Mplus Short Courses
Topic 4

Growth Modeling With Latent Variables Using Mplus: Advanced Growth Models, Survival Analysis, And Missing Data

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Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities
- Mplus versions
  - V1: November 1998
  - V2: February 2001
  - V3: March 2004
  - V4: February 2006
  - V5: November 2007
  - V5.21: May 2009
- Mplus team: Linda & Bengt Muthén, Thuy Nguyen, Tihomir Asparouhov, Michelle Conn, Jean Maninger

Statistical Analysis With Latent Variables A General Modeling Framework

Statistical Concepts Captured By Latent Variables

Continuous Latent Variables

- Measurement errors
- Factors
- Random effects
- Frailties, liabilities
- Variance components
- Missing data

Categorical Latent Variables

- Latent classes
- Clusters
- Finite mixtures
- Missing data

Statistical Analysis With Latent Variables
A General Modeling Framework (Continued)

Models That Use Latent Variables

Continuous Latent Variables

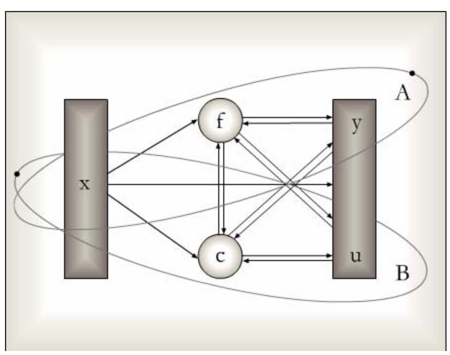
- Factor analysis models
- Structural equation models
- Growth curve models
- Multilevel models

Categorical Latent Variables

- Latent class models
- Mixture models
- Discrete-time survival models
- Missing data models

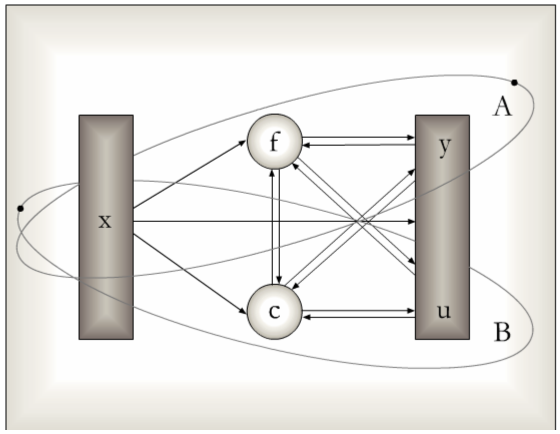
Mplus integrates the statistical concepts captured by latent variables into a general modeling framework that includes not only all of the models listed above but also combinations and extensions of these models.

General Latent Variable Modeling Framework

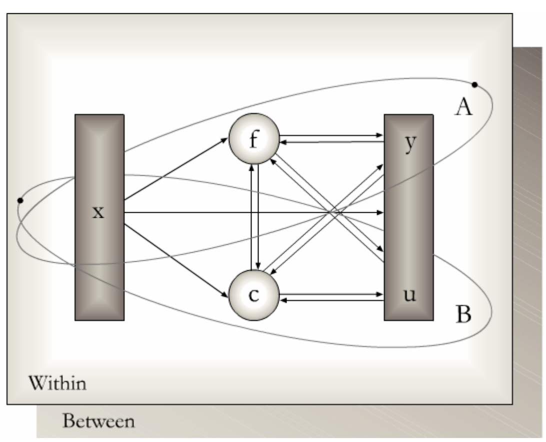


- Observed variables
  - x background variables (no model structure)
  - y continuous and censored outcome variables
  - u categorical (dichotomous, ordinal, nominal) and count outcome variables
- Latent variables
  - f continuous variables
    - – interactions among f's
  - c categorical variables
    - – multiple c's

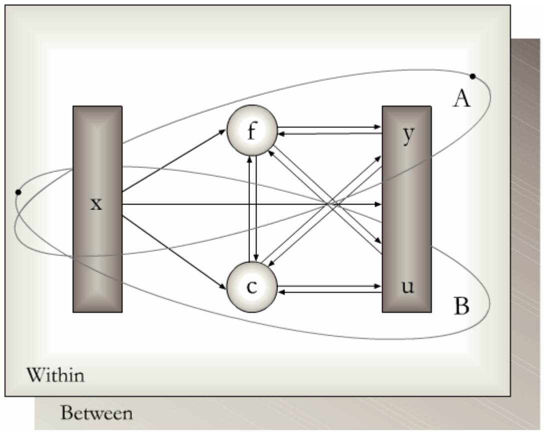
General Latent Variable Modeling Framework



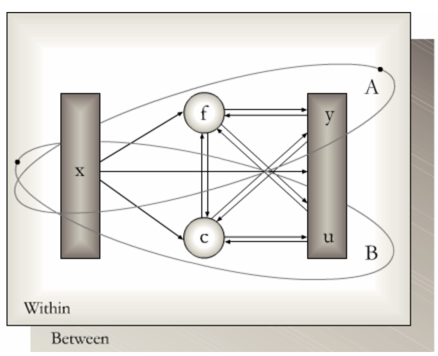
General Latent Variable Modeling Framework



General Latent Variable Modeling Framework



General Latent Variable Modeling Framework



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Mplus

Several programs in one

- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Latent class analysis
- Latent transition analysis
- Survival analysis
- Growth modeling
- Multilevel analysis
- Complex survey data analysis
- Monte Carlo simulation

Fully integrated in the general latent variable framework

Overview Of Mplus Courses

- **Topic 1.** August 20, 2009, Johns Hopkins University: Introductory - advanced factor analysis and structural equation modeling with continuous outcomes
- **Topic 2.** August 21, 2009, Johns Hopkins University: Introductory - advanced regression analysis, IRT, factor analysis and structural equation modeling with categorical, censored, and count outcomes
- **Topic 3.** March 22, 2010, Johns Hopkins University: Introductory and intermediate growth modeling
- **Topic 4.** March 23, 2010, Johns Hopkins University: Advanced growth modeling, survival analysis, and missing data analysis

Overview Of Mplus Courses (Continued)

- **Topic 5.** August 16, 2010, Johns Hopkins University: Categorical latent variable modeling with cross-sectional data
- **Topic 6.** August 17, 2010, Johns Hopkins University: Categorical latent variable modeling with longitudinal data
- **Extra Topic.** August 18, 2010, Johns Hopkins University: What's new in Mplus version 6?

- **Topic 7.** March, 2011, Johns Hopkins University: Multilevel modeling of cross-sectional data
- **Topic 8.** March, 2011, Johns Hopkins University: Multilevel modeling of longitudinal data

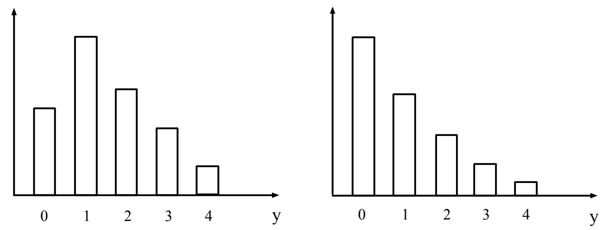
Advanced Growth Modeling

Modeling With Zeroes

Advantages Of Growth Modeling In A Latent Variable Framework

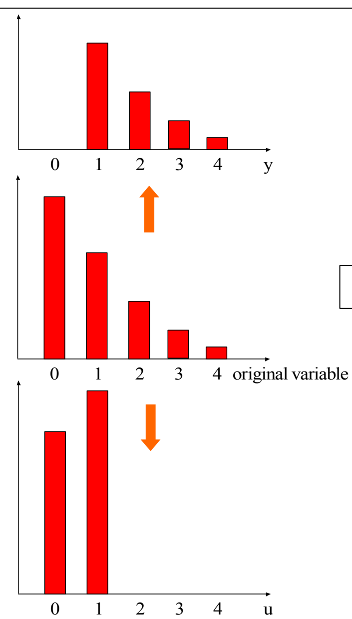
- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- **Modeling of zeroes**
- Multiple populations
- Multiple indicators
- Embedded growth models
- Categorical latent variables: growth mixtures

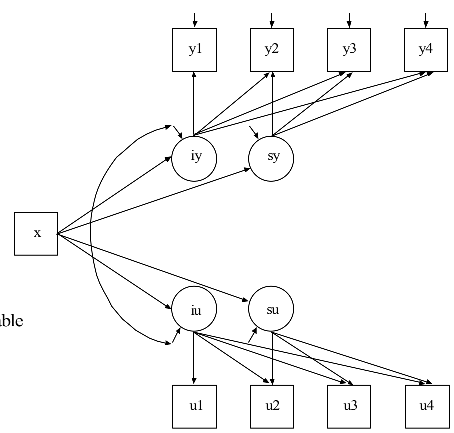
Modeling With A Preponderance Of Zeroes



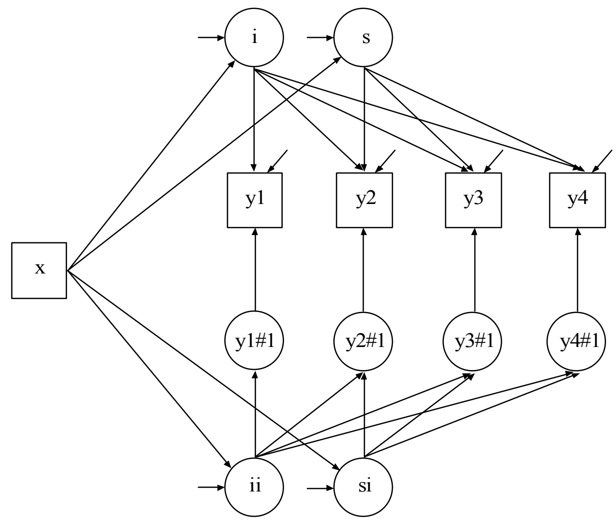
- Outcomes: non-normal continuous – count – categorical
- Censored-normal modeling
- Two-part (semicontinuous modeling): Duan et al. (1983), Olsen & Schafer (2001)
- Mixture models, e.g. zero-inflated (mixture) Poisson (Roeder et al., 1999), censored-inflated, mover-stayer latent transition models, growth mixture models
- Onset (survival) followed by growth: Albert & Shih (2003)

Two-Part (Semicontinuous) Growth Modeling

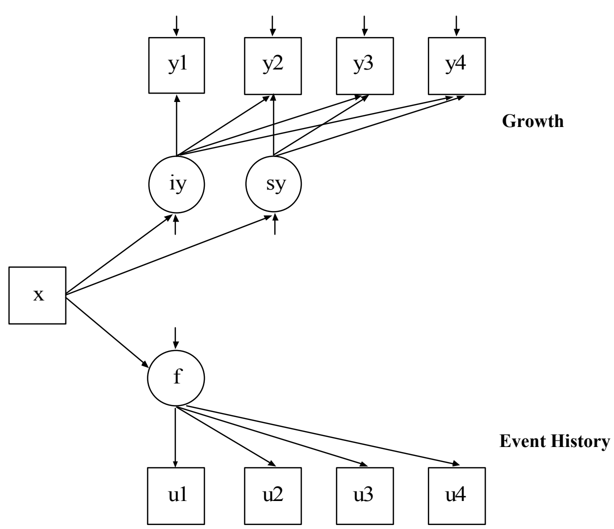




Inflated Growth Modeling (Two Classes At Each Time Point)



Onset (Survival) Followed By Growth



Two-Part Growth Modeling

AMPS Data

The data are taken from the Alcohol Misuse Prevention Study (AMPS). Forty-nine schools with a total of 2,666 students participated in the study. Students were measured seven times starting in the Fall of Grade 6 and ending in the Spring of Grade 12.

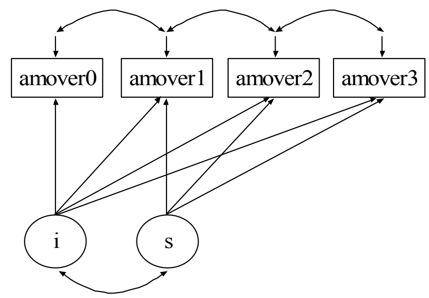
Data for the analysis include the average of three items related to alcohol misuse:

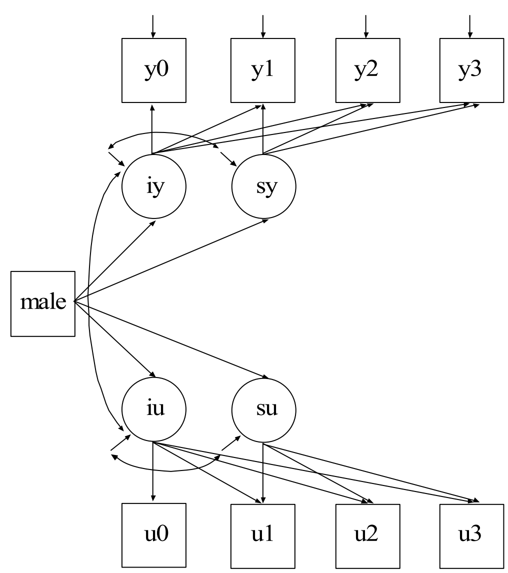
During the past 12 months, how many times did you

drink more than you planned to?
feel sick to your stomach after drinking?
get very drunk?

Responses: (0) never, (1) once, (2) two times, (3) three or more times

Four of the seven timepoints are studied: Fall Grade 6, Spring Grade 6, Spring Grade 7, and Spring Grade 8.


amover0
amover1
amover2
amover3
i
s


y0
y1
y2
y3
iy
sy
male
iu
su
u0
u1
u2
u3

Input For Two-Part Growth Model Using DATA TWOPART

Amover	u	y
>0	1	log Amover
0	0	999
999	999	999

```
TITLE:          This is an example of a two-part (semicontinuous)
                growth model for a continuous outcome
DATA:           FILE = amp.dat;
DATA TWOPART:   NAMES = amover0-amover3;
                BINARY = u0-u3;
                CONTINUOUS = y0-y3;
VARIABLE:       NAMES = amover0-amover3 male;
                USEVARIABLES = male u0-u3 y0-y3;
                CATEGORICAL = u0-u3;
                MISSING = ALL(999);
```


Input For Two-Part Growth Model (Continued)

```
ANALYSIS:   ESTIMATOR = ML;
            ALGORITHM = INTEGRATION;
            COVERAGE = .09;
MODEL:      iu su | u0@0 u1@0.5 u2@1.5 u3@2.5;
            iy sy | y0@0 y1@0.5 y2@1.5 y3@2.5;
            iu-sy ON male;
            ! estimate the residual covariances
            ! iu with su, iy with sy, and iu with iy
            iu WITH sy@0;
            su WITH iy-sy@0;
OUTPUT:     PATTERNS SAMPSTAT STANDARDIZED TECH1 TECH4 TECH8;
PLOT:       TYPE = PLOT3;
            SERIES = u0-u3(su) | y0-y3(sy);
```

Output Excerpts For A Two-Part Growth Model

Tests of Model Fit

```
Loglikelihood

          H0 Value                    -3277.101

Information Criteria

          Number of Free parameters         19
          Akaike (AIC)                6592.202
          Bayesian (BIC)              6689.444
          Sample-Size Adjusted BIC    6629.092
             (n* = (n + 2) / 24)
```


Output Excerpts For A Two-Part Growth Model (Continued)

```
                    Estimate  S.E.    Est./S.E.  Std     StdYX
  IU       ON
     MALE            0.569    0.234     2.433    0.200    0.100
  SU       ON
     MALE           -0.181    0.119    -1.518   -0.218   -0.109
  IY       ON
     MALE            0.149    0.061     2.456    0.279    0.139
  SY       ON
     MALE           -0.068    0.038    -1.790   -0.290   -0.145

  IU       WITH
     SU             -1.144    0.326    -3.509   -0.484   -0.484
     IY              1.193    0.134     8.897    0.788    0.788
     SY              0.000    0.000     0.000    0.000    0.000
  IY       WITH
     SY             -0.039    0.019    -2.109   -0.316   -0.316
  SU       WITH
     IY              0.000    0.000     0.000    0.000    0.000
     SY              0.000    0.000     0.000    0.000    0.000
```

Output Excerpts For A Two-Part Growth Model (Continued)

Intercepts

Y0	0.000	0.000	0.000	0.000	0.000
Y1	0.000	0.000	0.000	0.000	0.000
Y2	0.000	0.000	0.000	0.000	0.000
Y3	0.000	0.000	0.000	0.000	0.000
IU	0.000	0.000	0.000	0.000	0.000
SU	0.855	0.098	8.716	1.027	1.027
IY	0.232	0.059	3.901	0.435	0.435
SY	0.240	0.031	7.830	1.025	1.025

Thresholds

U0$1	2.655	0.206	12.877
U1$1	2.655	0.206	12.877
U2$1	2.655	0.206	12.877
U3$1	2.655	0.206	12.877

Output Excerpts For A Two-Part Growth Model (Continued)

