Reproducing results from Parker

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The functions in `stocins` were tested against (Parker, 1992) and (Parker, 1997).

This package implements the following:

- Functions for an Wiener process, Ornstein-Uhlenbeck process and Second order stochastic differential equation as shown in Chapter 2 and 3 of (Parker, 1992).
- Moments and density function for the present value of a term and endowment insurance on a single life as shown in Chapter 4 of (Parker, 1992).
- The first three moments for a portfolio of term and endowment contracts as shown in Chapter 5 of (Parker, 1992).
- The first two moments for a group of term and endowment contracts as shown in (Parker, 1997).

Various classes implement this functionality:

```
> library(stocins)
> g = expand.grid(x = seq(20, 70, 10), y = seq(1, 75, 10))
> g$z = 0
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> for(i in 1:nrow(g)) {
+   term = insurance(list(n = g$y[i], d = 1), "isingle", "term")
+   mort = mortassumptions(list(x = g$x[i], table = "MaleMort82"))
+   g$z[i] = z.moment(1, term, mort, oumodel)
+ }
> lattice::wireframe(z ~ x * y, data = g, drape = TRUE, col = 'black',
+         col.regions = 'white', aspect = c(1.0, 0.8), colorkey = FALSE,
+         xlab = "issue age", ylab = "n", zlab = "",
+         screen = list(z = 340, x = -70),
+     scales = list(arrows = FALSE, col="black", font = 10, cex= 1.0),
+         par.settings = list(regions=list(alpha = 0.3),
+                 axis.line = list(col = "transparent")),
+         zoom = 0.95, zlim = c(0,0.50))
```

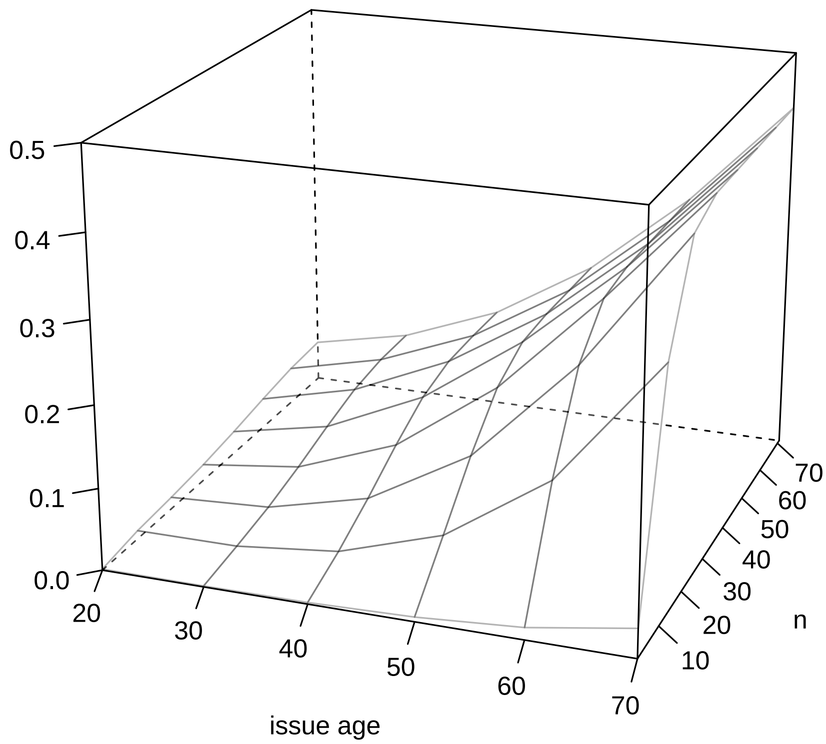

Figure 1: Figure 4.1 from Parker (1992)

Next, the standard deviation plot.

```
> g <- expand.grid(x = seq(1, 80, 5), y = seq(20, 70, 10))
> for(i in 1:nrow(g)) {
+   term = insurance(list(n = g$x[i], d = 1), "isingle", "term")
+   mort = mortassumptions(list(x = g$y[i], table = "MaleMort82"))
+   g$z[i] = z.sd(term, mort, oumodel)
+ }
> lattice::wireframe(z ~ x * y, data = g, drape = TRUE, col = 'black',
+                     col.regions = 'white', aspect = c(1, 0.8),
+                     colorkey = FALSE, xlab = "n", ylab = "issue age",
+                     zlab = "", screen = list(z = 340, x = -70, y = -20),
+                     scales = list(arrows = FALSE, col = 'black', font = 10),
+                     cex = 0.8, ylim = c(20, 70),
+                     zlim = c(0, 0.40),
+                     par.settings = list(regions=list(alpha = 0.3),
+                     axis.line = list(col = "transparent")), zoom = 0.95)
```

Next, the skewness plot.

```
> g <- expand.grid(x = seq(1, 70, 5), y = seq(20, 70, 10))
> for(i in 1:nrow(g)) {
+   term = insurance(list(n = g$x[i], d = 1), "isingle", "term")
```

