1 Parker (1992) Thesis

Some figures from Parker (1992) are reproduced here.

```
> library(stocins)
> g = expand.grid(x = seq(20, 70, 10), y = seq(1, 75, 10))
> g$z = 0
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> for(i in 1:nrow(g)) {
+   term = insurance(list(n = g$y[i], d = 1), "isingle", "term")
+   mort = mortassumptions(list(x = g$x[i], table = "MaleMort82"))
+   g$z[i] = z.moment(1, term, mort, oumodel)
+ }
> lattice::wireframe(z ~ x * y, data = g, drape = TRUE, col = 'black',
+         col.regions = 'white', aspect = c(1.0, 0.8), colorkey = FALSE,
+         xlab = "issue age", ylab = "n", zlab = "",
+         screen = list(z = 340, x = -70),
+     scales = list(arrows = FALSE, col="black", font = 10, cex= 1.0),
+         par.settings = list(regions=list(alpha = 0.3),
+                 axis.line = list(col = "transparent")),
+         zoom = 0.95, zlim = c(0,0.50))
```

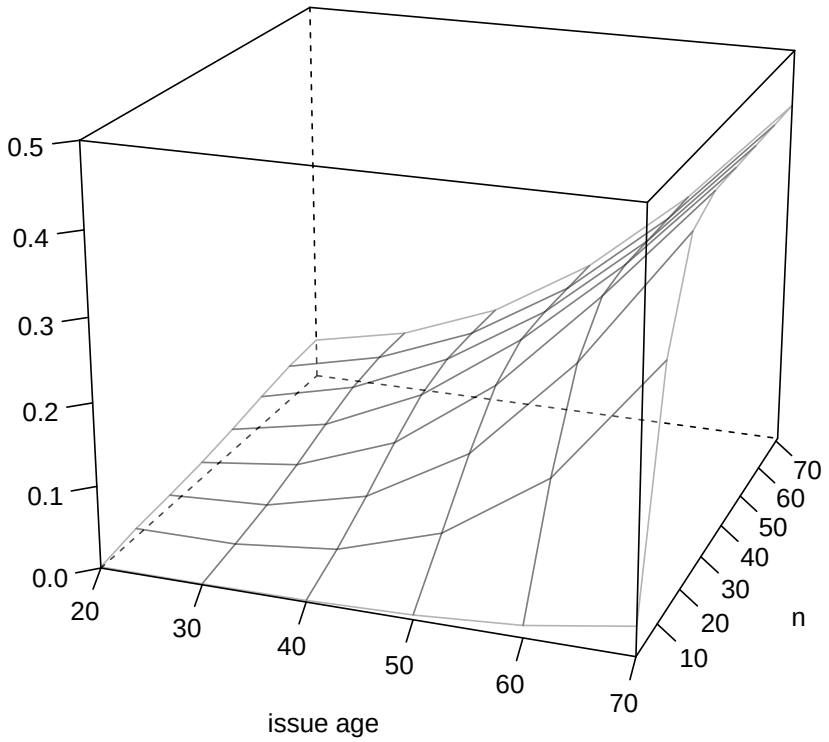

Figure 1: Figure 4.1 from Parker (1992)

Next, the standard deviation plot.

```
> g <- expand.grid(x = seq(1, 80, 5), y = seq(20, 70, 10))
> for(i in 1:nrow(g)) {
+   term = insurance(list(n = g$x[i], d = 1), "isingle", "term")
+   mort = mortassumptions(list(x = g$y[i], table = "MaleMort82"))
+   g$z[i] = z.sd(term, mort, oumodel)
+ }
> lattice::wireframe(z ~ x * y, data = g, drape = TRUE, col = 'black',
+                    col.regions = 'white', aspect = c(1, 0.8),
+                    colorkey = FALSE, xlab = "n", ylab = "issue age",
+                    zlab = "", screen = list(z = 340, x = -70, y = -20),
+                    scales = list(arrows = FALSE, col = 'black', font = 10),
+                    cex = 0.8, ylim = c(20, 70),
+                    zlim = c(0, 0.40),
+                    par.settings = list(regions=list(alpha = 0.3),
+                    axis.line = list(col = "transparent")), zoom = 0.95)
```