Residual Variances

Y0	0.175	0.032	5.470	0.175	0.380
Y1	0.266	0.029	9.159	0.266	0.509
Y2	0.238	0.027	8.810	0.238	0.457
Y3	0.269	0.054	5.014	0.269	0.392
IU	7.982	1.086	7.351	0.990	0.990
SU	0.685	0.202	3.400	0.988	0.988
IY	0.279	0.040	7.019	0.981	0.981
SY	0.054	0.017	3.224	0.979	0.979

Output Excerpts For A Two-Part Growth Model (Continued)

Observed Variable	R-Square
U0	0.710
U1	0.682
U2	0.650
U3	0.666
Y0	0.620
Y1	0.491
Y2	0.543
Y3	0.608

Latent Variable	R-Square
IU	0.010
SU	0.012
IY	0.019
SY	0.021

Output Excerpts For A Two-Part Growth Model (Continued)

Technical 4 Output

ESTIMATED MEANS FOR THE LATENT VARIABLES

	IU	SU	IY	SY	MALE
1	0.305	0.758	0.312	0.204	0.536

ESTIMATED COVARIANCE MATRIX FOR THE LATENT VARIABLES

	IU	SU	IY	SY	MALE
IU	8.062				
SU	-1.170	0.694			
IY	1.214	-0.007	0.285		
SY	-0.010	0.003	-0.042	0.055	
MALE	0.142	-0.045	0.037	-0.017	0.249

Output Excerpts For A Two-Part Growth Model (Continued)

ESTIMATED CORRELATION MATRIX FOR THE LATENT VARIABLES

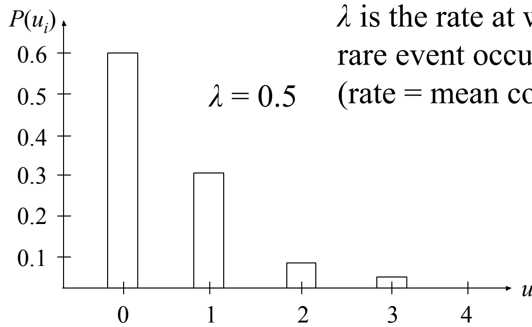
	IU	SU	IY	SY	MALE
IU	1.000				
SU	-0.495	1.000			
IY	0.801	-0.015	1.000		
SY	-0.014	0.016	-0.336	1.000	
MALE	0.100	-0.109	0.139	-0.145	1.000

Regression With A Count Dependent Variable

Poisson Regression

A Poisson distribution for a count variable u_i has

$$P(u_i = r) = \frac{\lambda_i^r e^{-\lambda_i}}{r!}, \text{ where } u_i = 0, 1, 2, \ldots$$



λ is the rate at which a rare event occurs (rate = mean count)

Regression equation for the log rate:

$$^{e}\log \lambda_i = \ln \lambda_i = \beta_0 + \beta_1 x_i$$

Zero-Inflated Poisson (ZIP) Regression

A Poisson variable has mean = variance.

Data often have variance > mean due to preponderance of zeros.

$\pi = P$ (being in the zero class where only $u = 0$ is seen)

$1 - \pi = P$ (not being in the zero class with u following a Poisson distribution)

A mixture at zero:	ZIP mean count:	ZIP variance of count:
$P(u = 0) = \pi + (1 - \pi)\underbrace{e^{-\lambda}}_{\text{Poisson part}}$	$\lambda (1 - \pi)$	$\lambda (1 - \pi)(1 + \lambda * \pi)$

The ZIP model implies two regressions:

$$\text{logit}(\pi_i) = \gamma_0 + \gamma_1 x_i,$$

$$\ln \lambda_i = \beta_0 + \beta_1 x_i$$

Negative Binomial Regression

Unobserved heterogeneity ε_i is added to the Poisson model

$$ln\,\lambda_i = \beta_0 + \beta_1 x_i + \varepsilon_i \text{ , where } exp\,(\varepsilon) \sim \Gamma$$

Poisson assumes

$$E(u_i \mid x_i) = \lambda_i$$
$$V(u_i \mid x_i) = \lambda_i$$

Negative binomial assumes

$$E(u_i \mid x_i) = \lambda_i$$
$$V(u_i \mid x_i) = \lambda_i (1 + \lambda_i\, \alpha)$$

NB with $\alpha = 0$ gives Poisson. When the dispersion parameter $\alpha > 0$, the NB model gives substantially higher probability for low counts and somewhat higher probability for high counts than Poisson.

Further variations are zero-inflated NB and zero-truncated NB (hurdle model or two-part model).

Zero-Inflated Poisson (ZIP) Growth Modeling Of Counts

$$u_{ti} = \begin{cases} 0 & \text{with probability } \pi_{ti} \\ \text{Poisson } (\lambda_{ti}) & \text{with probability } 1 - \pi_{ti} \end{cases}$$

$$ln\,\lambda_{ti} = \eta_{0i} + \eta_{1i} a_{ti} + \eta_{2i} a_{ti}^2$$
$$\eta_{0i} = \alpha_0 + \zeta_{0i}$$
$$\eta_{1i} = \alpha_1 + \zeta_{1i}$$
$$\eta_{2i} = \alpha_2 + \zeta_{2i}$$

In Mplus, $\pi_{ti} = P(u\#_{ti} = 1)$, where $u\#$ is a binary latent inflation variable

Philadelphia Crime Data ZIP Growth Modeling

- 13,160 males ages 4 - 26 born in 1958 (Moffitt, 1993; Nagin & Land, 1993)
- Annual counts of police contacts
- Individuals with more than 10 counts in any given year deleted (n=13,126)
- Data combined into two-year intervals

Input Excerpts Philadelphia Crime Data

```
DATA:        FILE = phillywide_zero_ln2006_del.dat;
VARIABLE:    NAMES = cohortid erace sesdummy sescomp1 juvtot
             adttot
             y10 y12 y14 y16 y18 y20 y22 y24
             sex race;
             MISSING = ALL (-9999);
             USEVAR = y10 y12 y14 y16 y18 y20 y22 y24;
             !y10 is ages 10-11, y12 is ages y12-13, etc
             COUNT = y10-y24(i);
             IDVAR = cohortid;
             USEOBS = y10 LE 10 AND y12 LE 10 AND y14 LE 10 AND
             y16 LE 10 AND y18 LE 10 AND y20 LE 10 AND y22 LE 10
             AND y24 LE 10;
```

Input Excerpts Philadelphia Crime Data (Continued)

```
ANALYSIS:
!             algorithm = integration;
              PROCESS = 4;
              INTERACTIVE = control.dat;

MODEL:        i s q | y10@0 y12@.1 y14@.2 y16@.3 y18@.4 y20@.5
              y22@.6 y24@.7;

OUTPUT:       TECH1 TECH10;

PLOT:         TYPE = PLOT3;
              SERIES = y10-y24(s);
```


Output Excerpts Philadelphia Crime Data

```
TESTS OF MODEL FIT

Loglikelihood
         H0 Value                          -40607.007
         H0 Scaling Correlation Factor          0.931
           for MLR

Information Criteria
         Number of Free Parameters                 17
         Akaike (AIC)                       81248.155
         Bayesian (BIC)                     81375.355
         Sample-Size Adjusted BIC           81321.330
           (n* = (n + 2) / 24)
```

Output Excerpts Philadelphia Crime Data (Continued)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means				
I	-4.689	0.145	32.402	0.000
S	14.003	0.749	18.700	0.000
Q	20.036	0.913	-21.942	0.000
Y10#1	0.768	0.123	6.268	0.000
Y12#1	-0.557	0.119	-4.679	0.000
Y14#1	-1.763	0.156	-11.322	0.000
Y16#1	-3.023	0.310	9.746	0.000
Y18#1	-0.284	0.061	-4.638	0.000
Y20#1	-0.319	0.074	-4.293	0.000
Y22#1	-1.521	0.166	-9.156	0.000
Y24#1	-13.723	9.974	-1.376	0.169

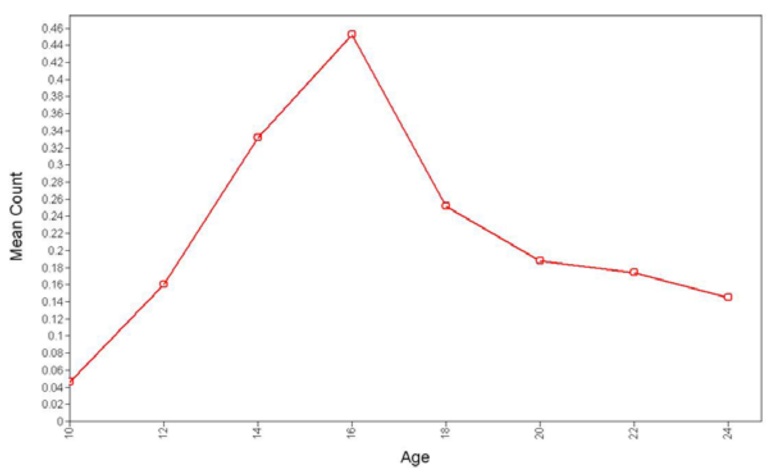
Output Excerpts Philadelphia Crime Data (Continued)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Intercepts				
Y10	0.000	0.000	999.000	999.000
Y12	0.000	0.000	999.000	999.000
Y14	0.000	0.000	999.000	999.000
Y16	0.000	0.000	999.000	999.000
Y18	0.000	0.000	999.000	999.000
Y20	0.000	0.000	999.000	999.000
Y22	0.000	0.000	999.000	999.000
Y24	0.000	0.000	999.000	999.000
Variances				
I	5.509	0.345	15.960	0.000
S	32.931	4.568	7.206	0.000
Q	59.745	7.603	7.858	0.000

Output Excerpts Philadelphia Crime Data (Continued)

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
S	WITH				
I		-8.320	1.206	-6.896	0.000
Q	WITH				
I		5.864	1.358	4.318	0.000
S		-35.766	5.594	-6.394	0.000

Estimated Mean Counts For Philadelphia Crime Data



Philadelphia Crime Data Model Fit To Counts For Most Frequent Response Patterns

Pattern	Observed	Estimated	Z Score
00000000	8021	7850	**3.04**
00010000	572	673	**-4.00**
00100000	378	433	**-2.72**
00001000	292	354	**-3.32**
00000010	203	233	-1.95
00000100	201	266	**-4.03**
20000000	181	173	0.60
00000001	141	157	-1.27
00110000	117	112	0.50
00020000	107	95	1.28

Growth Modeling With Multiple Populations

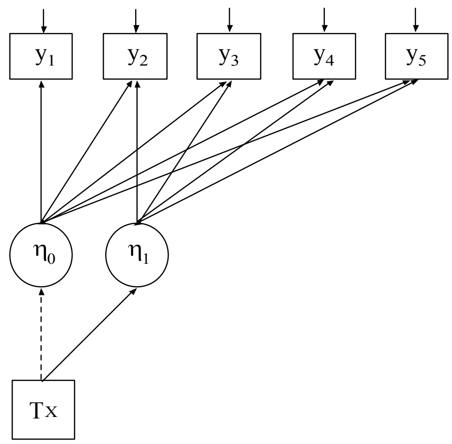
Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- **Multiple populations**
- Multiple indicators
- Embedded growth models
- Categorical latent variables: growth mixtures

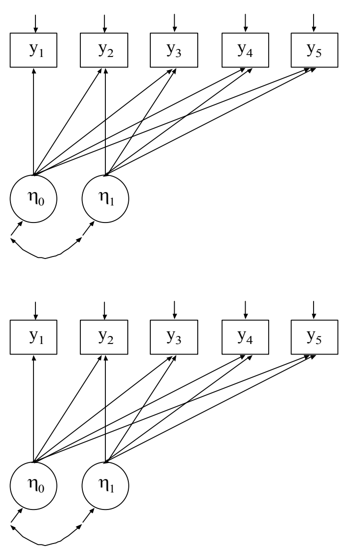
Multiple Population Growth Modeling

- Group as a dummy variable
- Multiple-group analysis
- Multiple-group analysis of randomized interventions

Group Dummy Variable As A Covariate



Two-Group Model



Multiple Population Growth Modeling Specifications

Let y_{git} denote the outcome for population (group) g, individual i, and timepoint t,

$$Level\ 1{:}\ y_{gti} = \eta_{g0i} + \eta_{g1i}\, x_t + \varepsilon_{gti}, \qquad (65)$$
$$Level\ 2a{:}\ \eta_{g0i} = \alpha_{g0} + \gamma_{g0}\, w_{gi} + \zeta_{g0i}, \qquad (66)$$
$$Level\ 2b{:}\ \eta_{g1i} = \alpha_{g1} + \gamma_{g1}\, w_{gi} + \zeta_{g1i}, \qquad (67)$$

Measurement invariance (level-1 equation): time-invariant intercept 0 and slopes 1, x_t

Structural differences (level-2): α_g , γ_g , $V(\zeta_g)$

Alternative parameterization:

$$Level\ 1{:}\ y_{gti} = v + \eta_{g0i} + \eta_{g1i}\, x_t + \varepsilon_{gti}, \qquad (68)$$

with α_{10} fixed at zero in level 2a.

Analysis steps:

1. Separate growth analysis for each group
2. Joint analysis of all groups, free structural parameters
3. Join analysis of all groups, tests of structural parameter invariance

NLSY: Multiple Cohort Structure

Birth	Age[a]																			
Year Cohort	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
57								82	83	84	85	86	87	88	89	90	91	92	93	94
58							82	83	84	85	86	87	88	89	90	91	92	93	94	
59						82	83	84	85	86	87	88	89	90	91	92	93	94		
60					82	83	84	85	86	87	88	89	90	91	92	93	94			
61				82	83	84	85	86	87	88	89	90	91	92	93	94				
62			82	83	84	85	86	87	88	89	90	91	92	93	94					
63		82	83	84	85	86	87	88	89	90	91	92	93	94						
64	82	83	84	85	86	87	88	89	90	91	92	93	94							

[a] Non-shaded areas represent years in which alcohol measures were obtained

Three Approaches To Cohort Structures

- Single group - using y18 - y37
- Single group - using 7 y's and AT with TSCORES to capture varying ages
- Multiple group - using 7 y's and 8 cohorts with s@xt

Multiple Group Modeling Of Multiple Cohorts

- Data – two cohorts born in 1961 and 1962 measured on the frequency of heavy drinking in the years 1983, 1984, 1988, and 1989
- Development of heavy drinking across chronological age, not year of measurement, is of interest

Cohort/Year	1983	1984	1988	1989
1961 (older)	**22**	23	**27**	28
1962 (younger)	21	**22**	26	**27**

Cohort/Age	21	22	23	24	25	26	27	28
1961 (older)		**83**	**84**				**88**	**89**
1962 (younger)	**83**	**84**				**88**	**89**	

Multiple Group Modeling Of Multiple Cohorts (Continued)

- Time scores calculated for age, not year of measurement

Age	**21**	**22**	**23**	24	25	**26**	**27**	**28**
Time score	**0**	**1**	**2**	3	4	**5**	**6**	**7**

Cohort 1961 time scores 1 2 6 7
Cohort 1962 time scores 0 1 5 6

- Can test the degree of measurement and structural invariance
  - Test of full invariance
    - Growth factor means, variances, and covariances held equal across cohorts
    - Residual variances of shared ages held equal across cohorts

Input For Multiple Group Modeling Of Multiple Cohorts

```
TITLE:        Multiple Group Modeling Of Multiple Cohorts
DATA:         FILE IS cohort.dat;
VARIABLE:     NAMES ARE cohort hd83 hd84 hd88 hd89;
              MISSING ARE *;
              USEV = hd83 hd84 hd88 hd89;
              GROUPING IS cohort (61 = older 62 = younger);
MODEL:        i s | hd83@0 hd84@1 hd88@5 hd89@6;
              [i] (1);
              [s] (2);
              i (3);
              s (4);
              i WITH s (5);
```

Input For Multiple Group Modeling Of Multiple Cohorts (Continued)

```
MODEL older:
              i s | hd83@1 hd84@2 hd88@6 hd89@7;
              hd83 (6);
              hd88 (7);
MODEL younger:
              hd84 (6);
              hd89 (7);

OUTPUT:       STANDARDIZED;
```


Output Excerpts Multiple Group Modeling Of Multiple Cohorts

Tests Of Model Fit

```
Chi-Square Test of Model Fit
          Value                              68.096
          Degrees of Freedom                     17
          P-Value                             .0000

Chi-Square Contributions From Each Group
          OLDER                              39.216
          YOUNGER                            28.880

Chi-Square Test of Model Fit for the Baseline Model
          Value                            3037.930
          Degrees of Freedom                     12
          P-Value                            0.0000

CFI/TLI
          CFI                                 0.983
          TLI                                 0.988
```

Output Excerpts Multiple Group Modeling Of Multiple Cohorts (Continued)

```
Loglikelihood
          H0 Value                      -18544.420
          H1 Value                      -18510.371

Information Criteria
          Number of Free Parameters             11
          Akaike (AIC)                   37110.839
          Bayesian (BIC)                 37175.770
          Sample-Size Adjusted BIC       37140.820
            (n* = (n + 2) / 24)

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.047
          90 Percent C.I.                    0.036   0.059

SRMR (Standardized Root Mean Square Residual)
          Value                              0.033
```


Output Excerpts Multiple Group Modeling Of Multiple Cohorts (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
Group OLDER					
I WITH					
S	-.111	.010	-11.390	-.537	-.537
Residual Variances					
HD83	1.141	.046	24.996	1.141	.445
HD84	1.062	.057	18.489	1.062	.453
HD88	1.028	.041	25.326	1.028	.455
HD89	.753	.053	14.107	.753	.358
Variances					
I	1.618	.068	23.651	1.000	1.000
S	.026	.002	13.372	1.000	1.000
Means					
I	1.054	.030	35.393	.828	.828
S	-.032	.005	-6.611	-.200	-.200

Output Excerpts Multiple Group Modeling Of Multiple Cohorts (Continued)

```
GROUP YOUNGER
Residual Variances
    HD83              1.049    .066   15.916      1.049    .393
    HD84              1.141    .046   24.996      1.141    .445
    HD88              1.126    .056   19.924      1.126    .491
    HD89              1.028    .041   25.326      1.028    .455
```


Preventive Interventions Randomized Trials

Prevention Science Methodology Group (PSMG)

Developmental Epidemiological Framework:

- Determining the levels and variation in risk and protective factors as well as developmental paths within a defined population in the absence of intervention
- Directing interventions at these risk and protective factors in an effort to change the developmental trajectories in a defined population
- Evaluating variation in intervention impact across risk levels and contexts on proximal and distal outcomes, thereby empirically testing the developmental model

Aggressive Classroom Behavior: The GBG Intervention

Muthén & Curran (1997, Psychological Methods)

The Johns Hopkins Prevention Center carried out a school-based preventive intervention randomized trial in Baltimore public schools starting in grade 1. One of the interventions tested was the Good Behavior Game intervention, a classroom based behavior management strategy promoting good behavior. It was designed specifically to reduce aggressive behavior of first graders and was aimed at longer term impact on aggression through middle school.

