Multilevel Mediation Analysis

Andrew F. Hayes
The Ohio State University
www.afhayes.com

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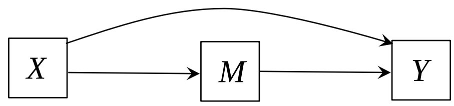
We will (and will not)…

- ☑ …do a brief review of principles of single-level statistical mediation analysis.
- ☑ …extend those principles to multilevel analysis, where components of the process modeled are allowed to vary between higher-order levels of analysis.
- ☑ …describe the equations and some Mplus code for estimating the models.
- ☑ …focus (with one exception) on "lower-level" mediation, with "nesting" of the data part nuisance but partly or mostly an opportunity
- ☑ …focus entirely on continuous mediators and outcome.
- ☐ …talk about latent variable models.
- ☐ …walk away multilevel mediation modeling experts.
- ☐ …walk away Mplus experts.
- ☐ …get all your questions answered.
- ☑ …leave with many new questions unanswered.

Statistical Mediation Analysis



Statistical mediation analysis is used to estimate and test hypotheses about the paths of causal influence from *X* to *Y*, one through a proposed "mediator" *M* and a second independent of the *X*→*M* →*Y* mechanism. It is widely used everywhere social science is conducted. It is one but not the only way to test mechanisms.

As a causal model…

- *M* must be causally located between *X* and *Y*: *X* affects *M*, which in turn affects *Y*
- the researcher carries all the design burdens on his or her shoulders. Statistics cannot be used to establish cause…it is only a part of the argument.
- the researcher must be careful in argument and language when the data don't deliver unequivocal causal interpretations, <u>as is often or even typically the case</u>.

The math doesn't care. Inference is a product of mind, not mathematics.

Empirical Example for Today

Data "inspired by" Cegala and Post (2009, *Patient Education and Counseling*), from a study of patient participation and physician communication.

n = 450 patient-doctor interactions

Patients are "nested" within 48 doctors, with the number of patients per doctor ranging between 4 and 12 (Mean = 9.375)

Our focus will be on examining the effect of completion of a training program to facilitate better communication with one's physician (*X*) on how much information the doctor volunteers (*Y*), with extent of patient participation in the clinical interview as the mediator variable (*M*).

These are not the actual data from this study. Although some of it was taken from the actual data file provided to me, some variables were added and findings embellished for pedagogical purposes.

The Data

docid	patient	volinfo	ppart	training	doccond	ppartmn	trainmn
1	9.00	57.00	28.00	-.50	.50	34.50	.00
1	3.00	57.00	29.00	-.50	.50	34.50	.00
1	8.00	109.00	32.00	-.50	.50	34.50	.00
1	2.00	109.00	34.00	-.50	.50	34.50	.00
1	4.00	124.00	36.00	.50	.50	34.50	.00
1	10.00	124.00	38.00	.50	.50	34.50	.00
1	7.00	136.00	29.00	-.50	.50	34.50	.00
1	1.00	136.00	31.00	-.50	.50	34.50	.00
1	5.00	137.00	35.00	.50	.50	34.50	.00
1	11.00	137.00	35.00	.50	.50	34.50	.00
1	12.00	180.00	43.00	.50	.50	34.50	.00
1	6.00	180.00	44.00	.50	.50	34.50	.00
2	3.00	15.00	23.00	-.50	-.50	35.92	.00
2	9.00	15.00	23.00	.50	-.50	35.92	.00
2	12.00	47.00	27.00	-.50	-.50	35.92	.00
2	6.00	47.00	28.00	.50	-.50	35.92	.00
2	5.00	49.00	48.00	-.50	-.50	35.92	.00
2	11.00	49.00	50.00	.50	-.50	35.92	.00
2	4.00	53.00	23.00	.50	-.50	35.92	.00
2	10.00	53.00	23.00	-.50	-.50	35.92	.00
2	8.00	80.00	36.00	.50	-.50	35.92	.00
2	2.00	80.00	37.00	-.50	-.50	35.92	.00
2	1.00	107.00	56.00	.50	-.50	35.92	.00
2	7.00	107.00	57.00	-.50	-.50	35.92	.00
3	11.00	8.00	23.00	-.50	.50	29.50	.00
3	5.00	8.00	24.00	-.50	.50	29.50	.00
3	1.00	11.00	41.00	.50	.50	29.50	.00
3	7.00	11.00	42.00	.50	.50	29.50	.00
3	4.00	40.00	25.00	-.50	.50	29.50	.00
3	10.00	40.00	26.00	-.50	.50	29.50	.00
3	12.00	40.00	27.00	.50	.50	29.50	.00
3	6.00	40.00	28.00	.50	.50	29.50	.00
3	2.00	73.00	26.00	-.50	.50	29.50	.00
3	8.00	73.00	26.00	-.50	.50	29.50	.00
3	3.00	95.00	33.00	.50	.50	29.50	.00
3	9.00	95.00	33.00	.50	.50	29.50	.00
4	4.00	23.00	38.00	-.50	.50	33.30	.00
4	9.00	23.00	39.00	.50	.50	33.30	.00
4	7.00	52.00	33.00	-.50	.50	33.30	.00
4	2.00	52.00	34.00	.50	.50	33.30	.00

Each row represents a doctor-patient interaction ("interview")

DOCID: A code identifying the doctor in the study.

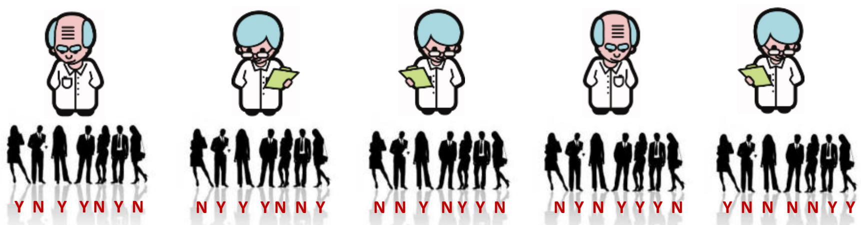
VOLINFO: Number of utterances volunteered by the doctor during the interview with a given patient. This is the outcome variable of interest *Y*.

PPART: Patient participation. Number of thought utterances from patient during the interview. This will be the mediator *M*.

TRAINING and DOCCOND: Whether or not a patient completed a Dr.-patient communication training seminar. Randomly assigned (+0.5 = yes, -0.5 = no). This is the independent variable *X*.

Two Design Variations

Patients randomly assigned to training, irrespective of which physician s/he sees. (TRAINING)



Patient assignment determined by which physician a patient sees. Random assignment occurs at the level of the physician. "Randomized cluster" trial. (DOCCOND)

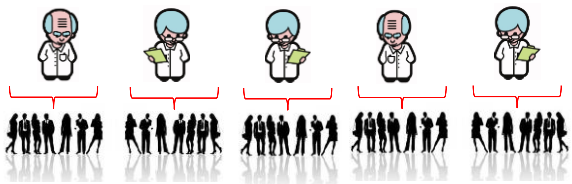
Design determines whether we estimate a "1-1-1" or a "2-1-1" model in this workshop.

"Nested" or "Clustered" Data

A feature of this study is "nesting" of the units of observation, where subsets of observations that are measured reside (are "nested") under a higher-level unit. Also called "clustered" data.

"Level-2" units:

"Level-1" units:



Other possibilities include

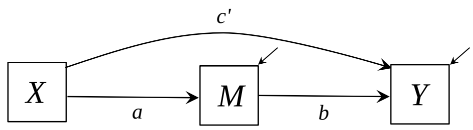
- Kids nested under classrooms
- Employees nested under companies
- Voters nested under neighborhoods/precincts
- Members of Congress nested under states
- Sets of stimuli administered to the same set of people, the features of which vary between stimuli.
- Measurements of the same thing repeatedly taken from the same people.

Such clustering of data introduces statistical challenges, but also provides analytical opportunities….in mediation analysis and any other kind of analysis.

Review: Single-Level Simple Mediation Analysis

Using OLS or ML, and treating *M* and *Y* as continuous:

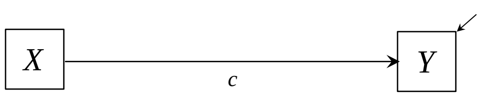
$$M_i = i_M + aX_i + e_{M_i}$$
$$Y_i = i_Y + c'X_i + bM_i + e_{Y_i}$$



c' = "direct effect" of X on Y

ab = "indirect effect" of X on Y through M

$$Y_i = i_Y + cX_i + e_{Y_i}$$



c = "total effect" of X on Y = $c' + ab$

Evidence of a total effect by a statistical significance criterion is not a requirement of mediation analysis in the 21st century. Correlation between X and Y is neither a sufficient ***nor a necessary*** condition of causality (!!!) The size and significance of c says little that is pertinent to the interpretation of direct and indirect effects. I don't concern myself with the total effect in general, or in this presentation.

