

# Package ‘stocins’

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**Type** Package

**Title** Stochastic Interest Rate Models for Life Insurance

**Version** 0.1.0

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**Description** This package can be used to analyze the risk of an insurance portfolio using stochastic interest rate models. The focus is on calculating the first three moments of the present value of benefit random variable for a portfolio of endowment or term insurance contracts. Several stochastic interest rate models are implemented including the Wiener process, the Ornstein-Uhlenbeck process and a Second Order Stochastic Differential equation for the force of interest. The references for this package are (1) Parker, Gary (1992) An application of stochastic interest rate models in life assurance and (2) Parker, Gary (1997) Stochastic analysis of the interaction between investment and insurance risks.

**License** GPL-2

**Encoding** UTF-8

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ann.ev	<i>Expected value of annuity</i>
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---

**Description**

Calculates the expected value of an n-period annuity. See the [iratemodel](#) class for details and examples.

**Usage**

```
ann.ev(n, irm)
```

**Arguments**

- |     |                                |
|-----|--------------------------------|
| n   | The length of the annuity      |
| irm | The interest rate model to use |

**Value**

Present value of annuity

---

ann.var	<i>Variance of annuity</i>
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**Description**

Calculates the variance of an n-period annuity. See the [iratemodel](#) class for details and examples.

**Usage**

```
ann.var(n, irm)
```

**Arguments**

n	The length of the annuity
irm	The interest rate model to use

**Value**

Variance of annuity

---

ar1	<i>The AR(1) Process</i>
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---

**Description**

A class used to model an AR(1) process. See the [iratemodel](#) class for details and examples.

**Usage**

```
## S3 method for class 'ar1'
delta.ev(t, irm)
```

```
## S3 method for class 'ar1'
delta.cov(s, t, irm)
```

```
## S3 method for class 'ar1'
delta.var(t, irm)
```

```
## S3 method for class 'ar1'
y.ev(t, irm)
```

```
## S3 method for class 'ar1'
y.var(t, irm)
```

```
## S3 method for class 'ar1'
y.cov(s, t, irm)
```

References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

---

arma	<i>The ARMA(2,1) Process</i>
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---

Description

A class used to model the ARMA(2,1) process. See the [iratemodel](#) class for details and examples.

Usage

```
## S3 method for class 'arma'
delta.ev(t, irm)

## S3 method for class 'arma'
delta.cov(s, t, irm)

## S3 method for class 'arma'
delta.var(t, irm)

## S3 method for class 'arma'
y.ev(t, irm)

## S3 method for class 'arma'
y.var(t, irm)

## S3 method for class 'arma'
y.cov(s, t, irm)
```

References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

---

CdnPop2016	<i>Canadian Population Table for 2016</i>
------------	---

---

Description

Canadian Population for 2016 from Statistics Canada census.

Usage

```
CdnPop2016
```

**Format**

Male and Female population counts and proportions for different age groups.

**Source**

Statistics Canada

---

determ	<i>Deterministic interest rate</i>
--------	------------------------------------

---

**Description**

A class used to model the deterministic interest rate. See the [iratemodel](#) class for details and examples.

**Usage**

```
## S3 method for class 'determ'  
delta.ev(t, irm)  
  
## S3 method for class 'determ'  
delta.cov(s, t, irm)  
  
## S3 method for class 'determ'  
delta.var(t, irm)  
  
## S3 method for class 'determ'  
y.ev(t, irm)  
  
## S3 method for class 'determ'  
y.var(t, irm)  
  
## S3 method for class 'determ'  
y.cov(s, t, irm)
```

**References**

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

---

endowport	<i>Endowment insurance portfolio (identical policies)</i>
-----------	---

---

### Description

A portfolio of  $c$  identical endowment policies. See the [insurance](#) class for details and examples on how to use this class.