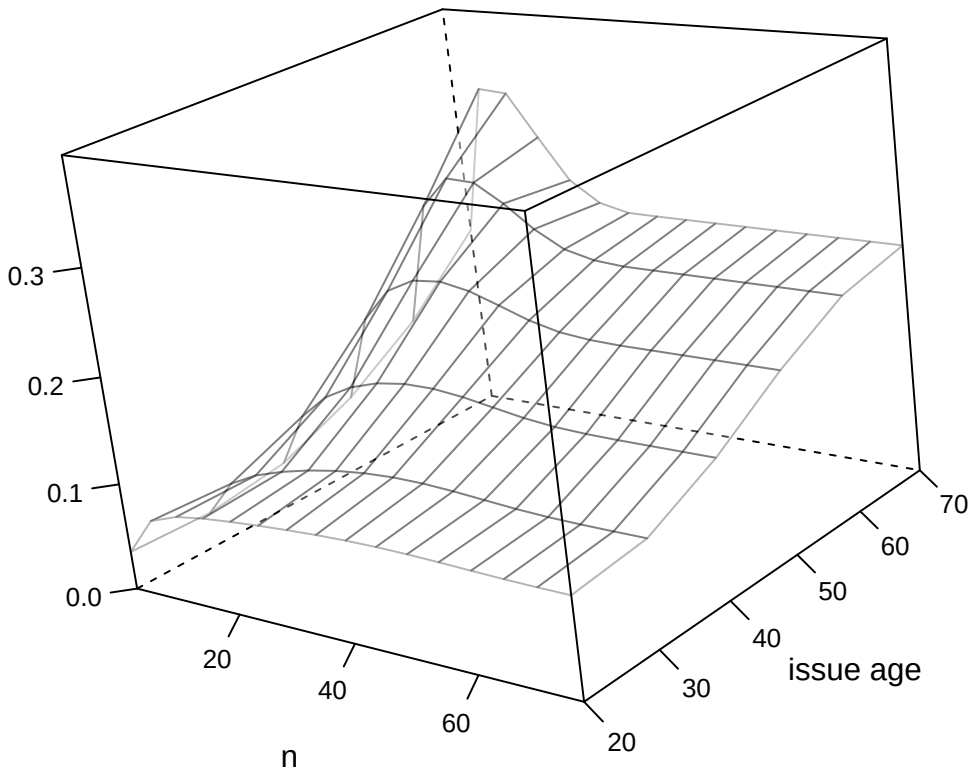

Figure 2: Figure 4.2 from Parker (1992)

Next, the skewness plot.

```
> g <- expand.grid(x = seq(1, 70, 5), y = seq(20, 70, 10))
> for(i in 1:nrow(g)) {
+   term = insurance(list(n = g$x[i], d = 1), "isingle", "term")
+   mort = mortassumptions(list(x = g$y[i], table = "MaleMort82"))
+   g$z[i] = z.sk(term, mort, oumodel)
+ }
> lattice::wireframe(z ~ y * x, data = g, drape = TRUE, col = 'black',
+                    col.regions = 'white', aspect = c(1, 0.8),
+                    colorkey = FALSE, xlab = "issue age", ylab = "n",
+                    zlab = "", screen = list(z = 340, x = -70, y = -20),
+                    scales = list(arrows = FALSE, col = 'black', font = 10),
+                    cex = 0.8, zlim = c(0, 30), ylim= c(0,70),
+                    par.settings = list(regions=list(alpha = 0.3),
+                    axis.line = list(col = "transparent")), zoom = 0.95)
```

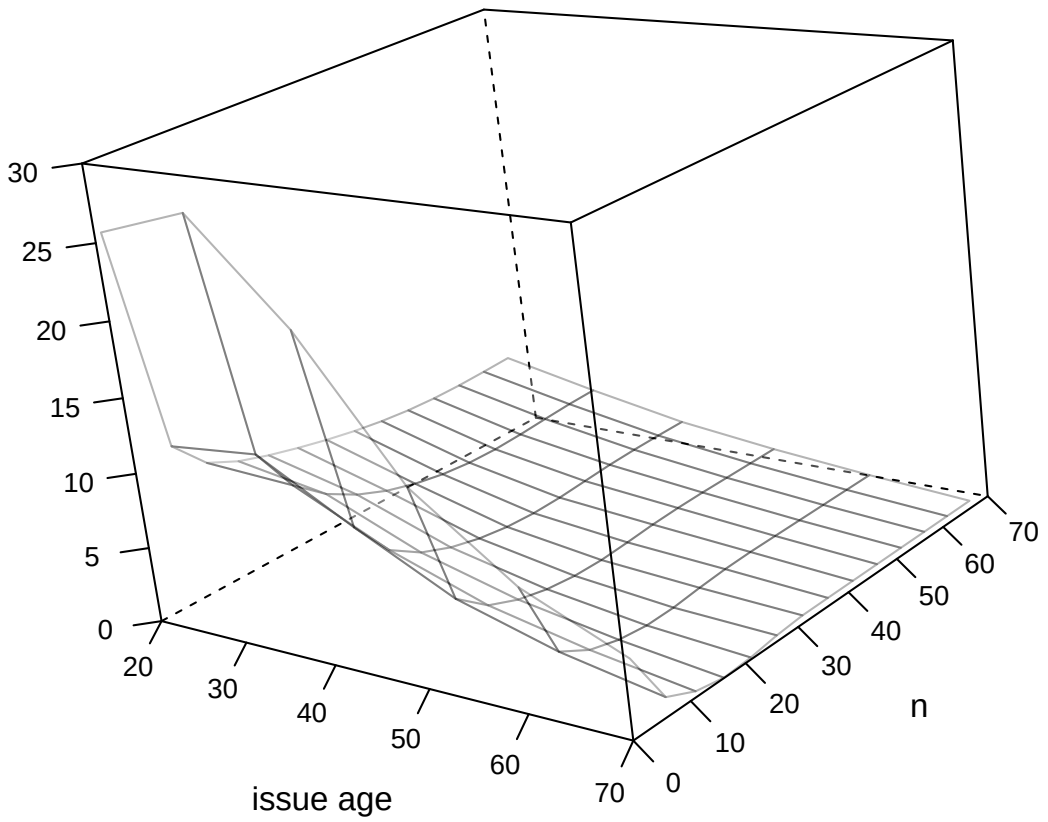

Figure 3: Figure 4.3 from Parker (1992)