One first grade classroom in a school was randomly assigned to receive the Good Behavior Game intervention and another matched classroom in the school was treated as control. After an initial assessment in fall of first grade, the intervention was administered during the first two grades.

Aggressive Classroom Behavior: The GBG Intervention (Continued)

The outcome variable of interest was teacher ratings (TOCA-R) of each child's aggressive behavior (breaks rules, harms property, fights, etc.) in the classroom through grades 1 – 6. Eight teacher ratings were made from fall and spring for the first two grades and every spring in grades 3 – 6.

The most important scientific question was whether the Good Behavior Game reduces the slope of the aggression trajectory across time. It was also of interest to know whether the intervention varies in impact for children who started out as high aggressive versus low aggressive.

Analyses in Muthén-Curran (1997) were based on data for 75 boys in the GBG group who stayed in the intervention condition for two years and 111 boys in the control group.

The GBG Aggression Example: Analysis Results

Muthén & Curran (1997):

- Step 1: Control group analysis
- Step 2: Treatment group analysis
- Step 3: Two-group analysis w/out interactions
- Step 4: Two-group analysis with interactions
- Step 5: Sensitivity analysis of final model
- Step 6: Power analysis

MUTHÉN AND CURRAN

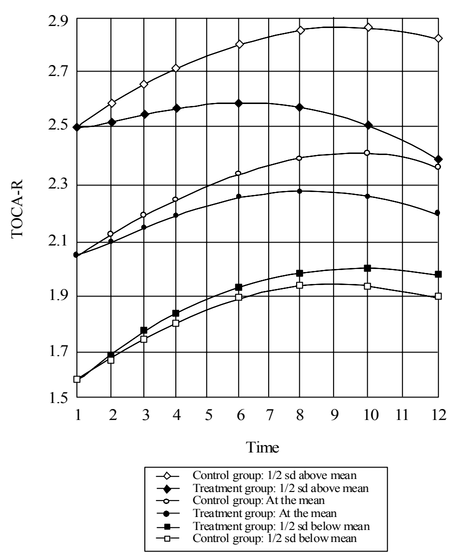
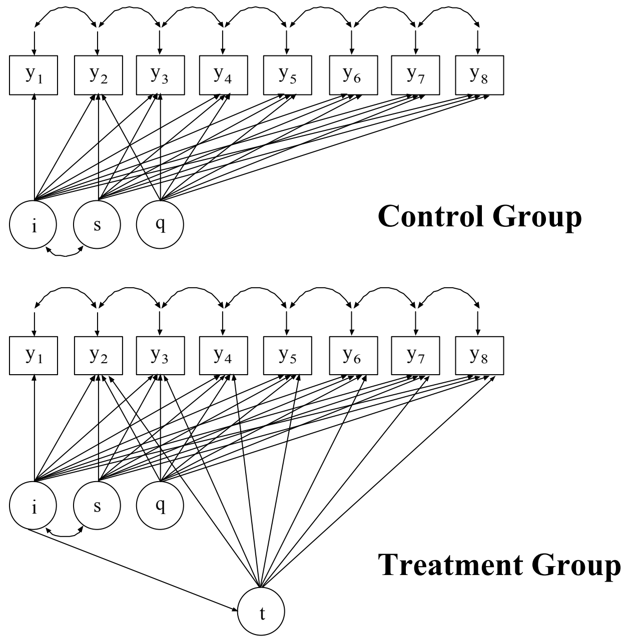


Figure 15. Model implied growth trajectories of Teacher Observation of Classroom Behavior—Revised (TOCA-R) scores as a function of initial status. Each timepoint represents one 6-month interval.



Input Excerpts For Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects

```
TITLE:           Aggressive behavior intervention growth model
                 n = 111 for control group
                 n = 75 for tx group
MODEL:           i s q | y1@0 y2@1 y3@2 y4@3 y5@5 y6@7 y7@9 y8@11;
                 i t | y1@0 y2@1 y3@2 y4@3 y5@5 y6@7 y7@9 y8@11;
                 [y1-y8] (1);   !alternative growth model
                 [i@0];         !parameterization
                 i (2);
                 s (3);
                 i WITH s (4);
                 [s] (5);
                 [q] (6);
                 t@0 q@0;
                 q WITH i@0 s@0 t@0; y1-y7 PWITH y2-y8;
                 t ON i;
```

Input Excerpts For Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

```
MODEL control:
                [s] (5);
                [q] (6);
                t ON i@0;
                [t@0];
```


Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects

Tests Of Model Fit

```
Chi-Square Test of Model Fit

          Value                         64.553
          Degrees of Freedom                50
          P-Value                        .0809

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                        .056
          90 Percent C.I.                 .000   .092
```

Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Control		Group Tx	
Observed Variable	R-Square	Observed Variable	R-Square
Y1	.644	Y1	.600
Y2	.642	Y2	.623
Y3	.663	Y3	.568
Y4	.615	Y4	.464
Y5	.637	Y5	.425
Y6	.703	Y6	.399
Y7	.812	Y7	.703
Y8	.818	Y8	.527

Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Control	Estimates	S.E.	Est./S.E.	Std	StdYX
T ON					
I	**.000**	**.000**	**.000**	**999.000**	**999.000**
Residual Variances					
Y1	.444	.088	5.056	.444	.356
Y2	.449	.079	5.714	.449	.358
Y3	.414	.069	6.026	.414	.337
Y4	.522	.080	6.551	.522	.385
Y5	.512	.079	6.469	.512	.363
Y6	.422	.074	5.677	.422	.297
Y7	.264	.083	3.186	.264	.188
Y8	.291	.094	3.097	.291	.182
T	.000	.000	.000	999.000	999.000
Variances					
I	.803	.109	7.330	1.000	1.000
S	.004	.001	3.869	1.000	1.000
Q	.000	.000	.000	999.000	999.000

Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Means					
I	.000	.000	.000	.000	.000
S	.086	.021	4.035	1.285	1.285
Q	-.005	.002	-3.005	999.000	999.000
Intercepts					
Y1	2.041	.078	26.020	2.041	1.828
Y2	2.041	.078	26.020	2.041	1.823
Y3	2.041	.078	26.020	2.041	1.841
Y4	2.041	.078	26.020	2.041	1.753
Y5	2.041	.078	26.020	2.041	1.718
Y6	2.041	.078	26.020	2.041	1.711
Y7	2.041	.078	26.020	2.041	1.724
Y8	2.041	.078	26.020	2.041	1.612
T	.000	.000	.000	999.000	999.000

Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

Group Tx

	Estimates	S.E.	Est./S.E.	Std	StdYX
T ON					
I	**-.052**	**.015**	**-3.347**	**-1.000**	**-1.000**
Residual Variances					
Y1	.535	.141	3.801	.535	.400
Y2	.439	.122	3.595	.439	.377
Y3	.501	.108	4.653	.501	.432
Y4	.701	.132	5.332	.701	.536
Y5	.736	.133	5.545	.736	.575
Y6	.805	.152	5.288	.805	.601
Y7	.245	.104	2.364	.245	.297
Y8	.609	.182	3.351	.609	.473
T	.000	.000	.000	.000	.000
Variances					
I	.803	.109	7.330	1.000	1.000
S	.004	.001	3.869	1.000	1.000
Q	.000	.000	.000	999.000	999.000

Output Excerpts Aggressive Behavior Intervention Using A Multiple Group Growth Model With A Regression Among Random Effects (Continued)

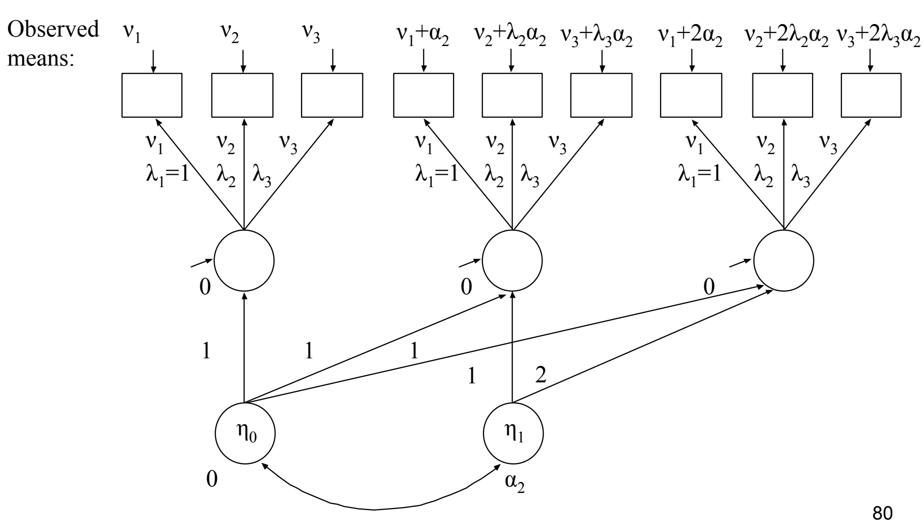
	Estimates	S.E.	Est./S.E.	Std	StdYX
Means					
I	.000	.000	.000	.000	.000
S	.086	.021	4.035	1.285	1.285
Q	-.005	.002	-3.005	999.000	999.000
Intercepts					
Y1	2.041	.078	26.020	2.041	1.764
Y2	2.041	.078	26.020	2.041	1.893
Y3	2.041	.078	26.020	2.041	1.895
Y4	2.041	.078	26.020	2.041	1.785
Y5	2.041	.078	26.020	2.041	1.805
Y6	2.041	.078	26.020	2.041	1.764
Y7	2.041	.078	26.020	2.041	2.248
Y8	2.041	.078	26.020	2.041	1.799
T	-.016	.013	-1.225	-.341	-.341

Growth Modeling With Multiple Indicators

Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- Multiple populations
- **Multiple indicators**
- Embedded growth models
- Categorical latent variables: growth mixtures

Growth Of Latent Variable Construct Measured By Multiple Indicators



Multiple Indicator Growth Modeling Specifications

Let y_{jti} denote the outcome for individual i, indicator j, and timepoint t, and let η_{ti} denote a latent variable construct,

Level 1*a (measurement part):*

$$y_{jti} = \nu_{jt} + \lambda_{jt}\, \eta_{ti} + \varepsilon_{jti}, \quad (44)$$

$$\textit{Level } 1b: \eta_{ti} = \eta_{0i} + \eta_{1i}\ x_t + \zeta_{ti}, \quad (45)$$

$$\textit{Level } 2a: \ \eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}, \quad (46)$$

$$\textit{Level } 2b: \ \eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}, \quad (47)$$

Measurement invariance: time-invariant indicator intercepts and slopes:

$$\nu_{j1} = \nu_{j2} = \ \ldots\ \nu_{jT} = \nu_j, \quad (48)$$

$$\lambda_{j1} = \lambda_{j2} = \ \ldots\ \lambda_{jT} = \lambda_j, \quad (49)$$

where $\lambda_1 = 1$, $\alpha_0 = 0$. $V(\varepsilon_{jti})$ and $V(\zeta_{ti})$ may vary over time. Structural differences: $E(\eta_{ti})$ and $V(\eta_{ti})$ vary over time.

Steps In Growth Modeling With Multiple Indicators

- Exploratory factor analysis of indicators for each timepoint
- Determine the shape of the growth curve for each indicator and the sum of the indicators
- Fit a growth model for each indicator—must be the same
- Confirmatory factor analysis of all timepoints together
  - Covariance structure analysis without measurement parameter invariance
  - Covariance structure analysis with invariant loadings
  - Mean and covariance structure analysis with invariant measurement intercepts and loadings
- Growth model with measurement invariance across timepoints

Advantages Of Using Multiple Indicators Instead Of An Average

- Estimation of unequal weights
- Partial measurement invariance—changes across time in individual item functioning
- No confounding of time-specific variance and measurement error variance
- Smaller standard errors for growth factor parameters (more power)

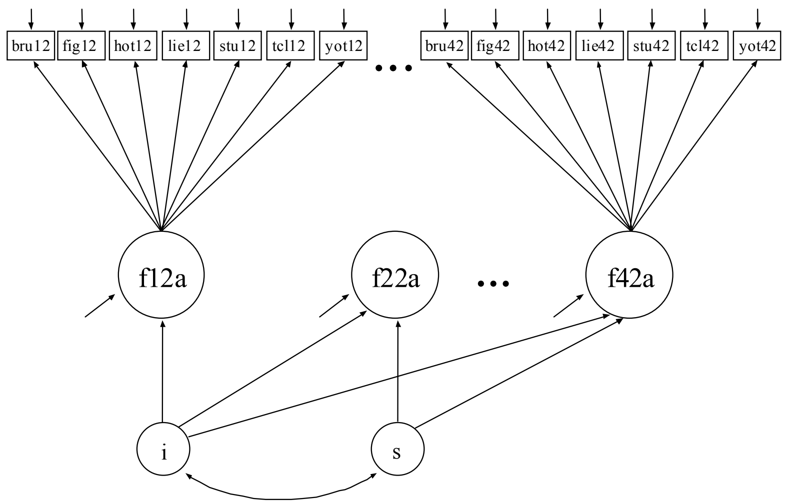
Classroom Aggression Data (TOCA)

The classroom aggression data are from an intervention study in Baltimore public schools carried out by the Johns Hopkins Prevention Research Center. Subjects were randomized into treatment and control conditions. The TOCA-R instrument was used to measure 10 aggression items at multiple timpoints. The TOCA-R is a teacher rating of student behavior in the classroom. The items are rated on a six-point scale from almost never to almost always.

Data for this analysis include the 342 boys in the control group. Four time points are examined: Spring Grade 1, Spring Grade 2, Spring Grade 3, and Spring Grade 4.