Statistical Inference

Direct effect

The standard error for the direct effect (c') is used for interval estimation or hypothesis testing in the usual way. This is noncontroversial.

Indirect effect

Old-school approaches

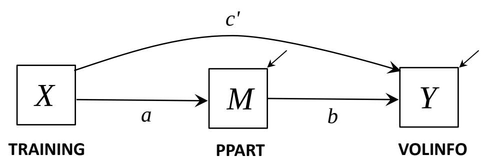
- **Causal steps** (a.k.a. test of "joint significance"): Both *a* and *b* statistically significant. Easy but "low tech"
- **Sobel test**: Estimate standard error of *ab* and then apply standard hypothesis test/confidence interval. Based on a faulty assumption about shape of sampling distribution of *ab* (i.e., it is normal).

Modern approaches

- **Monte Carlo confidence intervals**: Simulate the distribution of *ab* to produce a confidence interval for the indirect effect by making assumptions about the mean, standard deviation, and shape of the sampling distributions of *a* and *b* using estimates derived from the model.
- **Distribution of the product**: Like the MC confidence interval but analytical rather than simulation-based.
- **Bootstrap confidence intervals:** Empirically approximate the sampling distribution of *ab* by repeatedly resampling the data with replacement and estimating *a* and *b* in each resample. Use the empirical distribution to generate a confidence interval for the indirect effect.

For inference about the indirect effect, it often doesn't matter, but it can (see Hayes & Scharkow, 2013, *Psychological Science*). Use a modern approach when possible.

Example



Let's estimate the direct and indirect effects of the doctor-patient training seminar on how much information a patient is able to extract from the doctor during the doctor-patient interview. Patient participation is the proposed mediator.

We will ignore for this analysis the nesting of patients under doctors---the fact that subsets of patients share a physician.

Any OLS regression or SEM program could be used, but not all would give you a modern inferential test of the indirect effect. Most likely, you'd need to use something else to help your program do what it may not do out of the can… a macro (for SPSS or SAS, for example) or a package (in R, for instance). There are numerous options out there.

Output from PROCESS for SPSS

We are ignoring in this analysis the fact that subsets of patients share physician.

```
**************** PROCESS Procedure for SPSS Release 2.03 ***************

       Written by Andrew F. Hayes, Ph.D.    http://www.afhayes.com

**************************************************************************
Model = 4
    Y = volinfo
    X = training
    M = ppart

Sample size
        450

**************************************************************************
Outcome: ppart

Model Summary
          R       R-sq          F        df1        df2          p
      .2814      .0792    38.5257     1.0000   448.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant    33.3167      .4236    78.6443      .0000    32.4841    34.1493
training     5.2590      .8473     6.2069      .0000     3.5938     6.9241

**************************************************************************
Outcome: volinfo

Model Summary
          R       R-sq          F        df1        df2          p
      .3012      .0907    22.2976     2.0000   447.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant    18.2034     7.3275     2.4843      .0133     3.8027    32.6041
ppart        1.3561      .2124     6.3853      .0000      .9387     1.7735
training      .3151     3.9691      .0794      .9367    -7.4852     8.1155
```

a = 5.259 →

b = 1.356 →

← *c*′= 0.315

PROCESS is described in Hayes, A. F. (2013). ***Introduction to mediation, moderation, and conditional process analysis***. New York: Guilford Press.

Output from PROCESS for SPSS

```
************************** TOTAL EFFECT MODEL ****************************
Outcome: volinfo

Model Summary
          R       R-sq          F        df1        df2          p
      .0882      .0078     3.5112     1.0000   448.0000      .0616

Model
              coeff         se          t          p       LLCI       ULCI
constant    63.3842     1.9871    31.8983      .0000    59.4791    67.2893
training     7.4468     3.9741     1.8738      .0616     -.3634    15.2571

***************** TOTAL, DIRECT, AND INDIRECT EFFECTS *******************

Total effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
     7.4468     3.9741     1.8738      .0616     -.3634    15.2571

Direct effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .3151     3.9691      .0794      .9367    -7.4852     8.1155

Indirect effect of X on Y
           Effect    Boot SE   BootLLCI   BootULCI
ppart      7.1317     1.5342     4.4570    10.4869

******************* ANALYSIS NOTES AND WARNINGS ************************

Number of bootstrap samples for bias corrected bootstrap confidence intervals:
    10000

Level of confidence for all confidence intervals in output:
    95.00
```

c = 7.447 →

= *c*′ + *ab*

c = 7.447 →

← *c*′ = 0.315

ab →

← 95% bootstrap CI

PROCESS is described in Hayes, A. F. (2013). ***Introduction to mediation, moderation, and conditional process analysis***. New York: Guilford Press.

In Mplus

Mplus is a handy covariance structural modeling program useful for many kinds of statistical problems. It's advantage over HLM, SPSS, or SAS is its ability to estimate multiple models (i.e., *M* and *Y*) simultaneously. This is not always necessary, depending on the complexity of the model.

```
TITLE: Single level mediation analysis
DATA:
  file is doctors.txt;
VARIABLE:
  names are docid patient volinfo ppart length
        size training doccond ppartmn trainmn;
  usevariables are volinfo ppart training;
ANALYSIS:
  !bootstrap=10000;
MODEL:
  ppart on training (a);
  volinfo on ppart (b)
             training (cp);
MODEL CONSTRAINT:
  new (direct indirect total);
  indirect = a*b;
  direct = cp;
  total = cp+a*b;
OUTPUT:
  !cinterval (bcbootstrap);

!remove exclamation points above to generate bootstrap;
!confidence interval for the indirect effect;
```

Mplus Output

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
PPART ON					
TRAINING	5.259	0.845	6.221	0.000	*a* = 5.259, *p* < .001
VOLINFO ON					
PPART	1.356	0.212	6.407	0.000	*b* = 1.356, *p* < .001
TRAINING	0.316	3.956	0.080	0.936	*c'* = 0.316, *p* = 0.94
Intercepts					
VOLINFO	18.204	7.303	2.493	0.013	
PPART	33.317	0.423	78.819	0.000	
Residual Variances					
VOLINFO	1620.908	108.060	15.000	0.000	
PPART	80.397	5.360	15.000	0.000	
New/Additional Parameters					
DIRECT	0.316	3.956	0.080	0.936	
INDIRECT	7.132	1.598	4.463	0.000	*ab* = 7.132, *p* < .001
TOTAL	7.448	3.965	1.878	0.060	

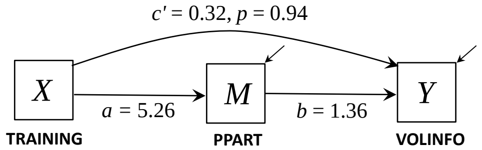
CONFIDENCE INTERVALS OF MODEL RESULTS

95% bootstrap CI

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
New/Additional Parameters							
DIRECT	-9.999	-7.466	-6.148	0.316	6.631	7.911	10.384
INDIRECT	3.651	4.448	4.837	7.132	9.917	10.424	11.569
TOTAL	-2.956	-0.441	0.754	7.448	14.008	15.251	17.723

ab

Single Level Mediation Analysis Summary



We have coded the groups so that they differ by one unit on *X*, so the direct and indirect effects can be interpreted as mean differences:

Those who attended the seminar received 7.13 units more information from their doctor, on average, than those who did not (bootstrap CI = 4.45 to 10.49) by increasing participation during the interview (because *a* is positive), and this increased participation was associated with more information provided by the doctor (because *b* is positive).

There is no evidence that, on average, attending the seminar influenced how much the doctor volunteered relative to those who did not attend independent of this patient participation mechanism ($c' = 0.32$, $p = 0.94$).

TIP: Get in the habit of coding a dichotomous variable by a one-unit difference (e.g., 0/1 or -0.5/0.5) so that its direct and indirect effects can be interpreted as a mean difference. Avoid reporting standardized effects involving a dichotomous predictor.

(Some) Problems With This Analysis

(1) Violation of the assumption of independence of errors

Standard error estimators used in OLS and ML programs typically assume independence of errors in the estimation of *M* and *Y*. This is likely violated with nested data when "context" is ignored. The standard error estimator is biased (usually downward) in this situation.

e.g., some physicians likely volunteer more than other physicians regardless of the behavior of his/her patients, or attract patients that are more inquisitive or communicative than are the patients of other doctors.

(2) All effects are assumed to be the same across doctors.