Next, the pdf plot.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> term5 = insurance(list(n = 5, d = 1), "isingle", "term")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> term25 = insurance(list(n = 25, d = 1), "isingle", "term")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> plot(function(z) z.pdf(z, term5, mort, oumodel), 0.01, 1.0,
+      ylim = c(0, 0.4), lty = 1, xlab = "z", ylab = "f(z)")
> plot(function(z) z.pdf(z, term25, mort, oumodel), 0.01, 1.0,
+      ylim = c(0, 0.4), add = TRUE, lty = 2)
> legend('topright', leg = c(paste0("term 25 [P(Z=0) = ",
+                                   round(kpx(25, mort),5), "]"),
+                            paste0("term 5   [P(Z=0) = ",
+                                   round(kpx(5, mort),5), "])),
+        lty = c(2,1), cex = 0.8)
```

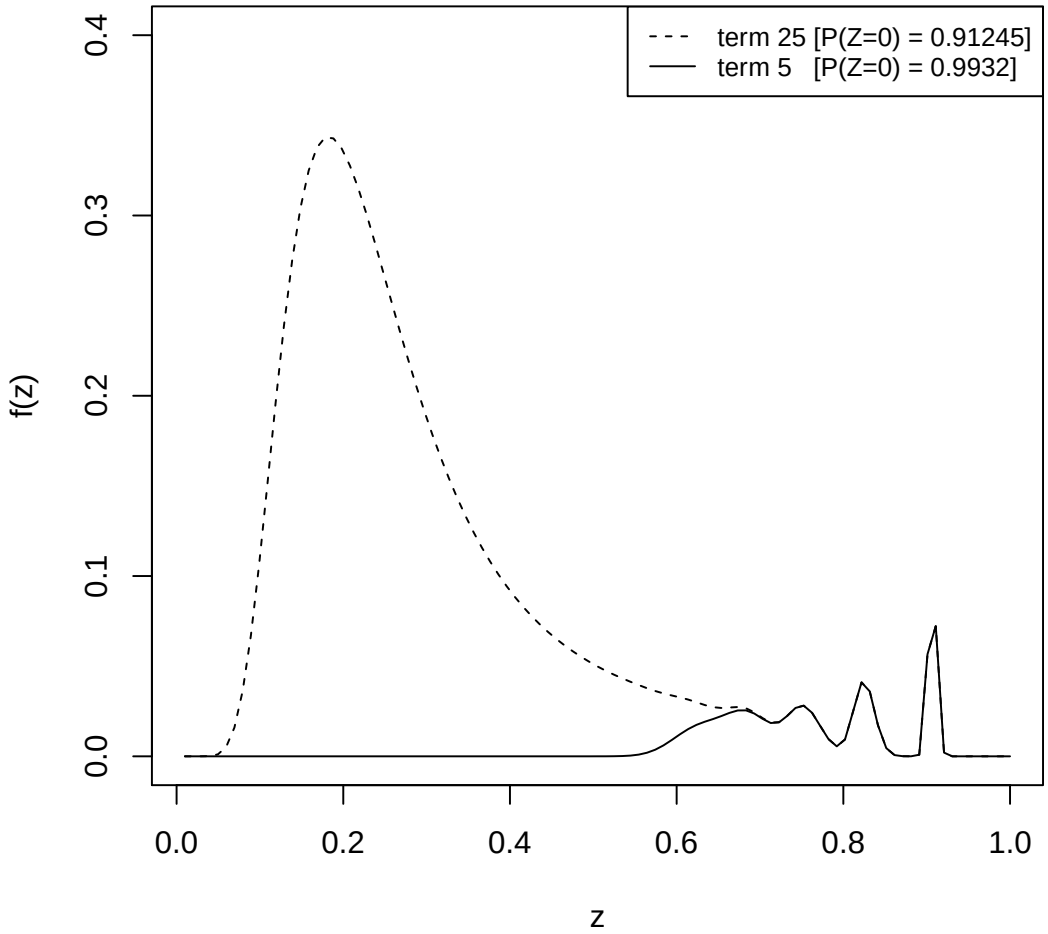

Figure 4: Figure 4.4 from Parker (1992)

For an endowment insurance, the expected value plot.

```
> library(stocins)
> g = expand.grid(x = seq(20, 70, 10), y = seq(1, 75, 10))
> g$z = 0
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> for(i in 1:nrow(g)) {
+   endow = insurance(list(n = g$y[i], d = 1, e = 1), "isingle", "endow")
+   mort = mortassumptions(list(x = g$x[i], table = "MaleMort82"))
+   g$z[i] = z.moment(1, endow, mort, oumodel)
+ }
> lattice::wireframe(z ~ x * y, data = g, drape = TRUE, col = 'black',
+         col.regions = 'white', aspect = c(1.0, 0.8), colorkey = FALSE,
+         xlab = "issue age", ylab = "n", zlab = "",
+         screen = list(z = 340, x = -70),
+     scales = list(arrows = FALSE, col="black", font = 10, cex= 1.0),
+         par.settings = list(regions=list(alpha = 0.3),
+                 axis.line = list(col = "transparent")),
+         zoom = 0.95, zlim = c(0,1.00))
```

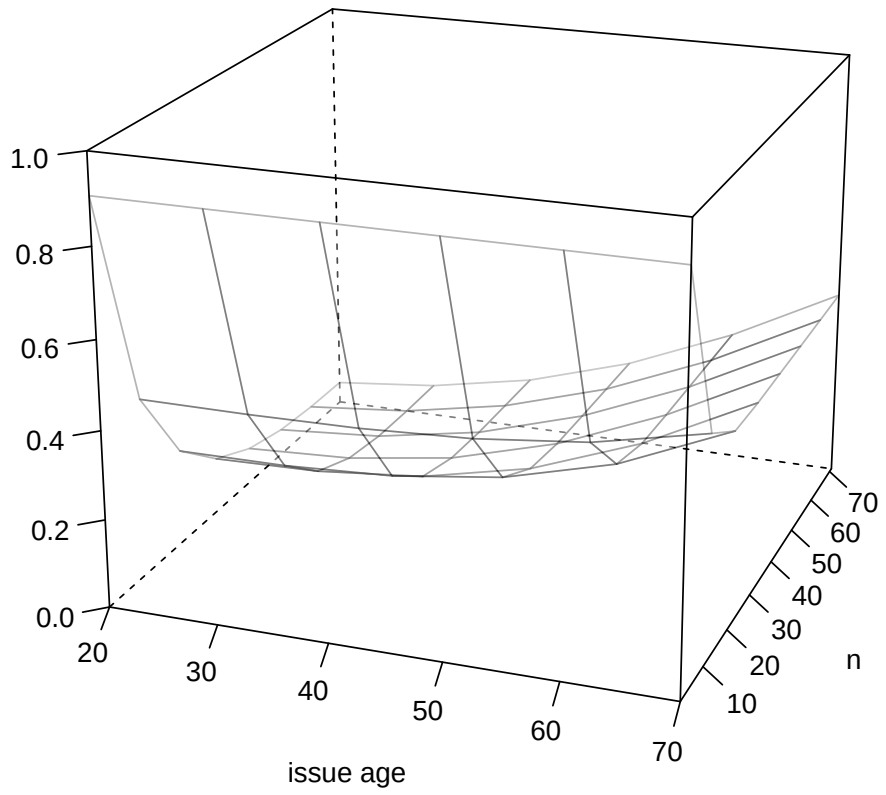

Figure 5: Figure 4.5 from Parker (1992)

Next, the standard deviation plot.

