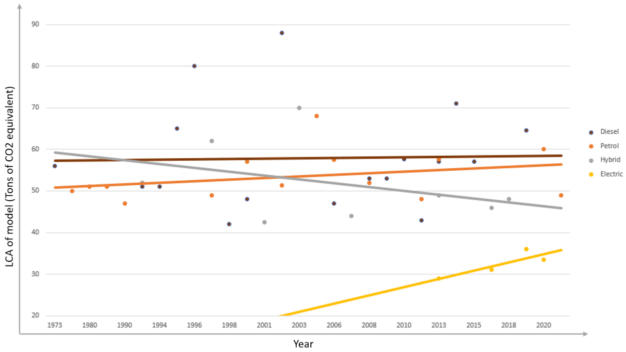
This matrix is crucial for two reasons. Firstly, it is important to differentiate cars across different years even when they are of the same type of power mechanism. This is because cars can vary in their LCA rates even when they are of the same power source (e.g. two EVs have different LCA rates because they are produced in two different years). Secondly, the demand for cars in Singapore across different years changes over time, making it necessary to differentiate the combination of cars from one year to another. These fluctuations often occur in cyclical patterns as in some years car owners have to change their cars due to expiring Certificate of Entitlements (COEs) (or other reasons) more so than in other years.

### C. Data

In order to proceed, we need to aggregate data as a starting point for our model. First, we need to collect LCA rates for the three different types of cars we selected. It is important to note that the LCA of a car may change over time due to various reasons. We have assumed, for the sake of simplicity, that the rate of change of LCA with respect to time is linear. By aggregating data for LCA from Europe (Green NCAP, 2022), earliest car model year (Back4App, 2022) and Singapore's Land Transport Authority (2022)

[2] From here onwards, the term LCA shall be used to denote the total carbon emission produced throughout the lifecycle of a car. Please do not confuse this term with the process of conducting a Life Cycle Assessment.

statistics for motor vehicle population and age distribution of car population, we can perform linear regression to estimate future changes in car efficiency in terms of LCA.




From this, we adjust for year[3] and car age to obtain the following time-dependent LCA functions:

$$LCA_1 = 34.375$$
$$LCA_2 = 0.3627839105t - 713.9791124$$
$$LCA_3 = -0.1743241405t + 382.6225211$$

It is also using this function that we interpolated $LCA_2$ for the years 2013 to 2015[4].

In addition, we also need to calculate the distribution of cars by age. The age distribution directly affects the demand for cars from the years 2023 and onwards and is therefore important to take into consideration. From LTA's (2022) report on the distribution of car age, we can find the following data:[5]

| AGE | CARS | AGE | CARS |
|---|---|---|---|
| 1 | 30,168 | 11 | 6,196 |
| 2 | 45,007 | 12 | 10,992 |
| 3 | 44,242 | 13 | 20,058 |
| 4 | 72,246 | 14 | 37,594 |
| 5 | 80,028 | 15 | 34,474 |
| 6 | 89,922 | 16 | 6,815 |
| 7 | 82,466 | 17 | 5,223 |
| 8 | 47,944 | 18 | 933 |
| 9 | 20,009 | 19 | 311 |
| 10 | 10,969 | 20 | 151 |

As there are a total of 127,000 cars aged 10 or above, there are 127,000 cars to be replaced by the year 2023 (which is the year when we start applying our model).

With these two sets of data, we are now well-equipped with all the necessary information to apply our model, and we thus proceed to the solutions section.

## III. SOLUTION

### A. Carbon Emissions Equation

To begin, we attempt to derive an equation to quantify the amount of $CO_2$ that a combination of cars manufactured within a year would produce throughout its 10-year lifespan. Conveniently, because we have adjusted the LCA rates for the cars to a 10-year lifespan, we can simply multiply the number of cars for every car type with its LCA and sum them up to calculate the total $CO_2$ produced by the said combination of cars. This process can be represented by the following equation:

$$\text{For the year } t:$$
$$CO_{2,t} = \sum_{i=1}^{n=3} x_{t,i} \cdot LCA_{t,i}$$

where $x_{t,i}$ is the number of the $i$-th type of car produced at the year $t$ and $LCA_{t,i}$ is the total $CO_2$ produced during the car's lifespan. We can now iterate this equation over the years $t$ = 2023 and $T$ = 2053. Thus, the total $CO_2$ produced over these 30 years can be represented as follows:

$$\sum_{t=2023}^{T=2053} CO_2 = \sum_{t=2023}^{T=2053} \sum_{i=0}^{n=3} x_{t,i} \cdot LCA_{t,i}$$