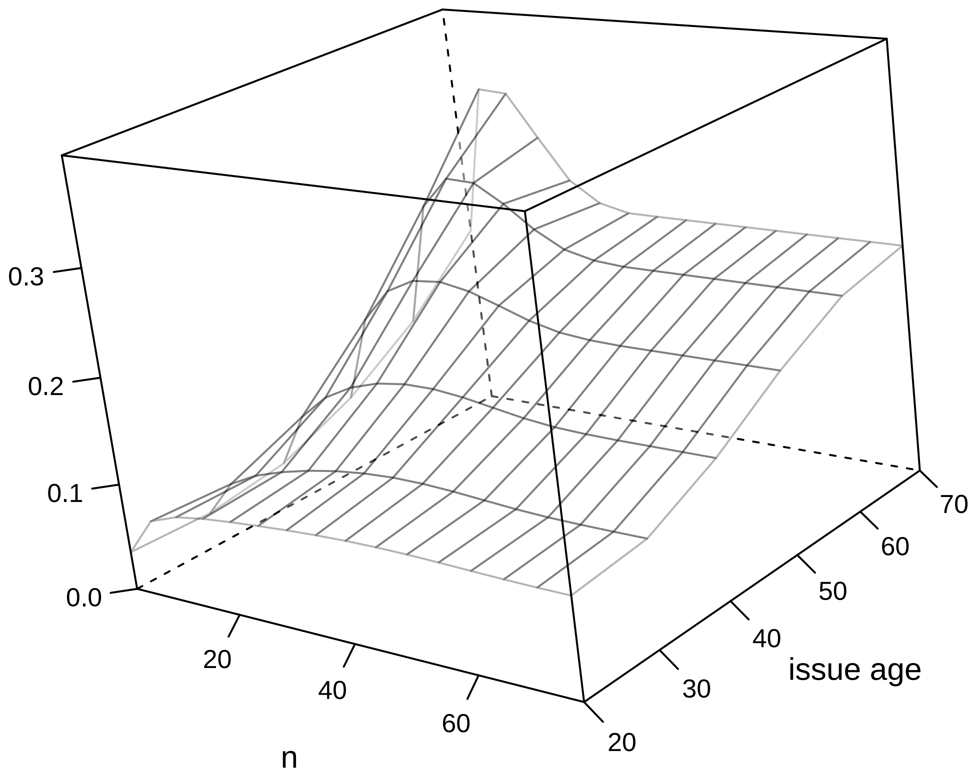

Figure 2: Figure 4.2 from Parker (1992)

```
+   mort = mortassumptions(list(x = g$y[i], table = "MaleMort82"))
+   g$z[i] = z.sk(term, mort, oumodel)
+ }
> lattice::wireframe(z ~ y * x, data = g, drape = TRUE, col = 'black',
+                    col.regions = 'white', aspect = c(1, 0.8),
+                    colorkey = FALSE, xlab = "issue age", ylab = "n",
+                    zlab = "", screen = list(z = 340, x = -70, y = -20),
+                    scales = list(arrows = FALSE, col = 'black', font = 10),
+                    cex = 0.8, zlim = c(0, 30), ylim= c(0,70),
+                    par.settings = list(regions=list(alpha = 0.3),
+                    axis.line = list(col = "transparent")), zoom = 0.95)
```

Next, the pdf plot.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> term5 = insurance(list(n = 5, d = 1), "isingle", "term")
```

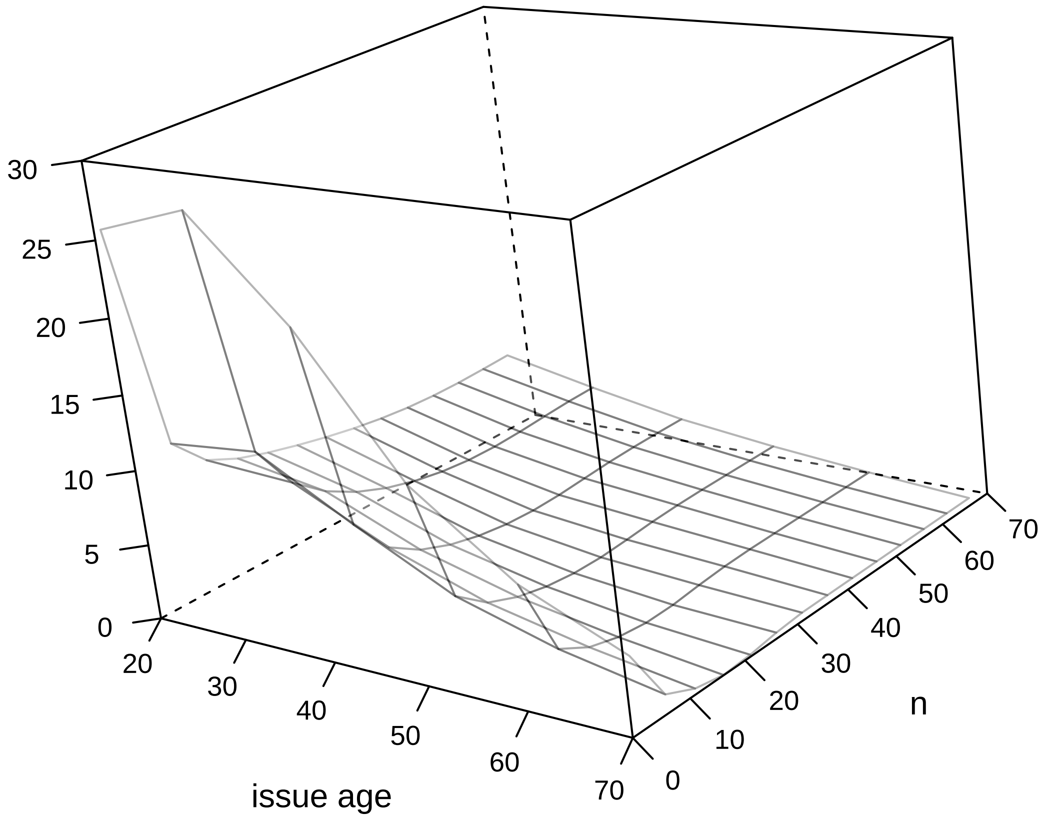

Figure 3: Figure 4.3 from Parker (1992)

```
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> term25 = insurance(list(n = 25, d = 1), "isingle", "term")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> plot(function(z) z.pdf(z, term5, mort, oumodel), 0.01, 1.0,
+      ylim = c(0, 0.4), lty = 1, xlab = "z", ylab = "f(z)")
> plot(function(z) z.pdf(z, term25, mort, oumodel), 0.01, 1.0,
+      ylim = c(0, 0.4), add = TRUE, lty = 2)
> legend('topright', leg = c(paste0("term 25 [P(Z=0) = ",
+                                   round(kpx(25, mort),5), "]"),
+                            paste0("term 5   [P(Z=0) = ",
+                                   round(kpx(5, mort),5), "]")),
+        lty = c(2,1), cex = 0.8)
```

For an endowment insurance, the expected value plot.