### Usage

```
## S3 method for class 'iport.endowport'
z.moment(moment, ins, mort, irm)
```

### References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

---

endowsingle	<i>Single endowment insurance product</i>
-------------	---

---

### Description

An endowment insurance pays a benefit of  $d$  if the policyholder dies within the term of the contract and  $e$  if the policyholder survives the term of the contract.

In addition to the functions described in the [insurance](#) class, other functions are available for the endowsingle class.

In particular, `z.ev.two.isingle.endowsingle` can be used to calculate  $E[Z_1 Z_2]$ , `z.ev.three.isingle.endowsingle` can be used to calculate  $E[Z_1 Z_2 Z_3]$  and `z.ev.twoone.isingle.endowsingle` can be used to calculate  $E[Z_1^2 Z_2]$  in Parker (1992).

### Usage

```
## S3 method for class 'isingle.endowsingle'
z.moment(moment, ins, mort, irm)

z.ev.two.isingle.endowsingle(ins, mort, irm)

z.ev.three.isingle.endowsingle(ins, mort, irm)

z.ev.twoone.isingle.endowsingle(ins, mort, irm)

## S3 method for class 'isingle.endowsingle'
z.insrisk(ins, mort, irm)

## S3 method for class 'isingle.endowsingle'
z.invrisk(ins, mort, irm)
```

### Details

The `z.insrisk` and `z.invrisk` functions for the `endowsingle` class are implemented such that we are conditioning on  $K$  not  $y$ . See Parker (1997) for details.

### References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

Parker, G. (1997). Stochastic analysis of the interaction between investment and insurance risks. North American actuarial journal, 1(2), 55–71.

### Examples

```
oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
alpha = 0.1, sigma = 0.01), "ou")
mort = mortassumptions(list(x = 40, table = "MaleMort91"))
endow = insurance(list(n = 10, d = 1, e = 1), "isingle", "endow")

z.ev(endow,mort,oumodel) # first moment
z.ev.two.isingle.endowsingle(endow, mort, oumodel)
z.ev.twoone.isingle.endowsingle(endow, mort, oumodel)
z.ev.three.isingle.endowsingle(endow, mort, oumodel)
```

---

FemaleMort82

CA Male Mortality rates from 1980-1982 \* 0.9

---

### Description

This data set contains the mortality rates used in Parker (1997).

### Usage

```
FemaleMort82
```

### Format

a `x` column for the age from 0 to 102 inclusive and a `qx` column for the mortality rate.

### References

Parker, G. (1997). Stochastic analysis of the interaction between investment and insurance risks. North American actuarial journal, 1(2), 55–71.

---

FemaleMort82Reduced	<i>CA Male Mortality rates from 1980-1982 * 0.75</i>
---------------------	--

---

### Description

This data set contains the mortality rates used in Parker.

### Usage

FemaleMort82Reduced

### Format

a x column for the age from 0 to 102 inclusive and a qx column for the mortality rate.

### References

Parker, G. (1997). Stochastic analysis of the interaction between investment and insurance risks. North American actuarial journal, 1(2), 55–71.

---

FemaleMort91	<i>CA Female Mortality rates from 1990-1992</i>
--------------	---

---

### Description

This data set contains the 1991 mortality rates from Statistics Canada.

### Usage

FemaleMort91

### Format

a x column for the age from 0 to 100 inclusive and a qx column for the mortality rate.

### Source

Statistics Canada



igroup

*Group of Insurance Portfolios***Description**

A class for calculating the moments, insurance risk and investment risk for a group of endowment and term insurance portfolios using the formulas described in Parker (1997).

In addition to the functions described in the [insurance](#) class some other functions are available for the `igroup` class.

In particular, `z.ev.two.igroup` calculates  $E[Z_{1,i}, Z_{i,r}]$  for  $i = \text{ind1}$  and  $r = \text{ind2}$ , `cashflow.ev` calculates the expected cashflow at time  $r$  and `cashflow.cov` calculates the covariance between the cashflows at time  $r$  and time  $s$  as described in Parker (1997).

