Mplus Textbook Examples
Introduction to Multilevel Modeling by Kreft and de Leeuw
Chapter 4: Analyses

This page uses the imm23.dat data file.

Table 4.2 on page 64.

- Input: ch4_p64.inp
- Output: ch4_p64.out

The variable **schid** identifies the schools. Using a Raudenbush and Bryk way of the describing the model, the null model is

Level 1

$MATH_{ij} = \beta_{0j} + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + u_{0j}$

Here is the Mplus setup for estimating this model.

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 64, Table 4.2
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
```

```
  cluster = schid;
  usevar  = math;
  within  = ;       ! level 1 variables here (none)
  between = ;       ! level 2 variables here (none)

analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math;      ! no fixed effects

  %between%
    math;      ! no predictors of intercept
```

and here is a selection of the output.

```
Estimated Intraclass Correlations for the Y Variables

                Intraclass
     Variable  Correlation
     MATH         0.234

Loglikelihood
          H0 Value                       -1900.388
          H1 Value                       -1900.388

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.

Within Level
 Variances
    MATH              81.237    5.158     15.750

Between Level
 Means
    MATH              50.756    1.127     45.044

 Variances
    MATH              24.855    8.588      2.894
```

Substituting the results yields

Level 1
$MATH_{ij} = \beta_{0j} + r_{ij}$
Level 2
$\beta_{0j} = 50.756 + u_{0j}$

$Var(r_{ij}) = 81.237$
$Var(u_{0j}) = 24.855$

Table 4.3 on page 65.

- Input: ch4_p65.inp
- Output: ch4_p65.out

This model is...

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_1 = \gamma_{10}$

Here is the Mplus setup for estimating this model.

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 65, Table 4.3
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework;
  within  = homework; ! level 1 variables here
  between = ;         ! level 2 variables here (none)
analysis:
  type = twolevel random;
  estimator = ml;
```

```
model:
  %within%
    math on homework; ! fixed effect
  %between%
    math ;            ! no predictors of intercept
```

Here is some of the output

```
Estimated Intraclass Correlations for the Y Variables

                Intraclass
     Variable  Correlation
     MATH          0.194

Loglikelihood
          H0 Value                       -1865.248
          H1 Value                       -1865.247

MODEL RESULTS
                    Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    HOMEWORK            2.391    0.255      9.393
 Residual Variances
    MATH               71.141    4.517     15.751

Between Level
 Means
    MATH               46.379    1.139     40.719
 Variances
    MATH               20.251    7.070      2.864
```

After substitution, the results are...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$

Level 2

$\beta_{0j} = 46.379 + u_{0j}$

$\beta_1 = 2.391$

$Var(r_{ij}) = 71.41$

$Var(u_0) = 20.251$

Table 4.4 on page 67.

- Input: ch4_p67.inp
- Output: ch4_p67.out

This model is...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_1 = \gamma_{10} + u_{1j}$

Here is the Mplus setup for estimating this model.

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 67, Table 4.4
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework;
  within  = homework; ! level 1 variables here
  between = ;         ! level 2 variables here (none)

analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math ;                  ! no fixed effects
    b1 | math on homework;  ! random slope for homework

  %between%
```

```
    math;                   ! nothing predicts intercept
    b1;                     ! nothing predicts slope

    math with b1;           ! covariance between intercept and slope
```

Here is some of the output

```
Estimated Intraclass Correlations for the Y Variables

                Intraclass              Intraclass
     Variable  Correlation   Variable  Correlation

     MATH          0.194

TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1819.518

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.
Within Level
 Residual Variances
    MATH              53.301    3.467     15.375