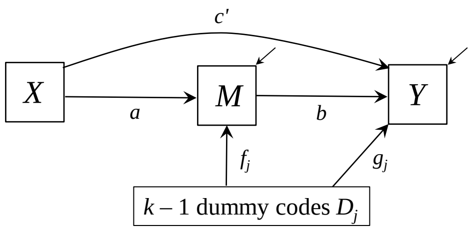
It completely ignores the real likelihood that physicians and their patients behave differently on average (see above), and that some doctors may be more (or less) affected by the training their patients receive ($X \rightarrow M$), or how they respond to the behavior of their patients as a result ($M \rightarrow Y$), relative to other doctors. Treats "context" as ignorable rather than interesting.

Although these aren't the only problems (e.g., we can't necessarily interpret the association between *M* and *Y* in causal terms), these ones we can at least deal with more rigorously using an alternative statistical approach that respects the nesting of patients under doctors.

A Partial Fix?

$$M_i = i_M + aX_i + \sum_{j=1}^{k-1} f_j D_{ij} + e_{M_i}$$

$$Y_i = i_Y + c'X_i + bM_i + \sum_{j=1}^{k-1} g_j D_{ij} + e_{Y_i}$$



This "fixed effects approach to clustering" models away covariation due to differences between the *k* level-2 units on the variables measured. All but one level-2 unit is coded with a dummy variable and these are included in the models of *M* and *Y*. The *a*, *b*, and *c′* paths can be interpreted as effects after partialing out variation due to level-2 "context" effects.

This helps with the independence problem, but still assumes *a*, *b*, and *c′* are constant across level-2 units. When the number of level-2 units is small, this may be the only option available to you. PROCESS has a simple option for doing it automatically when there are fewer than 20 level-2 units. This is probably better than ignoring the nesting entirely.

A Pure Multilevel Approach

- Allows for the estimation of individual-level data considering both individual and contextual processes at work simultaneously.
- Does not require (though still allows, if desired) assuming individual-level effects are constant across level-2 units.
- Independence in estimation errors still assumed, but much of the nonindependence in single-level analysis of multilevel data is captured by the estimation of certain effects as varying 'randomly' between level-2 units.
- Can "deconflate" individual and contextual effects that otherwise might be mistaken for each other.

We will consider some mediation models that explicitly consider that doctors differ on average in how much they volunteer, that patients with different doctors may participate differently on average, that doctors may be differentially sensitive to how much their patients participate, **and/or** that the effectiveness of the training program may differ between doctors.

Some Notation

see e.g., Bauer, Preacher, and Gil (2006)

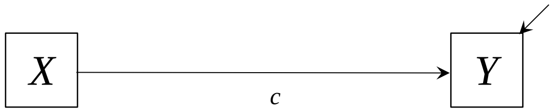
Level 2, units denoted j

Variables measured on/features of Level-2 units.
Constant for all level-1 units nested under a level-2 unit.

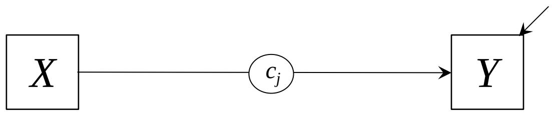
Level 1, units denoted i

Variables measured on/features of Level-1 units

c is a fixed effect of X on Y (i.e., estimated as constant across level-2 units)



c is a random effect of X on Y (i.e., estimating as varying between level-2 units)

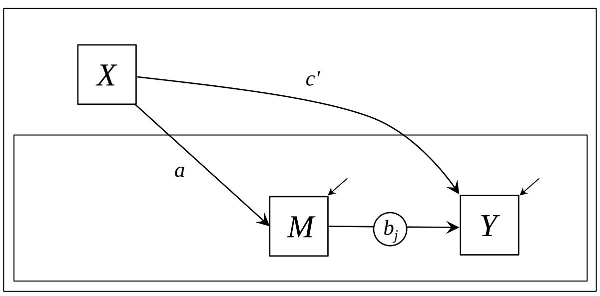


We will not denote intercepts. Assume they are always estimated as random effects---
It generally good practice to estimate intercepts as random in multilevel analysis.

A Handy Visual Representation

With this notation, a variety of multilevel mediation processes can be depicted.

This is a “2 – 1 – 1” Multilevel Mediation Model



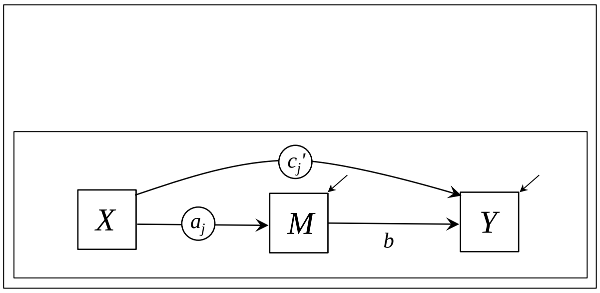
This model has a random indirect effect through the estimation of the $M \rightarrow Y$ effect as varying between level-2 units. An effect that “crosses” from a level-2 to a level-1 variable must be fixed. Thus, the effect of X on M and the direct effect of X on Y are fixed out of necessity in this model. We will estimate this model later.

This diagram is “conceptual”. It does not perfectly correspond to the model that is mathematically estimated---the “statistical model.”

A Handy Visual Representation

With this notation, a variety of multilevel mediation processes can be depicted.

This is a "1 – 1 – 1" Multilevel Mediation Model



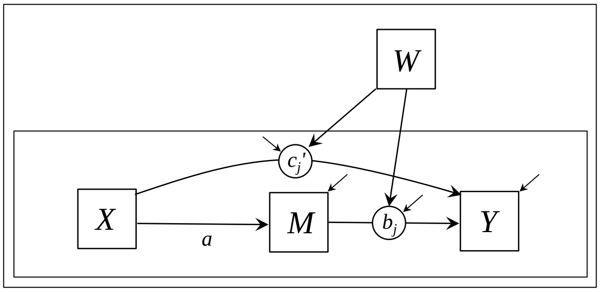
In this model, all three variables are measured at level-1. This model has a random indirect effect through the estimation of the $X \rightarrow M$ effect as varying between level-2 units. The direct effect of X on Y is also estimated as random. But the $M \rightarrow Y$ is estimated as constant across level-2 units.

This diagram is "conceptual". It does not perfectly correspond to the model that is mathematically estimated---the "statistical model."

A Handy Visual Representation

With this notation, a variety of multilevel mediation processes can be depicted.

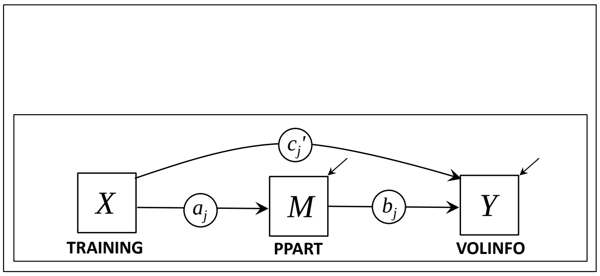
This is a "1 – 1 – 1" Multilevel Moderated Mediation Model



This model allows the effect of M on Y and the direct effect of X on Y to vary systematically as a function of W measured at level 2. Because one component of the indirect effect is modeled as a function of W, then the indirect effect is also a function of W. We wont consider multilevel "moderated mediation" here.

This diagram is "conceptual". It does not perfectly correspond to the model that is mathematically estimated---the "statistical model."

Example 1: A 1-1-1 Multilevel Mediation Model with All Effects Random

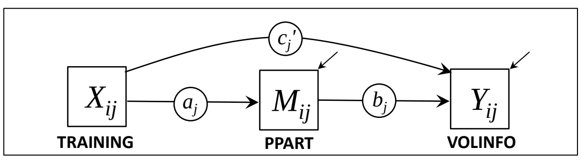


This is the most flexible multilevel mediation model with all variables measured at level-1. All causal paths are allowed to vary between level-2 units, meaning that the direct, indirect, and total effects can vary between level-2 units as well. We can estimate the average of each of these effects, as well as their variability, without too much hassle or difficulty.

This model allows the effect of training on how much patients participate and how much doctors volunteer to differ between doctors, while also allowing for between-doctor differences in how much patient participation influences how much information the doctor volunteers.

A 1-1-1 Multilevel Mediation Model with All Effects Random

j = doctor
i = patient



$$M_{ij} = d_{M_j} + a_j X_{ij} + e_{M_{ij}}$$

$$Y_{ij} = d_{Y_j} + c'_j X_{ij} + b_j M_{ij} + e_{Y_{ij}}$$

Level 1 model of how much a patient participates (M) from patient training (X) and how much a doctor volunteers (Y) from how much a patient participates and the patient's training. Think of this as "within-doctor" simple mediation analysis.

$$d_{M_j} = d_M + u_{d_{Mj}}$$

$$d_{Y_j} = d_Y + u_{Y_{Mj}}$$

$$a_j = a + u_{a_j}$$

$$b_j = b + u_{b_j}$$

$$c'_j = c' + u_{c'_j}$$

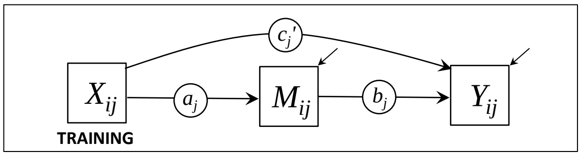
Level 2 model of the parameters of the level-1 model, allowing the intercepts and slopes to vary randomly between doctors around a common average. Our primary goal is estimate

$$a = \bar{a}_j \qquad b = \bar{b}_j \qquad c = \bar{c}'_j$$

Having done so, we can estimate the average direct and indirect effects and conduct an inference about these average effects.