For details and examples on how to use the `igroup` class see the [insurance](#) class.

**Usage**

```
z.ev.two.igroup(ind1, ind2, ins, mort, irm)
```

```
## S3 method for class 'igroup'
z.moment(moment, ins, mort, irm)
```

```
cashflow.ev(r, ins, mort, irm)
```

```
cashflow.cov(s, r, ins, mort, irm)
```

```
## S3 method for class 'igroup'
z.insrisk(ins, mort, irm)
```

```
## S3 method for class 'igroup'
z.invrisk(ins, mort, irm)
```

**References**

Parker, G. (1997). Stochastic analysis of the interaction between investment and insurance risks. *North American actuarial journal*, 1(2), 55–71.

**Examples**

```
oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
alpha = 0.1, sigma = 0.01), "ou")
mort = mortassumptions(list(x = 40, table = "MaleMort91"))
mort2 = mortassumptions(list(x = 50, table = "FemaleMort91"))

termins = insurance(list(n = 10, d = 1), "isingle", "term")
endowins = insurance(list(n = 10, e = 1, d = 1), "isingle", "endow")

termport = insurance(list(single = termins, c = 1000), "iport", "term")
```

```

endowport = insurance(list(single = endowins, c = 1000), "iport", "endow")

groupins = insurance(list(termport, endowport),
  "igroup") # 1000 term contracts, 1000 endow contracts
groupmort = list(mort, mort2) # term contracts are age 40, endow contracts are age 50
z.moment(1, groupins, groupmort, oumodel) / groupins$c # average cost per policy
z.ev.two.igroup(1, 2, groupins, groupmort, oumodel)
cashflow.ev(5, groupins, groupmort, oumodel)
cashflow.cov(3, 5, groupins, groupmort, oumodel)

```

insurance

*Insurance product*

## Description

A class used to describe the insurance product we are modeling. Three types of insurance policies can be modeled:

1. Insurance policies issued to a single life, i.e. the [isingle](#) subclasses. For these policies, the `params` argument should be a list with `n` for the term of the contract, `d` for the death benefit and `e` for the survival benefit. If `e` is not specified it is assumed to be 0. For a policy issued to a single life, the class argument should be "isingle" and the subclass argument should be either "term" for a term insurance or "endow" for an endowment insurance. See the examples below.
2. Identical policies issued to many lives, i.e. the [iport](#) subclasses. For these policies, the `params` argument should be a list with `single` being an [isingle](#) object and `c` being a number specifying how many identical policies are in the portfolio. For a portfolio of policies, the class argument should be "iport" and the subclass argument should either be "term" for a term insurance or "endow" for an endowment insurance. See the examples below.
3. A group of insurance portfolios, i.e. the [igroup](#) subclass. For a group of policies, the `params` argument should be a list of [iport](#) objects and the class argument should be "igroup". The subclass argument is not needed. See the examples below.

## Usage

```

insurance(params, class, subclass = NULL)

z.moment(moment, ins, mort, irm)

z.ev(ins, mort, irm)

z.sd(ins, mort, irm)

z.sk(ins, mort, irm)

z.insrisk(ins, mort, irm)

z.invrisk(ins, mort, irm)

```

```
z.pdf(z, ins, mort, irm)
```

## Details

For each class, several functions are available.

The `z.moment` function can be used to calculate the raw moments of the present value of benefit random variable. For the `isingle` classes all the moments are implemented, for the `iport` classes the first three moments are implemented and for the `igroup` class the first two moments are implemented. The formulas from Parker (1992) were used to implement these moments.

The `z.ev` function can be used to calculate the first moment, the `z.sd` function can be used to calculate the standard deviation and the `z.sk` function can be used to calculate the skewness of the present value of benefit random variable.

The `z.insrisk` function can be used to calculate the insurance risk arising from uncertain mortality and `z.invrisk` can be used to calculate the investment risk arising from uncertain investment returns. The formulas from Parker (1997) were used to implement these functions.