Between Level
 MATH     WITH
    B1               -26.111    9.839     -2.654
 Means
    MATH              46.322    1.720     26.934
    B1                 1.988    0.907      2.191
 Variances
    MATH              59.242   19.959      2.968
    B1                16.754    5.822      2.878
```

After substitution, the results are...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$

Level 2

$\beta_{0j} = 46.322 + u_{0j}$

$\beta_{1j} = 1.988 + u_{1j}$

$Var(r_{ij}) = 53.301$
$Var(u_0) = 59.242$
$Var(u_1) = 16.754$
$Cov(u_0,u_1) = -26.111$

Table 4.5 on page 69.

- Input: ch4_p69.inp
- Output: ch4_p69.out

This model is...

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(PARENTED) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10} + u_{1j}$
$\beta_2 = \gamma_{20}$

Here is the Mplus setup for estimating this model.

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 69, Table 4.5
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework parented;
  within  = homework parented; ! level 1 variables here
  between = ;                  ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
```

```
  %within%
    math on parented;       ! fixed effect
    b1 | math on homework;  ! random effect

  %between%
    math;                   ! nothing predicts intercept
    b1;                     ! nothing predicts slope

    math with b1;           ! covariance intercept and slope
```

and here are some of the results

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1801.181

MODEL RESULTS
                    Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    PARENTED           1.822    0.301      6.053
 Residual Variances
    MATH              50.674    3.302     15.348

Between Level
 MATH     WITH
    B1               -20.821    7.950     -2.619
 Means
    MATH              40.920    1.770     23.118
    B1                 1.889    0.813      2.325
 Variances
    MATH              45.419   15.921      2.853
    B1                13.158    4.712      2.792
```

After substitution, the results are...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(PARENTED) + r_{ij}$

Level 2

$\beta_{0j} = 40.920 + u_{0j}$
$\beta_{1j} = 1.889 + u_{1j}$
$\beta_2 = 1.822$

$Var(r_{ij}) = 50.674$
$Var(u_0) = 45.419$
$Var(u_1) = 13.158$
$Cov(u_0,u_1) = -20.821$

Table 4.6 on page 71.

A simple traditional regression analysis with HOMEWORK and PARENTED as predictors.

(We have skipped this example)

Table 4.7 on page 74.

- Input: ch4_p74.inp
- Output: ch4_p74.out

This model is...

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + \gamma_{01}(SCSIZE) + u_{0j}$
$\beta_{1j} = \gamma_{10} + u_{1j}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 74, Table 4.7
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework scsize;
```

```
  within  = homework;          ! level 1 variables here
  between = scsize;            ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math;                      ! no fixed effects
    b1 | math on homework;     ! random effect of homework
  %between%
    math on scsize;            ! scsize predicts intercept
    b1;                        ! nothing predicts homework slope

    math with b1;              ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1819.306

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.

Within Level
 Residual Variances
    MATH              53.303    3.467     15.374

Between Level
 MATH       ON
    SCSIZE             0.430    0.657      0.655
 MATH     WITH
    B1               -27.235   10.273     -2.651
 Means
    B1                 1.991    0.909      2.191
 Intercepts
    MATH              44.951    2.730     16.463
 Variances
    B1                16.817    5.842      2.879
```

```
 Residual Variances
    MATH               62.173    21.461       2.897
```

And here are the results substituted back into the model.

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$
Level 2
$\beta_{0j} = 44.951 + 0.430(SCSIZE) + u_{0j}$
$\beta_{1j} = 1.991 + u_{1j}$

$Var(r_{ij}) = 53.303$
$Var(u_0) = 62.173$
$Var(u_1) = 16.817$
$Cov(u_0,u_1) = -27.235$

Table 4.8 on page 75.

Variable **PUBLIC** is added as fixed effect and variable **SCSIZE** is taken out of the model.

- Input: ch4_p75.inp
- Output: ch4_p75.out

This model is...

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + \gamma_{01}(PUBLIC) + u_{0j}$
$\beta_{1j} = \gamma_{10} + u_{1j}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 75, Table 4.8
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
```

```
  cluster = schid;
  usevar  = math homework public;
  within  = homework;          ! level 1 variables here
  between = public;            ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math;                      ! no fixed effects
    b1 | math on homework;     ! random effect of homework

  %between%
    math on public;            ! public predicts intercept
    b1;                        ! nothing predicts homework slope

    math with b1;              ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood

          H0 Value                       -1817.421

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.