```
> library(stocins)
```

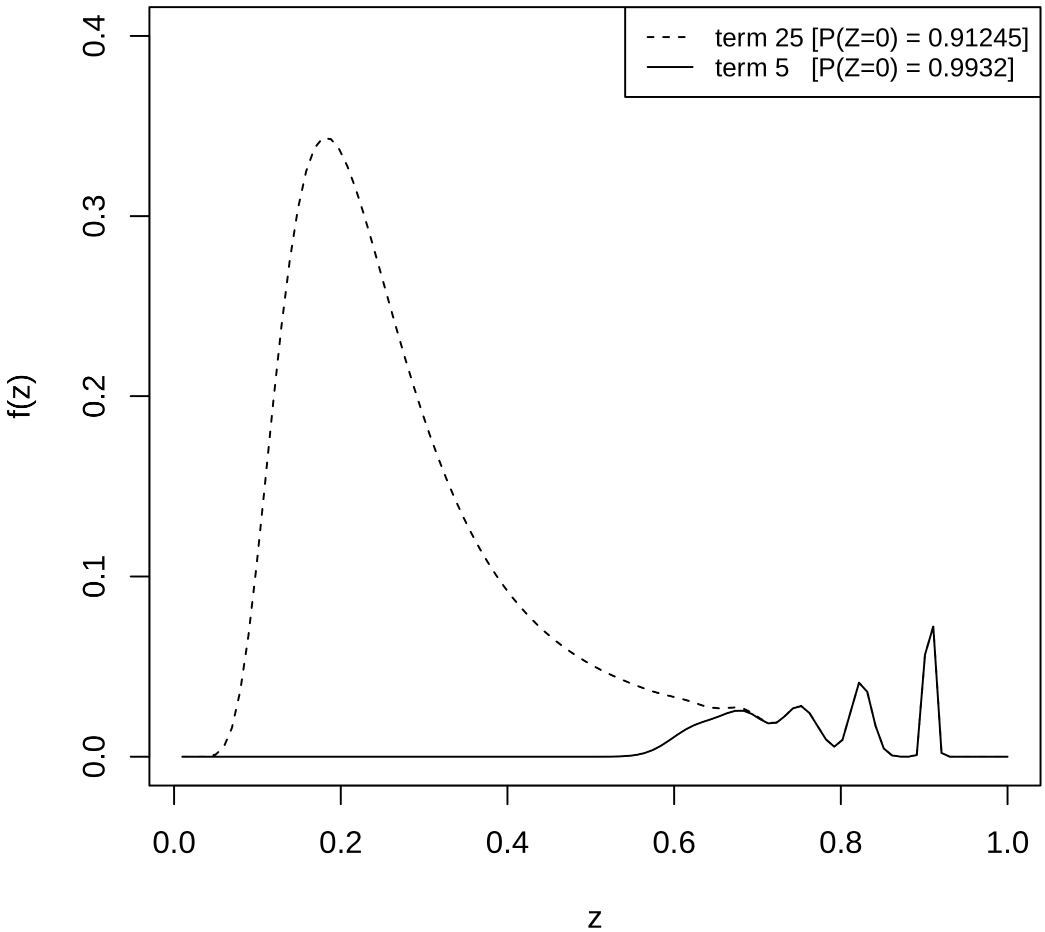

Figure 4: Figure 4.4 from Parker (1992)

```
> g = expand.grid(x = seq(20, 70, 10), y = seq(1, 75, 10))
> g$z = 0
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> for(i in 1:nrow(g)) {
+   endow = insurance(list(n = g$y[i], d = 1, e = 1), "isingle", "endow")
+   mort = mortassumptions(list(x = g$x[i], table = "MaleMort82"))
+   g$z[i] = z.moment(1, endow, mort, oumodel)
+ }
> lattice::wireframe(z ~ x * y, data = g, drape = TRUE, col = 'black',
+         col.regions = 'white', aspect = c(1.0, 0.8), colorkey = FALSE,
+         xlab = "issue age", ylab = "n", zlab = "",
+         screen = list(z = 340, x = -70),
+     scales = list(arrows = FALSE, col="black", font = 10, cex= 1.0),
+         par.settings = list(regions=list(alpha = 0.3),
+                   axis.line = list(col = "transparent")),
```