The `z.pdf` function can be used to calculate the density function of the present value of benefit random variable for the `isingle` classes. This function has not been implemented for the `iport` and `igroup` classes. For details on how this could be done, refer to Parker (1992) and Parker (1997).

Refer to the examples below for how to use these functions.

## References

- Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.
- Parker, G. (1997). Stochastic analysis of the interaction between investment and insurance risks. *North American actuarial journal*, 1(2), 55–71.

## Examples

```
oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
alpha = 0.1, sigma = 0.01), "ou")
mort = mortassumptions(list(x = 40, table = "MaleMort91"))
mort2 = mortassumptions(list(x = 50, table = "FemaleMort91"))

## isingle classes
termins = insurance(list(n = 10, d = 1), "isingle", "term")
endowins = insurance(list(n = 10, e = 1, d = 1), "isingle", "endow")

z.ev(termins, mort, oumodel) # first moment
z.moment(2, termins, mort, oumodel) # second moment
z.moment(3, termins, mort, oumodel) # third moment
z.sd(endowins, mort, oumodel) # standard deviation
z.sk(endowins, mort, oumodel) # skewness

plot(function(z) z.pdf(z, termins, mort, oumodel), 0.01, 1.0,
ylim = c(0, 0.15), lty = 1, xlab = "z", ylab = "f(z)")

legend('topleft', leg = c(paste0("P(Z=0) = ", round(kpx(1, mort), 5))),
```

```

lty = 1)

## iport classes
termport = insurance(list(single = termins, c = 1000), "iport", "term")
endowport = insurance(list(single = endowins, c = 1000), "iport", "endow")
z.moment(1, termport, mort, oumodel) / termport$c # average cost
z.sd(termport, mort, oumodel) / termport$c # average standard deviation

## igroup class
groupins = insurance(list(termport, endowport),
"igroup") # 1000 term contracts, 1000 endow contracts
groupmort = list(mort, mort2) # term contracts are age 40, endow contracts are age 50
z.moment(1, groupins, groupmort, oumodel) / groupins$c # average cost per policy
z.insrisk(groupins, groupmort, oumodel) / termport$c^2 # insrisk per policy
z.invrisk(groupins, groupmort, oumodel) / termport$c^2 # invrisk per policy
z.sd(groupins, groupmort, oumodel) / termport$c # sd per policy

```

---

iport

---

*Insurance portfolio (identical policies)*


---

## Description

A portfolio of  $c$  identical term or endowment policies. The [termport](#) and [endowport](#) classes implement the functions for the present value of benefit random variable. See the [insurance](#) class for details and examples on how to use this class.

## Usage

```

z.moment.iport(moment, ins, mort, irm)

## S3 method for class 'iport'
z.insrisk(ins, mort, irm)

## S3 method for class 'iport'
z.invrisk(ins, mort, irm)

## S3 method for class 'iport'
z.pdf(z, ins, mort, irm)

```

## References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

## Description

A class used to describe the interest rate we are modeling. Six types of interest rate models can be used to model the force of interest  $\delta_s$  (see Parker (1992)):