Seven aggression items are used in the analysis:

- Break rules
- Fights
- Harms others
- Lies
- Stubborn
- Teasing classmates
- Yells at others



Input Excerpts For TOCA Data Multiple Indicator CFA With No Measurement Invariance

```
TITLE:      Multiple indicator CFA with no measurement invariance
.
.
.
MODEL:      f12a BY bru12-yot12;
            f22a BY bru22-yot22;
            f32a BY bru32-yot32;
            f42a BY bru42-yot42;
```

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance

```
TITLE:      Multiple indicator CFA with factor loading invariance
.
.
.
MODEL:      f12a BY bru12
                    fig12-yot12 (1-6);
            f22a BY bru22
                    fig22-yot22 (1-6);
            f32a BY bru32
                    fig32-yot32 (1-6);
            f42a BY bru42
                    fig42-yot42 (1-6);
```


Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance

```
TITLE:      Multiple indicator CFA with factor loading and intercept
            incariance
.
.
.
MODEL:      f12a BY bru12
                    fig12-yot12 (1-6);
            f22a BY bru22
                    fig22-yot22 (1-6);
            f32a BY bru32
                    fig32-yot32 (1-6);
            f42a BY bru42
                    fig42-yot42 (1-6);
```

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance (Continued)

```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32 hot42] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22 stu32 stu42] (11);
[tcl12 tcl22 tcl32 tcl42] (12);
[yot12 yot22 yot32 yot42] (13);

[f12a@0 f22a f32a f42a];
```


Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance And Partial Intercept Invariance

```
TITLE:      Multiple indicator CFA with factor loading and partial
            intercept invariance

MODEL:      f12a BY bru12
                    fig12-yot12 (1-6);
            f22a BY bru22
                    fig22-yot22 (1-6);
            f32a BY bru32
                    fig32-yot32 (1-6);
            f42a BY bru42
                    fig42-yot42 (1-6);
```

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading Invariance And Partial Intercept Invariance (Continued)

```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32      ] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22            ] (11);
[tcl12 tcl22 tcl32      ] (12);
[yot12 yot22 yot32 yot42] (13);

[f12a@0 f22a f32a f42a];
```


Summary of Analysis Results For TOCA Measurement Invariance Models

Model	Chi-Square (d.f.)	Difference (d.f. diff.)
Measurement non-invariance	567.08 (344)	
Factor loading invariance	581.29 (362)	14.21 (18)
Factor loading and intercept invariance	654.59 (380)	73.30* (18)
Factor loading and partial intercept invariance	606.97 (376)	25.68* (14)
Factor loading and partial intercept invariance with a linear growth structure	614.74 (381)	7.77 (5)

Summary of Analysis Results For TOCA Measurement Invariance Models (Continued)

Explanation of Chi-Square Differences

Factor loading invariance (18)	6 factor loadings instead of 24
Factor loading and intercept invariance (18)	7 intercepts plus 3 factor means instead of 28 intercepts
Factor loading and partial intercept invariance (14)	4 additional intercepts
Factor loading and partial intercept invariance with a linear growth structure (5)	1 growth factor mean instead of 3 factor means 2 growth factor variances, 1 growth factor covariance, 4 factor residual variances instead of 10 factor variances/covariances

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure

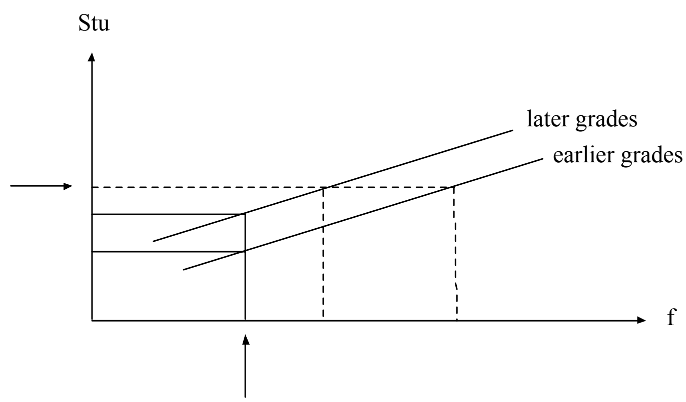
```
MODEL:      f12a BY bru12
                   fig12-yot12 (1-6);
            f22a BY bru22
                   fig22-yot22 (1-6);
            f32a BY bru32
                   fig32-yot32 (1-6);
            f42a BY bru42
                   fig42-yot42 (1-6);
```

Input Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure (Continued)

```
[bru12 bru22 bru32 bru42] (7);
[fig12 fig22 fig32 fig42] (8);
[hot12 hot22 hot32      ] (9);
[lie12 lie22 lie32 lie42] (10);
[stu12 stu22            ] (11);
[tcl12 tcl22 tcl32      ] (12);
[yot12 yot22 yot32 yot42] (13);

i s | f12a@0 f22a@1 f32a@2 f42a@3;



Alternative language:

i BY f12a-f42a@1;
s BY f12a@0 f22a@1 f32a@2 f42a@3;
[f12a-f42a@0 i@0 s];
```

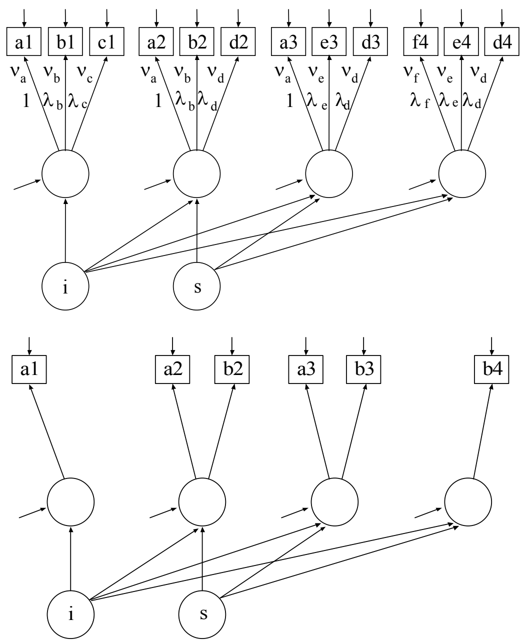

Output Excerpts For TOCA Data Multiple Indicator CFA With Factor Loading And Intercept Invariance With A Linear Growth Structure

	Estimates	S.E.	Est./S.E.	Std	StdYX
F12A |					
BRU12	1.000	.000	.000	.190	.786
FIG12	1.097	.039	28.425	.208	.868
HOT12	.986	.037	26.586	.187	.811
LIE12	.967	.041	23.769	.184	.742
STU12	.880	.041	21.393	.167	.667
TCL12	1.034	.039	26.206	.196	.786
YOT12	.932	.039	23.647	.177	.709
Intercepts					
STU12	.331	.013	25.408	.331	1.324
STU22	.331	.013	25.408	.331	1.231
STU32	.417	.017	24.345	.417	1.592
STU42	.390	.017	23.265	.390	1.496



Degrees Of Invariance Across Time

- Case 1
  - Same items
  - All items invariant
  - Same construct
- Case 2
  - Same items
  - Some items non-invariant
  - Same construct
- Case 3
  - Different items
  - Some items invariant
  - Same construct
- Case 4
  - Different items
  - Some items invariant
  - Different construct

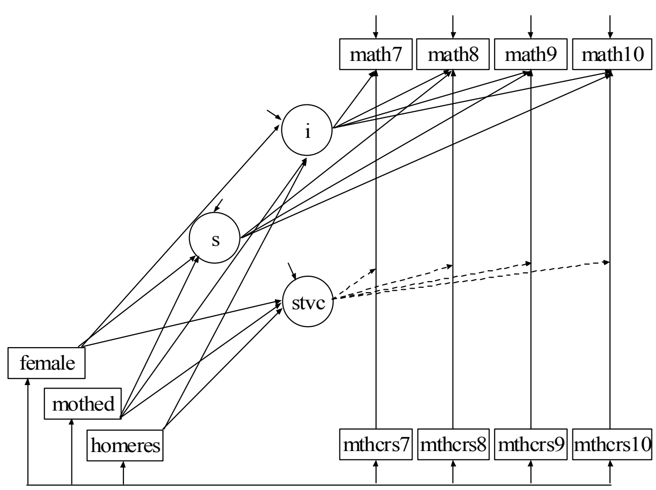


Embedded Growth Models

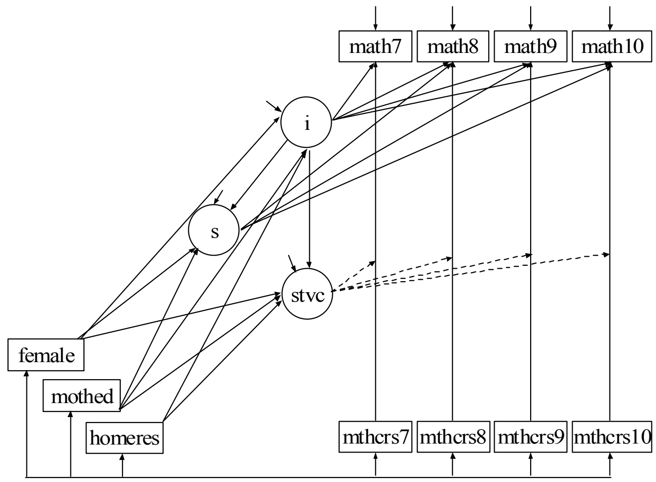
Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- Multiple populations
- Multiple indicators
- **Embedded growth models**
- Categorical latent variables: growth mixtures

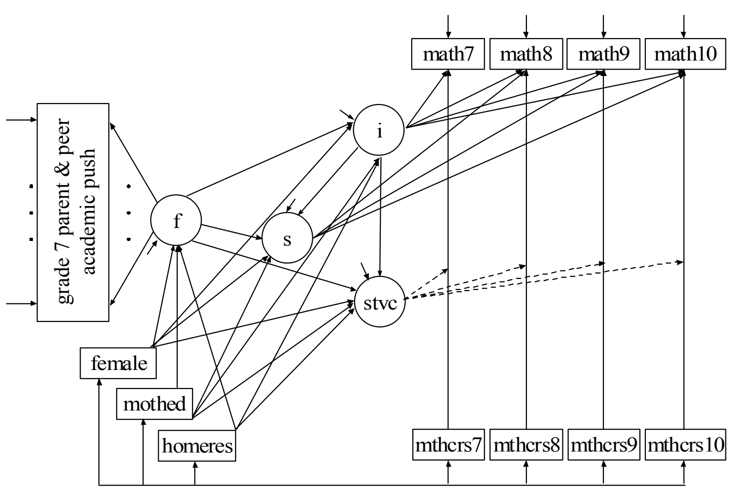
Growth Modeling With Time-Varying Covariates



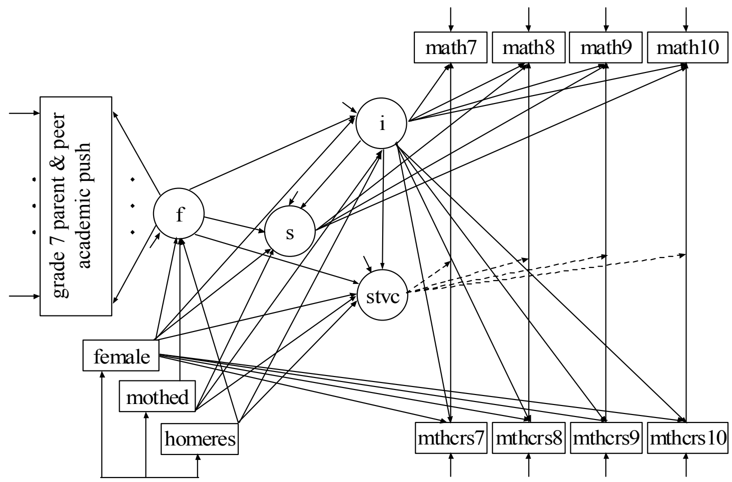
A Generalized Growth Model



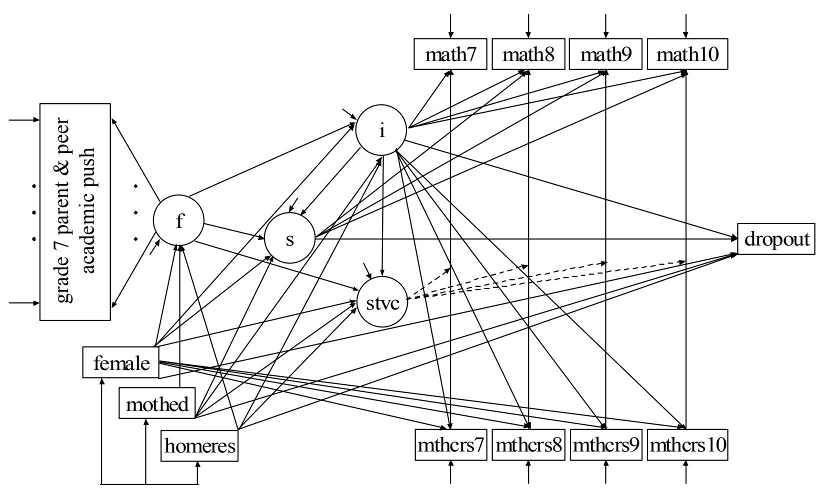
A Generalized Growth Model



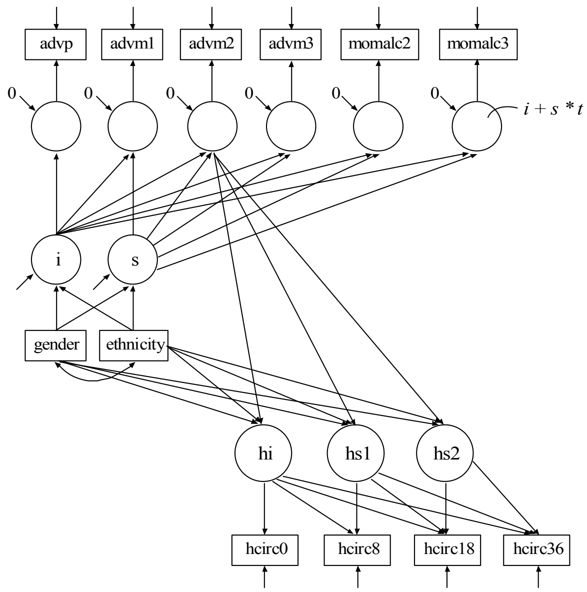
A Generalized Growth Model



A Generalized Growth Model



Two Linked Processes



Input Excerpts For Two Linked Processes With Measurement Error In The Covariates

```
TITLE:     Embedded growth model with measurement error in the
           covariates and sequential processes
           advp: mother's drinking before pregnancy
           advm1-advm3: drinking in first trimester
           momalc2-momalc3: drinking in 2nd and 3rd trimesters
           hcirc0-hcirc36; head circumference
MODEL:     fadvp    BY  advp;         fadvp@0;
           fadvm1   BY  advm1;       fadvm1@0;
           fadvm2   BY  advm2;       fadvm2@0;
           fadvm3   BY  advm3;       fadvm3@0;
           fmomalc2 BY  momalc2;   fmomalc2@0;
           fmomalc3 BY  momalc3;   fmomalc3@0;
           i BY fadvp-fmomalc3@1;
           s BY fadvp@0  fadvm1@1  fadvm2*2  fadvm3*3
                  fmomalc2-fmomalc3*5 (1);
           [advp-momalc3@0 fadvp-fmomalc3@0 i s];
```

Input Excerpts For Two Linked Processes With Measurement Error In The Covariates (Continued)

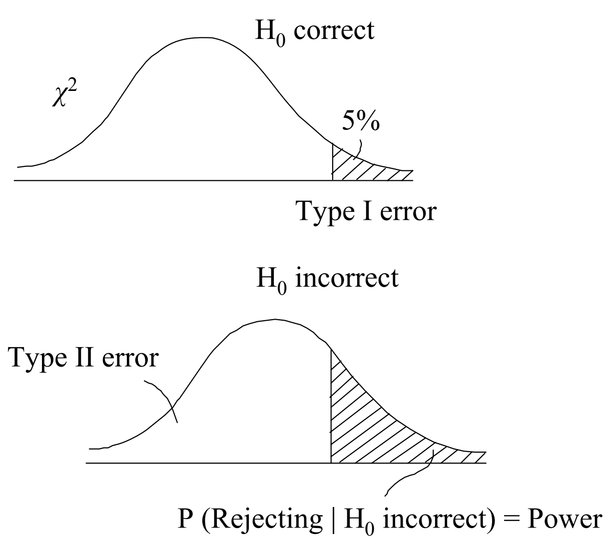
```
advp WITH advm1; advm1 WITH advm2; advm3 WITH advm2;

i s ON gender eth; s WITH i;


hi   BY  hcirc0-hcirc36@1;
hs1  BY  hcirc0@0 hcirc8@1.196 hcirc36@1.196 hcirc36@1.196;
hs2  BY  hcirc0@0 hcirc8@0 hcirc18@1 hcirc36*2;

[hcirc0-hcirc36@0 hi*34 hs1 hs2];

hs1 WITH hs2@0; hi WITH hs2@0; hi WITH hs1@0;
hi  WITH i@0; hi   WITH s@0; hs1  WITH i@0;
hi1 WITH s@0; hs2  WITH i@0; hs2  WITH s@0;

hi-hs2 ON gender eth fadvm2;
```


Power For Growth Models

Designing Future Studies: Power

- Computing power for growth models using Satorra-Saris (Muthén & Curran, 1997; examples)
- Computing power using Monte Carlo studies (Muthén & Muthén, 2002)
- Power calculation web site – PSMG
- Multilevel power (Miyazaki & Raudenbush, 2000; Moerbeek, Breukelen & Berger, 2000; Raudenbush, 1997; Raudenbush & Liu, 2000)
- School-based studies (Brown & Liao, 1999: Principles for designing randomized preventive trials)
- Multiple- (sequential-) cohort power
- Designs for follow-up (Brown, Indurkhia, & Kellam, 2000)

Designing Future Studies: Power



Power Estimation For Growth Models Using Satorra & Saris (1985)

- Step 1: Create mean vector and covariance matrix for hypothesized parameter values
- Step 2: Analyze as if sample statistics and check that parameter values are recovered
- Step 3: Analyze as if sample statistics, misspecifying the model by fixing treatment effect(s) at zero
- Step 4: Use printed x^2 as an appropriate noncentrality parameter and computer power.

