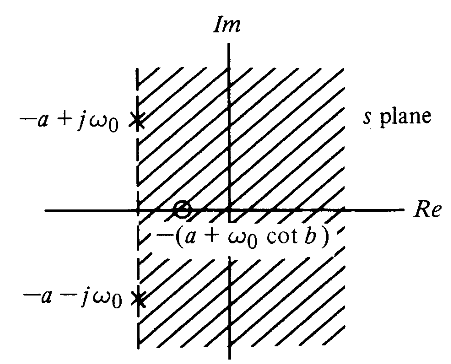

Figure S20.9-7

S20.10

(a) $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$

Consider

$$X_1(s) = \int_{-\infty}^{\infty} x(-t)e^{-st}\,dt$$

Letting $t = -t'$, we have

$$X_1(s) = \int_{-\infty}^{\infty} x(t')e^{st'}\,dt'$$
$$= X(-s),$$

but $X_1(s) = X(s)$ since $x(t) = x(-t)$. Therefore, $X(s) = X(-s)$.

(b) $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$

Consider

$$X_1(s) = \int_{-\infty}^{\infty} -x(-t)e^{-st}\,dt,$$
$$X_1(s) = \int_{-\infty}^{\infty} -x(t')e^{st'}\,dt'$$
$$= -X(s),$$

but $X_1(s) = X(s)$ since $x(t) = -x(-t)$. Therefore, $X(s) = -X(-s)$.

(c) We note that if $X(s)$ has poles, then it must be two-sided in order for $x(t) = x(-t)$.

(i) $$X(s) = \frac{Ks}{(s+1)(s-1)},$$
$$X(-s) = \frac{-Ks}{(-s+1)(-s-1)} = \frac{-Ks}{(s-1)(s+1)} \neq X(s),$$

so $x(t) \neq x(-t)$.

(ii) $$X(s) = \frac{K(s+1)(s-1)}{s},$$

$$X(-s) = \frac{K(-s+1)(-s-1)}{-s} \neq X(s)$$

Also, this pole pattern cannot have a two-sided ROC.

(iii) $$X(s) = \frac{K(s+j)(s-j)}{(s+1)(s-1)},$$

$$X(-s) = \frac{K(-s+j)(-s-j)}{(-s+1)(-s-1)} = \frac{K(s-j)(s+j)}{(s-1)(s+1)} = X(s),$$

so this can correspond to an even $x(t)$. The corresponding ROC must be two-sided, as shown in Figure S20.10-1.

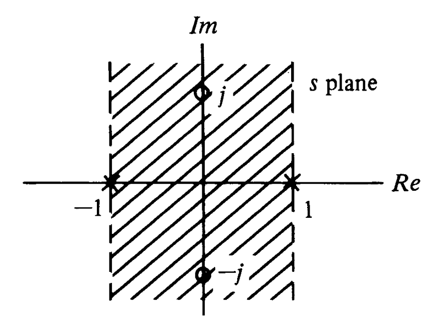

Figure S20.10-1

(iv) This does not have any possible two-sided ROCs.

(d) We see from the results in part (c)(i) that $X(s) = -X(-s)$, so the result in part (c)(i) corresponds to an odd $x(t)$ with an ROC as given in Figure S20.10-2.

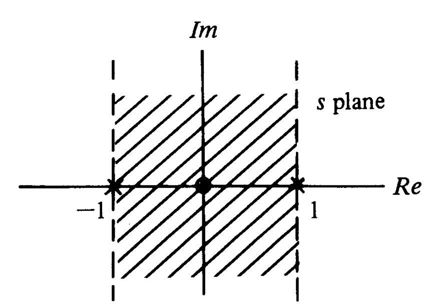

Figure S20.10-2

Parts (c)(ii) and (c)(iv) do not have any possible two-sided ROCs. Part (c)(iii) is even, as previously shown, and therefore cannot be odd.