1. Deterministic interest rate, i.e. the [determ](#) class. For the deterministic interest rate, the params argument should be a list with delta for the interest rate. The class argument should be "determ". See the examples below.
2. Wiener process, i.e. the [wiener](#) class. For the wiener process, the params argument should be a list with delta for the long term mean and sigma for the local volatility. The class argument should be "wiener". See the examples below.
3. Ornstein-Uhlenbeck process, i.e. the [ou](#) class. For the Ornstein-Uhlenbeck process, the params should be a list with delta for the long term mean, alpha for the friction parameter, sigma for the local volatility and delta0 for the initial value of the process. The class argument should be "ou". See the examples below.
4. Second Order Stochastic Differential Equation, i.e. the [second](#) class. For the second order process, the params should be a list with delta for the long term mean, alpha1 for the first friction parameter, alpha2 for the second friction parameter, sigma for the local volatility, delta0prime for the initial derivative of the process and delta0 for the initial value of the process. Note that Parker uses the notation of alpha0 instead of alpha2 but note that we are referring to the same parameter. The class argument should be "second". See the examples below.
5. AR(1) process, i.e. the [ar1](#) class. For the AR(1) process, the params should be a list with delta for the long term mean, delta0 for the initial value of the process, phi1 for the AR(1) coefficient and sigma for the local volatility. The class argument should be "ar1". See the examples below.
6. ARMA(2,1) process, i.e. the [arma](#) class. For the ARMA(2,1) process, the params should be a list with delta for the long term mean, delta0 for the initial value of the process, phi1 for the AR(1) coefficient, phi2 for the AR(2) coefficient, theta1 for the MA(1) coefficient and sigma for the local volatility. The class argument should be "arma".

## Usage

```
iratemodel(params, class)

iratemodel.convert(from, to, irm, Delta = 1)

delta.ev(t, irm)

delta.cov(s, t, irm)

delta.var(t, irm)

y.ev(t, irm)
```

```

y.var(t, irm)

y.cov(s, t, irm)

pv.moment(t, moment, irm)

pv.ev(t, irm)

pv.var(t, irm)

pv.cov(s, t, irm)

```

### Details

Define:

$$y(t) = \int_0^t \delta_s$$

$$pv(t) = \exp(-y(t))$$

For each class, several functions are available.

The `delta.ev` function can be used to calculate the expected value of the interest rate process at time  $t$ , the `delta.var` function can be used to calculate the variance of the interest rate process at time  $t$  and `delta.cov` can be used to calculate the covariance of the interest rate process at two times  $s$  and  $t$ .

The `y.ev` function can be used to calculate the expected value of the  $y$  at time  $t$ , the `y.var` function can be used to calculate the variance of  $y$  at time  $t$  and `y.cov` can be used to calculate the covariance of  $y$  at two times  $s$  and  $t$ .

The `pv.moment` can be used to calculate the raw moments of the present value random variable, `pv.ev` can be used to calculate the expected value of the present value random variable, `pv.var` can be used to calculate the variance of the present value random variable and `pv.cov` can be used to calculate the covariance of the present value random variable at two times  $s$  and  $t$ .

The `ann.ev` function can be used to calculate the expected value of an  $n$ -year annuity and `ann.var` can be used to calculate the variance of an  $n$ -year annuity.

The `iratemodel.convert` function can be used to convert between these interest rate models using the principle of covariance equivalence.

Refer to the examples below for how to use these functions.

### References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

### Examples

```

## 1. Deterministic interest rate
determodel = iratemodel(list(delta = 0.06), "determ")
delta.ev(5, determodel) # expected value

```

```

## 2. Wiener process
wienermodel = iratemodel(list(delta = 0.05, sigma = 0.01), "wiener")
pv.ev(10, wienermodel) # expected present value
pv.var(10, wienermodel) # variance of present value
pv.cov(5,10,wienermodel) # covariance of present value

## 3. Ornstein-Uhlenbeck process
oumodel = iratemodel(list(delta0 = 0.08, delta = 0.05,
alpha = 0.1, sigma = 0.01), "ou")
delta.ev(5, oumodel)
delta.var(5, oumodel)
y.ev(10, oumodel)
y.var(10, oumodel)
y.cov(5, 10, oumodel)
pv.ev(10, oumodel)
pv.var(10, oumodel)
pv.cov(5, 10, oumodel)

## 4. Second Order Stochastic Differential Equation
secondmodel = iratemodel(params = list(alpha1 = -0.50, alpha2 = -0.04,
delta0prime = 0.05, delta0 = 0.10, delta = 0.06, sigma = 0.01), "second")

## 5. AR(1) Process
ar1model = iratemodel(params = list(delta = 0.05, delta0 = 0.08,
phi1 = 0.90, sigma = 0.01), "ar1")
delta.ev(5, ar1model)
delta.var(5, ar1model)
delta.cov(5, 10, ar1model)
ann.ev(5, ar1model)
ann.var(5, ar1model)

## 6. ARMA(2,1) Process
armamodel = iratemodel(params = list(delta = 0.05, delta0 = 0.08,
phi1 = 1.05, phi2 = -0.095, theta1 = -0.05, sigma = 0.01), "arma")

## Convert from one model to another
sdemodel = iratemodel.convert("arma", "second", armamodel, 5)
armamodel = iratemodel.convert("second", "arma", sdemodel, 5)

oumodel = iratemodel.convert("ar1", "ou", ar1model, 1/12)