Muthén & Curran (1997): Artificial and real data situations.

Input For Step 1 Of Power Calculation

```
TITLE:     Power calculation for a growth model
           Step 1: Computing the population means and
           covariance matrix
DATA:      FILE IS artific.dat;
           TYPE IS MEANS COVARIANCE;
           NOBSERVATIONS = 500;
VARIABLE:  NAMES ARE y1-y4;
MODEL:     i s | y1@0 y2@1 y3@2 y4@3;
           i@.5;
           s@.1;
           i WITH s@0;
           y1-y4@.5;
OUTPUT:    STANDARDIZED RESIUDAL;
```

Data For Step 1 Of Power Calculation (Continued)

```
0 0 0 0
1
0 1
0 0 1
0 0 0 1
```


Input For Step 2 Of Power Calculation

```
TITLE:     Power calculation for a growth model
           Step 2: Analyzing the population means and
           covariance matrix to check that parameters are
           recovered
DATA:      FILE IS pop.dat;
           TYPE IS MEANS COVARIANCE;
           NOBSERVATIONS = 500;
VARIABLE:  NAMES ARE y1-y4;
MODEL:     i s | y1@0 y2@1 y3@2 y4@3;
OUTPUT:    STANDARDIZED RESIUDAL;
```

Data For Step 2
Of Power Calculation (Continued)

Data From Step 1 Residual Output

```
0   .2   .4   .6
1
.5   1.1
.5   .7   1.4
.5   .8   1.1   1.9
```


Input For Step 3
Of Power Calculation

```
TITLE:      Power calculation for a growth model
            Step 3: Analyzing the population means and
            covariance matrix with a misspecified model
DATA:       FILE IS pop.dat;
            TYPE IS MEANS COVARIANCE;
            NOBSERVATIONS = 50;
VARIABLE:   NAMES ARE y1-y4;
MODEL:      i s | y1@0 y2@1 y3@2 y4@3;
OUTPUT:     STANDARDIZED RESIUDAL;
```

Step 4 Of Power Calculation

Output Excerpt From Step 3

```
Chi-Square Test of Model Fit

          Value                        9.286
          Degrees of Freedom               6
          P-Value                      .1580
```

Power Algorithm in SAS

```
DATA POWER;
DF=1; CRIT=3.841459;
LAMBDA=9.286;
Power=(1 - (PROBCHI(CRIT, DF, LAMBDA)));
RUN;
```


Step 4 Of Power Calculation (Continued)

Results From Power Algorithm

SAMPLE SIZE	POWER
44	0.80
50	0.85
100	0.98
200	0.99

Note: **Non-centrality parameter = printed chi-square value from Step 3 = 2*sample size*F**

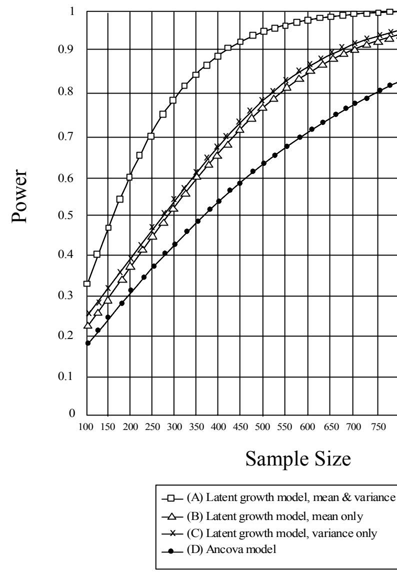


Figure 6. Power to detect a main effect of *ES* = .20 assessed at Time 5.

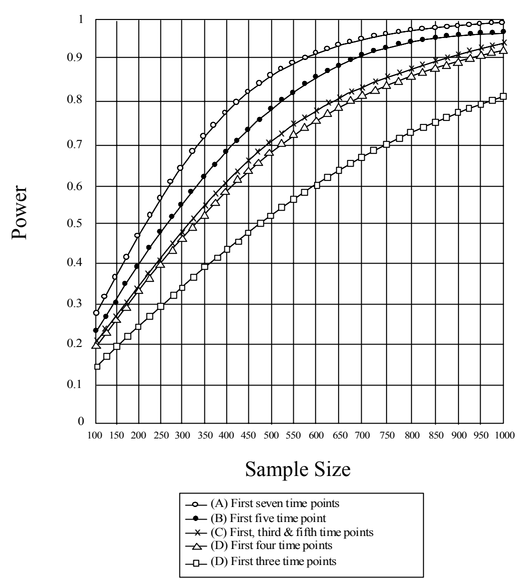


Figure 7. Power to detect a main effect of *ES* = .20 assessed at Time 5 varying as a function of total number of measurement occasions.

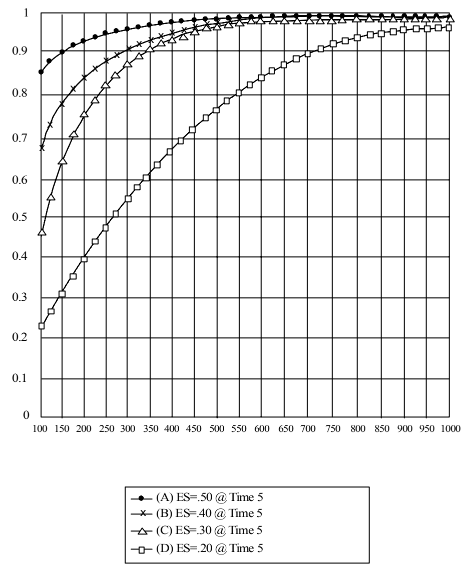


Figure 8. Power to detect various effect sizes assessed at Time 5 based on the first five measurement occasions

Power Estimation For Growth Models Using Monte Carlo Studies

Muthén, L.K. and Muthén, B.O. (2002). How to use a Monte Carlo study to decide on sample size and determine power. Structural Equation Modeling, 4, 599-620.

Input Power Estimation For Growth Models Using Monte Carlo Studies

```
TITLE:        This is an example of a Monte Carlo
              simulation study for a linear growth model
              for a continuous outcome with missing data
              where attrition is predicted by time-
              invariant covariates (MAR)

MONTECARLO:   NAMES ARE y1-y4 x1 x2;
              NOBSERVATIONS = 500;
              NREPS = 500;
              SEED = 4533;
              CUTPOINTS = x2(1);
              MISSING = y1-y4;
```


Input Power Estimation For Growth Models Using Monte Carlo Studies (Continued)

```
MODEL POPULATION:    x1-x2@1;
                     [x1-x2@0];
                     i s | y1@0 y2@1 y3@2 y4@3;
                     [i*1 s*2];
                     i*1; s*.2; i WITH s*.1;
                     y1-y4*.5;
                     i ON x1*1 x2*.5;
                     s ON x1*.4 x2*.25;

MODEL MISSING:       [y1-y4@-1];
                     y1 ON x1*.4 x2*.2;
                     y2 ON x1*.8 x2*.4;
                     y3 ON x1*1.6 x2*.8;
                     y4 ON x1*3.2 x2*1.6;
```

Input Power Estimation For Growth Models Using Monte Carlo Studies (Continued)

```
MODEL:        i s | y1@0 y2@1 y3@2 y4@3;
              [i*1 s*2];
              i*1; s*.2; i WITH s*.1;
              y1-y4*.5;
              i ON x1*1 x2*.5;
              s ON x1*.4 x2*.25;
OUTPUT:       TECH9;
```


Output Excerpts Power Estimation For Growth Models Using Monte Carlo Studies

Model Results

	Population	ESTIMATES Average	Std. Dev.	S.E. Average	M. S. E.	95% Cover	%Sig Coeff
I ON							
X1	1.000	1.0032	0.0598	0.0579	0.0036	0.936	1.000
X2	0.500	0.5076	0.1554	0.1570	0.0241	0.952	0.908
S ON							
X1	0.400	0.3980	0.0366	0.0349	0.0013	0.936	1.000
X2	0.250	0.2469	0.0865	0.0877	0.0075	0.938	0.830

Cohort-Sequential Designs and Power

Considerations:

- Model identification
- Number of timepoints needed substantively
- Number of years of the study
- Number of cohorts: More gives longer timespan but greater risk of cohort differences
- Number of measurements per individual
- Number of individuals per cohort
- Number of individuals per age

Tentative conclusion:

Power most influenced by total timespan, not the number of measures per cohort

Survival Analysis

Survival Analysis

- Discrete-time
  - Infrequent measurement (monthly, annually)
  - Limited number of time periods
- Continuous-time
  - Frequent measurement (hourly, daily)
  - Large number of time points

Discrete-Time Survival Analysis

Discrete-Time Survival Analysis

Other terms: event history analysis, time-to-event.

References: Allison (1984), Singer & Willet (1993), Vermunt (1997).

- Setting
  - Discrete time periods (e.g. grade), non-repeatable event (e.g. onset of drug use)
  - Uncensored and censored individuals
  - Time-invariant and time-varying covariates
- Aim
  - Estimate survival curves and hazard probabilities
  - Relate survival to covariates
- Generalized models using multiple latent classes of survival
  - Long-term survivors with zero hazard
  - Growth mixture modeling in combination with survival analysis
- Application: School removal and aggressive behavior in the classroom (Muthén & Masyn, 2005). Grade 1 sample, n = 403 control group children.

Data For Discrete-Time Survival Analysis

- Single non-repeatable event – data collection ends for individual i when the event has been observed, where j_i is the last time period of data collection for individual i
- u_j (j = 1, 2,…, j_i) are binary 0/1 event history indicators, where u_{ij} = 1 if individual i experiences an event in time period j

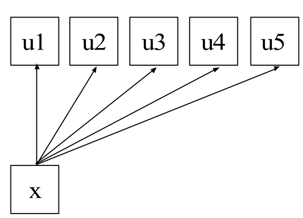
Event history information entered into an r x 1 data vector $\mathbf{u}_i$ where r denotes the maximum value of j_i over all individuals and where u = 999 denotes missing data.

Data For Discrete-Time Survival Analysis (Continued)

Examples:

- An individual who is censored after time period five ($j_i = 6$)
$$(0\ \ 0\ \ 0\ \ 0\ \ 0),$$
- An individual who experiences the event in period four ($j_i = 4$)
$$(0\ \ 0\ \ 0\ \ 1\ \ 999),$$
- An individual who drops out after period three, i.e. is censored during period four before the study ends ($j_i = 4$)
$$(0\ \ 0\ \ 0\ \ 999\ \ 999).$$

Model For Discrete-Time Survival Analysis



Hazard, Survival, Likelihood

The hazard is the probability of experiencing the event in the time period j given that it was not experienced prior to j. Letting the time of the event for individual i be denoted T_i, the logistic hazard function with q covariates x is

$$P\,(u_{ij} = 1) = P\,(T_i = j \mid T_i \geq j\,) = h_{ij} = \frac{1}{1 + e^{-(-\tau_j + \kappa_j x_i)}}\,, \qquad (49)$$

where a proportional-odds assumption is obtained by dropping the j subscript for κ_j. The survival function is

$$S_{ij} = \prod_{k=1}^{j} (1 - h_{ik}). \qquad (50)$$

Hazard, Survival, Likelihood (Continued)

The likelihood $L = \prod_{i=1}^{n} l_i$, where

$$l_i = \prod_{j=1}^{j_i} h_{ij}^{u_{ij}}\, (1 - h_{ij})^{\;1-u_{ij}}. \qquad (51)$$

A censored individual is observed with probability

$$l_i = \prod_{j=1}^{j_i} (1 - h_{ij}). \qquad (52)$$

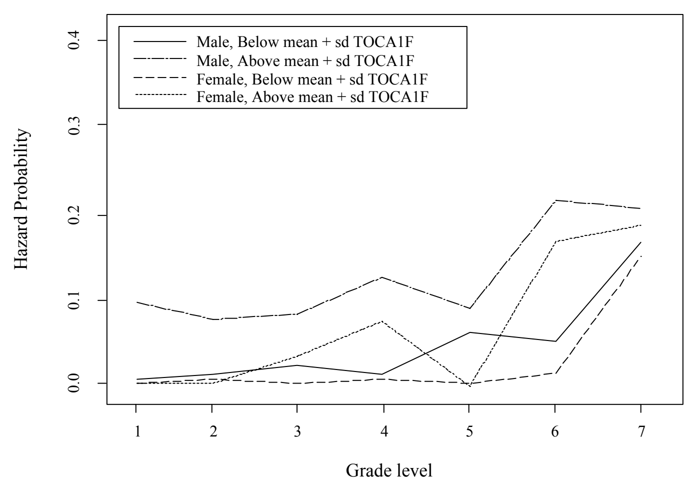
An uncensored individual experiences the event in time period j_i with probability

$$l_i = h_{ij_i} \prod_{j=1}^{j_i - 1} (1 - h_{ij}). \qquad (53)$$

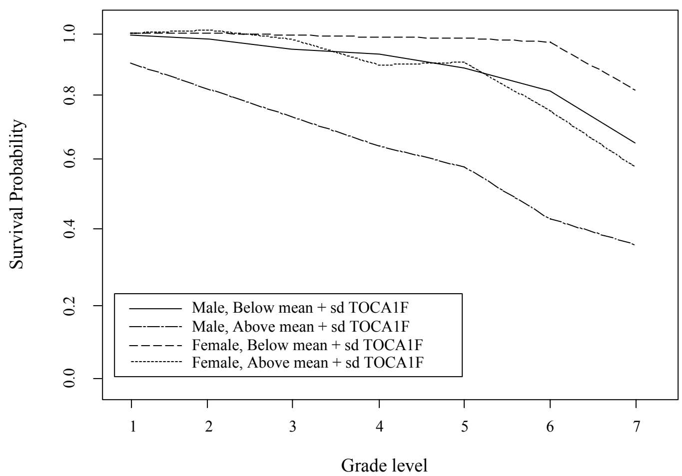
School Removal Data (n = 403)

Gender	Grade	No School removal	At least one school removal	Sample Hazard
Male	1	0	4	4/200 = 0.02
	2	0	5	5/196 = 0.03
	3	0	7	7/191 = 0.04
	4	0	6	6/184 = 0.03
	5	0	14	14/178 = 0.08
	6	0	14	14/164 = 0.08
	7	122	28	28/136 = 0.19
	Total	122	78	200
Female	1	0	0	0/203 = 0.00
	2	0	1	1/203 = 0.005
	3	0	1	1/202 = 0.005
	4	0	3	3/201 = 0.01
	5	0	1	1/198 = 0.005
	6	0	9	9/197 = 0.05
	7	157	31	31/188 = 0.16
	Total	157	46	203

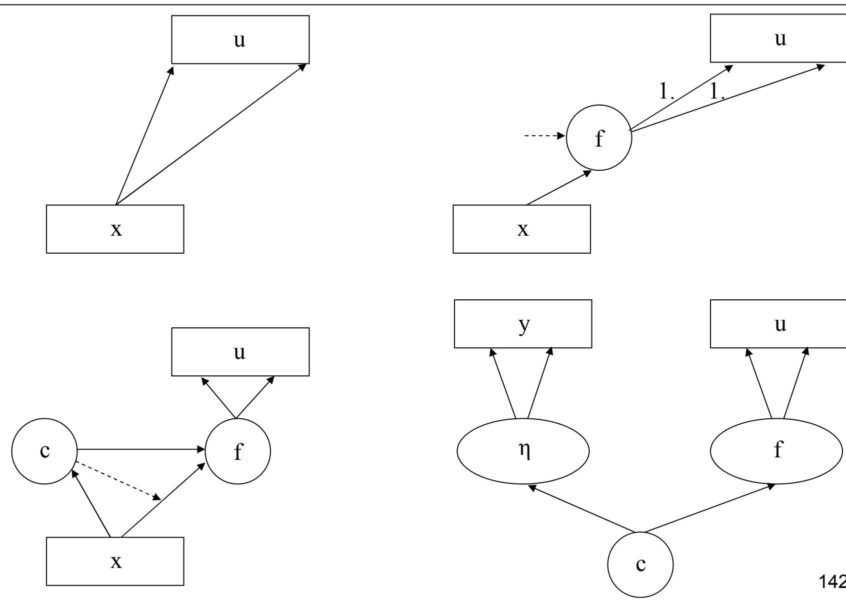
Hazard Sample Estimates



Survival Sample Estimates



Discrete-Time Survival Models



Input For A Discrete-Time Survival Analysis

```
TITLE:        A discrete-time survival analysis
DATA:         FILE IS survival.dat;
VARIABLE:     NAMES ARE u1-u7 race lunch cavtoca cavlunch
              cntrlg y1 gender;
              MISSING are all (999);
              CATEGORICAL ARE u1-u7;
MODEL:        f BY u1-u7@1;
              f ON race-gender;
              f@0;
OUTPUT:       TECH1 TECH8;
```


Output Excerpts
A Discrete-Time Survival Analysis

Tests Of Model Fit

```
Loglikelihood
              H0 Value                          -388.074

Information Criteria
              Number of Free Parameters              14
              Akaike (AIC)                      804.147
              Bayesian (BIC)                    860.132
              Sample-Sized Adjusted BIC         815.709
                (n* = (n + 2)/24)
```

Output Excerpts
A Discrete-Time Survival Analysis (Continued)

Model Results

```
                    Estimates       S.E.     Est./S.E.
   Thresholds
      U1$1              4.707      0.694        6.782
      U2$1              4.118      0.782        5.269
      U3$1              3.764      0.658        5.725
      U4$1              3.588      0.648        5.537
      U5$1              2.958      0.677        4.371
      U6$1              2.382      0.625        3.809
      U7$1              1.048      0.609        1.721

   F          ON
      RACE             -0.449      0.379       -1.183
      LUNCH            -0.136      0.268       -0.506
      CAVTOCA          -1.104      0.295       -3.738
      CAVLUNCH          1.571      0.476        3.302
      CNTRLG           -0.336      0.213       -1.578
      Y1                0.783      0.119        6.566
      GENDER           -0.700      0.206       -3.402
```


Multiple Latent Classes

Muthén & Masyn (2005)

Unobserved heterogeneity in hazard and survival

- Long-term survivors (one class has zero hazards, non-zero long-term survival probability)
- Latent classes of survival
- Growth mixtures and survival

Example: Long-term survivors

Individuals who are not censored, i.e. who experience the event within the observation period, are not long-term survivors (known latent class membership).