Ultimately, we will obtain an equation that represents an objective function $Z$: a function to compute the total carbon emissions produced throughout the lifecycle of every car in Singapore from 2023 to 2053. When expanded, this equation looks like the following:

$$\begin{aligned} Z = & [x_1 LCA_1 + \ldots + x_n LCA_n]_{2023} \\ & + [x_1 LCA_1 + \ldots + x_n LCA_n]_{2024} \\ & + \ldots \\ & + [x_1 LCA_1 + \ldots + x_n LCA_n]_T \end{aligned}$$

This optimisation problem essentially takes the form of a single objective, three-dimensional linear programming problem as $x_1$, $x_2$, and $x_3$ are single-degree variables.

### B. Constraints

Tackling the problem statement would require us to minimise the above function. However, before doing so, it would be wise to define the region of optimisation as without such a region, any optimisation efforts would simply produce results that are meaningless albeit mathematically valid. Thus, in this study we have selected three scenarios to tackle our problem statement:

- **Scenario 1**

In scenario 1, we imagine a Singapore where most private cars are powered by internal combustion engines. There are several electric vehicles and hybrid cars, but these two types of cars make up a minority of the cars produced. One way to represent this situation is by defining the following constraints:

$$\text{For } t = 2023, 2024, \ldots, 2052, 2053$$
$$x_{t,1} + x_{t,2} + x_{t,3} = D_t$$
$$x_{t,1} \geq 70\% D_t$$
$$x_{t,2} \geq 10\% D_t$$
$$x_{t,3} \geq 10\% D_t$$

Note that the equality constraint represents the fact that the number of cars produced in the year $t$ has to match the $t$-th demand in order to keep the total number of cars in Singapore at a constant 650,000. This equality applies to the other two scenarios too.

[3] The European LCA data assume a car lifespan of 16 years, which leads to a higher LCA rate. The median lifespan of Singaporean cars is 10 years (LTA, 2022), so we divide by 16 years and multiply by 10 years to get an adjusted LCA. While this produces some errors in our future calculations, we can consider these errors to have a negligible effect on our results. Thus, we are confident to proceed with these adjusted LCA rates.
[4] There were few to no electric vehicles in Singapore before 2017.
[5] Cars 0 to 1 year old are considered 1 year old, cars 1 to 2 years old are considered 2 years old, and so on. This means that 10-year-old cars can range from 9 to 10 years old.

- **Scenario 2**

In scenario 2, we imagine a world where most cars are powered by electric motors. ICE and hybrid-powered cars still exist, but like in scenario 1, they only make up a small minority of all the cars used in Singapore. This scenario can thus be represented by the following region:

$$\text{For } t = 2023, 2024, \dots, 2052, 2053$$
$$x_{t,1} + x_{t,2} + x_{t,3} = D_t$$
$$x_{t,1} \geq 10\% D_t$$
$$x_{t,2} \geq 10\% D_t$$
$$x_{t,3} \geq 10\% D_t$$
$$3x_{t,2} \geq 7x_{t,3}$$

Do note here that the fourth constraint mandates that there are at least $\frac{7}{3}$ times more EVs than hybrid cars.

- **Scenario 3**

In scenario 3, we imagine a similar situation as in scenario 2, but now car owners prioritise using hybrid cars over electric vehicles and ICE-powered cars. Again, EVs and ICE cars still exist, but they only make up a comparatively small portion of all cars in Singapore. This scenario can be represented by the following region:

$$\text{For } t = 2023, 2024, \dots, 2052, 2053$$
$$x_{t,1} + x_{t,2} + x_{t,3} = D_t$$
$$x_{t,1} \geq 10\% D_t$$
$$x_{t,2} \geq 10\% D_t$$
$$x_{t,3} \geq 10\% D_t$$
$$3x_{t,3} \geq 7x_{t,2}$$

By performing the optimisation across these three different scenarios, we will be able to compare how $Z$ varies when (1) we do nothing about our current state of private transportation (i.e. let most cars be ICE-powered until 2053), (2) we prioritise using EVs while still using a small minority of ICE and hybrid cars, and (3) we prioritise using hybrid cars while still using a small minority of ICE cars and EVs.