```
> g <- expand.grid(x = seq(1, 80, 5), y = seq(20, 70, 10))
> for(i in 1:nrow(g)) {
+   endow = insurance(list(n = g$x[i], d = 1, e = 1), "isingle", "endow")
+   mort = mortassumptions(list(x = g$y[i], table = "MaleMort82"))
+   g$z[i] = z.sd(endow, mort, oumodel)
+ }
> lattice::wireframe(z ~ x * y, data = g, drape = TRUE, col = 'black',
+                    col.regions = 'white', aspect = c(1, 0.8),
+                    colorkey = FALSE, xlab = "n", ylab = "issue age",
+                    zlab = "", screen = list(z = 340, x = -70, y = -20),
+                    scales = list(arrows = FALSE, col = 'black', font = 10),
+                    cex = 0.8, ylim = c(20, 70),
+                    zlim = c(0, 0.25),
+                    par.settings = list(regions=list(alpha = 0.3),
+                    axis.line = list(col = "transparent")), zoom = 0.95)
```

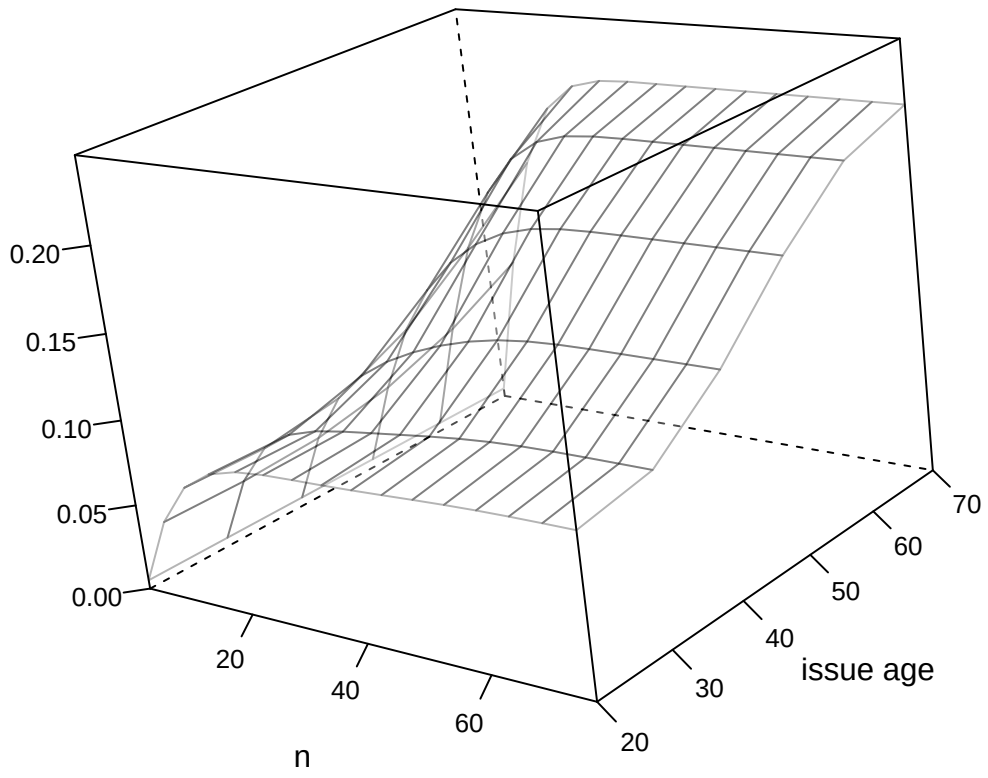

Figure 6: Figure 4.6 from Parker (1992)

Next, the skewness plot.