```

## Description

A class used to describe insurance policies issued to a single life. The [termsingle](#) and [endowsingle](#) classes can be used to model term and endowment insurance contracts respectively. See the [insurance](#) class for details and examples.

**Usage**

```

z.moment.isingle(moment, ins, mort, irm)

z.insrisk.isingle(ins, mort, irm)

z.invrisk.isingle(ins, mort, irm)

z.pdf.isingle(z, ins, mort, irm)

## S3 method for class 'isingle.endowsingle'
z.pdf(z, ins, mort, irm)

```

**References**

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

---

MaleMort82

---

*CA Male Mortality rates from 1980-1982*


---

**Description**

This data set contains the mortality rates used in Parker (1992).

**Usage**

```
MaleMort82
```

**Format**

a x column for the age from 0 to 102 inclusive and a qx column for the mortality rate.

**Source**

Appendix B in Parker (1992).

**References**

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.



---

MaleMort82Reduced	<i>CA Male Mortality rates from 1980-1982 * 0.8</i>
-------------------	---

---

**Description**

This data set contains the mortality rates used in Parker (1992).

**Usage**

MaleMort82Reduced

**Format**

a x column for the age from 0 to 102 inclusive and a qx column for the mortality rate.

**References**

Parker, G. (1997). Stochastic analysis of the interaction between investment and insurance risks. North American actuarial journal, 1(2), 55–71.

---

MaleMort91	<i>CA Male Mortality rates from 1990-1992</i>
------------	---

---

**Description**

This data set contains the 1991 mortality rates from Statistics Canada.

**Usage**

MaleMort91

**Format**

a x column for the age from 0 to 100 inclusive and a qx column for the mortality rate.

**Source**

Statistics Canada

---

mortassumptions	<i>Mortality Assumptions</i>
-----------------	------------------------------

---

### Description

A class which describes the mortality for a policyholder. The params argument should be a list with x for the age and table for the table. See the examples below.

### Usage

```
mortassumptions(params)

kpx(k, mort)

kdeferredqx(k, mort)

## S3 method for class 'mortassumptions'
kpx(k, mort)

## S3 method for class 'mortassumptions'
kdeferredqx(k, mort)
```

### Details

The kpx function can be used to calculate survival probabilities and the kdeferredqx function can be used to calculate probability of death in a given year. See the examples below.

### Examples

```
malemort = mortassumptions(list(x = 50, table = "MaleMort91"))
femalemort = mortassumptions(list(x = 30, table = "FemaleMort91"))

# calculate probabilities
kpx(5, malemort)
kdeferredqx(2, femalemort)
```

---

ou	<i>The Ornstein-Uhlenbeck Process</i>
----	---------------------------------------

---

### Description

A class used to model the Ornstein-Uhlenbeck process. See the [iratemodel](#) class for details and examples.