### C. Summary

Our linear program can be summarised as follows:

$$\min . Z$$
$$\text{s.t. } x_{t,1} + x_{t,2} + x_{t,3} = D_t$$
$$h_k(x_{t,1}, x_{t,2}, x_{t,3}) \geq 0$$

where $k$ is the number of inequality constraints and $h_k$ is the $k$-th inequality depending on the scenario of concern.

### D. Executing the Linear Program

Since we are dealing with hundreds of linear equations to optimise, it would be most efficient to execute the linear program using Python. With the aid of the scipy and pandas packages, we managed to write a script to iterate the linear program across $t$ = 2023 and $T$ = 2053. Please refer to Appendix A to view the full script.

## IV. RESULTS

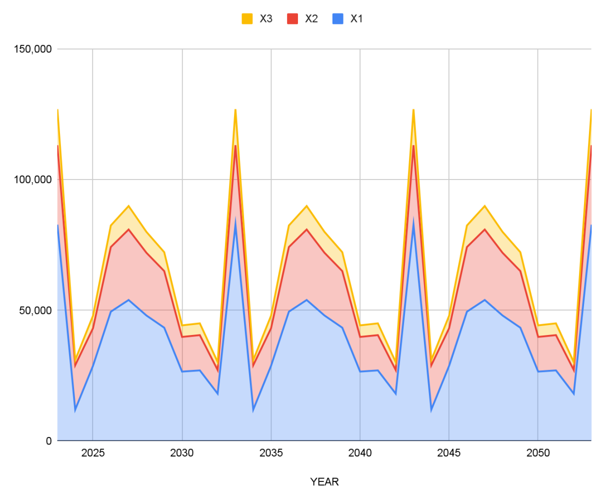
### A. Solution Matrix

The following table summarises the solution matrix for all three scenarios. To ease readers in analysing the data, we have also applied some colour coding, causing high values to be shaded in dark blue.