```
> g <- expand.grid(x = seq(1, 70, 5), y = seq(20, 70, 10))
> for(i in 1:nrow(g)) {
+   endow = insurance(list(n = g$x[i], d = 1, e = 1), "isingle", "endow")
+   mort = mortassumptions(list(x = g$y[i], table = "MaleMort82"))
+   g$z[i] = z.sk(endow, mort, oumodel)
+ }
> lattice::wireframe(z ~ y * x, data = g, drape = TRUE, col = 'black',
+                    col.regions = 'white', aspect = c(1, 0.8),
+                    colorkey = FALSE, xlab = "issue age", ylab = "n",
+                    zlab = "", screen = list(z = 340, x = -70, y = -20),
+                    scales = list(arrows = FALSE, col = 'black', font = 10),
+                    cex = 0.8, zlim = c(0, 6), ylim= c(0,70),
+                    par.settings = list(regions=list(alpha = 0.3),
+                    axis.line = list(col = "transparent")), zoom = 0.95)
```

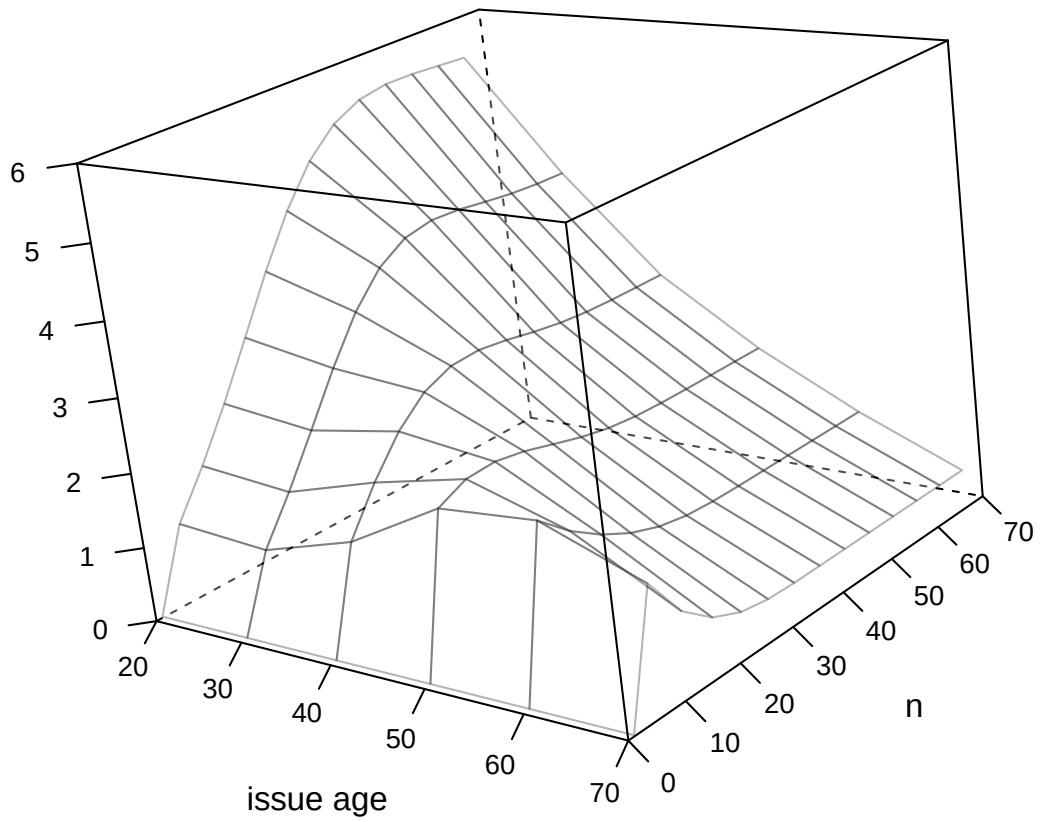

Figure 7: Figure 4.7 from Parker (1992)

Next, the pdf plot.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> endow5 = insurance(list(n = 5, d = 1, e = 1), "isingle", "endow")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> endow25 = insurance(list(n = 25, d = 1, e = 1), "isingle", "endow")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> plot(function(z) z.pdf(z, endow5, mort, oumodel), 0.01, 1.0,
+      ylim = c(0, 12), lty = 1, xlab = "z", ylab = "f(z)")
> plot(function(z) z.pdf(z, endow25, mort, oumodel), 0.01, 1.0,
+      ylim = c(0, 12), add = TRUE, lty = 2)
> legend('topright', leg = c("endow 25", "endow 5"),
+        lty = c(2,1), cex = 0.8)
```