```
+           zoom = 0.95, zlim = c(0,1.00))
```

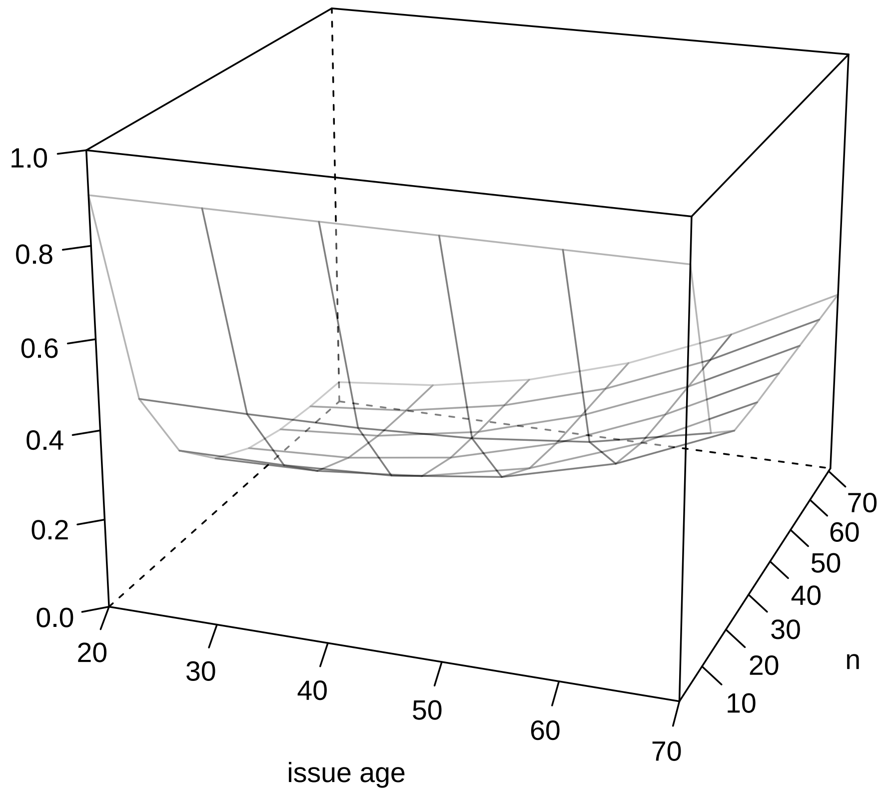

Figure 5: Figure 4.5 from Parker (1992)

Next, the standard deviation plot.

```
> g <- expand.grid(x = seq(1, 80, 5), y = seq(20, 70, 10))
> for(i in 1:nrow(g)) {
+   endow = insurance(list(n = g$x[i], d = 1, e = 1), "isingle", "endow")
+   mort = mortassumptions(list(x = g$y[i], table = "MaleMort82"))
+   g$z[i] = z.sd(endow, mort, oumodel)
+ }
> lattice::wireframe(z ~ x * y, data = g, drape = TRUE, col = 'black',
+                     col.regions = 'white', aspect = c(1, 0.8),
+                     colorkey = FALSE, xlab = "n", ylab = "issue age",
+                     zlab = "", screen = list(z = 340, x = -70, y = -20),
+                     scales = list(arrows = FALSE, col = 'black', font = 10),
+                     cex = 0.8, ylim = c(20, 70),
+                     zlim = c(0, 0.25),
+                     par.settings = list(regions=list(alpha = 0.3),
+                     axis.line = list(col = "transparent")), zoom = 0.95)
```

Next, the skewness plot.

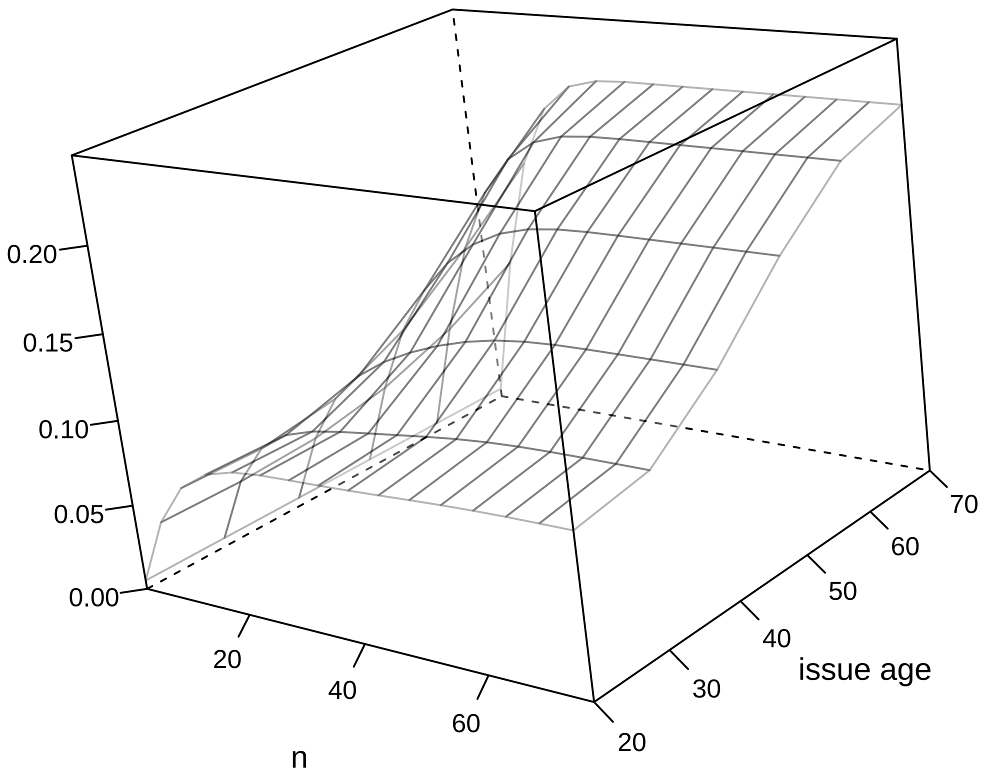

Figure 6: Figure 4.6 from Parker (1992)