Within Level
 Residual Variances
    MATH              53.347    3.472     15.364

Between Level
 MATH       ON
    PUBLIC            -4.085    1.901     -2.150
 MATH     WITH
    B1               -25.957    9.623     -2.698
 Means
    B1                 1.984    0.897      2.212
```

```
 Intercepts
    MATH               49.067     2.113     23.220
 Variances
    B1                 16.338     5.675      2.879
 Residual Variances
    MATH               56.214    19.221      2.925
```

And here are the results substituted back into the model.

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$
Level 2
$\beta_{0j} = 49.067 + -4.085(PUBLIC) + u_{0j}$
$\beta_{1j} = 1.984 + u_{1j}$

$Var(r_{ij}) = 53.347$
$Var(u_0) = 56.214$
$Var(u_1) = 16.338$
$Cov(u_0,u_1) = -25.957$

Table 4.10 on page 77.

This model asks whether **PUBLIC** can predict the relationship between **MATH** and **HOMEWORK** (i.e. **B1**).

- Input: ch4_p77.inp
- Output: ch4_p77.out

This model is...

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + \gamma_{01}(PUBLIC) + u_{0j}$
$\beta_{1j} = \gamma_{10} + \gamma_{11}(PUBLIC) + u_{1j}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 77, Table 4.10
data:
```

```
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework public;
  within  = homework;    ! level 1 variables here
  between = public;      ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math;                  ! no fixed effects
    b1 | math on homework;  ! random slope for homework
  %between%
    math on public;        ! intercept predicted by public
    b1 on public;          ! slope predicted by public

    math with b1;          ! covariance of intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1817.386

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.

Within Level
 Residual Variances
    MATH              53.349    3.472     15.364

Between Level
 B1         ON
    PUBLIC            -0.498    1.874     -0.266
 MATH       ON
    PUBLIC            -3.291    3.547     -0.928
```

```
MATH     WITH
   B1               -25.886     9.592     -2.699
Intercepts
   MATH              48.548     2.882     16.845
   B1                 2.308     1.514      1.525
Residual Variances
   MATH              56.175    19.181      2.929
   B1                16.268     5.655      2.877
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$

Level 2

$\beta_{0j} = 48.548 + -3.291(PUBLIC) + u_{0j}$

$\beta_{1j} = 2.308 + -0.498(PUBLIC) + u_{1j}$

$Var(r_{ij}) = 53.349$

$Var(u_0) = 56.175$

$Var(u_1) = 16.268$

$Cov(u_0,u_1) = -25.886$

Table 4.11 on page 80. This model uses full NELS-88 data (we dont have this data, so this is omitted).

Table 4.12 on page 82.

- Input: ch4_p82.inp
- Output: ch4_p82.out

This model is...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + \gamma_{01}(PUBLIC) + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

$\beta_2 = \gamma_{20}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 82, Table 4.12
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework white public ;
  within  = homework white; ! level 1 variables here
  between = public;         ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math on white;          ! fixed effect of white
    b1 | math on homework;  ! random effect of homework

  %between%
    math on public;         ! public predicts intercept
    b1;                     ! nothing predicts homework slope

    math with b1;           ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1811.629

MODEL RESULTS
                    Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    WHITE              3.283    0.976      3.364
 Residual Variances
    MATH              52.627    3.427     15.355
```

```
Between Level
 MATH       ON
    PUBLIC            -3.914     1.727     -2.267
 MATH     WITH
    B1               -25.360     9.286     -2.731
 Means
    B1                 1.908     0.884      2.158
 Intercepts
    MATH              46.679     2.126     21.954
 Variances
    B1                15.851     5.520      2.871
 Residual Variances
    MATH              52.354    18.079      2.896
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

$\beta_{0j} = 46.679 + -3.914(PUBLIC) + u_{0j}$

$\beta_{1j} = 1.908 + u_{1j}$

$\beta_2 = 3.283$

$Var(r_{ij}) = 52.627$

$Var(u_0) = 52.354$

$Var(u_1) = 15.851$

$Cov(u_0,u_1) = -25.360$

Table 4.13 on page 83.

Variable **WHITE** is now made a random effect.

- Input: ch4_p83.inp
- Output: ch4_p83.out

This model is…

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + \gamma_{01}(PUBLIC) + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

$\beta_{2j} = \gamma_{20} + u_{2j}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 83, Table 4.13
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework white public;
  within  = homework white;   ! level 1 variables here
  between = public;           ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math;                     ! no fixed effects
    b1 | math on homework;    ! random effect homework predicting math
    b2 | math on white;       ! random effect white    predicting math

  %between%
    math on public;           ! public predicts intercept
    b1;                       ! nothing predicts b1 (homework slope)
    b2;                       ! nothing predicts b2 (white slope)

    math with b1;             ! covariance intercept and b1
    math with b2;             ! covariance intercept and b2
    b1 with b2;               ! covariance b1 and b2
```