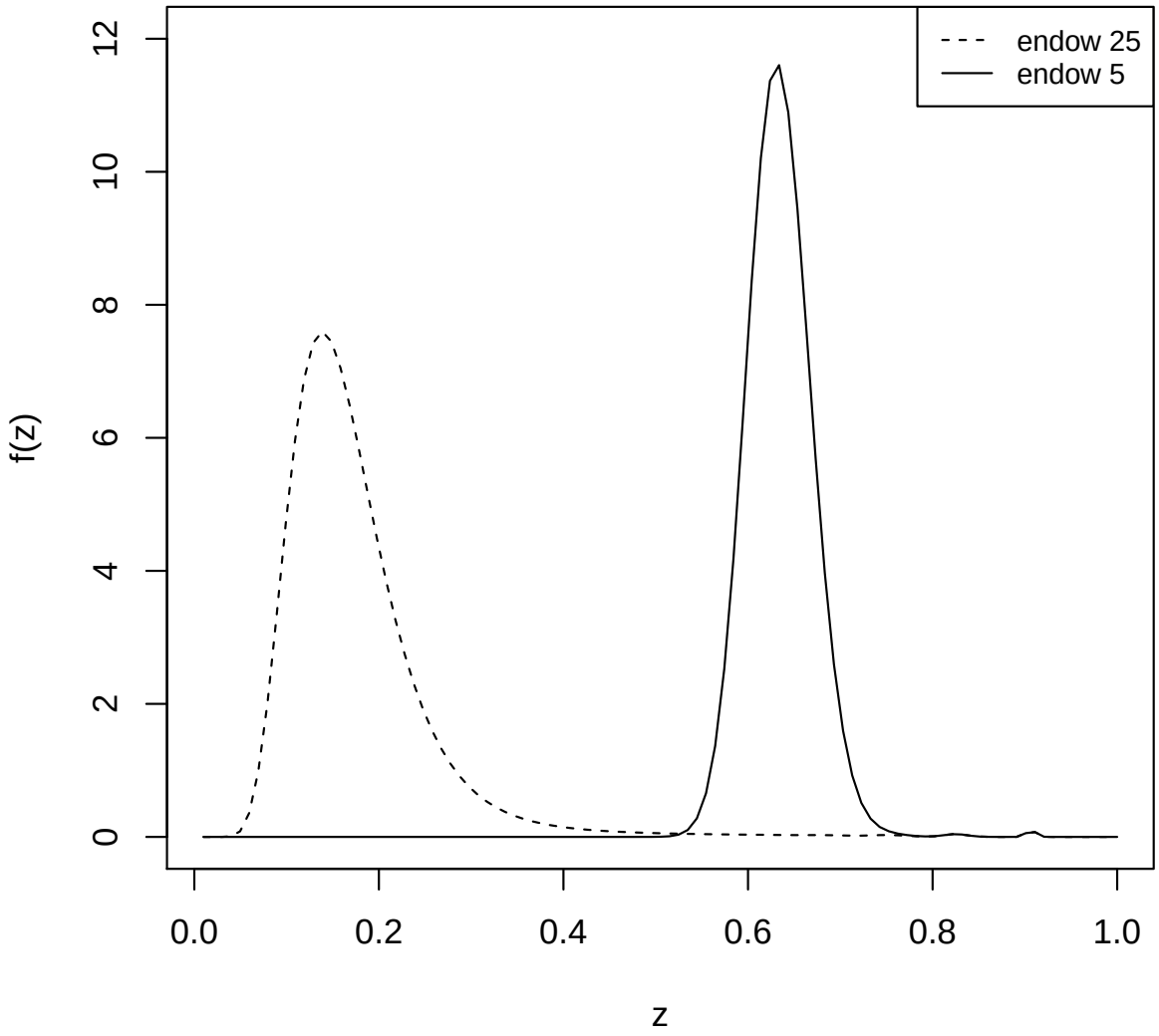

Figure 8: Figure 4.8 from Parker (1992)

Some numerical results are reproduced below.

```
> library(stocins)
> library(xtable)
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> wholelife = insurance(list(n = 100, d = 1), "isingle", "term")
> x = c(20,30,40,50,60,70,80,90,100)
> first = numeric(length(x))
> sdev = numeric(length(x))
> skew = numeric(length(x))
> for(i in 1:length(x)) {
+   mort = mortassumptions(list(x = x[i], table = "MaleMort82"))
+   first[i] = z.moment(1, wholelife, mort, oumodel)
+   sdev[i] = z.sd(wholelife, mort, oumodel)
+   skew[i] = z.sk(wholelife, mort, oumodel)
+ }
> table41 = data.frame(Age = x, Mean = first,
+                      "Standard Deviation" = sdev,
+                      "Skewness" = skew,
+                      "Coefficient of Variation" = sdev/first)
> print(xtable(table41,
+       digits = c(0,0,6,6,6,6),
+       caption = "Table 4.1 from Parker (1992)"),
+       include.rownames = FALSE)
```

Age	Mean	Standard.Deviation	Skewness	Coefficient.of.Variation
20	0.051187	0.090805	5.411855	1.773981
30	0.076342	0.097460	3.915181	1.276631
40	0.123992	0.127706	2.632900	1.029956
50	0.199394	0.167886	1.783115	0.841979
60	0.303412	0.200298	1.100983	0.660153
70	0.432234	0.213380	0.523390	0.493667
80	0.573185	0.200033	-0.009555	0.348985
90	0.698856	0.161555	-0.388251	0.231171
100	0.883526	0.041424	-1.502273	0.046885

Table 1: Table 4.1 from Parker (1992)

The pdf for a whole life insurance is shown below.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> whole = insurance(list(n = 100, d = 1, e = 1), "isingle", "term")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> plot(function(z) z.pdf(z, whole, mort, oumodel), 0.00001, 1.0,
```

