




Agenda

- Measurement equivalence / invariance
 - Why we worry about it
 - What it is
 - How to test it

Measurement Equivalence/Invariance

- When we want to **compare groups** on a latent construct, we need to be sure we are measuring the latent construct the same way in both groups.
 - Otherwise, our comparison may not mean much.
- Cross-cultural research provides strong examples of cases where we might worry about this:
 - E.g., studying marital satisfaction among U.S. and Afghan women.
 - Why should you be cautious about comparing means here?

Challenges in Comparison

- When is this an issue?
 - **Translated** measures
 - Culture (even when language is not a barrier).
 - Gender (sometimes)
 - Translated media – e.g., online vs. paper-and-pencil
 - And... ?
- We can't *assume* groups are different... and we can't assume they aren't. 
 - Need a way to separate true differences on the underlying construct from differences in the way the groups use or interpret the measurement instrument.
 - Two major strategies: MGCFA and IRT

MGCFA



- Multiple Group Confirmatory Factor Analysis
 - Goes beyond the individual item-level bias indices we talked about last week (and works better, too!).
 - Considers all items together in a factor analytic framework.
- Essentially, tests whether the same CFA model holds for two (or more) groups.
 - We do this by **constraining** parameters to be equal across groups – does the model still fit?

Formally

- Vandenberg & Lance (2000) use a very formal notation for CFA. The **model** is the same, just the notation is different!
 - This notation comes from LISREL (Jöreskog & Sörbom).
- We can write the model for item responses as:
 - $\mathbf{X}_k^g = \boldsymbol{\tau}_k^g + \boldsymbol{\Lambda}_k^g \boldsymbol{\xi}^g + \boldsymbol{\delta}_k^g$
 - \mathbf{X}_k^g are the item responses (observed data)
 - $\boldsymbol{\tau}_k^g$ are the item means (also called *thresholds*). We don't always model these in normal CFA because they don't affect the covariance matrix – but we may care about them in measurement equivalence because they are our item difficulty parameters!
 - $\boldsymbol{\Lambda}_k^g$ are the factor loadings
 - $\boldsymbol{\xi}^g$ are the true scores (note that there isn't an item subscript here)
 - $\boldsymbol{\delta}_k^g$ are the uniquenesses (residual variances)

Formally, cont.

- o So if our model is:
 - o $\mathbf{X}_k^g = \boldsymbol{\tau}_k^g + \boldsymbol{\Lambda}_k^g \boldsymbol{\xi}^g + \boldsymbol{\delta}_k^g$
- o We can then write the covariance matrix as:
 - o $\boldsymbol{\Sigma}^g = \boldsymbol{\Lambda}_X^g \boldsymbol{\Phi}^g \boldsymbol{\Lambda}_X^{g'} + \boldsymbol{\Theta}_{\delta k}^g$
- o Why are we bothering?
 - o It's good to be able to translate!
 - o Also – all the *gs* in this model indicate that these are parameters that *might* be specific to a particular *group*.
 - o **Every parameter matrix with a *g* is a set of parameters that could potentially vary from one group to another.**



ME/I Hypotheses

- o In ME/I analysis, we can (and will!) test whether each piece of the model is equivalent across groups.
 - o Each of these tests has a name.
- o $\boldsymbol{\xi}^g$ is the same across groups – the construct has the same number of factors in all groups.
 - o **Configural invariance.**
- o $\boldsymbol{\Lambda}_k^g$ is the same across groups – the items load on the same factors and to the same extent across groups.
 - o **Metric invariance.**

More ME/I Hypotheses

- o τ_k^g is the same across groups – the items have the same intercepts (difficulty) across groups.
 - o **Scalar invariance.**
- o $\Theta_{\delta k}^g$ is the same across groups – the item uniquenesses or residual variances are the same in all groups.
 - o **Invariance of uniquenesses.**
- o Φ^g is the same across groups – the factor variances and covariances are the same in all groups.
 - o Actually, we can break this into two tests (variances & covariances).
- o And we can also test whether the **factor means** are equal.