and some of the output is shown below.

```
Loglikelihood
          H0 Value                       -1809.432
```

```
MODEL RESULTS
                    Estimates     S.E.  Est./S.E.
Within Level
 Residual Variances
    MATH              51.189    3.389     15.105

Between Level
 MATH       ON
    PUBLIC            -4.856    1.728     -2.811
 MATH     WITH
    B1               -27.060   11.291     -2.397
    B2               -18.912   18.400     -1.028
 B1       WITH
    B2                 2.807    7.059      0.398
 Means
    B1                 1.944    0.880      2.208
    B2                 2.712    1.510      1.796
 Intercepts
    MATH              48.061    2.455     19.581
 Variances
    B1                15.712    5.468      2.873
    B2                21.995   19.825      1.109
 Residual Variances
    MATH              64.017   28.215      2.269
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

$\beta_{0j} = 48.061 + -4.856(PUBLIC) + u_{0j}$

$\beta_{1j} = 1.944 + u_{1j}$

$\beta_{2j} = 2.712 + u_{2j}$

$Var(r_{ij}) = 51.189$

$Var(u_0) = 64.017$

$Var(u_1) = 15.172$

$Var(u_2) = 21.995$

$Cov(u_0,u_1) = -27.060$

$Cov(u_0,u_2) = -19.912$

$Cov(u_1,u_2)$= 2.807

Table 4.14 on page 85.

- Input: ch4_p85.inp
- Output: ch4_p85.out

This model is...

Level 1

$MATH_{ij}= \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + \gamma_{01}(PUBLIC) + \gamma_{02}(MEANSES) + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

$\beta_{2j} = \gamma_{20}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 85, Table 4.14
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework white public meanses;
  within  = homework white;      ! level 1 variables here
  between = public meanses;      ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math on white;               ! fixed effect of white
    b1 | math on homework;       ! random effect for homework

  %between%
```

```
    math on public meanses;      ! intercept predicted from public, meanses
    b1;                          ! no predictors of b1, homework random slope

    math with b1;                ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1808.416

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.

Within Level
 MATH       ON
    WHITE             3.072    0.957      3.210
 Residual Variances
    MATH             52.710    3.437     15.335

Between Level
 MATH       ON
    PUBLIC            0.180    2.121      0.085
    MEANSES           5.052    1.831      2.759
 MATH     WITH
    B1              -25.531    9.161     -2.787
 Means
    B1                1.936    0.873      2.216
 Intercepts
    MATH             44.637    2.140     20.861
 Variances
    B1               15.462    5.393      2.867
 Residual Variances
    MATH             50.158   17.467      2.872
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

β_{0j} = 44.637 + 0.180(PUBLIC) +5.052(MEANSES) + u_{0j}
β_{1j} =1.936 + u_{1j}
β_{2j} = 3.072

Var(r_{ij}) = 52.710
Var(u_0) = 50.158
Var(u_1) = 15.462
Cov(u_0,u_1)= -25.531

Table 4.15 on page 86.

- Input: ch4_p86.inp
- Output: ch4_p86.out

This model is...

Level 1
$MATH_{ij}$= β_{0j} + β_1(HOMEWORK) + β_2(WHITE) + r_{ij}
Level 2
β_{0j} = γ_{00} + γ_{02}(MEANSES) + u_{0j}
β_{1j} = γ_{10} + u_{1j}
β_{2j} = γ_{20}

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 86, Table 4.15
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework white meanses;
  within  = homework white;     ! level 1 variables here
  between =  meanses;           ! level 2 variables here
analysis:
  type = twolevel random;
```

```
  estimator = ml;
model:
  %within%
    math on white;              ! fixed effect of white
    b1 | math on homework;      ! random effect of homework

  %between%
    math on meanses;            ! intercept predicted from meanses
    b1;                         ! no predictors of b1, homework random slope

    math with b1;               ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1808.419

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    WHITE              3.079    0.954      3.226
 Residual Variances
    MATH              52.710    3.437     15.334

Between Level
 MATH       ON
    MEANSES            4.942    1.291      3.828
 MATH     WITH
    B1               -25.519    9.156     -2.787
 Means
    B1                 1.935    0.873      2.216
 Intercepts
    MATH              44.742    1.743     25.667
 Variances
    B1                15.455    5.389      2.868
 Residual Variances
    MATH              50.144   17.464      2.871
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

$\beta_{0j} = 44.742 + 4.942(MEANSES) + u_{0j}$

$\beta_{1j} = 1.935 + u_{1j}$

$\beta_{2j} = 3.079$

$Var(r_{ij}) = 52.710$

$Var(u_0) = 50.144$

$Var(u_1) = 15.455$

$Cov(u_0,u_1) = -25.519$

Table 4.16 on page 88.

