

# 20 The Laplace Transform

## Recommended Problems

### P20.1

✓ Consider the signal  $x(t) = 3e^{2t}u(t) + 4e^{3t}u(t)$ .

- (a) Does the Fourier transform of this signal converge?
- (b) For which of the following values of  $\sigma$  does the Fourier transform of  $x(t)e^{-\sigma t}$  converge?
  - (i)  $\sigma = 1$
  - (ii)  $\sigma = 2.5$
  - (iii)  $\sigma = 3.5$
- (c) Determine the Laplace transform  $X(s)$  of  $x(t)$ . Sketch the location of the poles and zeros of  $X(s)$  and the ROC.

### P20.2

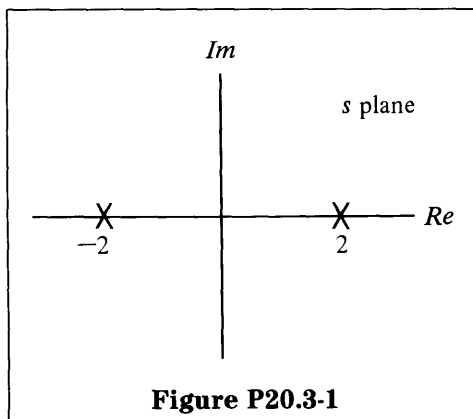
✓ Determine the Laplace transform, pole and zero locations, and associated ROC for each of the following time functions.

- (a)  $e^{-at}u(t)$ ,  $a > 0$
- (b)  $e^{-at}u(t)$ ,  $a < 0$
- (c)  $-e^{-at}u(-t)$ ,  $a < 0$

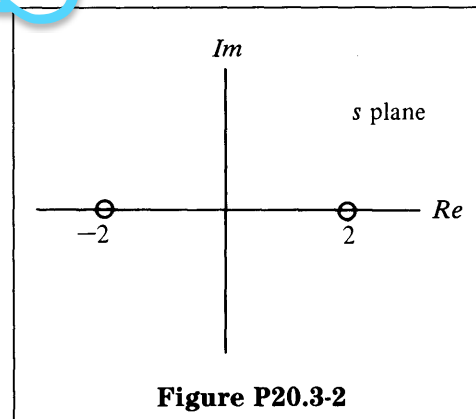
### P20.3

✓ Shown in Figures P20.3-1 to P20.3-4 are four pole-zero plots. For each statement in Table P20.3 about the associated time function  $x(t)$ , fill in the table with the corresponding constraint on the ROC.

(a)



(b)



(c)

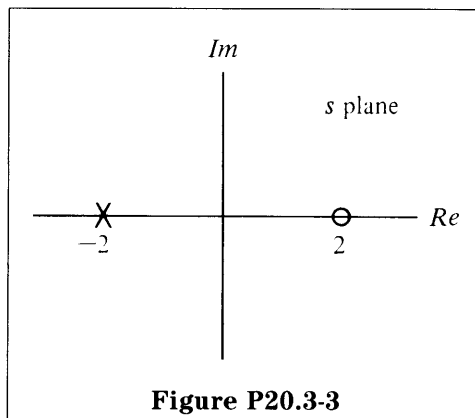


Figure P20.3-3

(d)

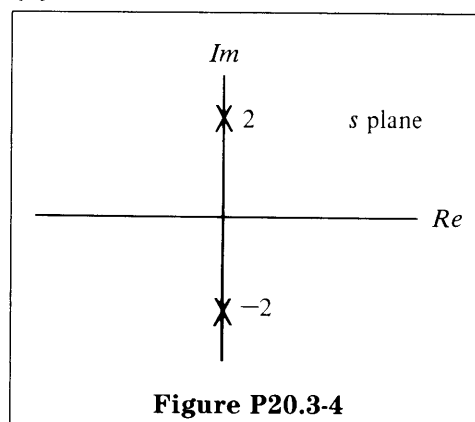


Figure P20.3-4

Constraint on ROC for Pole-Zero Pattern

$x(t)$	(a)	(b)	(c)	(d)
(i) Fourier transform of $x(t)e^{-t}$ converges				
(ii) $x(t) = 0, t > 10$				
(iii) $x(t) = 0, t < 0$				

Table P20.3

**P20.4**

Determine  $x(t)$  for the following conditions if  $X(s)$  is given by

$$X(s) = \frac{1}{(s+1)(s+2)}$$

(a)  $x(t)$  is right-sided

(b)  $x(t)$  is left-sided

(c)  $x(t)$  is two-sided

**P20.5**

An LTI system has an impulse response  $h(t)$  for which the Laplace transform  $H(s)$  is

$$H(s) = \int_{-\infty}^{+\infty} h(t)e^{-st}dt = \frac{1}{s+1}, \quad \text{Re}\{s\} > -1$$

Determine the system output  $y(t)$  for all  $t$  if the input  $x(t)$  is given by

$$x(t) = e^{-t/2} + 2e^{-t/3} \quad \text{for all } t.$$

**P20.6**

(a) From the expression for the Laplace transform of  $x(t)$ , derive the fact that the Laplace transform of  $x(t)$  is the Fourier transform of  $x(t)$  weighted by an exponential.



(b) Derive the expression for the inverse Laplace transform using the Fourier transform synthesis equation.

## Optional Problems

**P20.7**

Determine the time function  $x(t)$  for each Laplace transform  $X(s)$ .