```
+      ylim = c(0, 15), lty = 1, xlab = "z", ylab = "f(z)")
```

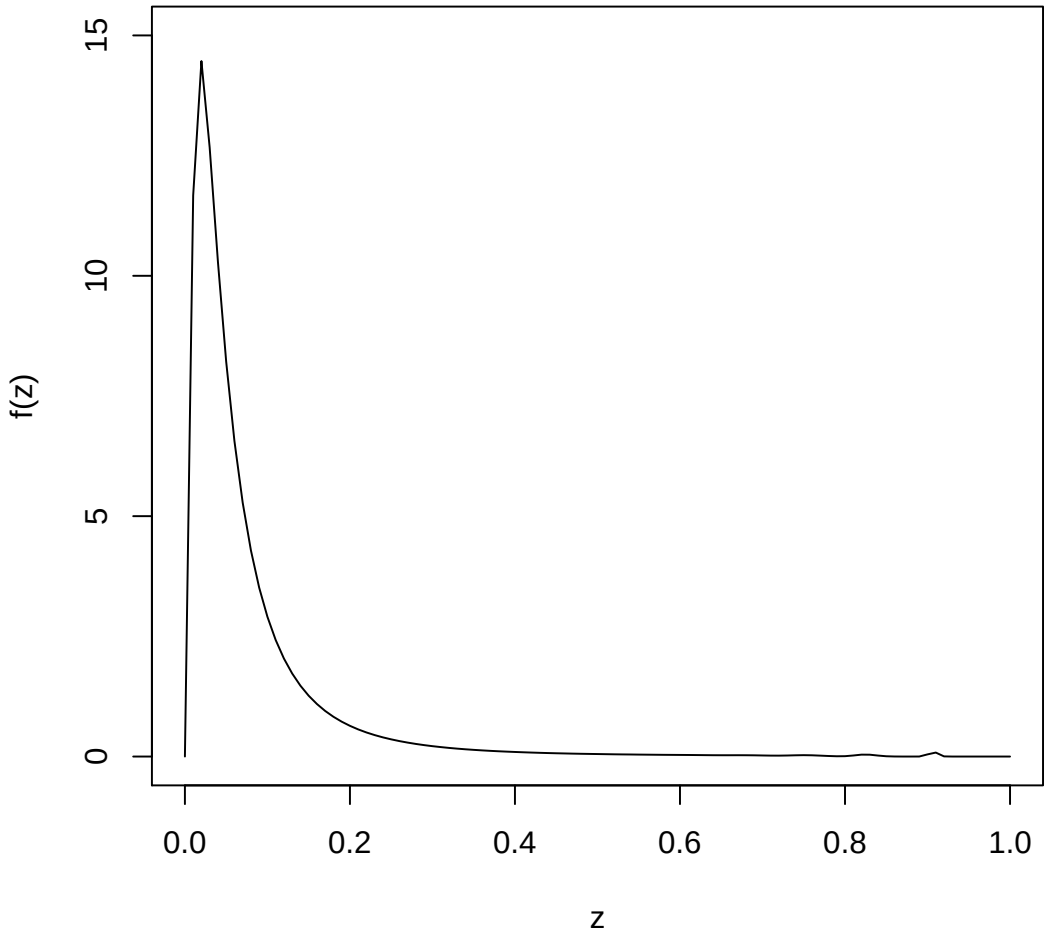

Figure 9: Figure 4.9 from Parker (1992)

Some results for a portfolio of policies are reproduced below.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> n = seq(1,70,1)
> sdev1 = numeric(length(n))
> sdev10 = numeric(length(n))
> sdev100 = numeric(length(n))
> sdev1000 = numeric(length(n))
> sdevInf = numeric(length(n))
> for(i in 1:length(n))
+ {
+   term = insurance(list(n=n[i], d=1), "isingle", "term")
+   port1 = insurance(list(single = term, c = 1), "iport", "term")
+   port10 = insurance(list(single = term, c = 10), "iport", "term")
```

```
+   port100 = insurance(list(single = term, c = 100), "iport", "term")
+   port1000 = insurance(list(single = term, c = 1000), "iport", "term")
+   portInf = insurance(list(single = term, c = 1e18), "iport", "term")
+
+   sdev1[i] = z.sd(port1,mort,oumodel)
+   sdev10[i] = z.sd(port10,mort,oumodel)
+   sdev100[i] = z.sd(port100,mort,oumodel)
+   sdev1000[i] = z.sd(port1000,mort,oumodel)
+   sdevInf[i] = z.sd(portInf,mort,oumodel)
+ }
> plot(x = n, y = sdev1, type = 'l', ylab = "sd", xlab = "n",
+      ylim = c(0, 0.15))
> lines(x = n, y = sdev10/10, type = 'l', lty = 2)
> lines(x = n, y = sdev100/100, type = 'l', lty = 3)
> lines(x = n, y = sdev1000/1000, type = 'l', lty = 4)
> lines(x = n, y = sdevInf/1e18, type = 'l', lty = 5)
> legend('topright', leg = c("c=1","c=10","c=100","c=1000","c=Inf"),
+        lty = c(1,2,3,4,5), ncol = 5, cex = 0.9)
```