```
> g <- expand.grid(x = seq(1, 70, 5), y = seq(20, 70, 10))
> for(i in 1:nrow(g)) {
+   endow = insurance(list(n = g$x[i], d = 1, e = 1), "isingle", "endow")
+   mort = mortassumptions(list(x = g$y[i], table = "MaleMort82"))
+   g$z[i] = z.sk(endow, mort, oumodel)
+ }
> lattice::wireframe(z ~ y * x, data = g, drape = TRUE, col = 'black',
+                    col.regions = 'white', aspect = c(1, 0.8),
+                    colorkey = FALSE, xlab = "issue age", ylab = "n",
+                    zlab = "", screen = list(z = 340, x = -70, y = -20),
+                    scales = list(arrows = FALSE, col = 'black', font = 10),
+                    cex = 0.8, zlim = c(0, 6), ylim= c(0,70),
+                    par.settings = list(regions=list(alpha = 0.3),
+                    axis.line = list(col = "transparent")), zoom = 0.95)
```

Next, the pdf plot.

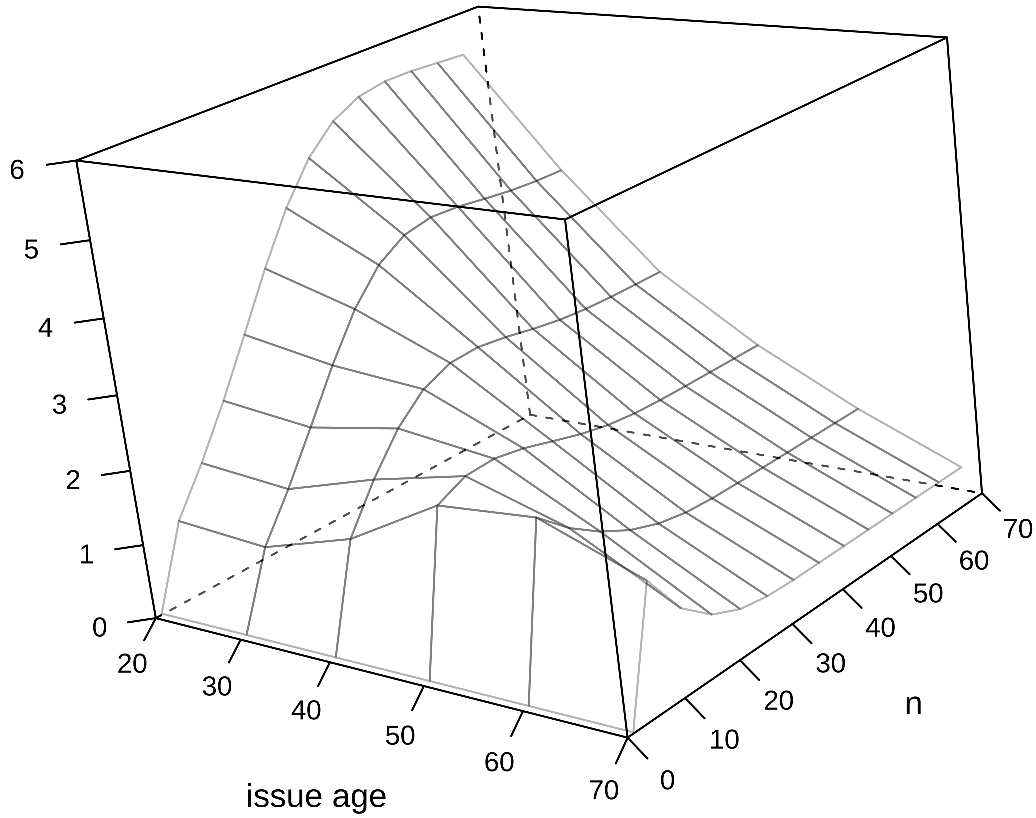

Figure 7: Figure 4.7 from Parker (1992)

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                            alpha = 0.1, sigma = 0.01), "ou")
> endow5 = insurance(list(n = 5, d = 1, e = 1), "isingle", "endow")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> endow25 = insurance(list(n = 25, d = 1, e = 1), "isingle", "endow")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> plot(function(z) z.pdf(z, endow5, mort, oumodel), 0.01, 1.0,
+      ylim = c(0, 12), lty = 1, xlab = "z", ylab = "f(z)")
> plot(function(z) z.pdf(z, endow25, mort, oumodel), 0.01, 1.0,
+      ylim = c(0, 12), add = TRUE, lty = 2)
> legend('topright', leg = c("endow 25", "endow 5"),
+        lty = c(2,1), cex = 0.8)
```

