PSY792F SEM

Week 14 – Multilevel SEM
Mediation/Moderation

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MSEM Mediation/Moderation

- Approaches to MSEM mediation and moderation vary by hypothesis.
  - I will review some, but not all approaches
- Read:
  - Preacher, Zyphur, Zhang, 2010 – Multilevel Mediation
  - Preacher, Zhang, Zyphur, 2016 – Multilevel Moderation
    - There are other good references as well, but these are good to start with
- I'm serious this time… Read these articles.

MULTILEVEL MEDIATION

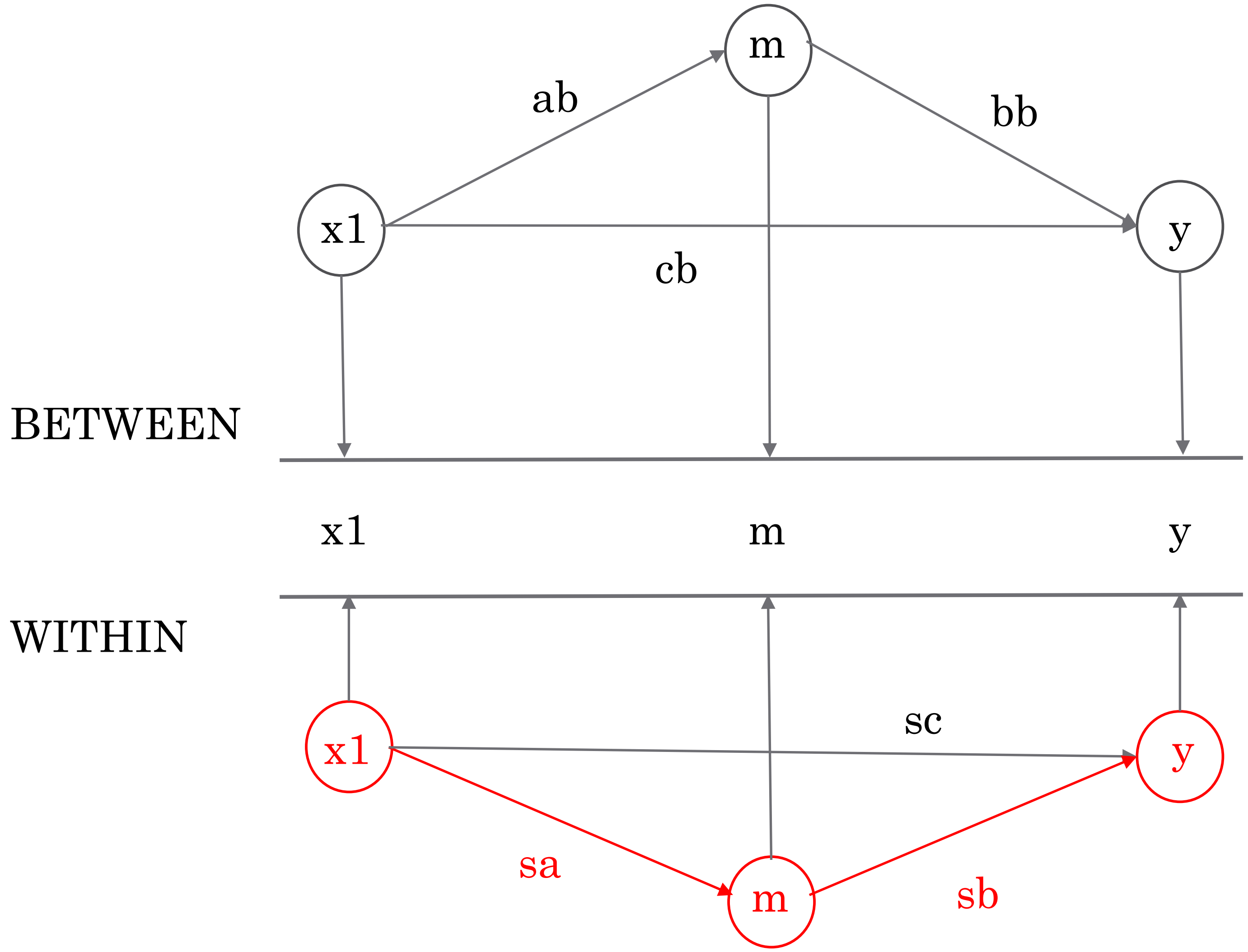
Multilevel Mediation

- Mediation: when the effect of Y on X is transmitted by M.
- Traditional mediation methods are inappropriate because the nesting of the data violates independence assumption and downwardly biases the standard errors.
- Terminology:
- Variables on the within level are notated with a 1
- Variables on the between level are notated with a 2
- Examples:
  - 1-1-1 – all variables on the within level
    - Daily levels of MPS use, Perceived Helpfulness, and Marijuana use
  - 2-1-1 - the IV is on the between level and the mediator and DV are on the within level
    - Treatment condition, Daily MPS use, Marijuana use
  - 2-2-2 – all variables on the between level
    - This is regular SEM or path analysis

Random slopes

- In MLM random slopes can only be dependent variables
- MSEM random slopes can be
  - Independent variables
  - Mediators
  - Dependent variables
- This is an example of the flexibility of MSEM
- *note, random slopes models do not have model fit

1-1-1 Mediation
Random slopes



1-1-1 Mediation Example

```
TITLE:    1-1-1 mediation
DATA:     FILE IS ex9.10.dat;
VARIABLE:
NAMES ARE x1 m y x2 m2 w clus;

usevariables are clus x1 m y;

CLUSTER = clus; !grouping variable

ANALYSIS:
TYPE = TWOLEVEL RANDOM;
MODEL:
%WITHIN%
sa | m on x1; !create random slope sa from m on x1
sb | y on m; !create random slope sb from y on m
sc | y on x1; !create random slope sc from y on x1

%BETWEEN%
sa sb sc x1 m y; !estimate level-2 residual variances
sa with sc x1 m y; !estimate level-2 covariances of sa with x1 m y
sa with sb (cab); !estimate level-2 covariances of sa with sb call it cab
sb with sc x1 m y; !estimate level-2 covariances of sb with sc x1 m y
m on x1 (ab); !ab = contextual effect, not the between slope
y on m (bb); !bb = contextual effect, not the between slope
y on x1;
[sa] (aw);   !estimate mean of sa call it aw