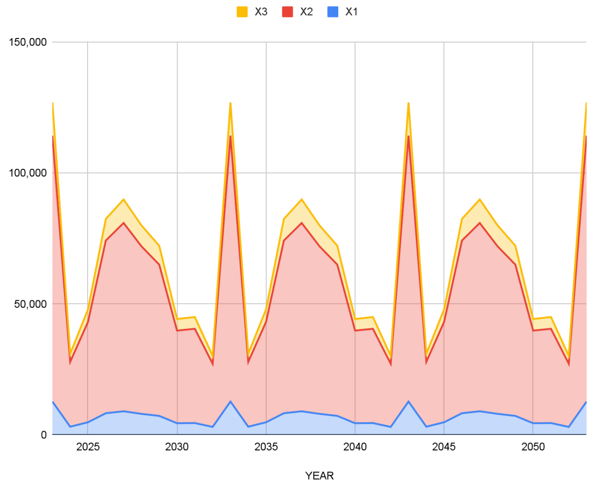
| | Scenario 1 | | | Scenario 2 | | | Scenario 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| YEAR | $x_1$ | $x_2$ | $x_3$ | $x_1$ | $x_2$ | $x_3$ | $x_1$ | $x_2$ | $x_3$ |
| 2023 | 82,781 | 30,422 | 13,797 | 12,700 | 101,600 | 12,700 | 12,700 | 12,700 | 101,600 |
| 2024 | 12,005 | 6,003 | 2,001 | 3,098 | 24,782 | 3,098 | 3,098 | 3,098 | 24,782 |
| 2025 | 28,766 | 14,384 | 4,794 | 4,794 | 38,356 | 4,794 | 4,794 | 4,794 | 38,356 |
| 2026 | 49,480 | 24,739 | 8,247 | 8,247 | 65,972 | 8,247 | 8,247 | 8,247 | 65,972 |
| 2027 | 53,953 | 26,977 | 8,992 | 8,992 | 71,938 | 8,992 | 8,992 | 8,992 | 71,938 |
| 2028 | 48,017 | 24,008 | 8,003 | 8,003 | 64,022 | 8,003 | 8,003 | 8,003 | 64,022 |
| 2029 | 43,348 | 21,673 | 7,225 | 7,225 | 57,796 | 7,225 | 7,225 | 7,225 | 57,796 |
| 2030 | 26,545 | 13,273 | 4,424 | 4,424 | 35,394 | 4,424 | 4,424 | 4,424 | 35,394 |
| 2031 | 27,004 | 13,502 | 4,501 | 4,501 | 36,005 | 4,501 | 4,501 | 4,501 | 36,005 |
| 2032 | 18,101 | 9,050 | 3,017 | 3,017 | 24,134 | 3,017 | 3,017 | 3,017 | 24,134 |
| 2033 | 82,781 | 41,391 | 13,797 | 12,700 | 101,600 | 12,700 | 12,700 | 12,700 | 101,600 |
| 2034 | 12,005 | 6,003 | 2,001 | 3,098 | 24,782 | 3,098 | 3,098 | 3,098 | 24,782 |
| 2035 | 28,766 | 14,384 | 4,794 | 4,794 | 38,356 | 4,794 | 4,794 | 4,794 | 38,356 |
| 2036 | 49,480 | 24,739 | 8,247 | 8,247 | 65,972 | 8,247 | 8,247 | 8,247 | 65,972 |
| 2037 | 53,953 | 26,977 | 8,992 | 8,992 | 71,938 | 8,992 | 8,992 | 8,992 | 71,938 |
| 2038 | 48,017 | 24,008 | 8,003 | 8,003 | 64,022 | 8,003 | 8,003 | 8,003 | 64,022 |
| 2039 | 43,348 | 21,673 | 7,225 | 7,225 | 57,796 | 7,225 | 7,225 | 7,225 | 57,796 |
| 2040 | 26,545 | 13,273 | 4,424 | 4,424 | 35,394 | 4,424 | 4,424 | 4,424 | 35,394 |
| 2041 | 27,004 | 13,502 | 4,501 | 4,501 | 36,005 | 4,501 | 4,501 | 4,501 | 36,005 |
| 2042 | 18,101 | 9,050 | 3,017 | 3,017 | 24,134 | 3,017 | 3,017 | 3,017 | 24,134 |
| 2043 | 82,781 | 41,391 | 13,797 | 12,700 | 101,600 | 12,700 | 12,700 | 12,700 | 101,600 |
| 2044 | 12,005 | 6,003 | 2,001 | 3,098 | 24,782 | 3,098 | 3,098 | 3,098 | 24,782 |
| 2045 | 28,766 | 14,384 | 4,794 | 4,794 | 38,356 | 4,794 | 4,794 | 4,794 | 38,356 |
| 2046 | 49,480 | 24,739 | 8,247 | 8,247 | 65,972 | 8,247 | 8,247 | 8,247 | 65,972 |
| 2047 | 53,953 | 26,977 | 8,992 | 8,992 | 71,938 | 8,992 | 8,992 | 8,992 | 71,938 |
| 2048 | 48,017 | 24,008 | 8,003 | 8,003 | 64,022 | 8,003 | 8,003 | 8,003 | 64,022 |
| 2049 | 43,348 | 21,673 | 7,225 | 7,225 | 57,796 | 7,225 | 7,225 | 7,225 | 57,796 |
| 2050 | 26,545 | 13,273 | 4,424 | 4,424 | 35,394 | 4,424 | 4,424 | 4,424 | 35,394 |
| 2051 | 27,004 | 13,502 | 4,501 | 4,501 | 36,005 | 4,501 | 4,501 | 4,501 | 36,005 |
| 2052 | 18,101 | 9,050 | 3,017 | 3,017 | 24,134 | 3,017 | 3,017 | 3,017 | 24,134 |
| 2053 | 82,781 | 41,391 | 13,797 | 12,700 | 101,600 | 12,700 | 12,700 | 12,700 | 101,600 |