Some numerical results are reproduced below.

```
> library(stocins)
```

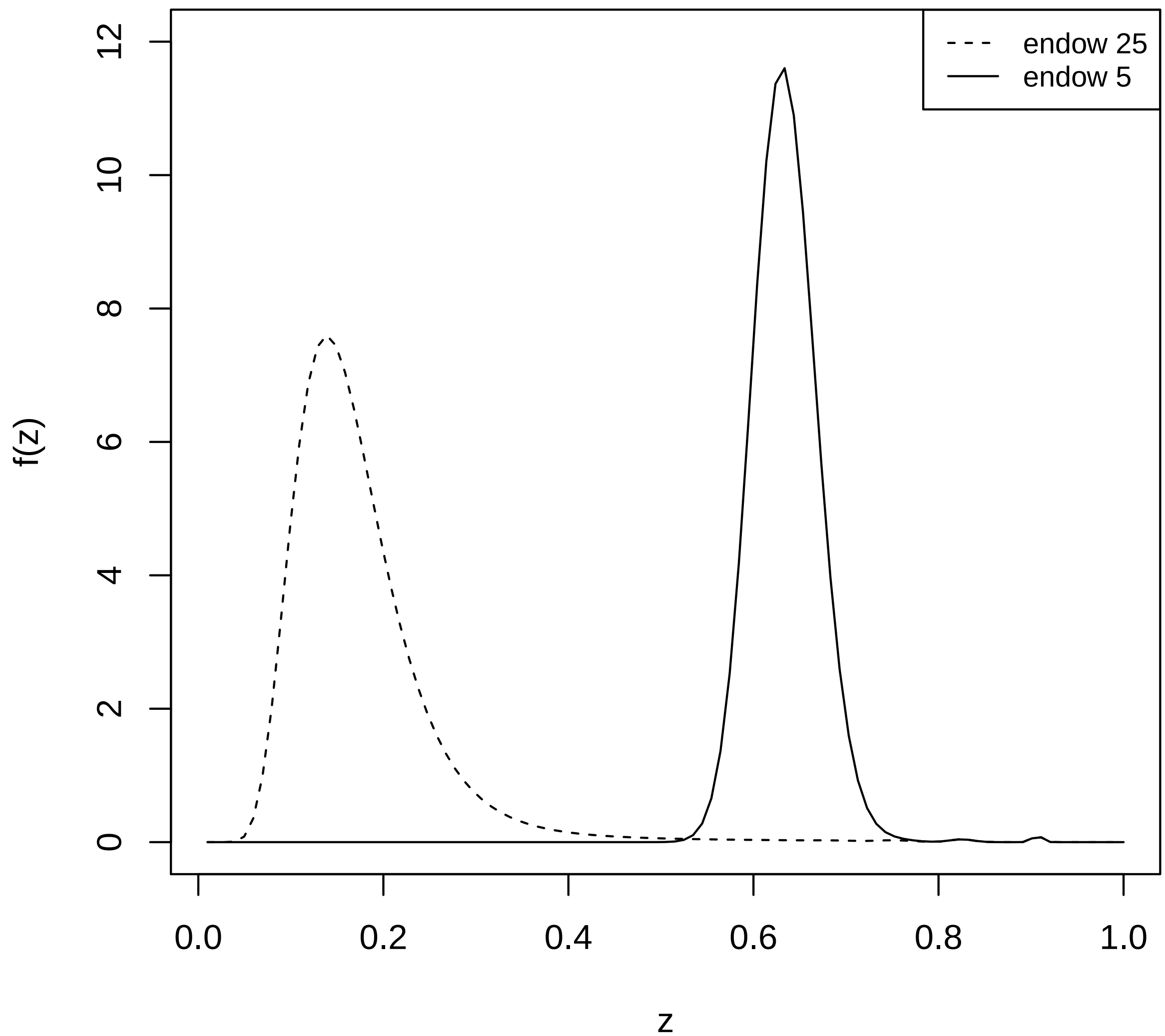

Figure 8: Figure 4.8 from Parker (1992)

```
> library(xtable)
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> wholelife = insurance(list(n = 100, d = 1), "isingle", "term")
> x = c(20,30,40,50,60,70,80,90,100)
> first = numeric(length(x))
> sdev = numeric(length(x))
> skew = numeric(length(x))
> for(i in 1:length(x)) {
+   mort = mortassumptions(list(x = x[i], table = "MaleMort82"))
+   first[i] = z.moment(1, wholelife, mort, oumodel)
+   sdev[i] = z.sd(wholelife, mort, oumodel)
+   skew[i] = z.sk(wholelife, mort, oumodel)
```

```
+ }
> table41 = data.frame(Age = x, Mean = first,
+                      "Standard Deviation" = sdev,
+                      "Skewness" = skew,
+                      "Coefficient of Variation" = sdev/first)
> print(xtable(table41,
+       digits = c(0,0,6,6,6,6),
+       caption = "Table 4.1 from Parker (1992)"),
+       include.rownames = FALSE)
```

Age	Mean	Standard.Deviation	Skewness	Coefficient.of.Variation
20	0.051187	0.090805	5.411855	1.773981
30	0.076342	0.097460	3.915181	1.276631
40	0.123992	0.127706	2.632900	1.029956
50	0.199394	0.167886	1.783115	0.841979
60	0.303412	0.200298	1.100983	0.660153
70	0.432234	0.213380	0.523390	0.493667
80	0.573185	0.200033	-0.009555	0.348985
90	0.698856	0.161555	-0.388251	0.231171
100	0.883526	0.041424	-1.502273	0.046885

Table 1: Table 4.1 from Parker (1992)

The pdf for a whole life insurance is shown below.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> whole = insurance(list(n = 100, d = 1, e = 1), "isingle", "term")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> plot(function(z) z.pdf(z, whole, mort, oumodel), 0.00001, 1.0,
+      ylim = c(0, 15), lty = 1, xlab = "z", ylab = "f(z)")
```

Some results for a portfolio of policies are reproduced below.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> n = seq(1,70,1)
> sdev1 = numeric(length(n))
> sdev10 = numeric(length(n))
> sdev100 = numeric(length(n))
> sdev1000 = numeric(length(n))
> sdevInf = numeric(length(n))
> for(i in 1:length(n))
+ {
+   term = insurance(list(n=n[i], d=1), "isingle", "term")
+   port1 = insurance(list(single = term, c = 1), "iport", "term")
```