A 1-1-1 Multilevel Mediation Model with All Effects Random

j = doctor
i = patient



$$M_{ij} = d_{M_j} + a_j X_{ij} + e_{M_{ij}}$$

$$Y_{ij} = d_{Y_j} + c_j' X_{ij} + b_j M_{ij} + e_{Y_{ij}}$$

$$d_{M_j} = d_M + u_{d_{Mj}}$$

Random intercept for M. This allows for differences between the 48 doctors in how much their patients participate on average.

$$d_{Y_j} = d_Y + u_{Y_{Mj}}$$

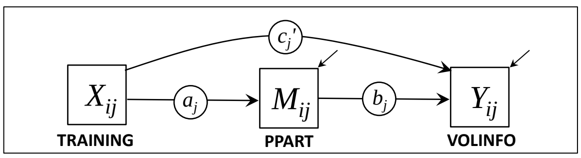
Random intercept for Y. This allows for differences between 48 doctors in how much they volunteer to their patients, on average.

$$\left.\begin{aligned} a_j &= a + u_{a_j} \\ b_j &= b + u_{b_j} \\ c_j' &= c' + u_{c_j'} \end{aligned}\right\}$$

Random effects. Each path in the system allowed to differ between doctors. We will estimate the average of each effect across all 48 doctors. Thus, we don't assume the effects are homogeneous across doctors.

A 1-1-1 Multilevel Mediation Model with All Effects Random

j = doctor
i = patient



$$M_{ij} = d_{M_j} + a_j X_{ij} + e_{M_{ij}}$$

$$Y_{ij} = d_{Y_j} + c_j' X_{ij} + b_j M_{ij} + e_{Y_{ij}}$$

$$d_{M_j} = d_M + u_{d_{Mj}}$$

$$d_{Y_j} = d_Y + u_{Y_{Mj}}$$

$$a_j = a + u_{a_j}$$

$$b_j = b + u_{b_j}$$

$$c_j' = c' + u_{c_j'}$$

<u>Average direct effect</u>

$$c = \overline{c_j'}$$

<u>Average indirect effect</u>

$$ab + COV_{a_j b_j} \quad \text{where } a = \overline{a}_j, b = \overline{b}_j$$

$COV_{a_j b_j}$ is the estimated covariance between a_j and b_j. This covariance is zero if a_j **or** b_j (or both) is estimated as fixed rather than random

Mplus: 1-1-1 MLM with All Random Effects

```
DATA:
  file is doctors.txt;
VARIABLE:
  names are docid patient volinfo ppart length size training
          doccond ppartmn trainmn;
  usevariables are docid volinfo ppart training;
  cluster is docid;
  within are training;
ANALYSIS:
  TYPE = twolevel random;
MODEL:
  %WITHIN%
  s_a|ppart on training;
  s_b|volinfo on ppart;
  s_cp|volinfo on training;
  %BETWEEN%
  [s_a] (a);
  [s_b] (b);
  [s_cp] (cp);
  !freely estimate covariance between random intercepts;
  ppart with volinfo;
  !freely estimate covariances between random intercepts and slopes;
  ppart with s_a s_b s_cp;
  volinfo with s_a s_b s_cp;
  !freely estimate covariances between random slopes;
  s_a with s_b (covab);
  s_a with s_cp;
  s_b with s_cp;
MODEL CONSTRAINT:
  new (direct indirect);
  indirect = a*b + covab;
  direct = cp;
```

cluster is docid; / within are training;: Specifies "docid" as variable defining level-2 unit and tells Mplus that "training" has no level 2 model.

TYPE = twolevel random;: Requests a two-level model. Random intercepts are specified by default.

%WITHIN% model: The level-1 model, specifying each of the three paths as "latent" random effects. Leaving off | and text to the left of | would fix the effect.

%BETWEEN% [s_a] (a); [s_b] (b); [s_cp] (cp);: Labels the mean of paths across doctors as "a", "b", and "cp" for use in MODEL CONSTRAINT.

s_a with s_b (covab);: Labels the covariance of the random a_j and b_j paths as "covab" for use in MODEL CONSTRAINT

MODEL CONSTRAINT: Defines indirect (and direct) effect in terms of model parameters and puts output for these in one place (tidy, convenient)

...all Mplus code with thanks to Preacher, Zypher, and Zhang (2010), *Psychological Methods*

Mplus Abbreviated Output

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Residual Variances
    VOLINFO          797.635     44.424     17.955      0.000
    PPART             47.313      2.141     22.094      0.000

Between Level

 PPART    WITH
    S_A                7.420     12.779      0.581      0.561
    S_B               -0.212      3.942     -0.054      0.957
    S_CP               0.276     49.476      0.006      0.996

 VOLINFO  WITH
    S_A              -62.041    132.289     -0.469      0.639
    S_B              -57.950     30.238     -1.916      0.055
    S_CP              25.833    789.028      0.033      0.974

 S_A      WITH
    S_B                1.629      3.898      0.418      0.676
    S_CP              -1.795     96.209     -0.019      0.985

 S_B      WITH
    S_CP              -0.827     21.621     -0.038      0.969

 PPART    WITH
    VOLINFO          -15.902    129.814     -0.122      0.903

 Means
    VOLINFO           11.277     19.294      0.584      0.559
    PPART             33.882      1.420     23.861      0.000
    S_A                5.463      1.871      2.919      0.004
    S_B                1.475      0.482      3.063      0.002
    S_CP              -0.270      9.019     -0.030      0.976

 Variances
    VOLINFO         2228.819   1096.050      2.034      0.042
    PPART             29.780     11.965      2.489      0.013
    S_A               24.105     20.388      1.182      0.237
    S_B                2.156      1.075      2.005      0.045
    S_CP               1.182    188.595      0.006      0.995

 New/Additional Parameters
    DIRECT            -0.270      9.019     -0.030      0.976
    INDIRECT           9.689      5.534      1.751      0.080
```

S_A WITH S_B: COV_{ajbj} = 1.629

Means S_A: a = 5.463, p < .01

Means S_B: b = 1.475, p < .01

Means S_CP: c' = -0.270, p = 0.98

Variances S_A: $V(a_j)$ = 24.105, p = .24

Variances S_B: $V(b_j)$ = 2.156, p < .05

Variances S_CP: $V(c'_j)$ = 1.182, p > .99

INDIRECT: $ab + COV_{ajbj}$ = 9.689, p = .08

Inference

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
New/Additional Parameters				
DIRECT	-0.270	9.019	-0.030	0.976
INDIRECT	9.689	5.534	1.751	0.080

- Inference for the direct effect is noncontroversial and comes right out of Mplus

c' = -0.270, p = 0.98, 95% CI = -17.947 to 17.407

- A few choices available for the indirect effect, but not as many as in single-level mediation analysis.
  - A Sobel-type test: Derive an estimate of the standard error. The formula is complicated. Mplus implements it.

$ab + COV_{a_j b_j}$ = 9.689, Z = 1.751, p = 0.08, 95% CI = -1.158 to 20.536

Inference

- Bootstrap confidence interval

  In principle, this is possible. Mplus can't do it with random effects models such as this. And there is some uncertainty in just how to properly resample the data.

- Monte Carlo confidence interval

  This can be done but it is tedious because a complicated covariance matrix is needed, and you'd have to hard program the simulation.

$$\begin{array}{ccc} V(a) & COV(a,b) & COV(a,COV(a_j,b_j)) \\ COV(a,b) & V(b) & COV(b,COV(a_j,b_j)) \\ COV(a,COV(a_j,b_j)) & COV(b,COV(a_j,b_j)) & V(COV(a_j,b_j)) \end{array}$$

  Once we construct this, we can take many random samples from a multivariate normal population with this covariance matrix and means equal to the estimates of a, b, and $COV_{a_j b_j}$ from the data. Then use this simulated sampling distribution for the construction of a confidence interval.

From Mplus TECH1 Output

```
OUTPUT:
   tech1;tech3;
```

Tech1 output gives us the Mplus parameter numbers for the diagonal elements

$$\begin{array}{ccc} V(a)\ \ 3 & COV(a,b) & COV(a,COV(a_j,b_j)) \\ COV(a,b) & V(b)\ \ 4 & COV(b,COV(a_j,b_j) \\ COV(a,COV(a_j,b_j)) & COV(b,COV(a_j,b_j) & V(COV(a_j,b_j))\ \ 9 \end{array}$$