(a)  $\frac{1}{s+1}$ ,  $\text{Re}\{s\} > -1$

(b)  $\frac{1}{s+1}$ ,  $\text{Re}\{s\} < -1$

(c)  $\frac{s}{s^2+4}$ ,  $\text{Re}\{s\} > 0$

(d)  $\frac{s+1}{s^2+5s+6}$ ,  $\text{Re}\{s\} > -2$

(e)  $\frac{s+1}{s^2+5s+6}$ ,  $\text{Re}\{s\} < -3$

(f)  $\frac{s^2-s+1}{s^2(s-1)}$ ,  $0 < \text{Re}\{s\} < 1$

(g)  $\frac{s^2-s+1}{(s+1)^2}$ ,  $-1 < \text{Re}\{s\}$

(h)  $\frac{s+1}{(s+1)^2+4}$ ,  $\text{Re}\{s\} > -1$

*Hint:* Use the result from part (c).

**P20.8**

The Laplace transform  $X(s)$  of a signal  $x(t)$  has four poles and an unknown number of zeros.  $x(t)$  is known to have an impulse at  $t = 0$ . Determine what information, if any, this provides about the number of zeros.

**P20.9**

Determine the Laplace transform, pole-zero location, and associated ROC for each of the following time functions.

(a)  $e^{-at}u(t)$ ,  $a < 0$

(b)  $-e^{at}u(-t)$ ,  $a > 0$

(c)  $e^{at}u(t)$ ,  $a > 0$

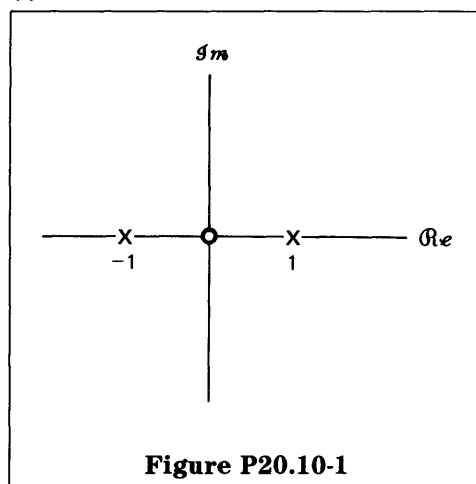
(d)  $e^{-a|t|}$ ,  $a > 0$

- (e)  $u(t)$
- (f)  $\delta(t - t_0)$
- (g)  $\sum_{k=0}^{\infty} a^k \delta(t - kT), \quad a > 0$
- (h)  $\cos(\omega_0 t + b)u(t)$
- (i)  $\sin(\omega_0 t + b)e^{-at}u(t), \quad a > 0$

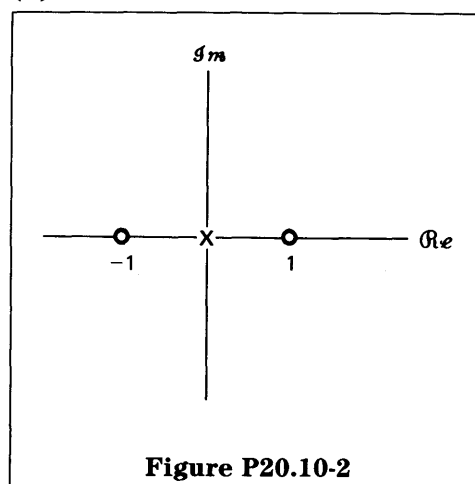
**P20.10**

- (a) If  $x(t)$  is an even time function such that  $x(t) = x(-t)$ , show that this requires that  $X(s) = X(-s)$ .
- (b) If  $x(t)$  is an odd time function such that  $x(t) = -x(-t)$ , show that  $X(s) = -X(-s)$ .
- (c) Determine which, if any, of the pole-zero plots in Figures P20.10-1 to P20.10-4 could correspond to an even time function. For those that could, indicate the required ROC.

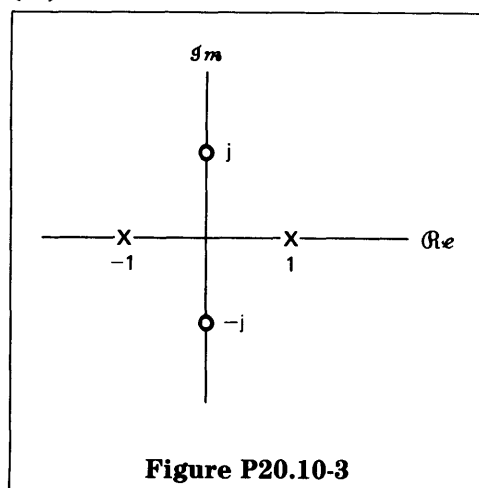
(i)



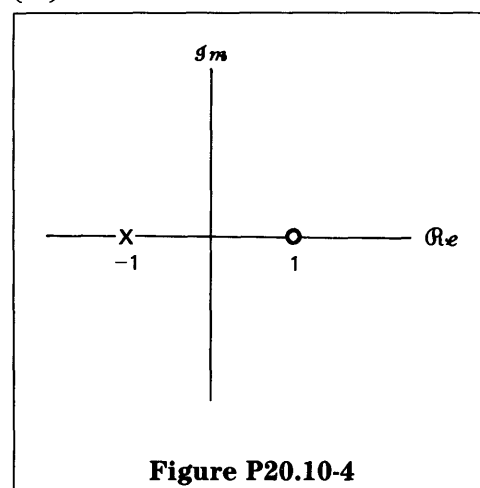
(ii)



(iii)



(iv)



- (d) Determine which, if any, of the pole-zero plots in part (c) could correspond to an odd time function. For those that could, indicate the required ROC.

MIT OpenCourseWare  
<http://ocw.mit.edu>

Resource: Signals and Systems  
Professor Alan V. Oppenheim

The following may not correspond to a particular course on MIT OpenCourseWare, but has been provided by the author as an individual learning resource.

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.