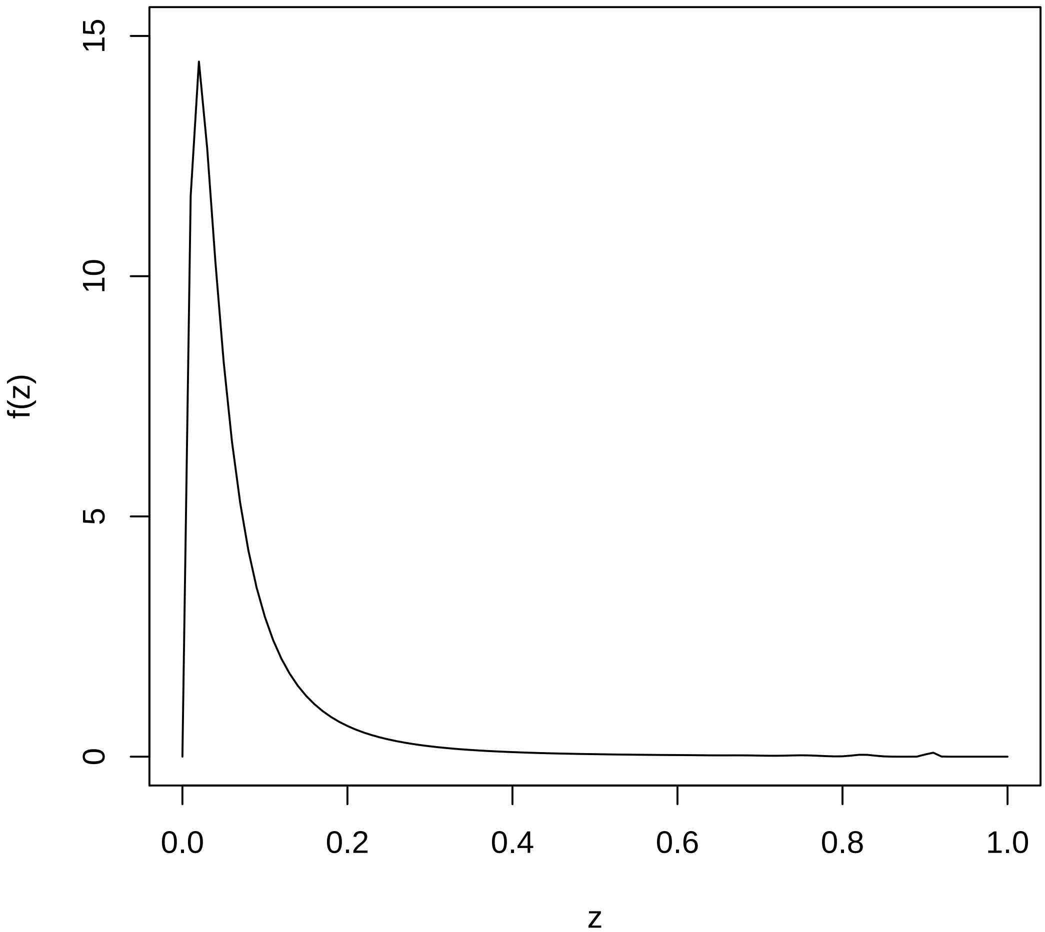

Figure 9: Figure 4.9 from Parker (1992)

```
+   port10 = insurance(list(single = term, c = 10), "iport", "term")
+   port100 = insurance(list(single = term, c = 100), "iport", "term")
+   port1000 = insurance(list(single = term, c = 1000), "iport", "term")
+   portInf = insurance(list(single = term, c = 1e18), "iport", "term")
+
+   sdev1[i] = z.sd(port1,mort,oumodel)
+   sdev10[i] = z.sd(port10,mort,oumodel)
+   sdev100[i] = z.sd(port100,mort,oumodel)
+   sdev1000[i] = z.sd(port1000,mort,oumodel)
+   sdevInf[i] = z.sd(portInf,mort,oumodel)
+ }
> plot(x = n, y = sdev1, type = 'l', ylab = "sd", xlab = "n",
+      ylim = c(0, 0.15))
> lines(x = n, y = sdev10/10, type = 'l', lty = 2)
> lines(x = n, y = sdev100/100, type = 'l', lty = 3)
> lines(x = n, y = sdev1000/1000, type = 'l', lty = 4)
```

```
> lines(x = n, y = sdevInf/1e18, type = 'l', lty = 5)
> legend('topright', leg = c("c=1","c=10","c=100","c=1000","c=Inf"),
+        lty = c(1,2,3,4,5), ncol = 5, cex = 0.9)
```

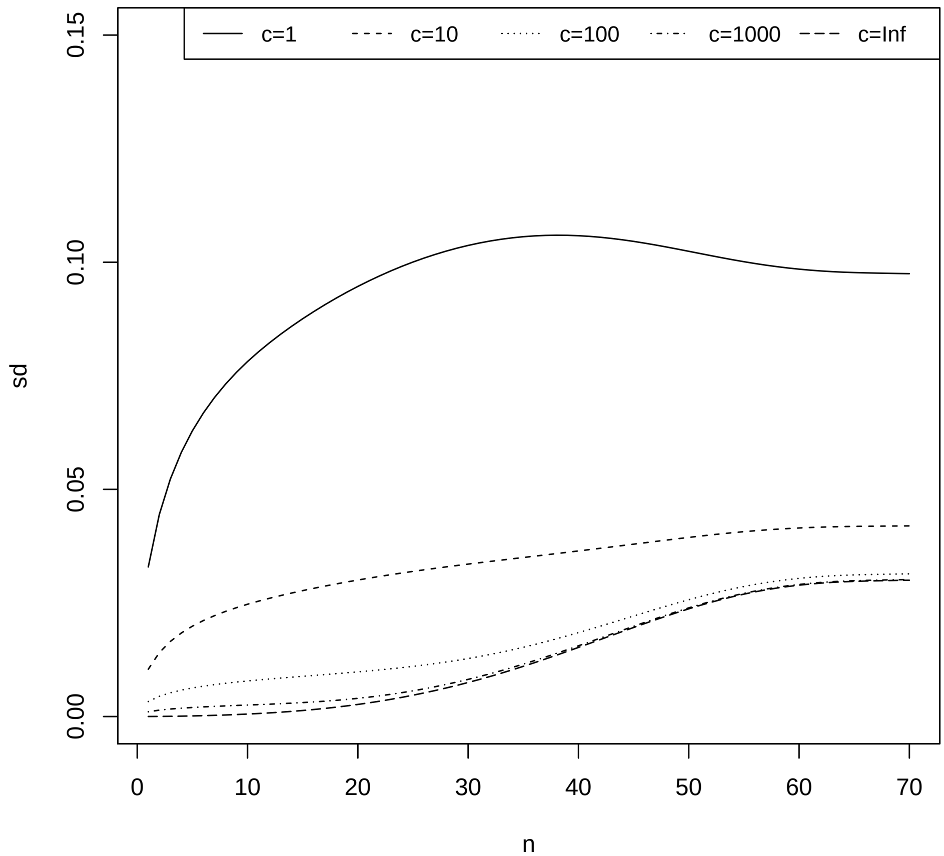

Figure 10: Figure 5.1 from Parker (1992)