- Input: ch4_p88.inp
- Output: ch4_p88.out

This model is...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + \gamma_{02}(MEANSES) + u_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{12}(MEANSES) + u_{1j}$

$\beta_{2j} = \gamma_{20}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 88, Table 4.16
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework white meanses;
```

```
  within  = homework white;     ! level 1 variables here
  between = meanses;            ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math on white;              ! fixed effect of white
    b1 | math on homework;      ! random effect of homework

  %between%
    math on meanses;            ! intercept predicted from meanses
    b1 on meanses;              ! slope predicted from meanses

    math with b1;               ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1808.353

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    WHITE              3.084    0.955      3.231
 Residual Variances
    MATH              52.720    3.439     15.332

Between Level
 B1         ON
    MEANSES            0.565    1.560      0.362
 MATH       ON
    MEANSES            4.011    2.878      1.394
 MATH     WITH
    B1               -25.350    9.100     -2.786
 Intercepts
    MATH              44.646    1.761     25.348
```

```
    B1                  1.990      0.883      2.255
 Residual Variances
    MATH               49.945     17.371      2.875
    B1                 15.316      5.356      2.860
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + r_{ij}$

Level 2

$\beta_{0j} = 44.646 + 4.011(MEANSES) + u_{0j}$

$\beta_{1j} = 1.990 + 0.565(MEANSES) + u_{1j}$

$\beta_{2j} = 3.084$

$Var(r_{ij}) = 52.720$

$Var(u_0) = 49.945$

$Var(u_1) = 15.316$

$Cov(u_0,u_1) = -25.350$

Table 4.17 on page 89.

- Input: ch4_p89.inp
- Output: ch4_p89.out

This model is...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + \beta_3(SES) + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + \gamma_{02}(MEANSES) + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

$\beta_2 = \gamma_{20}$

$\beta_3 = \gamma_{30}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 89, Table 4.17
data:
```

```
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework white ses meanses;
  within  = homework white ses;   ! level 1 variables here
  between =  meanses;             ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math on white ses;            ! fixed effect of white and ses
    b1 | math on homework;        ! random effect of homework

  %between%
    math on meanses;              ! intercept predicted from meanses
    b1 ;                          ! no predictors of homework slope

    math with b1;                 ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1800.043

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    WHITE              2.254    0.974      2.314
    SES                2.192    0.536      4.087
 Residual Variances
    MATH              51.124    3.335     15.330

Between Level
 MATH       ON
```

```
    MEANSES              2.997      1.377      2.176
 MATH     WITH
    B1                 -23.058      8.398     -2.746
 Means
    B1                   1.834      0.830      2.210
 Intercepts
    MATH                45.610      1.710     26.677
 Variances
    B1                  13.802      4.870      2.834
 Residual Variances
    MATH                46.648     16.363      2.851
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + \beta_2(WHITE) + \beta_3(SES) + r_{ij}$

Level 2

$\beta_{0j} = 45.610 + 2.997(MEANSES) + u_{0j}$

$\beta_{1j} = 1.834 + u_{1j}$

$\beta_2 = 2.254$

$\beta_3 = 2.192$

$Var(r_{ij}) = 51.124$

$Var(u_0) = 46.648$

$Var(u_1) = 13.802$

$Cov(u_0,u_1) = -23.058$

Table 4.19 on page 91.

- Input: ch4_p91.inp
- Output: ch4_p91.out

This model is...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(SES) + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 91, Table 4.19
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math ses;
  within  = ses;       ! level 1 variables here
  between = ;          ! level 2 variables here (none)
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math on ses;       ! fixed effect of ses

  %between%
    math ;             ! no predictors of intercept
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood

          H0 Value                       -1874.178
          H1 Value                       -1874.178

MODEL RESULTS
                    Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    SES                4.346    0.580      7.495
 Residual Variances
```

```
    MATH               75.190     4.774     15.749

Between Level
 Means
    MATH               51.200     0.831     61.649
 Variances
    MATH               11.866     4.685      2.533
```