### B. Stacked Area Charts for $x_1$, $x_2$, and $x_3$

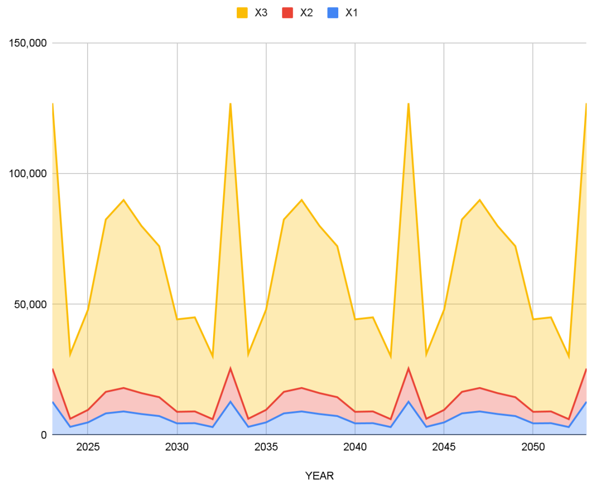
**Scenario 1**



**Scenario 2**




**Scenario 3**




### C. $CO_2$ Comparison

The following table summarises the *Z* values (i.e. the total carbon emissions due to cars manufactured from 2023 to 2053) for the three different scenarios.

| | SCENARIO 1 | SCENARIO 2 | SCENARIO 3 |
|---|---|---|---|
| *Z* (tons $CO_2$ eq) | 64,433,637 | 54,895,744 | 57,887,568 |

## V. INTERPRETATION AND EVALUATION

### A. Interpretation

- Scenario 1 produces the highest $CO_2$ because of the excessive fossil fuel usage.
- Scenario 2 performs slightly better than scenario 3 in terms of $CO_2$ emission. However, it is important to note the LCA of EVs in this study is high because these EVs are charged using 98% fossil fuels. When renewable energy becomes more prevalent, scenario 2 should produce a smaller *Z* value compared to scenario 3.
- Interestingly, all three scenarios produce a solution that contains a cyclical pattern of 10 years. This is because of our assumption that there is always 650,000 cars in Singapore every year and that every car is used for exactly 10 years before being replaced.
- All in all, the study shows that simply transitioning to EVs for private transportation is not a sufficient nor sustainable approach to tackle climate change. If we want to effectively tackle the climate crisis, at the very least we have to revolutionise our energy systems (i.e. shift to cleaner and renewable energy such that we emit even less $CO_2$ in every single activity).

### B. Evaluation

**Strengths**

1.) The $CO_2$ emission function is tested using multiple scenarios which allow for a higher certainty of estimates and validation of the naïve assumptions made in section **II**.

2.) The model allows easy inclusion of additional types of cars, provided trends in LCA are available.

3.) External constraints such as policy (e.g. 100% EV by 2040) can be added as scenarios.

4.) Changes in car efficiency over time can be taken into account.

**Weaknesses**

1.) The model relies on the extrapolation of a fixed dataset from the past 10 years. However due to assumed relationships when extrapolating a data set beyond its range, such as the rate of improvement in the efficiency of the vehicles, the likelihood of biased conclusions increases. (Hahn, 2018) It is possible that the efficiency of electric vehicles and hybrid cars plateau in the future, instead of increasing infinitely.

2.) The large number of assumptions due to the lack of data can lead to inaccuracies. More resources to collect data will improve the model's accuracy.

3.) The model assumes that the LCA data is a function of time. In actuality, LCA for EVs can also be functions of the EV population as existing infrastructure needs to be upgraded to support a larger EV population.

4.) For EVs specifically, this model does not account for possible changes in energy sources that provide electricity to charging stations in Singapore. The model assumes that Singapore continues using 98% fossil fuels indefinitely. Given the possibility that Singapore might replace natural gas with greener sources of energy in the future, the LCA for the EV might give a drastically lower lifetime emission and the actual total carbon emission may be lower than the model predicts.