```
          ALPHA
             S_A           S_B           S_CP          VOLINFO       PPART
             ________      ________      ________      ________      ________
 1                  3             4             5             6             7

           PSI
              S_A           S_B           S_CP          VOLINFO       PPART
              ________      ________      ________      ________      ________
 S_A                8
 S_B                9            10
 S_CP              11            12            13
 VOLINFO           14            15            16            17
 PPART             18            19            20            21            22
 TRAINING           0             0             0             0             0
```

From Mplus TECH3 Output

Tech3 output provides the entries of the matrix using the parameter numbers

$$\begin{array}{c} \\ \\ 3,9 \end{array}\left[\begin{array}{ccc} 3\ \ V(a) & COV(a,b)\ 3,4 & COV(a,COV(a_j,b_j)) \\ 3,4\ \ COV(a,b) & V(b)\ \ 4 & COV(b,COV(a_j,b_j) \\ COV(a,COV(a_j,b_j)) & COV(b,COV(a_j,b_j)\ 4,9 & V(COV(a_j,b_j))\ \ 9 \end{array}\right]\begin{array}{c} 3,9 \\ 4,9 \\ \\ \end{array}$$

```
     ESTIMATED COVARIANCE MATRIX FOR PARAMETER ESTIMATES
                  1             2             3             4             5
              ________      ________      ________      ________      ________
   1        1973.519
   2         -15.544         4.586
   3           8.968         0.954         3.502 3
   4           7.643        -0.103   3,4   0.131         0.232 4
   5         -38.023         2.586         1.117        -1.079        81.339
   6        -431.347         8.853        -7.101        -8.538        45.091
   7          -3.173        -0.542         0.411        -0.094         0.482
   8          23.501        -6.116       -15.861         0.317       -39.049
   9           4.854        -1.048   3,9  -0.549         0.036 4,9    -3.952
  10           3.315         0.129        -0.117         0.132        -1.594
  11         603.345        24.203        36.651         9.728      -293.870

     ESTIMATED COVARIANCE MATRIX FOR PARAMETER ESTIMATES
                  6             7             8             9            10
              ________      ________      ________      ________      ________
   6         372.244
   7           2.192         2.016
   8          -8.728        -9.190       415.654
   9          -4.390        -2.306        49.661        15.192 9
  10          -5.876        -0.176         4.305         0.361         1.155
```

An SPSS Program for a Monte Carlo CI

The SPSS code below generates a Monte Carlo confidence interval for the indirect effect based on 100,000 replications.

$a, b, COV(a_j, b_j)$

$$\begin{matrix} 3.502 & 0.131 & -0.549 \\ 0.131 & 0.232 & 0.036 \\ -0.549 & 0.036 & 15.192 \end{matrix}$$

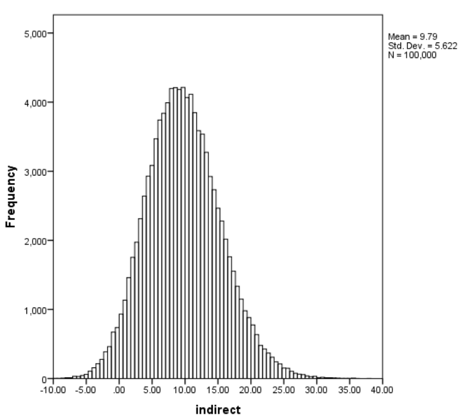
```
define mcmedr ().
matrix.
!let !samples = 100000.
!let !a = 5.463.
!let !b = 1.475
!let !covab = 1.629.
compute r =
{3.502, 0.131, -0.549;
0.131,  0.232,  0.036;
-0.549, 0.036,  15.192}.
compute rn = nrow(r).
compute mns={make(!samples,1,!a), make(!samples, 1, !b), make(!samples,1,!covab)}.
compute x1 = sqrt(-2*ln(uniform(!samples,rn)))&*cos((2*3.14159265358979)*uniform(!samples,rn)).
compute x1=(x1*chol(r))+mns.
save x1/outfile = */variables = a,b,covab.
end matrix.
/* Activate data file that results from above before running this below */.
compute indirect=a*b+covab.
frequencies variables = indirect/format = notable/statistics mean stddev/percentiles = 2.5 5 95 97.5.
graph/histogram=indirect.
!enddefine.
mcmedr.
```

Output

Statistics

indirect

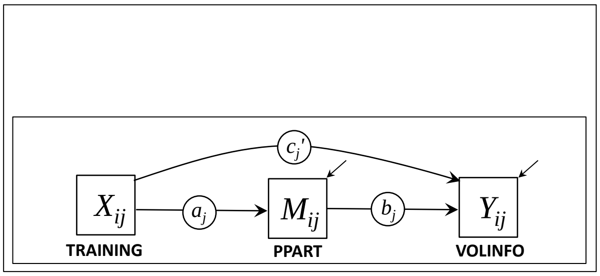
N	Valid	100000
	Missing	0
Mean		9.7942
Std. Deviation		5.62232
Percentiles	2.5	-.5597
	5	1.0419
	95	19.4883
	97.5	21.5435



MC confidence intervals:

90%: 1.042 to 19.489
95%: -0.560 to 21.544

Summary of Findings



On average, patients who experienced the training participated more during the doctor-patient interview than those without training (a = 5.463, p < 0.01), and there was no evidence that this effect differed between doctors, V(a_j) = 24.105, p = .24.

On average, patients who participated more elicited more information from their doctors (b = 1.475, p < 0.01), but this relationship varied between doctors, V(b_j) = 2.156, p < .05.

A formal test of the indirect effect revealed a "marginally significant" indirect effect of training on information elicited through patient participation (9.689, Z = 1.751, p < 0.10, 90% Monte Carlo CI = 1.042 to 19.489). There was no direct effect of training on information elicited (c' = -0.270, p = 0.98), and no evidence of between-doctor variation in the direct effect, V(c'_j) = 1.182, p > .99

Example 2:
A "Deconflated" 1-1-1 MLM with All Random Effects

$$\tilde{M}_{ij} = d_{M_j} + a_j \tilde{X}_{ij} + e_{M_{ij}}$$

$$Y_{ij} = d_{Y_j} + c'_j \tilde{X}_{ij} + b_j \tilde{M}_{ij} + g_1 \bar{X}_j + g_2 \bar{M}_j + e_{Y_{ij}}$$

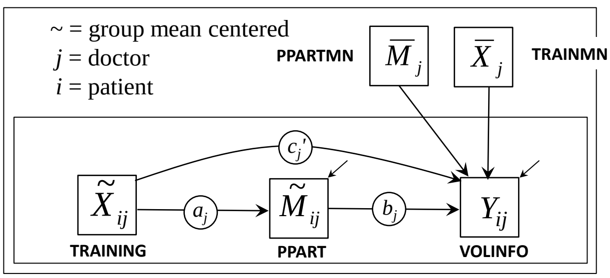
$$d_{M_j} = d_M + u_{d_{Mj}}$$

$$d_{Y_j} = d_Y + u_{Y_{Mj}}$$

$$a_j = a + u_{a_j}$$

$$b_j = b + u_{b_j}$$

$$c'_j = c' + u_{c'_j}$$



When there are differences between level-2 units on X and/or M, quantification of lower-level (level-1) effects contain a level-2 component. Often such differences do exist, and frequently this is one reason we use MLM in the first place. Mean centering X and M within level-2 and use of group levels means on X and M in the level-2 model of Y can "deconflate" the level-1 effects with their level-2 component (see Zhang, Zypher, and Preacher, 2009).

The level-2 components here are the effects of "average training" of a doctor's patients on how much the doctor volunteers on average, and how much a doctor's patients participate on average on how much the doctor volunteers on average.

Mplus: "Deconflated" 1-1-1 MLM with All Random Effects