And here are the results substituted back into the model.

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(SES) + r_{ij}$
Level 2
$\beta_{0j} = 51.200$
$\beta_{1j} = 4.346$

$Var(r_{ij}) = 75.190$
$Var(u_0) = 11.866$

Table 4.20 on page 92.

- Input: ch4_p92.inp
- Output: ch4_p92.out

This model is...

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(SES) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10} + u_{1j}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 92, Table 4.20
data:
  file = imm23.dat ;
variable:
```

```
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math ses ;
  within  = ses;           ! level 1 variables here
  between = ;              ! level 2 variables here (none)
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math ;                 ! no fixed effects
    b1 | math on ses;      ! random effect of ses
  %between%
    math ;                 ! no predictors of intercept
    b1   ;                 ! no predictors of ses slope

    math with b1;          ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1874.197

MODEL RESULTS
                    Estimates     S.E.  Est./S.E.
Within Level
 Residual Variances
    MATH              74.962    4.949     15.148

Between Level
 MATH     WITH
    B1                -0.764    4.062     -0.188
 Means
    MATH              51.245    0.853     60.071
    B1                 4.340    0.608      7.144
 Variances
    MATH              12.201    5.120      2.383
```

```
    B1                    0.380     3.926       0.097
```

And here are the results substituted back into the model.

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(SES) + r_{ij}$
Level 2
$\beta_{0j} = 51.245 + u_{0j}$
$\beta_{1j} = 4.340 + u_{1j}$

$Var(r_{ij}) = 74.962$
$Var(u_0) = 12.201$
$Var(u_1) = 0.380$
$Cov(u_0,u_1) = -0.764$

Table 4.21 on page 93.

- Input: ch4_p93.inp
- Output: ch4_p93.out

This model is…

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(SES) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + \gamma_{01}(PERCMIN) + u_{0j}$
$\beta_{1j} = \gamma_{10}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 93, Table 4.21
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math ses percmin;
```

```
  within  = ses;              ! level 1 variables here
  between = percmin;          ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math on ses;              ! fixed effect of ses ;

  %between%
    math on percmin;          ! intercept predicted from percmin
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood

          H0 Value                       -1871.697
          H1 Value                       -1871.696

MODEL RESULTS
                    Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    SES                4.329    0.573      7.550
 Residual Variances
    MATH              75.010    4.753     15.781

Between Level
 MATH       ON
    PERCMIN           -0.804    0.350     -2.295
 Intercepts
    MATH              53.119    1.130     47.024
 Residual Variances
    MATH               9.496    3.792      2.504
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(SES) + r_{ij}$

Level 2
β_{0j} = 53.119 + -0.804(PERCMIN) + u_{0j}
β_{1j} = 4.329

Var(r_{ij}) = 75.010
Var(u_0) = 9.496

Table 4.22 on page 95.

- Input: ch4_p95.inp
- Output: ch4_p95.out

This model is...

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(SES) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + \gamma_{01}(PERCMIN) + \gamma_{02}(MEANSES) + u_{0j}$
$\beta_{1j} = \gamma_{10}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 95, Table 4.22
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math ses percmin meanses;
  within  = ses;                    ! level 1 variables here
  between = percmin meanses;        ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
```

```
    math on ses;                    ! fixed effect of ses

  %between%
    math on percmin meanses;        ! intercept predicted from percmin and meanses
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1869.804
          H1 Value                       -1869.803

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.
Within Level
 MATH       ON
    SES               3.865    0.609      6.345
 Residual Variances
    MATH             75.083    4.763     15.766

Between Level
 MATH       ON
    PERCMIN          -0.683    0.323     -2.113
    MEANSES           2.905    1.397      2.079
 Intercepts
    MATH             53.085    1.031     51.490
 Residual Variances
    MATH              7.215    3.177      2.271
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(SES) + r_{ij}$

Level 2

$\beta_{0j} = 53.085 + -0.683(PERCMIN) + 2.905(MEANSES) + u_{0j}$

$\beta_{1j} = 3.865$

$Var(r_{ij}) = 75.083$

$Var(u_0) = 7.215$

Table 4.23 and Table 4.24 on page 97.

Analyses with NELS-88, models 2 and 3 (we do not have this data, so these analyses are omitted).

Table 4.25 on page 99.

- Input: ch4_p99.inp
- Output: ch4_p99.out

This model is...

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$

Level 2

$\beta_{0j} = \gamma_{00} + \gamma_{01}(RATIO) + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 99, Table 4.25
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework ratio;
  within  = homework;          ! level 1 variables here
  between = ratio;             ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math ;                    ! no fixed effects
    b1 | math on homework;    ! random effect of homework

  %between%
```

```
    math on ratio;            ! intercept predicted from ratio
    b1 ;                      ! no predictors of homework slope

    math with b1;             ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1819.409

MODEL RESULTS
                   Estimates     S.E.  Est./S.E.

Within Level
 Residual Variances
    MATH              53.304    3.467     15.373

Between Level
 MATH       ON
    RATIO             -0.095    0.204     -0.468
 MATH     WITH
    B1               -26.220    9.870     -2.657
 Means
    B1                 1.988    0.908      2.190
 Intercepts
    MATH              47.973    3.922     12.232
 Variances
    B1                16.776    5.831      2.877
 Residual Variances
    MATH              59.268   20.000      2.963
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$

Level 2

$\beta_{0j} = 47.973 + -0.095(RATIO) + u_{0j}$

$\beta_{1j} = 1.988 + u_{1j}$

$Var(r_{ij})$ = 53.304
$Var(u_0)$ = 59.268
$Var(u_1)$ = 16.776
$Cov(u_0,u_1)$= -26.220

Table 4.26 on page 100.

- Input: ch4_p100.inp
- Output: ch4_p100.out

This model is...

Level 1
$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$
Level 2
$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10} + \gamma_{01}(RATIO) + u_{1j}$

The Mplus setup is shown below