# REFERENCES


Gerald J. Hahn (1977) *The Hazards of Extrapolation in Regression Analysis*, Journal of Quality Technology, 9:4, 159-165, DOI: 10.1080/00224065.1977.11980791

Anthony, A. (2022, May 25). *Electric Cars vs Petrol Cars: The Real Costs*. SingSaver. Retrieved February 27, 2023, from https://www.singsaver.com.sg/blog/cost-comparison-of-electric-cars-vs-petrol-cars

*Car Models by Manufacturer, Category, and Year | Database Hub*. (n.d.). Back4App. Retrieved February 28, 2023, from https://www.back4app.com/database/back4app/car-make-model-dataset

Green NCAP. (n.d.). *European Life Cycle Assessment Results & Fact Sheets*. Green NCAP. Retrieved February 28, 2023, from https://www.greenncap.com/european-lca-results/

Land Transport Authority. (2022). *AGE DISTRIBUTION OF CAR POPULATION 1*. Land Transport Authority. Retrieved February 28, 2023, from https://www.lta.gov.sg/content/dam/ltagov/who_we_are/statistics_and_publications/statistics/pdf/MVP01-3-1_Cars_by_age.pdf

Land Transport Authority. (2022). *MOTOR VEHICLE POPULATION BY TYPE OF FUEL USED 1*. Land Transport Authority. Retrieved February 28, 2023, from https://www.lta.gov.sg/content/dam/ltagov/who_we_are/statistics_and_publications/statistics/pdf/MVP01-4_MVP_by_fuel.pdf

Land Transport Authority. (2022, April 8). *LTA | Electric Vehicles*. Land Transport Authority. Retrieved February 27, 2023, from https://www.lta.gov.sg/content/ltagov/en/industry_innovations/technologies/electric_vehicles.html

MIT Climate. (n.d.). *How much CO2 is emitted by manufacturing batteries?* MIT Climate Portal. Retrieved February 27, 2023, from https://climate.mit.edu/ask-mit/how-much-co2-emitted-manufacturing-batteries

SciPy. (n.d.). *scipy.optimize.linprog — SciPy v1.10.1 Manual*. Numpy and Scipy Documentation. Retrieved February 28, 2023, from https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.linprog.html

Stojiljković, M. (n.d.). *Hands-On Linear Programming: Optimization With Python – Real Python*. Real Python. Retrieved February 28, 2023, from https://realpython.com/linear-programming-python/#linear-programming-solvers

# APPENDIX A

### Excel File

To view the file used in the code, click here:
[https://docs.google.com/spreadsheets/d/1CuOWOIACC9QaUsz9FS8r7GHCm3eJ7WWd/edit?usp=sharing&ouid=107420744130848768585&rtpof=true&sd=true](https://docs.google.com/spreadsheets/d/1CuOWOIACC9QaUsz9FS8r7GHCm3eJ7WWd/edit?usp=sharing&ouid=107420744130848768585&rtpof=true&sd=true)

### Script for Scenario 1

```
import scipy
from scipy.optimize import linprog
import pandas as pd

#ALL THE DATA
df = pd.read_excel('Data.xlsx',
sheet_name='Scenario1')
t = df.iloc[:,0]
D = df.iloc[:,1]
LCA1 = df.iloc[:,2]
LCA2 = df.iloc[:,3]
LCA3 = df.iloc[:,4]
c1 = df.iloc[:,5]
c2 = df.iloc[:,6]
c3 = df.iloc[:,7]

# EXECUTION
n = len(t)
Solution_Matrix = []

for i in range(n):
    # t1 = t[i]
    L1 = LCA1[i]
    L2 = LCA2[i]
    L3 = LCA2[i]
    Demand = D[i]
    C1 = -1 * c1[i]
    C2 = -1 * c2[i]
    C3 = -1 * c3[i]

    # PART 1: CO2 FUNCTION
    coeff_CO2 = [L1, L2, L3]

    # PART 2: INEQUALITIES
    # ICE, EV, then H
    lhs_ineq = [
        [-1, 0, 0],
        [0, -1, 0],
        [0, 0, -1]
    ]
    rhs_ineq = [
        C1,
        C2,
        C3
    ]

    # PART 3: EQUALITIES
    lhs_eq = [[1, 1, 1]]
    rhs_eq = [Demand]

    opt = linprog(c=coeff_CO2, A_ub=lhs_ineq,
b_ub=rhs_ineq, A_eq=lhs_eq, b_eq=rhs_eq,
method="highs")
    Solution_Matrix.append(list(opt.x))

for x in Solution_Matrix:
    print('\t'.join([str(n) for n in x]))
```