```
DATA:
  file is doctors.txt;
VARIABLE:
  names are docid patient volinfo ppart length size training
            doccond ppartmn trainmn;
  usevariables are docid volinfo ppart training ppartmn trainmn;
  cluster is docid;
  within are training ppart;
  between are trainmn ppartmn;
  centering is groupmean(training ppart); !group mean center
ANALYSIS:
  TYPE = twolevel random;
MODEL:
  %WITHIN%
  s_a|ppart on training;
  s_b|volinfo on ppart;
  s_cp|volinfo on training;
  [training@0];
  [ppart@0];
  %BETWEEN%
  volinfo on ppartmn trainmn;
  [s_a] (a);
  [s_b] (b);
  [s_cp] (cp);
  !freely estimate covariances between random volinfo intercepts and slopes;
   volinfo with s_a s_b s_cp;
  !freely estimate covariances beween random slopes;
   s_a with s_b (covab);
   s_a with s_cp;
   s_b with s_cp;
MODEL CONSTRAINT:
  new (direct indirect);
  indirect = a*b+covab;
  direct = cp;
```

Within-group means of patient participation and condition are now variables in the model. These are in the data file.

No level-2 model for these.

No variance within doctor, so no level-1 model for these.

Mplus mean centers these variables within doctor. So doctor means are now all equal to zero, and patient measurements are deviation from the doctor mean.

As we have group mean centered these variables, we fix intercepts within doctor to zero to take them out of the model.

Estimates how much doctor volunteers on average from how much his/her patients participate on average, as well as training mean. This is the "deconflation" component of the model.

Mplus Abbreviated Output

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Means
    TRAINING           0.000      0.000    999.000    999.000

 Intercepts
    PPART              0.000      0.000    999.000    999.000

 Variances
    TRAINING           0.241      0.069      3.502      0.000

 Residual Variances
    VOLINFO          783.538     38.845     20.171      0.000
    PPART             41.735      1.531     27.260      0.000

Between Level

 VOLINFO    ON
    PPARTMN            0.684      0.826      0.829      0.407
    TRAINMN          -25.033     57.442     -0.436      0.663

 VOLINFO  WITH
    S_A                7.438     39.997      0.186      0.852
    S_B               17.742     15.980      1.110      0.267
    S_CP              -5.439    260.735     -0.021      0.983

 S_A      WITH
    S_B                2.120      3.740      0.567      0.571
    S_CP              -2.520     84.953     -0.030      0.976

 S_B      WITH
    S_CP              -1.213     16.358     -0.074      0.941

 Means
    S_A                5.382      1.621      3.319      0.001
    S_B                1.433      0.473      3.032      0.002
    S_CP              -0.337      9.762     -0.035      0.972

 Intercepts
    VOLINFO           37.936     30.480      1.245      0.213

 Variances
    S_A               24.716     18.347      1.347      0.178
    S_B                2.809      0.998      2.816      0.005
    S_CP               1.385    163.901      0.008      0.993

 Residual Variances
    VOLINFO          700.297    249.523      2.807      0.005

 New/Additional Parameters
    DIRECT            -0.337      9.762     -0.035      0.972
    INDIRECT           9.832      4.775      2.059      0.039
```

$g_1 = 1.433$

$g_2 = 1.433$

$COV_{ajbj} = 2.120$

$b = 1.433, p < .01$

$a = 5.382, p < .01$

$c' = -0.337, p = 0.97$

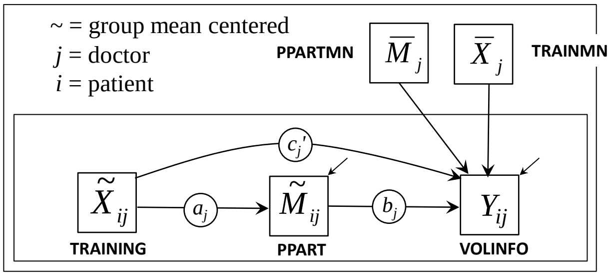
$V(b_j) = 2.809, p < .01$

$V(a_j) = 24.716, p = .18$

$V(c'_j) = 1.385, p > .99$

$ab + COV_{ajbj} = 9.832, p = .04$

Summary of Findings



On average, patients who experienced the training participated more during the doctor-patient interview than those without training (a = 5.382, $p < 0.01$), and there was no evidence that this effect differed between doctors, V(a_j) = 24.716, p = .18.

On average, patients who participated more elicited more information from their doctors (b = 1.433, $p < 0.01$), but this relationship varied between doctors, V(b_j) = 2.809, $p < .01$.

A formal test of the indirect effect revealed a statistically significant indirect effect of training on information elicited through patient participation (9.832, Z = 2.059, $p < 0.05$, 95% Monte Carlo CI = 0.748 to 19.789). There was no direct effect of training on information elicited (c' = -0.337, p = 0.97), and no evidence of between-doctor variation in the direct effect, V(c'_j) = 1.385, $p > .99$

Example 3:
A "Deconflated" 1-1-1 MLM with only *b* Random

$$\tilde{M}_{ij} = d_{M_j} + a\tilde{X}_{ij} + e_{M_{ij}}$$

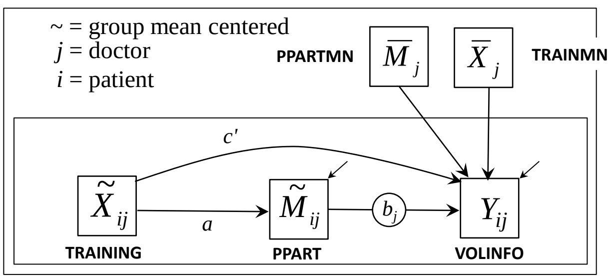
$$Y_{ij} = d_{Y_j} + c'\tilde{X}_{ij} + b_j\tilde{M}_{ij} + g_1\bar{X}_j + g_2\bar{M}_j + e_{Y_{ij}}$$

$$d_{M_j} = d_M + u_{d_{Mj}}$$

$$d_{Y_j} = d_Y + u_{Y_{Mj}}$$

$$b_j = b + u_{b_j}$$

Observe that only b_j is allowed to vary randomly between doctors.



With no evidence from the prior analysis that the effect of training on patient participation or the direct effect of training varies between patients with different doctors, we might choose to fix those effects. Doing so makes the covariance component of the indirect effect disappear.

Direct effect	Average indirect effect
c'	ab where $b = \bar{b}_j$

Mplus: "Deconflated" 1-1-1 MLM with only *b* Random

```
DATA:
  file is doctors.txt;
VARIABLE:
  names are docid patient volinfo ppart length size training
           doccond ppartmn trainmn;
  usevariables are docid volinfo ppart training ppartmn trainmn;
  cluster is docid;
  within are training ppart;
  between are trainmn ppartmn;
  centering is groupmean(training ppart);
ANALYSIS:
  TYPE = twolevel random;
MODEL:
  %WITHIN%
  s_a|ppart on training;
  s_b|volinfo on ppart;
  s_cp|volinfo on training;
  [training@0];
  [ppart@0];
  %BETWEEN%
  volinfo on ppartmn trainmn;
  [s_a] (a);
  [s_b] (b);
  [s_cp] (cp);
  !fix variance of random a and c' effects to zero;
  s_a@0;
  s_cp@0;
  !freely estimate covariance between volinfo random intercept and random b;
  volinfo with s_b;
MODEL CONSTRAINT:
  new (direct indirect);
  indirect = a*b;
  direct = cp;
```

Constrain the variance of these two random effects specified in %WITHIN% to be zero (i.e., fix them)

Because the covariance of the random effects disappears when *a* and/or *b* paths are estimated as fixed, the indirect effect simplifies to just *ab*.

An Alternative Form in Mplus

```
DATA:
  file is doctors.txt;
VARIABLE:
  names are docid patient volinfo ppart length size training
           doccond ppartmn trainmn;
  usevariables are docid volinfo ppart training ppartmn trainmn;
  cluster is docid;
  within are training ppart;
  between are trainmn ppartmn;
  centering is groupmean(training ppart);
ANALYSIS:
  TYPE = twolevel random;
MODEL:
  %WITHIN%
  ppart on training (a);
  s_b|volinfo on ppart;
  volinfo on training (cp);
  [training@0];
  [ppart@0];
  %BETWEEN%
  volinfo on ppartmn trainmn;
  [s_b] (b);
  !freely estimate covariance between volinfo intercept and b;
  volinfo with s_b;
MODEL CONSTRAINT:
  new (direct indirect);
  indirect = a*b;
  direct = cp;
```

The *b* path is estimated as a random effect. The other two paths are fixed and labeled "a" and "cp"

Mplus Abbreviated Output

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Means
    TRAINING           0.000      0.000    999.000    999.000

 Intercepts
    PPART              0.000      0.000    999.000    999.000

 Variances
    TRAINING           0.241      0.005     51.174      0.000

 Residual Variances
    VOLINFO          783.822    170.879      4.587      0.000
    PPART             47.980      7.231      6.636      0.000

Between Level

 VOLINFO    ON
    PPARTMN            0.697      0.607      1.148      0.251
    TRAINMN          -25.438     23.811     -1.068      0.285

 VOLINFO    WITH
    S_B               16.734      9.234      1.812      0.070

 Means
    S_A                5.465      1.035      5.282      0.000
    S_B                1.479      0.342      4.324      0.000
    S_CP              -0.539      1.355     -0.397      0.691

 Intercepts
    VOLINFO           37.512     20.662      1.815      0.069

 Variances
    S_A                0.000      0.000    999.000    999.000
    S_B                2.728      1.615      1.689      0.091
    S_CP               0.000      0.000    999.000    999.000