How to Do It

- o **Free parameters vs. constrained parameters.**
 - o Fit the model to both groups simultaneously, but allow the particular parameters you are interested in to be **different** in each group (free).
 - o Then fit the model again, but require that those parameters be **equal** (constrained) across the groups.
 - o Not specifying a value for those parameters, just saying they need to be equal.
- o One-item example:
 - o Test 1 (free):
 - o $x = .735F + .543$ in Group 1, $x = .541F + .489$ in Group 2 
 - o Test 2 (constrained):
 - o $x = .601F + .543$ in Group 1, $x = .601F + .489$ in Group 2 

Model Comparisons

- We constrain one set of parameters at a time.
 - Adding constraints as we go – keep constraints that worked from previous steps.
- At each step, we test whether the fit of the constrained model is significantly worse than the fit of the free model.
 - Remember that the free model will always fit better... the question is how much better.
 - Is it reasonable to use 1 set of parameters to describe both groups?
- How do we test it?
 - Chi-square difference test
 - Change in CFI of .01 or greater (Cheung & Rensvold, 2001).
 - Implies a big enough difference to care about!
 - Based on simulation data – not totally arbitrary.

Order of Operations

- #1. First, test a **fully restricted model** – that covariance matrices are equal.
 - If so, you have ME/I! You're done.
 - If not, proceed to Step 2.
- #2. Test **configural invariance**.
 - Same # of factors, same items on each factor.
 - If you can't fit the same configural model to each group, **stop**. This implies that the constructs are fundamentally different for the different groups.
 - You cannot compare groups **at all** without configural invariance.

Next Steps & Partial Invariance

- o #3. After configural invariance, test **metric invariance**.
 - o **Equal** factor loadings across groups.
- o If you don't get full metric invariance, you **can** consider **partial invariance**.
 - o Allowing **some** loadings to vary across groups.
 - o There are pros and cons to this.
- o V & L recommend allowing individual loadings to vary **only** if:
 - o **Most** loadings are invariant (you're only relaxing a few constraints).
 - o It is theoretically reasonable that those particular items might measure differently in different groups.
 - o You have cross-validation or replication data to support the different loadings.



More Steps

- o #4. Test **scalar** invariance – **if** it's appropriate.
 - o Equal item intercepts (means).
 - o **Add** this constraint to your metric invariance model.
 - o Meaning of this depends on underlying theory!
 - o If the groups should not differ on the construct, lack of scalar invariance can signal response bias (e.g., leniency).
 - o If the groups should differ on the construct, we don't *expect* scalar invariance! Differences are real, not measurement bias.
- o #5. See V & L's flow chart – different study goals require different kinds of ME/I.
 - o We often want to compare **latent means**.

Partial Invariance

- To reiterate: **partial invariance can be ok.**
 - As long as you are within reasonable bounds and not totally capitalizing on chance (per V & L).
- If you have partial invariance (and you know where that invariance lies), you can estimate and compare latent means.
 - You also have the option to **drop** items that don't behave well across groups.
- The flow chart in V & L is really, really handy.



IRT Methods

- We can use IRT to test for differential item functioning (DIF) across groups.
- Literally, comparing the item characteristic curves to see whether they are equivalent.
- But this is not necessarily better than MGCFA!
 - MGCFA actually makes it *easier* to tell *where* and *how* items lack invariance because we test loadings, thresholds, & uniquenesses separately.
- Up-and-coming methods:
 - Multiple indicators multiple causes (MIMIC) models
 - IRT with covariates models
- Right now, MGCFA is the standard for most applications.

Questions?

NO CLASS THURS OR LAB FRI!

For next time: Validity scales / detecting faking.

Read: Schmitt & Oswald (2006); Piedmont et al. (2000).

Reading Response: According to Piedmont et al., how are validity scales *supposed* to work? Do they?