Two different latent classes of censored individuals:

$$\textit{Eventually experiences the event} : 0 \ldots 0 | 0 \ldots 0\ 1$$
$$\textit{Long-term survivor} : 0 \ldots 0 | 0 \ldots 0$$

Input For A Two-Class Discrete-Time Survival Analysis

```
TITLE:      A 2-class discrete-time survival analysis in a
            mixture modeling framework including long-term
            survivors
DATA:       FILE IS long.sav;
VARIABLE:   NAMES ARE u1-u7 race lunch cavtoca cavlunch
            cntrlg y1 gender t1 t2;

            MISSING ARE ALL (999);

            CATEGORICAL ARE u1-u7;
            CLASSES = c(2);

            TRAINING = t1 t2;
ANALYSIS:   TYPE = MIXTURE;
```


Input For A Two-Class Discrete-Time Survival Analysis (Continued)

```
MODEL:      %OVERALL%

            f BY u1-u7@1;
            f ON race-gender;
            [f@0];
            c#1 ON race-gender;

            %c#1% ! class of non-long-term survivors

            [u1$1*4 u2$1*3 u3$1*3 u4$1*3.5 u5$1*2.5 u6$1*4 u7$1*1];
            [f@0];
            f ON race-gender;

            %c#2% ! class of long-term survivors

            [u1$1-u7$1@10];
            f ON race-gender@0;
OUTPUT:     PATTERNS TECH1 TECH8;
```

Output Excerpts A Two-Class Discrete-Time Survival Analysis

Tests Of Model Fit

```
Loglikelihood
             H0 Value                            -375.951

Information Criteria
             Number of Free Parameters                 22
             Akaike (AIC)                         795.903
             Bayesian (BIC)                       883.879
             Sample-Sized Adjusted BIC            814.071
               (n* = (n + 2)/24)
             Entropy                                0.644
```


Output Excerpts A Two-Class Discrete-Time Survival Analysis (Continued)

Classification Information

```
FINAL CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE

  Class 1          201.96062          0.50114
  Class 2          201.03938          0.49886

CLASSIFICATION OF INDIVIDUALS BASED ON THEIR MOST LIKELY CLASS
MEMBERSHIP

Class Counts and Proportions

  Class 1                190          0.47146
  Class 2                213          0.52854

Average Latent Class Probabilities for Most Likely Latent Class
Membership (Row) by Latent Class (Column)

                1            2

  Class 1      0.916      0.084
  Class 2      0.131      0.869
```

Output Excerpts A Two-Class Discrete-Time Survival Analysis (Continued)

Model Results

```
                    Estimates     S.E.     Est./S.E.

 Class 1

 Thresholds
    U1$1               4.590      0.809        5.673
    U2$1               4.060      0.928        4.376
    U3$1               3.654      0.804        4.542
    U4$1               3.402      0.756        4.498
    U5$1               2.656      0.764        3.477
    U6$1               1.866      0.708        2.634
    U7$1              -0.026      0.841       -0.030
```


Output Excerpts A Two-Class Discrete-Time Survival Analysis (Continued)

```
 C#1          ON
    RACE              -1.099      0.552       -1.990
    LUNCH              0.446      0.712        0.627
    CAVTOCA           -2.459      0.824       -2.983
    CAVLUNCH           2.907      1.470        1.977
    CNTRLG            -0.101      0.498       -0.204
    Y1                 0.913      0.301        3.036
    GENDER             0.150      0.665        0.226
 Intercepts
    C#1                1.773       1.411       1.257

 Class 1
 F            ON
    RACE               0.882      0.511        1.728
    LUNCH             -0.679      0.550       -1.236
    CAVTOCA           -0.234      0.642       -0.365
    CAVLUNCH           0.540      0.809        0.667
    CNTRLG            -0.441      0.355       -1.242
    Y1                 0.605      0.218        2.774
    GENDER            -1.141      0.510       -2.237
```

Further Readings On Discrete-Time Survival Analysis

Allison, P.D. (1984). Event history analysis. Regression for Longitudinal Event Data. Quantitative Applications in the Social Sciences, No. 46. Thousand Oaks: Sage Publications.

Masyn, K. E. (2008). Modeling measurement error in event occurrence for single, non-recurring events in discrete-time survival analysis. In Hancock, G. R., & Samuelsen, K. M. (Eds.), Advances in latent variable mixture models, pp. 105-145. Charlotte, NC: Information Age Publishing, Inc.

Muthén, B. & Masyn, K. (2005). Discrete-time survival mixture analysis. Journal of Educational and Behavioral Statistics, 30, 27-28.

Singer, J.D., & Willett, J.B. (1993). It's about time: Using discrete-time survival analysis to study duration and the timing of events. Journal of Educational Statistics, 18(2), 155-195.

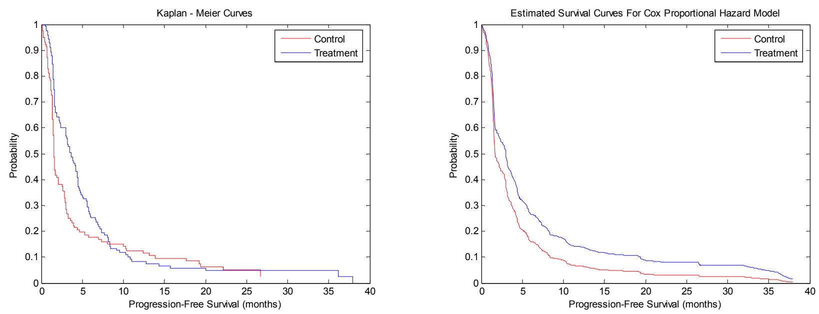
Singer, J. D. & Willett, J. B. (2003). Applied longitudinal analysis.

Modeling change and event occurrence. Oxford, UK: Oxford University Press.

Vermunt, J.K. (1997). Log-linear models for event histories. Advanced quantitative techniques in the social sciences, vol 8. Thousand Oaks: Sage Publications.

Continuous-Time Survival Analysis

Survival Curves For Kaplan-Meier Versus Cox Proportional Hazard Model



Continuous-Time Survival Analysis

- T_0: time-to-event such as time to death
- I: time of censoring
- The survival variable T and the censoring indicator c are defined by

$$T = \min\{T_0, I\} \tag{1}$$

$$c = \begin{cases} 1 & \text{if } T_0 > I \\ 0 & \text{if } T_0 \leq I \end{cases} \tag{2}$$

For example, $c = 1$ implies $T = I$, the time the individual leaves the sample

The Proportional Hazard Model

The proportional hazard (PH) model specifies that the hazard function is proportional to the baseline hazard function,

$$h(t) = \lambda(t)\, Exp(\beta X) \qquad (3)$$

Two proportional hazard models:

- Nonparametric shape for the baseline hazard function $\lambda(t)$: Cox regression
- Parametric model for the baseline hazard function $\lambda(t)$: parametric PH model

Example 6.21: Continuous-Time Survival Analysis Using The Cox Regression Model

```
TITLE:     this is an example of a continuous-time survival
           analysis using the Cox regression model
DATA:      FILE = ex.6.21.dat;
VARIABLE:  NAMES = t x tc;
           SURVIVAL = t (ALL);
           TIMECENSORED = tc (0 = NOT 1 = RIGHT);
ANALYSIS:  BASEHAZARD = OFF;
MODEL:     t ON x;
```

Continuous-Time Survival Data

t	x	c
7.330493	-0.378137	1.000000
0.894182	-0.880031	1.000000
1.219113	0.369423	0.000000
0.134073	1.886903	0.000000
0.598567	1.118025	0.000000
0.725646	0.642068	0.000000
1.637967	-0.324017	0.000000
5.534057	-0.760867	0.000000
3.316749	0.194822	1.000000
4.176435	-0.311791	1.000000

Event occurred at time 1.219113

Translating Continuous-Time Survival Data To Discrete-Time Survival Data

```
VARIABLE:  NAMES = t x c;
           ! t =time of death or censoring
           ! c = not censored (0), censored (1)
           CATEGORICAL = u1-u8(*);
           USEVAR = x u1-u8;
DATA:      FILE = surveq1.dat;
           VARIANCE = NOCHECK;
DEFINE:    IF (t>2) THEN u1=0;
           IF ((t>0) .AND. (t<2)) THEN u1=1-c;
           ! u1 = 0 if c = 1, i.e. censoring time between 0 and 2
           ! u1 = 1 if person died then

           IF (t>4) THEN u2=0;
           IF ((t>2) .AND. (t<4)) THEN u2=1-c;
           IF (t<2) THEN u2=_missing;
           ! u2 is missing either because u1 = 1 or because
           ! u1 = 0 and c = 1
```

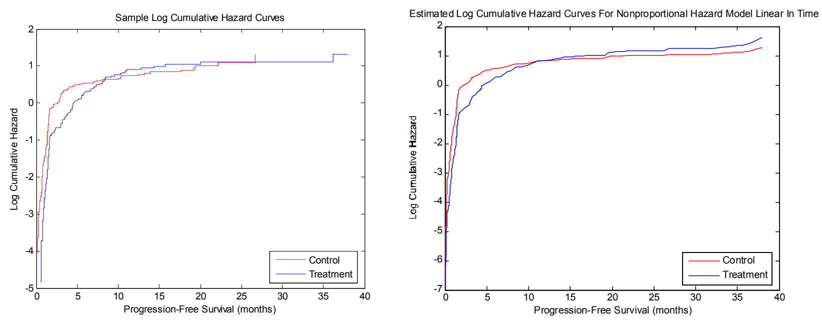
Translating Continuous-Time Survival Data To Discrete-Time Survival Data (Continued)

```
IF (t>6) THEN u3=0;
IF ((t>4) .AND. (t<6)) THEN u3=1-c;
IF (t<4) THEN u3=_missing;

IF (t>8) THEN u4=0;
IF ((t>6) .AND. (t<8)) THEN u4=1-c;
IF (t<6) THEN u4=_missing;

IF (t>10) THEN u5=0;
IF ((t>8) .AND. (t<10)) THEN u5=1-c;
IF (t<8) THEN u5=_missing;

IF (t>12) THEN u6=0;
IF ((t>10) .AND. (t<12)) THEN u6=1-c;
IF (t<10) THEN u6=_missing;
```


Translating Continuous-Time Survival Data To Discrete-Time Survival Data (Continued)

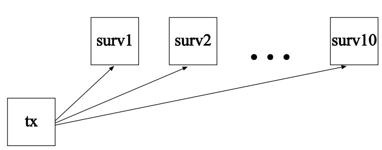
```
            IF (t>14) THEN u7=0;
            IF ((t>12) .AND. (t<14)) THEN u7=1-c;
            IF (t<12) THEN u7=_missing;

            IF (t>16) THEN u8=0;
            IF ((t>14) .AND. (t<16)) THEN u8=1-c;
            IF (t<14) THEN u8=_missing;
MODEL:      u1-u8 ON x*1 (1);
ANALYSIS:   ESTIMATOR = MLR;
            TYPE = MISSING;
```

Log Cumulative Hazard Curves For Kaplan-Meier Versus Nonproportional Hazard Model

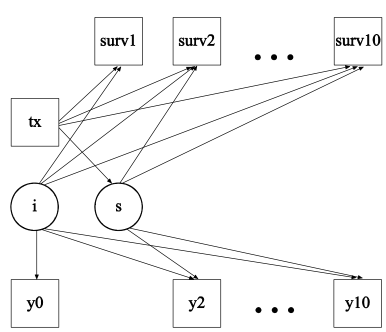


Survival Analysis With Non-Proportional Hazard Models

Non-proportional hazard modeling



Growth and non-proportional hazard modeling



Further Readings For Continuous-Time Survival Analysis

Asparouhov, T., Masyn, K. & Muthen, B. (2006). Continuous time survival in latent variable models. Proceedings of the Joint Statistical Meeting in Seattle, August 2006. ASA section on Biometrics, 180-187.

Larsen, K. (2004). Joint analysis of time-to-event and multiple binary indicators of latent classes, Biometrics, 60(1), 85–92.

Larsen, K. (2005). The Cox proportional hazards model with a continuous latent variable measured by multiple binary indicators, Biometrics, 61(4), 1049–1055.

Singer, J.D., & Willett, J.B. (1993). It's about time: Using discrete-time survival analysis to study duration and the timing of events. Journal of Educational Statistics, 18(2), 155-195.

Singer, J. D. & Willett, J. B. (2003). Applied longitudinal analysis. Modeling change and event occurrence. Oxford, UK: Oxford University Press.

Analysis With Missing Data

Analysis With Missing Data

Used when individuals are not observed on all outcomes in the analysis to make the best use of all available data and to avoid biases in parameter estimates, standard errors, and tests of model fit.