 Residual Variances
    VOLINFO          700.131    162.404      4.311      0.000

 New/Additional Parameters
    DIRECT            -0.539      1.355     -0.397      0.691
    INDIRECT           8.085      2.622      3.083      0.002
```

a = 5.465, *p* < .001

b = 1.479, *p* < .001

c' = -0.539, *p* = 0.69

V(b_j) = 2.728, *p* < .10

V(a_j) = 0 by constraint

V(c'_j) = 0 by constraint

ab = 8.085, *p* < .01

Monte Carlo Confidence Interval Construction

When the *a* or *b* path is fixed, construction of a Monte Carlo confidence interval for the indirect effect is a little bit easier because there is a simple tool already available to construct one once you have the covariance between *a* and *b*.

From Mplus tech1 output, *a* and *b* are parameters 4 and 5:

```
ALPHA
   S_A              S_B
   ________         ________
          4                5
```

From Mplus tech3 output, the covariance between parameters 4 and 5 is 0.064:

```
     ESTIMATED COVARIANCE MATRIX FOR PARAMETER ESTIMATES
                1             2             3             4             5
           ________      ________      ________      ________      ________
 1        29199.665
 2         -111.600        52.283
 3            0.040        -0.017         0.000
 4           -3.166         0.249         0.000         1.071
 5          -16.608         0.057         0.000         0.064         0.117
 6           53.175        -0.950         0.001        -0.326        -0.227
```

MCMED macro for SPSS and SAS

The MCMED macro available at www.afhayes.com (and described in Appendix B of *Introduction to Mediation, Moderation, and Conditional Process Analysis*) can be used to construct a Monte Carlo confidence interval.

```
mcmed a=5.465/sea=1.035/b=1.479/seb=0.342/covab=0.064/samples=100000.
```

From Mplus output:

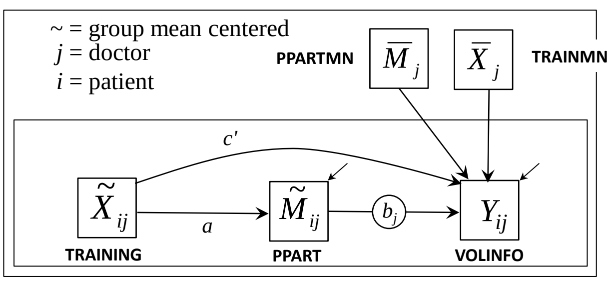
	Estimate	S.E.
Means		
S_A	5.465	1.035
S_B	1.479	0.342
S_CP	-0.539	1.355

```
Run MATRIX procedure:

*** Input Data ***
a:           5.4650
SE(a):       1.0350
b:           1.4790
SE(b):        .3420
COV(ab):      .0640
Samples: 100000.0
Conf:       95.0000

**** Monte Carlo Confidence Interval ****
       ab      LLCI      ULCI
   8.0827    3.5502   13.8541
```

Summary of Findings

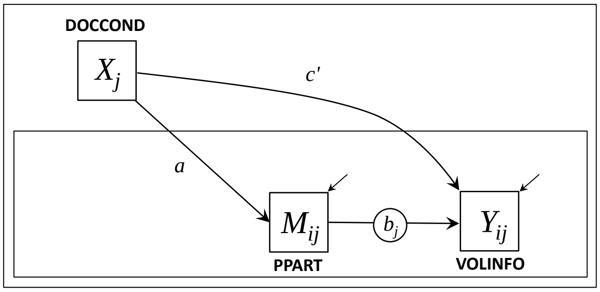


Patients who experienced the training participated more during the doctor-patient interview than those without training (a = 5.465, p < 0.01).

On average, patients who participated more elicited more information from their doctors (b = 1.479, p < 0.01). The evidence that this relationship varied between doctors was only marginally significant, $V(b_j) = 2.728$, p < .10.

A formal test of the indirect effect revealed a statistically significant indirect effect of training on information elicited through patient participation (8.085, Z = 3.083, p < 0.01, 95% Monte Carlo CI = 3.550 to 13.854). There was no direct effect of training on information elicited (c' = -0.539, p = 0.69).

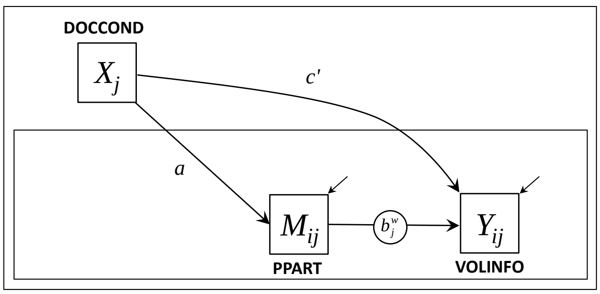
Example 4:
A 2-1-1 MLM with a Random Level-1 Effect



If X is measured at Level-2 (e.g., all of doctor j's patients trained or not trained) but the mediator and outcome are measured at the individual level, a 2-1-1 model can be estimated where the first stage of the mechanism crosses levels. In such a model, the effect of X on M and the direct effect of X on Y must be fixed, as there is no variation in X within Level-2 unit.

A 2-1-1 MLM with a Random Level-1 Effect

j = doctor
i = patient



$$M_{ij} = d_{M_j} + e_{M_{ij}}$$
$$Y_{ij} = d_{Y_j} + b_j^w M_{ij} + e_{Y_{ij}}$$

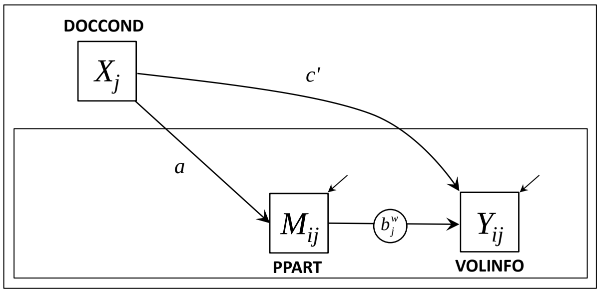
The level-1 model is just an intercept for M and Y, and a within-doctor effect of M on Y.

$$d_{M_j} = d_M + aX_j + u_{d_{Mj}}$$
$$d_{Y_j} = d_Y + c'X_j + b^b d_{M_j} + u_{Y_{Mj}}$$
$$b_j = b + u_{b_j}$$

The level-2 model estimates a random M intercept from X (this is a mean difference---path a), a random Y intercept from level-2 X (an adjusted mean difference and the direct effect of X, path c') and the M intercept for level-2 unit j (a between-doctor effect of M on Y), and allows the within-doctor effect of M on Y to vary between doctors.

A 2-1-1 MLM with a Random Level-1 Effect

j = doctor
i = patient



$$M_{ij} = d_{M_j} + e_{M_{ij}}$$
$$Y_{ij} = d_{Y_j} + b_j^w M_{ij} + e_{Y_{ij}}$$
$$d_{M_j} = d_M + aX_j + u_{d_{Mj}}$$
$$d_{Y_j} = d_Y + c'X_j + b^b d_{M_j} + u_{Y_{Mj}}$$
$$b_j = b + u_{b_j}$$

<u>Direct effect</u>

c'

<u>Average indirect effect</u>

$$a(b^b + b^w) = a(b^b + \overline{b}_j^w)$$

Mplus: A 2-1-1 MLM with a Random Level-1 Effect