```
title:
  Introducing Multilevel Modeling by Kreft and de Leeuw.
  Page 100, Table 4.26
data:
  file = imm23.dat ;
variable:
  names = schid stuid ses meanses homework white parented public
          ratio percmin math sex race sctype cstr scsize urban region;
  cluster = schid;
  usevar  = math homework ratio;
  within  = homework;          ! level 1 variables here
  between = ratio;             ! level 2 variables here
analysis:
  type = twolevel random;
  estimator = ml;
model:
  %within%
    math ;                     ! no fixed effects
    b1 | math on homework;     ! random effect of homework
```

```
  %between%
    math ;                    ! no predictors of intercept
    b1 on ratio;              ! homework slope predicted from ratio

    math with b1;             ! covariance intercept and slope
```

and some of the output is shown below.

```
TESTS OF MODEL FIT

Loglikelihood
          H0 Value                       -1819.397

MODEL RESULTS
                    Estimates     S.E.  Est./S.E.
Within Level
 Residual Variances
    MATH              53.307    3.468     15.372

Between Level
 B1         ON
    RATIO             -0.053    0.107     -0.495
 MATH     WITH
    B1               -26.232    9.857     -2.661
 Means
    MATH              46.320    1.721     26.917
 Intercepts
    B1                 2.909    2.065      1.408
 Variances
    MATH              59.318   19.984      2.968
 Residual Variances
    B1                16.760    5.824      2.878
```

And here are the results substituted back into the model.

Level 1

$MATH_{ij} = \beta_{0j} + \beta_1(HOMEWORK) + r_{ij}$

Level 2

$\beta_{0j} = 46.320 + u_{0j}$

$\beta_{1j} = 2.909 + -0.053(RATIO) + u_{1j}$

$Var(r_{ij})$ = 53.307
$Var(u_0)$ = 59.318
$Var(u_1)$ = 16.760
$Cov(u_0,u_1)$= -26.232

Table 4.27 on page 101.

Analyses with NELS-88, (we do not have this data, so these analyses are omitted).