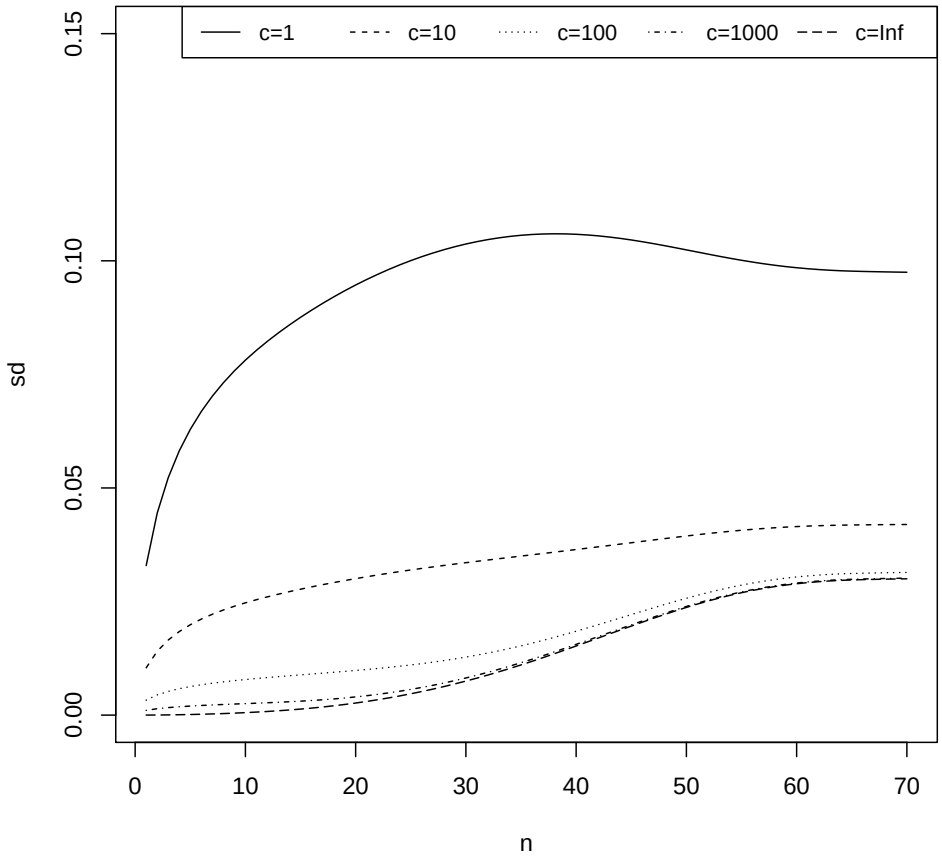

Figure 10: Figure 5.1 from Parker (1992)

The standard deviation for an endowment policy is below.

```
> oumodel = iratemodel(list(delta0 = 0.1, delta = 0.06,
+                           alpha = 0.1, sigma = 0.01), "ou")
> mort = mortassumptions(list(x = 30, table = "MaleMort82"))
> n = seq(1,70,1)
> sdev1 = numeric(length(n))
> sdev10 = numeric(length(n))
> sdev100 = numeric(length(n))
> sdev1000 = numeric(length(n))
> sdevInf = numeric(length(n))
> for(i in 1:length(n))
+ {
+   endow = insurance(list(n=n[i], d=1, e=1), "isingle", "endow")
+   port1 = insurance(list(single = endow, c = 1), "iport", "endow")
+   port10 = insurance(list(single = endow, c = 10), "iport", "endow")
+   port100 = insurance(list(single = endow, c = 100), "iport", "endow")
+   port1000 = insurance(list(single = endow, c = 1000), "iport", "endow")
+   portInf = insurance(list(single = endow, c = 1e18), "iport", "endow")
+
+   sdev1[i] = z.sd(port1,mort,oumodel)
+   sdev10[i] = z.sd(port10,mort,oumodel)
+   sdev100[i] = z.sd(port100,mort,oumodel)
+   sdev1000[i] = z.sd(port1000,mort,oumodel)
+   sdevInf[i] = z.sd(portInf,mort,oumodel)
+ }
> plot(x = n, y = sdev1, type = 'l', ylab = "sd", xlab = "n",
+      ylim = c(0, 0.15))
> lines(x = n, y = sdev10/10, type = 'l', lty = 2)
> lines(x = n, y = sdev100/100, type = 'l', lty = 3)
> lines(x = n, y = sdev1000/1000, type = 'l', lty = 4)
> lines(x = n, y = sdevInf/1e18, type = 'l', lty = 5)
> legend('topright', leg = c("c=1","c=10","c=100","c=1000","c=Inf"),
+        lty = c(1,2,3,4,5), ncol = 5, cex = 0.9)
```

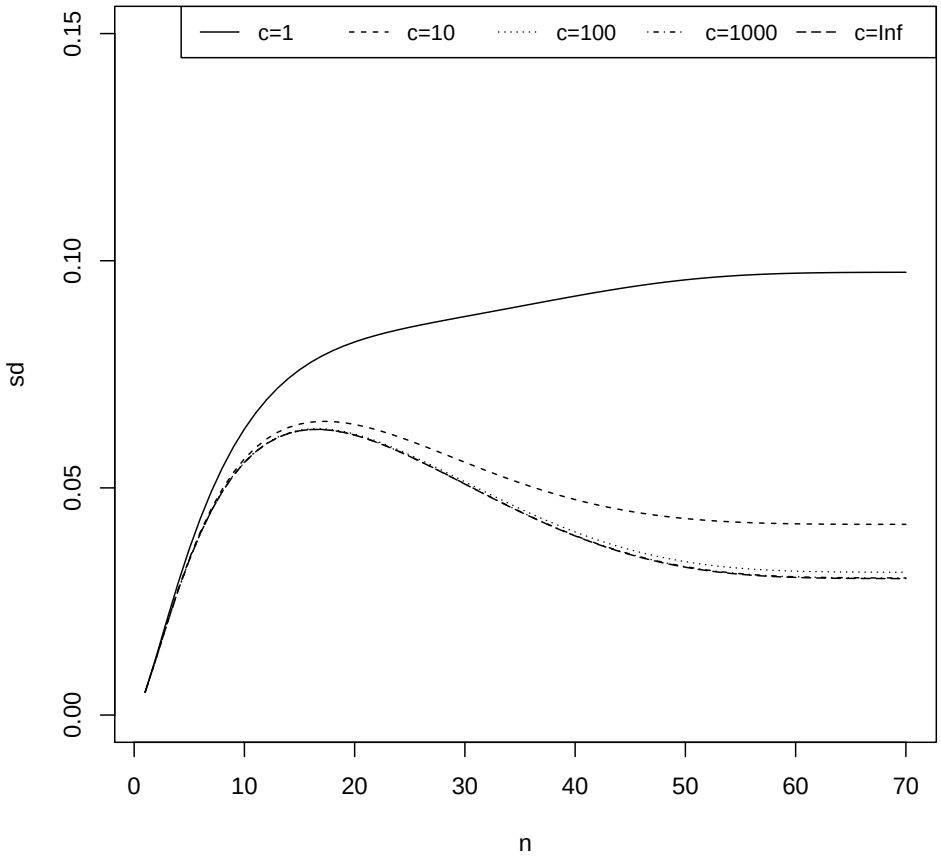

Figure 11: Figure 5.3 from Parker (1992)