Types of Missingness

- MCAR -- missing completely at random
  - Variables missing by chance
  - Missing by randomized design
  - Multiple cohorts assuming a single population
- MAR -- missing at random
  - Missingness related to observed variables
  - Missing by selective design
- Non-Ignorable (NMAR)
  - Missingness related to values that would have been observed
  - Missingness related to latent variables

Estimation With Missing Data

Types of Estimation (Little & Rubin, 2002)

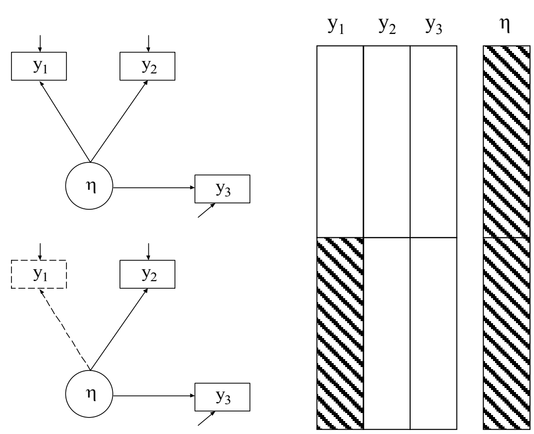
- Estimation using listwise deleted sample
  - When MCAR is true, parameter estimates and s.e.'s are consistent but estimates are not efficient
  - When MAR is true but not MCAR, parameter estimates and s.e.'s are not consistent
- Maximum likelihood using all available data
  - When MCAR or MAR is true, parameter estimates and s.e.'s are consistent and estimates are efficient
- Selection and pattern-mixture modeling – used for non-ignorable missingness
- Imputation
  - Mean and regression imputation – underestimation of variances and covariances
  - Multiple imputation using all available data – a Bayesian approach – credibility intervals are Bayesian justifiable under MCAR and MAR

Weighted Least Squares Estimation With Missing Data

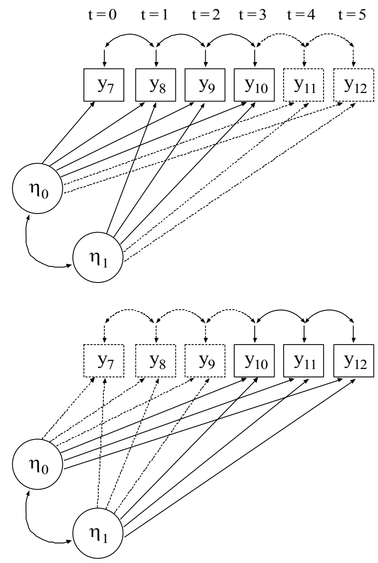
Weighted least squares for categorical and censored outcomes

- Assumes MCAR when there are no covariates
- Allows MAR when missingness is a function of covariates

MCAR: Missing By Design

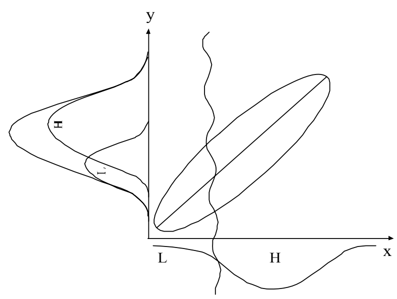


Two-Cohort Growth Model



MAR

MAR: Bivariate, Monotone Missing Case

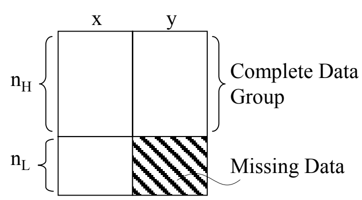


$y_i = \alpha + \beta x_i + \zeta_i$

$E(\zeta) = 0$, $V(\zeta) = \sigma_\zeta^2$

$E(x) = \mu_x$, $V(x) = \sigma_x^2$

- Data Matrix:



Missing At Random (MAR): Missing On y In Bivariate Normal Case

$$\hat{\mu}_x = \sum_{i=1}^{n_L + n_H} x_i / (n_L + n_H) = \frac{n_L \bar{x}_L + n_H \bar{x}_H}{n_L + n_H} , \tag{52}$$

$$\hat{\sigma}_{xx} = \sum_{i=1}^{n_L + n_H} (x_i - \hat{\mu}_x)^2 / (n_L + n_H). \tag{53}$$

Missing At Random (MAR): Missing On y In Bivariate Normal Case (Continued)

Consider the regression

$$y_i = \alpha + \beta\, x_i + \zeta_i \qquad (54)$$

estimated by the complete-data (listwise present) sample (sample size n_H)

$$\hat{\alpha} = \bar{y} - \hat{\beta}\,\bar{x}\,, \qquad (55)$$

$$\hat{\beta} = s_{yx} / s_{xx}\,, \qquad (56)$$

$$\hat{\sigma}_{\zeta\zeta} = s_{yy} - s^2_{yx} / s_{xx}\,. \qquad (57)$$

This gives the ML estimates of μ_y and σ_{yy}, adjusting the complete-data sample statistics:

$$\hat{\mu}_y = \hat{\alpha} + \hat{\beta}\,\hat{\mu}_x = \bar{y} + \hat{\beta}\,(\hat{\mu}_x - \bar{x}), \qquad (58)$$

$$\hat{\sigma}_{yy} = \hat{\sigma}_{\zeta\zeta} + \hat{\beta}^2\,\hat{\sigma}_{xx} = s_{yy} + \hat{\beta}^2\,(\hat{\sigma}_{xx} - s_{xx}). \qquad (59)$$

Correlates Of Missing Data

- MAR is more plausible when the model includes covariates influencing missing data
- Correlates of missing data may not have a "causal role" in the model, i.e. not influencing dependent variables, in which case including them as covariates can bias model estimates
  - Multiple imputation (Bayes; Schafer, 1997) with two different sets of observed variables
    - Imputation model
    - Analysis model
  - Modeling (ML)
    - Including missing data correlates not as x variables but as "y variables," freely correlated with all other observed variables

Recent overview in Schafer & Graham (2002).

Missing On X

- Regular modeling concerns the conditional distribution

  $[y \mid x]$ (1)

  that is, as in regular regression the marginal distribution of $[x]$ is not involved. This is fine if there is no missing on x in which case considering

  $[y \mid x]$

  gives the same estimates as (Joreskog & Goldberger, 1975) considering the joint distribution

  $[y, x] = [y \mid x]\ [x]$

Missing On X (Continued)

- With missing on x, ML under MAR must make a distributional assumption about $[x]$, typically normality. The modeling then concerns

  $[y, x] = [y \mid x]\ [x]$ (2)

  which with missing on $[x]$ is an expanded model that makes stronger assumptions as compared to (1).

- The LHS of (2) shows that y and x are treated the same - they are both "*y* variables" in Mplus terminology. This is the default in Mplus when all *y*'s are continuous. In other cases, *x*'s can be turned into "*y*'s" e.g. by the model statement

  x1-xq;

Technical Aspects Of Ignorable Missing Data: ML Under MAR

$$Likelihood: \sum_{i=1}^{n} log\,[\mathbf{y_i} \mid \mathbf{x_i}]. \qquad (87)$$

With missing data on $\mathbf{y}$, the i^{th} term of (87) expands into

$$[\mathbf{y}_i^{obs}, \mathbf{y}_i^{mis}, \mathrm{m}_i \mid \mathbf{x}_i], \qquad (88)$$

where $\mathbf{m}_i$ is a 0/1 indicator vector of the same length as $\mathbf{y}_i$.
The likelihood focuses on the observed variables,

$$[\mathbf{y}_i^{obs}, \mathbf{m}_i \mid \mathbf{x}_i] = \int [\mathbf{y}_i^{obs}, \mathbf{y}_i^{mis} \mid \mathbf{x}_i]\,[\mathbf{m}_i \mid \mathbf{y}_i^{obs}\ \mathbf{y}_i^{mis}, \mathrm{x}_i]\, d\mathbf{y}_i^{mis}, \qquad (89)$$

which, when assuming that missingness is not a function of $\mathbf{y}_i^{mis}$ (that is, assuming MAR),

Technical Aspects Of Ignorable Missing Data: ML Under MAR (Continued)

$$= \int [\mathbf{y}_i^{obs}, \mathbf{y}_i^{mis} \mid \mathbf{x}_i]\ d\mathbf{y}_i^{mis}\ \ [\mathbf{m}_i \mid \mathbf{y}_i^{obs}, \mathbf{x}_i], \qquad (90)$$

$$= [\mathbf{y}_i^{obs} \mid \mathbf{x}_i]\,[\mathbf{m}_i \mid \mathbf{y}_i^{obs}, \mathbf{x}_i]. \qquad (91)$$

With distinct parameter sets in (91), the last term can be ignored and maximization can focus on the $[\mathbf{y}_i^{obs} \mid \mathbf{x}_i]$ term. This leads to the standard MAR ignorable missing data procedure.

AMPS Data

The data are taken from the Alcohol Misuse Prevention Study (AMPS). Forty-nine schools with a total of 2,666 students participated in the study. Students were measured seven times starting in the Fall of Grade 6 and ending in the Spring of Grade 12.

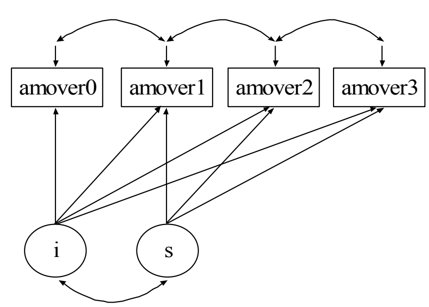
Data for the analysis include the average of three items related to alcohol misuse:

During the past 12 months, how many times did you

drink more than you planned to?
feel sick to your stomach after drinking?
get very drunk?

Responses: (0) never, (1) once, (2) two times,
(3) three or more times

Four of the seven timepoints are studied: Fall Grade 6, Spring Grade 6, Spring Grade 7, and Spring Grade 8.



Input For AMPS Growth Model With Missing Data

```
TITLE:     AMPS growth model with missing data
DATA:      FILE IS amps.dat;
VARIABLE:  NAMES ARE caseid
           amover0 ovrdrnk0 illdrnk0 vrydrn0
           amover1 ovrdrnk1 illdrnk1 vrydrn1
           amover2 ovrdrnk2 illdrnk2 vrydrn2
           amover3 ovrdrnk3 illdrnk3 vrydrn3
           amover4 ovrdrnk4 illdrnk4 vrydrn4
           amover5 ovrdrnk5 illdrnk5 vrydrn5
           amover6 ovrdrnk6 illdrnk6 vrydrn6;
           USEV = amover0 amover1 amover2 amover3;
           MISSING = ALL (999);
```


Input For AMPS Growth Model With Missing Data (Continued)

```
MODEL:     i s | amover0@0 amover1@1 amover2@3 amover3*5;
           amover1-amover3 PWITH amover0-amover2;

OUTPUT:    PATTERNS SAMPSTAT MODINDICES STANDARDIZED;
```

Output Excerpts AMPS Growth Model With Missing Data

Summary of Data

Number of patterns 15

SUMMARY OF MISSING DATA PATTERNS

MISSING DATA PATTERNS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AMOVER0	x	x	x	x	x	x	x	x							
AMOVER1	x	x	x	x					x	x	x	x			
AMOVER2	x	x			x	x			x	x			x	x	
AMOVER3	x		x		x		x		x		x		x		x

MISSING DATA PATTERN FREQIENCIES

Pattern	Frequency	Pattern	Frequency	Pattern	Frequency
1	685	6	29	11	104
2	143	7	11	12	237
3	73	8	64	13	6
4	164	9	866	14	1
5	65	10	208	15	3

Output Excerpts AMPS Growth Model With Missing Data (Continued)

COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value 0.100

PROPORTION OF DATA PRESENT

Covariance Coverage

	AMOVER0	AMOVER1	AMOVER2	AMOVER3
AMOVER0	0.464			
AMOVER1	0.401	0.933		
AMOVER2	0.347	0.715	0.753	
AMOVER3	0.314	0.650	0.610	0.682

Output Excerpts AMPS Growth Model With Missing Data (Continued)

Tests Of Model Fit

```
Chi-square Test of Model Fit

            Value                        0.011
            Degrees of Freedom               1
            P-Value                     0.9177

RMSEA (Root Mean Square Error Of Approximation)

            Estimate                     0.000
            90 Percent C.I.              0.000  0.019
            Probability RMSEA <= .05     0.997
```


Output Excerpts AMPS Growth Model With Missing Data (Continued)

Model Results

```
                Estimates    S.E.  Est./S.E.    Std     StdYX
I          |
   AMOVER0        1.000     0.000    0.000     0.426    0.921
   AMOVER1        1.000     0.000    0.000     0.426    0.774
   AMOVER2        1.000     0.000    0.000     0.426    0.645
   AMOVER3        1.000     0.000    0.000     0.426    0.529
S          |
   AMOVER0        0.000     0.000    0.000     0.000    0.000
   AMOVER1        1.000     0.000    0.000     0.109     .198
   AMOVER2        3.000     0.000    0.000     0.327    0.494
   AMOVER3        6.244     0.426   14.645     0.680    0.843
S          I
    WITH         -0.007     0.003   -2.278    -0.146   -0.146
AMOVER1  WITH
   AMOVER0       -0.022     0.011   -2.010    -0.022   -0.085
AMOVER2  WITH
   AMOVER1        0.017     0.007    2.505     0.017    0.047
AMOVER3  WITH
   AMOVER2       -0.001     0.027   -0.050    -0.001   -0.003
```

Output Excerpts AMPS Growth Model With Missing Data (Continued)

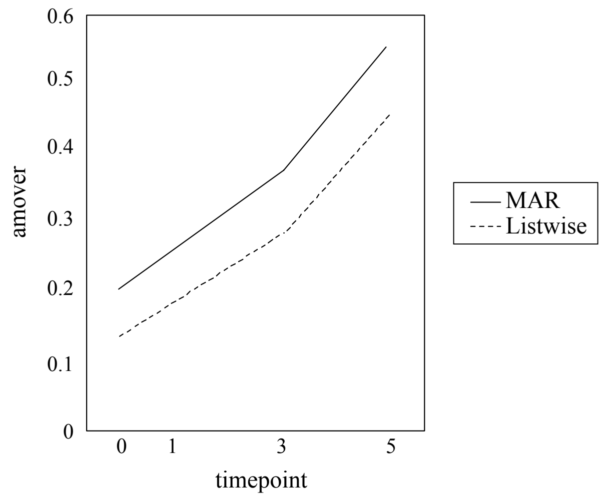
Residual Variances					
AMOVER0	0.033	0.013	2.509	0.033	0.152
AMOVER1	0.123	0.011	10.950	0.123	0.406
AMOVER2	0.190	0.017	11.461	0.190	0.433
AMOVER3	0.091	0.068	1.340	0.091	0.140
Variances					
I	0.182	0.014	12.891	1.000	1.000
S	0.012	0.002	5.378	1.000	1.000
Means					
I	0.200	0.010	19.391	0.469	0.469
S	0.057	0.005	11.858	0.520	0.520
Intercept					
AMOVER0	0.000	0.000	0.000	0.000	0.000
AMOVER1	0.000	0.000	0.000	0.000	0.000
AMOVER2	0.000	0.000	0.000	0.000	0.000
AMOVER3	0.000	0.000	0.000	0.000	0.000

Output Excerpts AMPS Growth Model With Missing Data (Continued)

R-SQUARE

Observed Variable	R-Square
AMOVER0	0.848
AMOVER1	0.594
AMOVER2	0.567
AMOVER3	0.860

AMPS: Estimated Growth Curves



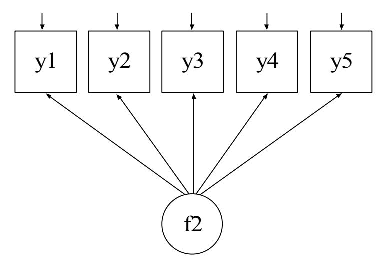
Missing Data Correlates Using ML

Missing Data Correlates

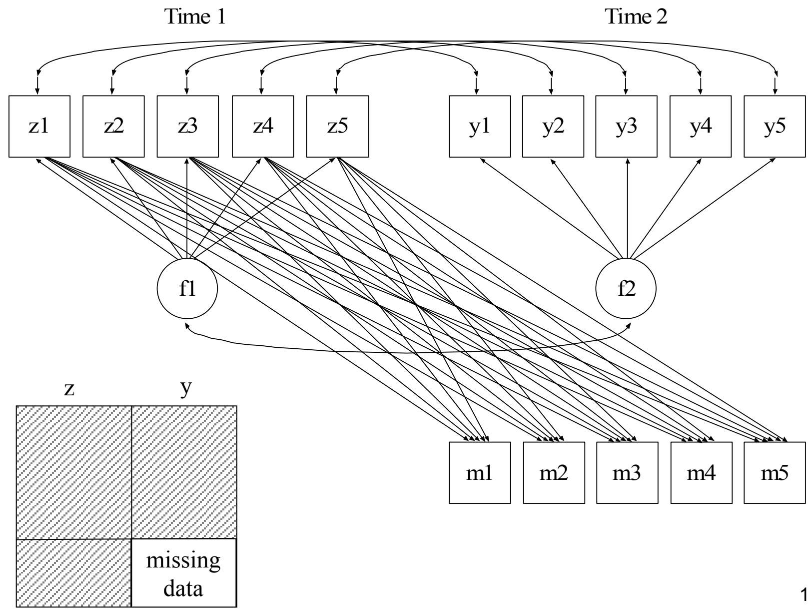
Handling missing-data-related variables that are different from analysis variables:

- Multiple imputation
- ML using missing data correlates
- References:
  - Collins, Schafer, Kam (2001) in Psych Methods
  - Graham (2003) in SEM
  - Enders and Peugh (2004) SEM
  - Savalei and Bentler (2007)