**Usage**

```
## S3 method for class 'ou'  
delta.ev(t, irm)  
  
## S3 method for class 'ou'  
delta.cov(s, t, irm)  
  
## S3 method for class 'ou'  
delta.var(t, irm)  
  
## S3 method for class 'ou'  
y.ev(t, irm)  
  
## S3 method for class 'ou'  
y.var(t, irm)  
  
## S3 method for class 'ou'  
y.cov(s, t, irm)
```

**References**

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

---

returns91

*Investment Returns from 1991 to 2017*

---

**Description**

Company returns observed on monthly basis from 1991 to 2017

**Usage**

```
returns91
```

**Format**

A column for month and column for return (annualized). Rates are continuously compounded.

---

second

*Second Order Stochastic Differential Equation*


---

### Description

A class used to model the Second Order Stochastic Differential Equation for the force of interest. See the [iratemodel](#) class for details and examples.

### Usage

```
## S3 method for class 'second'
delta.ev(t, irm)

## S3 method for class 'second'
delta.cov(s, t, irm)

## S3 method for class 'second'
delta.var(t, irm)

## S3 method for class 'second'
y.ev(t, irm)

## S3 method for class 'second'
y.var(t, irm)
```

### References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

---

termport

*Term insurance portfolio (identical policies)*


---

### Description

A portfolio of  $c$  identical term policies. See the [insurance](#) class for details and examples on how to use this class.

### Usage

```
## S3 method for class 'iport.termport'
z.moment(moment, ins, mort, irm)
```

### References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

---

termsingle

Single term insurance product

---

## Description

A term insurance pays a benefit of  $d$  if the policyholder dies within the term of the contract and 0 if the policyholder survives the term of the contract.

In addition to the functions described in the [insurance](#) class, other functions are available for the termsingle class.

In particular, `z.ev.two.isingle.termsingle` can be used to calculate  $E[Z_1 Z_2]$ , `z.ev.three.isingle.termsingle` can be used to calculate  $E[Z_1 Z_2 Z_3]$  and `z.ev.twoone.isingle.termsingle` can be used to calculate  $E[Z_1^2 Z_2]$  in Parker (1992).

## Usage

```
## S3 method for class 'isingle.termsingle'
z.moment(moment, ins, mort, irm)

z.ev.two.isingle.termsingle(ins, mort, irm)

z.ev.three.isingle.termsingle(ins, mort, irm)

z.ev.twoone.isingle.termsingle(ins, mort, irm)

## S3 method for class 'isingle.termsingle'
z.insrisk(ins, mort, irm)

## S3 method for class 'isingle.termsingle'
z.invrisk(ins, mort, irm)

## S3 method for class 'isingle.termsingle'
z.pdf(z, ins, mort, irm)
```

## Details

The `z.insrisk` and `z.invrisk` functions for the [termsingle](#) class are implemented such that we are conditioning on  $K$  not  $y$ . See Parker (1997) for details.

## References

- Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.
- Parker, G. (1997). Stochastic analysis of the interaction between investment and insurance risks. North American actuarial journal, 1(2), 55–71.

## Examples

```

oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
alpha = 0.1, sigma = 0.01), "ou")
mort = mortassumptions(list(x = 40, table = "MaleMort91"))
term = insurance(list(n = 10, d = 1), "isingle", "term")

z.ev(term,mort,oumodel) # first moment
z.ev.two.isingle.termsingle(term, mort, oumodel)
z.ev.twoone.isingle.termsingle(term, mort, oumodel)
z.ev.three.isingle.termsingle(term, mort, oumodel)

```

---

wiener

---

*The Wiener Process*


---

## Description

A class used to model the Wiener process. See the [iratemodel](#) class for details and examples.

## Usage

```

## S3 method for class 'wiener'
delta.ev(t, irm)

## S3 method for class 'wiener'
delta.cov(s, t, irm)

## S3 method for class 'wiener'
delta.var(t, irm)

## S3 method for class 'wiener'
y.ev(t, irm)

## S3 method for class 'wiener'
y.cov(s, t, irm)

## S3 method for class 'wiener'
y.var(t, irm)

```

## References

Parker, Gary. An application of stochastic interest rate models in life assurance. Diss. Heriot-Watt University, 1992.

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