```
DATA:
  file is doctors.txt;
VARIABLE:
  names are docid patient volinfo ppart length size training
           doccond ppartmn trainmn;
  usevariables are docid volinfo ppart doccond;
  cluster is docid;
  between is doccond;
ANALYSIS:
  TYPE = random twolevel;
MODEL:
  %WITHIN%
  s_b|volinfo on ppart;
  %BETWEEN%
  [s_b] (bw);
  ppart on doccond (a);
  volinfo on ppart (bb);
  volinfo on doccond (cp);
  !freely estimate covariances between intercepts and b slopes;
  s_b with ppart volinfo;
MODEL CONSTRAINT:
  new (direct indirect);
  indirect = a*(bw+bb);
  direct = cp;
```

between is doccond; ← No variation between doctors in training of his or her patients.

s_b|volinfo on ppart; ← This is the level-1 random effect of patient participation on how much the doctor volunteers to the patient.

[s_b] (bw); ← Label the average level-1 *b* path “bw” for use later

ppart on doccond (a); ← The effect of patient training on how much the doctor’s patient’s participate. Call it “a”

volinfo on ppart (bb); ← Doctor-level effect of how much a doctor’s patient’s participates on how much the doctor volunteers. Call it “bb”

volinfo on doccond (cp); ← The direct effect of patient training on how much the doctor volunteers. Call it “cp”

indirect = a*(bw+bb); ← The effect of *M* on *Y* now has two components, between doctor (level-2) and average within doctor (level-1). We add them up and then multiply by the *a* path to get the average indirect effect.

Mplus Abbreviated Output

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
Within Level					
Variances					
PPART	62.052	8.735	7.104	0.000	
Residual Variances					
VOLINFO	799.432	172.825	4.626	0.000	
Between Level					
PPART ON					
DOCCOND	2.428	1.803	1.347	0.178	a = 2.428, p = .18
VOLINFO ON					
PPART	-0.121	2.619	-0.046	0.963	b^b = -0.121, p = .96
DOCCOND	-1.238	9.573	-0.129	0.897	c' = -1.238, p = .90
S_B WITH					
PPART	-0.788	1.823	-0.432	0.665	
VOLINFO	-55.966	43.170	-1.296	0.195	
Means					
S_B	1.531	0.305	5.017	0.000	b^w = 1.531, p < .001
Intercepts					
VOLINFO	13.745	84.241	0.163	0.870	
PPART	33.892	0.868	39.068	0.000	
Variances					
S_B	2.076	1.291	1.608	0.108	V(b^w_j) = 2.076, p = .11
Residual Variances					
VOLINFO	2175.110	1549.706	1.404	0.160	
PPART	27.000	7.175	3.763	0.000	
New/Additional Parameters					
DIRECT	-1.238	9.573	-0.129	0.897	
INDIRECT	3.424	7.401	0.463	0.644	$a(b^w+b^b)$ = 3.424, p = .64

Monte Carlo Confidence Interval Construction

Mplus tech1 output:

a and b^w and b^b are parameters 8, 3, and 6, respectively.

ALPHA

	S_B	VOLINFO	PPART	DOCCOND
1	3	4	5	0

BETA

	S_B	VOLINFO	PPART	DOCCOND
S_B	0	0	0	0
VOLINFO	0	0	6	7
PPART	0	0	0	8
DOCCOND	0	0	0	0

Mplus tech3 output:

ESTIMATED COVARIANCE MATRIX FOR PARAMETER ESTIMATES

	1	2	3	4	5
1	29868.641				
2	-152.625	76.302			
3	-12.079	0.141	0.093		
4	-3981.635	259.478	3.499	7096.590	
5	8.521	3.917	-0.003	21.803	0.753
6	144.731	-8.209	-0.187	-219.365	-0.664
7	85.781	35.690	0.503	464.046	2.998
8	35.008	-4.980	0.023	-44.765	0.105

ESTIMATED COVARIANCE MATRIX FOR PARAMETER ESTIMATES

	6	7	8	9	10
6	6.861				
7	-14.334	91.635			
8	1.363	-4.851	3.253		
9	1.641	-4.646	-0.008	1.666	
10	-50.437	147.983	1.001	-54.211	1863.633

$$\begin{bmatrix} V(b^w)\ \text{3} & COV(b^w,b^b)\ \text{3,6} & COV(a,b^w)\ \text{3,8} \\ COV(b^w,b^b)\ \text{3,6} & V(b^b)\ \text{6} & COV(a,b^b)\ \text{6,8} \\ COV(a,b^w)\ \text{3,8} & COV(a,b^b)\ \text{6,8} & V(a)\ \text{8} \end{bmatrix}$$

$$\begin{bmatrix} 0.093 & -0.187 & 0.023 \\ -0.187 & 6.861 & 1.363 \\ 0.023 & 1.363 & 3.253 \end{bmatrix}$$

An SPSS Program for a Monte Carlo CI

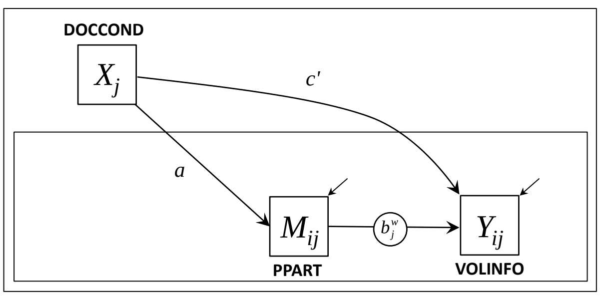
The SPSS code below generates a Monte Carlo confidence interval for the indirect effect based on 100,000 replications.

```
define mcmedr ().
matrix.
!let !samples = 100000.
!let !bw = 1.531.
!let !bb = -0.121.
!let !a = 2.428.
compute r =
{0.093, -0.187, 0.023;
-0.187,  6.861, 1.363;
0.023, 1.363,  3.253}.
compute rn = nrow(r).
compute mns={make(!samples,1,!bw), make(!samples, 1, !bb), make(!samples,1,!a)}.
compute x1 = sqrt(-2*ln(uniform(!samples,rn)))&*cos((2*3.14159265358979)*uniform(!samples,rn)).
compute x1=(x1*chol(r))+mns.
save x1/outfile = */variables = bw,bb,a.
end matrix.
/* Activate data file that results from above before running this below */.
compute indirect=a*(bb+bw).
frequencies variables = indirect/format = notable/statistics mean stddev/percentiles = 2.5 5 95 97.5.
graph/histogram=indirect.
!enddefine.
mcmedr.
```

b^w, b^b, a

$$\begin{bmatrix} 0.093 & -0.187 & 0.023 \\ -0.187 & 6.861 & 1.363 \\ 0.023 & 1.363 & 3.253 \end{bmatrix}$$

Summary of Findings



Patients who experienced the training did not participate more during the doctor-patient interview than those without training, on average (a = 2.428, p = 0.18).

Patients who participated relatively more elicited more information from their doctors, b^w = 1.531, p < 0.001. This relationship did not vary between doctors, V(b_j) = 2.728, p < .10. Doctor's whose patients participated relatively more on average relative to other doctors did not volunteer more to their patients on average relative to other doctors, b^b = -0.121, p = 0.96

A formal test of the indirect effect revealed no evidence of an indirect effect of training on information elicited through patient participation (3.424, Z = 0.463, p = .64, 95% Monte Carlo CI = -5.234 to 22.569). There was also no direct effect of training on how much information a doctor volunteered on average (c' = -1.238, p = 0.90).

Common Sources of Estimation Hassles

Multilevel models are computationally complex to estimate, and the mathematics can break down producing various errors in even good MLM programs. The most common errors are convergence failures, and a "nonpositive definite" matrix.

- **Insufficient data**

- **Two few level-2 units, or two few level-1 units given the number of predictors.**

- **Estimating a level-1 effect as random when there is little variance in the effect between level-2 units.**

- **Complex covariance structures in models with many random effects, or too many covariances to estimate.**

- **Constraints in the model which are highly unrealistic given the data available.**

Before giving up, try a different program, make sure you are using the program correctly, understand what it is estimating and what it is constraining, and not specifying a model that is "unidentified." In general, simpler models are easier for programs to estimate. Master simple models before trying anything complex.

Advice

- **Allocate your resources to both level-1 and level-2 sampling and measurement.**

  Quality of inference at level-1 and level-2 will be determined by quality and amount of data at both levels.

- **Keep your models simple and consistent with how much data you have at each level.**

  Complex models with little data are bound to produce problems at estimation or yield results that are hard to trust.

  Models with more than a few random effects are exceeding more likely to yield estimation problems. These are complex methods!

- **Be patient, stay organized in your thinking, and don't be overly ambitious in your modeling. This is not point-and-click analysis.**

Some of the key (and readable) literature

Bauer, D. J., Preacher, K. J., & Gil, K. M. (2006). Conceptualizing and testing random indirect effects and moderate mediation in multilevel models: New procedures and recommendations. *Psychological Methods, 11*, 142-163.

Kenny, D., Korchmaros, J. D., & Bolger, N. (2003). Lower level mediation in multilevel models. *Psychological Methods, 8*, 115-128.

Krull, J. L., & MacKinnon, D. P. (1999). Multilevel mediation modeling in group-based intervention studies. *Evaluation Revew, 23*, 418-444.

Krull, J. L., & MacKinnon, D. P. (2001). Multilevel modeling of individual and group level mediated effects. *Multivariate Behavioral Research, 36*, 249-277.

Pituch, K. A., Whittaker, T. A., & Stapleton, L. M. (2005). A comparison of methods to test for mediation in multisite experiments. *Multivariate Behavioral Research, 40*, 1-23.

Pituch, K. A., Stapleton, L. M., & Kang, J. Y. (2006). A comparison of single sample and bootstrap methods to assess mediation in cluster randomized trials. *Multivariate Behavioral Research, 41*, 367-400.

Preacher, K. J., Zypher, M. J., & Zhang, Z. (2010). A general multilevel SEM framework for assessing multilevel mediation. *Psychological Methods, 15*, 209-233.

Zhang, Z., Zypher, M. J., & Preacher, K. J. (2009). Testing multilevel mediation using hierarchical linear models: Problems and solutions. *Organizational Research Methods, 12*, 695-719.