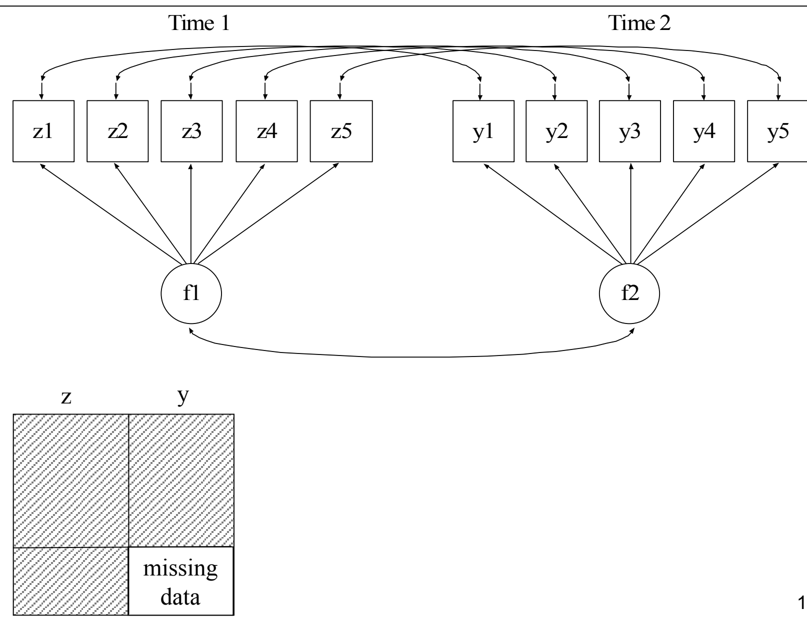
Model Of Interest



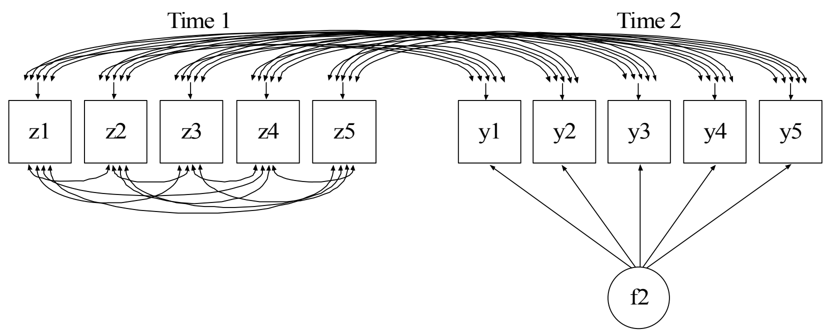
Data-Generating Model

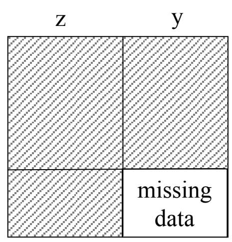


Full Analysis Model (ML Under MAR)



Simple Analysis Model (ML Under MAR)






```
VARIABLE:   NAMES = y1-y5 z1-z5;
            USEV = y1-y5;
            AUXILIARY = (M) z1-z5;
            MISSING = ALL(999);

MODEL:      f2 BY y1-y5;
```


Monte Carlo Simulation Results (N = 200)

Y with z as aux

	ESTIMATES Population	Average	Std. Dev.	S.E. Average	M.S.E.	95% Cover
F2 BY						
Y1	0.7000	0.6914	0.0820	0.0766	0.0067	0.9400
Y2	0.7000	0.6854	0.0829	0.0766	0.0070	0.9300
Y3	0.7000	0.6947	0.0650	0.0767	0.0042	0.9600
Y4	0.7000	0.6934	0.0719	0.0767	0.0052	0.9600
Y5	0.7000	0.6911	0.0804	0.0761	0.0065	0.9300

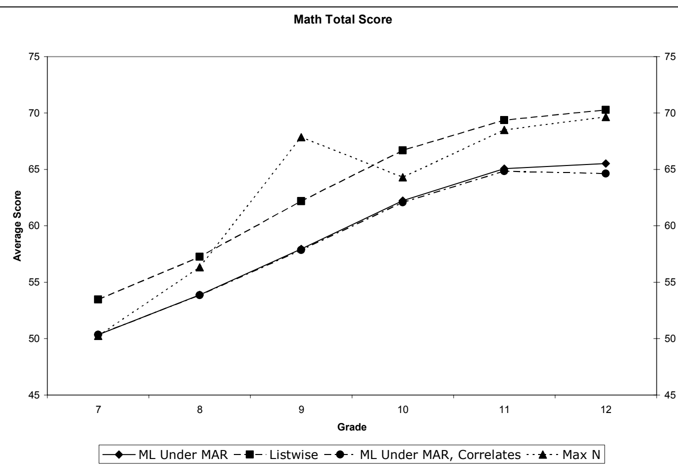
Y alone

	ESTIMATES Population	Average	Std. Dev.	S.E. Average	M.S.E.	95% Cover
F2 BY						
Y1	0.7000	0.6482	0.0869	0.0797	0.0102	0.890
Y2	0.7000	0.6392	0.0861	0.0803	0.0110	0.850
Y3	0.7000	0.6450	0.0751	0.0802	0.0086	0.900
Y4	0.7000	0.6474	0.0741	0.0802	0.0082	0.910
Y5	0.7000	0.6463	0.0825	0.0796	0.0096	0.890

References

Asparouhov & Muthen (2008). Auxiliary variables predicting missing data. Technical report. www.statmodel.com.

LSAY Estimated Means



MAR sample size = 3102
Listwise sample size = 782
Max N sample size = 3065, 2581, 2241, 2040, 1593, 1168

Input Excerpts LSAY Math Mean Using Missing Data Correlates

```
            USEV = math7 math8 math9 math10 math11 math12;
            AUXILIARY = (M) female mothed homeres expel arrest
            hisp black hsdrop
            expect droptht7
            lunch mstrat;
DEFINE:     lunch = lunch/100;
            mstrat = mstrat/1000;
MODEL:      math7 WITH math8-math12;
            math8 WITH math9-math12;
            math9 WITH math10-math12;
            math10 WITH math11-math12;
            math11 WITH math12;
PLOT:       TYPE = PLOT3;
            SERIES = math7-math12(*);
```


Non-Ignorable Missing Data

Non-Ignorable Missing Data Modeling Approaches And References

Selection modeling: [y | x] [m | y, x]. Different approaches to [m | y, x]:

Little & Rubin (2002) book, Little (2008) handbook chapter: overview
Diggle & Kenward (1994) in Applied Statistics:
predicting from y, y* (y* is non-ignorable dropout)
Wu & Carroll (1988), Wu & Bailey (1989) in Biometrics:
predicting from the slope s

Pattern-mixture modeling: [m | x] [y | m, x]. Different approaches to [y | m, x]:

Little & Rubin (2002), Little (2008): overview
Conventional approach: predicting i, s from m
Roy (2003) in Biometrics:
predicting i, s from a latent class variable c (missing data patterns)

Non-Ignorable Missing Data Modeling Approaches And References (Continued)

Shared-parameter modeling: $[y, m] = \sum_c [c]\,[m \mid c]\,[y \mid c]$:

Albert & Follman (2008) handbook chapter: overview
Beunckens et al (2008) in Biometrics: using a latent class variable c and i, s

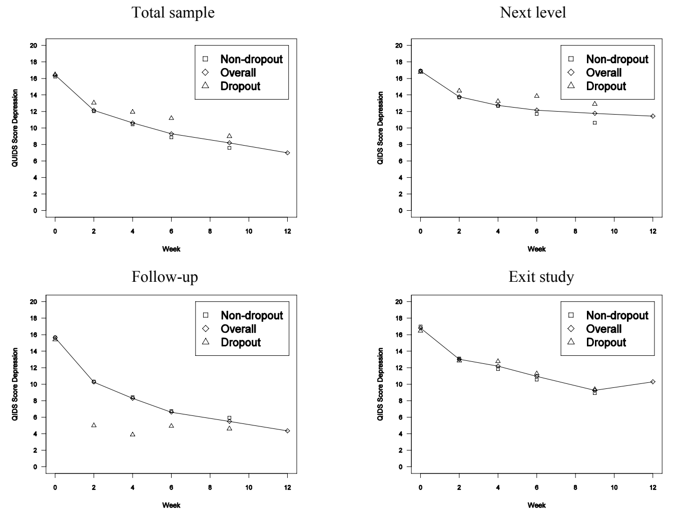
Non-Ignorable Missing Data Modeling In Longitudinal Studies

- Intermittent missingness versus dropout
- Muthén, B., Asparouhov, T., Hunter, A. & Leuchter, A. (2010). Growth modeling with non-ignorable dropout: Alternative analyses of the STAR*D antidepressant trial. Submitted for publication.
- Applied to STAR*D antidepressant trial data (n=4041)

STAR*D Antidepressant Trial

- STAR*D multi-site NIMH antidepressant trial with $n = 4041$
- Subjects treated with citalopram (Level 1). No placebo group
- Clinician-rated QIDS depression score measured at baseline and weeks 2, 4, 6, 9, and 12
- 25% have complete data, 60% have monotone (dropout) missing data patterns, 14% have non-monotone missing data patterns
- Coverage at baseline and weeks 2, 4, 6, 9, and 12: 1.00, 0.79, 0.69, 0.68, 0.57, and 0.39
- Reasons for leaving Level 1: Remission (moved to follow-up), medication inefficient or not tolerated (moved to Level 2), study exit

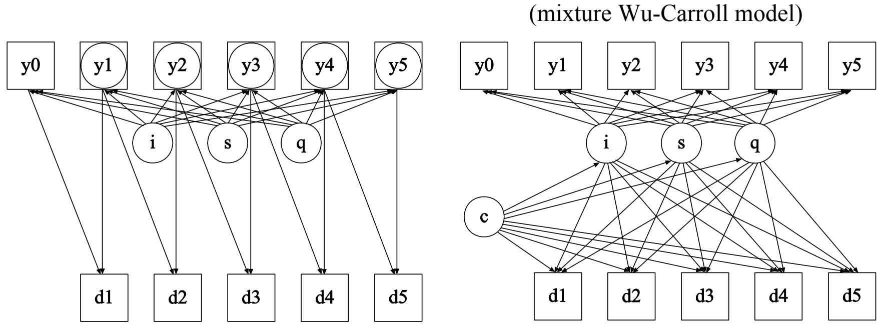
STAR*D Antidepressant Trial



Selection Modeling

Diggle-Kenward selection modeling

Beunckens mixture model (mixture Wu-Carroll model)



Selection modeling (d's are survival indicators)

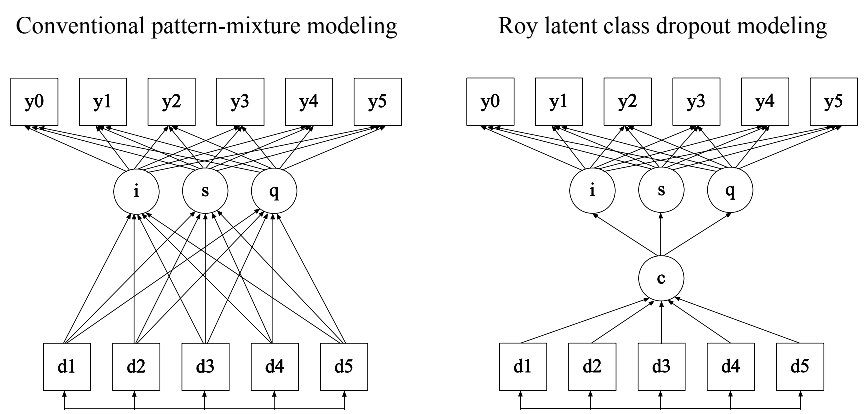
Input Excerpts Diggle-Kenward Selection Modeling

```
DATA:          FILE = StarD Ratings 1-23-09.dat;
VARIABLE:      NAMES = …
               … ;
               MISSING = ALL (-9999);
               USEV = y0 y1 y2 y3 y4 y5
               d1 d2 d3 d4 d5;
               CATEGORICAL = d1 d2 d3 d4 d5;
DATA MISSING:  NAMES = y1 y2 y3 y4 y5;
               TYPE = SDROPOUT;
               BINARY = d1-d5;
ANALYSIS:      PROCESS = 4;
               ALGORITHM = INTEGRATION;
               INTEGRATION = MONTECARLO;
               INTERACTIVE = control.dat;
```


Input Excerpts Diggle-Kenward Selection Modeling (Continued)

```
MODEL:         i s q | y0@0 y1@.2 y2@.4 y3@.6 y4@.9 y5@1.2;
               d1 ON y0 (beta2)
               y1 (beta1);
               d2 ON y1 (beta2)
               y2 (beta1);
               d3 ON y2 (beta2)
               y3 (beta1);
               d4 ON y3 (beta2)
               y4 (beta1);
               d5 ON y4 (beta2)
               y5 (beta1);
PLOT:          TYPE = PLOT3;
               SERIES = y0-y5 (s);
OUTPUT:        TECH1 SAMPSTAT RESIDUAL STANDARDIZED;
```

Pattern-Mixture Modeling



Pattern-mixture modeling (d's are dropout dummy variables)

Input Excerpts Pattern-Mixture Modeling

```
TITLE:
DATA:           FILE = StarD Ratings 1-23-09.dat;
VARIABLE:       NAMES = …
                … ;
                MISSING = ALL (-9999);
                USEV = y0 y1 y2 y3 y4 y5
                d1 d2 d3 d4 d5;
DATA MISSING:   NAMES = y1 y2 y3 y4 y5;
                TYPE = DDROPOUT;
                BINARY = d1-d5;
```

Input Excerpts Pattern-Mixture Modeling (Continued)

```
MODEL:          i s q | y0@0 y1@.2 y2@.4 y3@.6 y4@.9 y5@1.2;
                i-q ON d4-d5;
                i ON d1 d2 d3;
                s ON d3;
                s ON d1 (1);
                s ON d2 (1);
                q ON d1 (2);
                q ON d2 (2);
                q ON d3 (2);
PLOT:           TYPE = PLOT3;
                SERIES = y0-y5 (s);
OUTPUT:         TECH1 SAMPSTAT RESIDUAL STANDARDIZED;
```


Input Excerpts Roy 4-Class Latent Dropout Modeling

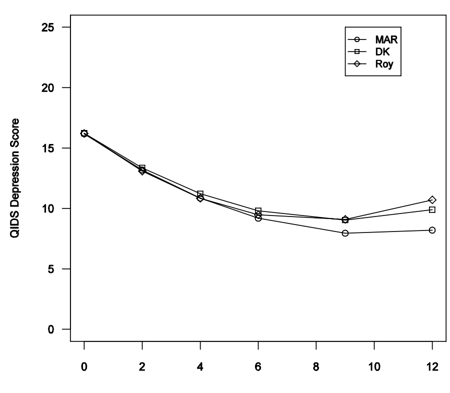
```
TITLE:
DATA:           FILE = StarD Ratings 1-23-09.dat;
VARIABLE:       NAMES = …
                … ;
                MISSING = ALL (-9999);
                USEV = y0 y1 y2 y3 y4 y5
                d1-d5;
                CLASSES = c(4);
DATA MISSING:   NAMES = y1 y2 y3 y4 y5;
                TYPE = DDROPOUT;
                BINARY = d1-d5;
ANALYSIS:       TYPE = MIXTURE;
                PROCESS = 4(STARTS);
                INTERACTIVE = control.dat;
                STARTS = 200 40;
```

Input Excerpts Roy 4-Class Latent Dropout Modeling (Continued)

```
MODEL:          %OVERALL%
                i s q | y0@0 y1@.2 y2@.4 y3@.6 y4@.9 y5@1.2;
                c ON d1-d5;
PLOT:           TYPE = PLOT3;
                SERIES = y0-y5 (s);
OUTPUT:         TECH1 SAMPSTAT RESIDUAL STANDARDIZED;
```

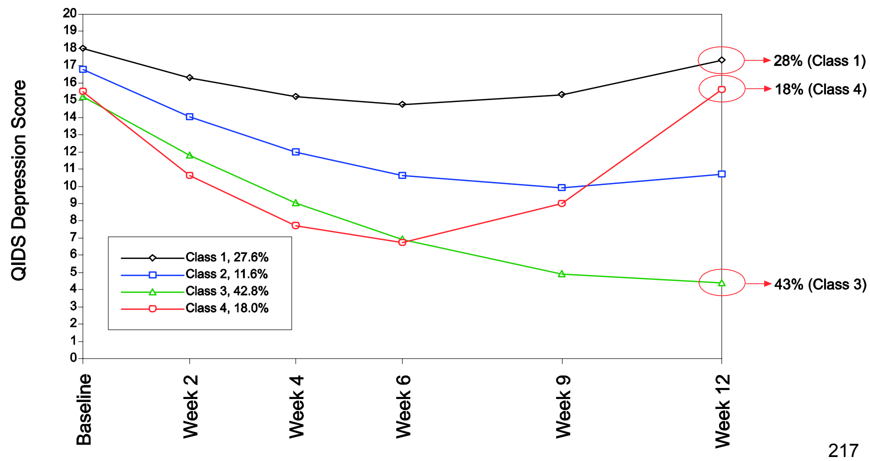

STAR*D Estimated Mean Growth Curves

Depression mean curves estimated under MAR, Diggle-Kenward selection modeling, and Roy latent dropout modeling



STAR*D Estimated Mean Growth Curves Under 4-Class Roy Model

4-class Roy latent dropout model



Multiple Imputation Analysis In Mplus

- Multiply imputed data an input data alternative
- Estimates and SEs aggregated over the analyses
- Model testing

Further Readings On Missing Data Analysis

Albert, P.S. & Follman, D.A. (2008). Shared-parameter models. In Fitzmaurice, G., Davidian, M., Verbeke, G. & Molenberghs, G. (eds.), Longitudinal Data Analysis, pp. 433-452. Boca Raton: Chapman & Hall/CRC Press.

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Further Readings On Missing Data Analysis (Continued)

Muthén, B., Jo, B. & Brown, H. (2003). Comment on the Barnard, Frangakis, Hill & Rubin article, Principal stratification approach to broken randomized experiments: A case study of school choice vouchers in New York City. Journal of the American Statistical Association, 98, 311-314.

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