The standard deviation for an endowment policy is below.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                            alpha = 0.1, sigma = 0.01), "ou")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> n = seq(1,70,1)
> sdev1 = numeric(length(n))
> sdev10 = numeric(length(n))
> sdev100 = numeric(length(n))
> sdev1000 = numeric(length(n))
> sdevInf = numeric(length(n))
> for(i in 1:length(n))
+ {
+   endow = insurance(list(n=n[i], d=1, e=1), "isingle", "endow")
+   port1 = insurance(list(single = endow, c = 1), "iport", "endow")
+   port10 = insurance(list(single = endow, c = 10), "iport", "endow")
```

```
+   port100 = insurance(list(single = endow, c = 100), "iport", "endow")
+   port1000 = insurance(list(single = endow, c = 1000), "iport", "endow")
+   portInf = insurance(list(single = endow, c = 1e18), "iport", "endow")
+
+   sdev1[i] = z.sd(port1,mort,oumodel)
+   sdev10[i] = z.sd(port10,mort,oumodel)
+   sdev100[i] = z.sd(port100,mort,oumodel)
+   sdev1000[i] = z.sd(port1000,mort,oumodel)
+   sdevInf[i] = z.sd(portInf,mort,oumodel)
+ }
> plot(x = n, y = sdev1, type = 'l', ylab = "sd", xlab = "n",
+      ylim = c(0, 0.15))
> lines(x = n, y = sdev10/10, type = 'l', lty = 2)
> lines(x = n, y = sdev100/100, type = 'l', lty = 3)
> lines(x = n, y = sdev1000/1000, type = 'l', lty = 4)
> lines(x = n, y = sdevInf/1e18, type = 'l', lty = 5)
> legend('topright', leg = c("c=1","c=10","c=100","c=1000","c=Inf"),
+        lty = c(1,2,3,4,5), ncol = 5, cex = 0.9)
```

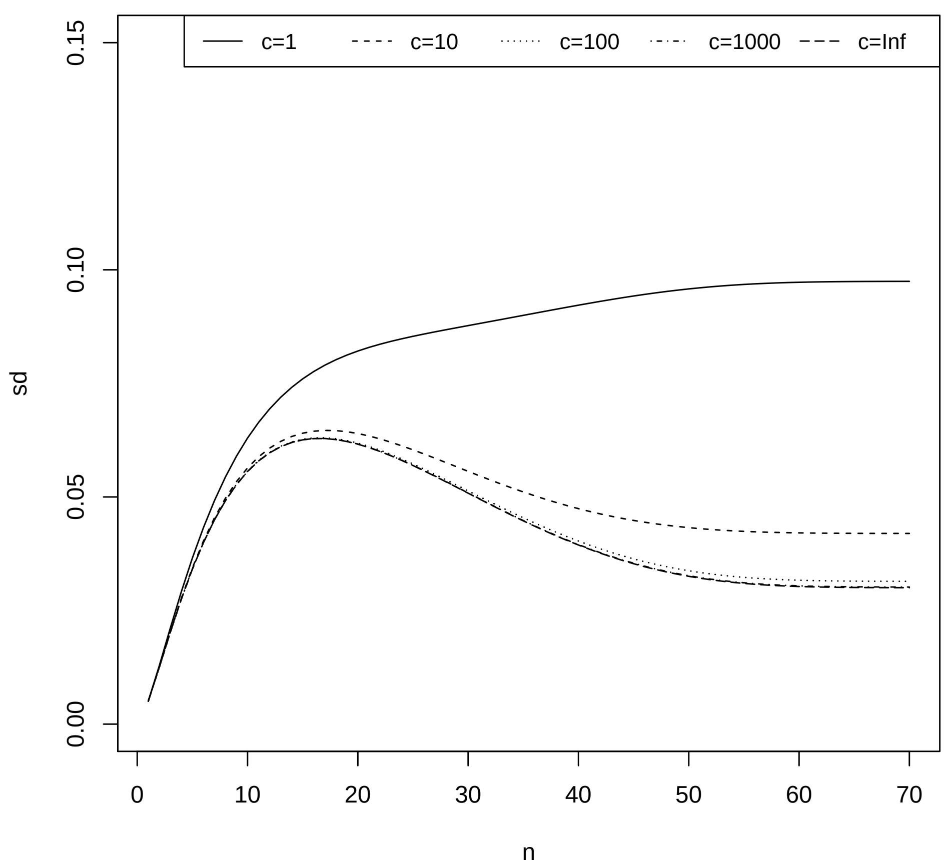

Figure 11: Figure 5.3 from Parker (1992)

References

Parker, G. (1992). *An application of stochastic interest rate models in life assurance* (Unpublished doctoral dissertation). Heriot-Watt University.

Parker, G. (1997). Stochastic analysis of the interaction between investment and insurance risks. *North American actuarial journal*, *1*(2), 55–71.