### Script for Scenario 2

```
import scipy
from scipy.optimize import linprog
import pandas as pd

#ALL THE DATA
df = pd.read_excel('Data.xlsx',
sheet_name='Scenario2')
t = df.iloc[:,0]
D = df.iloc[:,1]
LCA1 = df.iloc[:,2]
LCA2 = df.iloc[:,3]
LCA3 = df.iloc[:,4]
c1 = df.iloc[:,5]
c2 = df.iloc[:,6]
c3 = df.iloc[:,7]

# EXECUTION
n = len(t)
Solution_Matrix = []

for i in range(n):
    # t1 = t[i]
    L1 = LCA1[i]
    L2 = LCA2[i]
    L3 = LCA2[i]
    Demand = D[i]
    C1 = -1 * c1[i]
    C2 = -1 * c2[i]
    C3 = -1 * c3[i]

    # PART 1: CO2 FUNCTION
    coeff_CO2 = [L1, L2, L3]

    # PART 2: INEQUALITIES
    # ICE, EV, then H
    lhs_ineq = [
        [-1, 0, 0],
        [0, -1, 0],
        [0, 0, -1],
        [0, -3, 7]
    ]
    rhs_ineq = [
        C1,
        C2,
        C3,
        0
    ]

    # PART 3: EQUALITIES
    lhs_eq = [[1, 1, 1]]
    rhs_eq = [Demand]

    opt = linprog(c=coeff_CO2, A_ub=lhs_ineq,
b_ub=rhs_ineq, A_eq=lhs_eq, b_eq=rhs_eq,
method="highs")
```

```
    Solution_Matrix.append(list(opt.x))

for x in Solution_Matrix:
    print('\t'.join([str(n) for n in x]))
```

**Script for Scenario 3**

```
import scipy
from scipy.optimize import linprog
import pandas as pd

#ALL THE DATA (NEW)
df = pd.read_excel('Data.xlsx',
sheet_name='Scenario3')
t = df.iloc[:,0]
D = df.iloc[:,1]
LCA1 = df.iloc[:,2]
LCA2 = df.iloc[:,3]
LCA3 = df.iloc[:,4]
c1 = df.iloc[:,5]
c2 = df.iloc[:,6]
c3 = df.iloc[:,7]

# EXECUTION
n = len(t)
Solution_Matrix = []

for i in range(n):
    # t1 = t[i]
    L1 = LCA1[i]
    L2 = LCA2[i]
    L3 = LCA2[i]
    Demand = D[i]
    C1 = -1 * c1[i]
    C2 = -1 * c2[i]
    C3 = -1 * c3[i]

    # PART 1: CO2 FUNCTION
    coeff_CO2 = [L1, L2, L3]

    # PART 2: INEQUALITIES
    # ICE, EV, then H
    lhs_ineq = [
        [-1, 0, 0],
        [0, -1, 0],
        [0, 0, -1],
        [0, 7, -3]
    ]
    rhs_ineq = [
        C1,
        C2,
        C3,
        0
    ]

    # PART 3: EQUALITIES
    lhs_eq = [[1, 1, 1]]
    rhs_eq = [Demand]

    opt = linprog(c=coeff_CO2, A_ub=lhs_ineq,
b_ub=rhs_ineq, A_eq=lhs_eq, b_eq=rhs_eq,
method="highs")
    Solution_Matrix.append(list(opt.x))

for x in Solution_Matrix:
    print('\t'.join([str(n) for n in x]))
```

# APPENDIX B

**Google Sheets**

This was where we did most of our rough working: https://docs.google.com/spreadsheets/d/1vMukl7axAgHxQT6QTpUv0b8su2Q0t2IwzOAe6GpqR9k/edit?usp=sharing

**10-Minute YouTube Video**

https://youtu.be/iwfikPAdOHw