[sb] (bw);   !estimate mean of sb call it bw

MODEL CONSTRAINT:
NEW (a b indb indw);

a = aw+ab; !compute a path
b = bw+bb; !compute b path
indw = aw*bw+cab;    !compute within indirect effect
indb = a*b; !compute between indirect effect

OUTPUT:  TECH1 tech3 sampstat CINTERVAL;
```

Note this can also be run with Estimator = bayes

<u>To give this meaning</u>

X – skill level assessed daily

M – effort assessed daily

Y – performance assessed daily

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Variances
    X1                 1.756      0.135     12.990      0.000

 Residual Variances
    M                  1.642      0.097     16.844      0.000
    Y                  1.255      0.099     12.679      0.000

Between Level

 M          ON
    X1                 0.393      0.119      3.303      0.001

 Y          ON
    M                  0.521      0.033     15.677      0.000
    X1                 0.116      0.086      1.359      0.174

 SA       WITH
    SC                 0.004      0.032      0.123      0.902
    X1                 0.002      0.027      0.073      0.942
    M                 -0.006      0.035     -0.157      0.875
    Y                 -0.005      0.040     -0.127      0.899
    SB                 0.000      0.026     -0.002      0.998

 SB       WITH
    SC                -0.002      0.017     -0.098      0.922
    X1                -0.036      0.028     -1.301      0.193
    M                  0.006      0.028      0.228      0.820
    Y                 -0.003      0.022     -0.156      0.876

 Means
    X1                -0.115      0.097     -1.186      0.236
    SA                 0.523      0.050     10.544      0.000
    SB                 0.387      0.054      7.180      0.000
    SC                 0.331      0.059      5.609      0.000

 Intercepts
    M                  0.037      0.075      0.491      0.623
    Y                  0.064      0.062      1.028      0.304

 Variances
    X1                 0.477      0.117      4.072      0.000
    SA                 0.008      0.037      0.213      0.831
    SB                 0.009      0.022      0.415      0.678
    SC                 0.007      0.019      0.381      0.703

 Residual Variances
    M                  0.025      0.050      0.499      0.618
    Y                  0.010      0.053      0.192      0.847

New/Additional Parameters
    A                  0.916      0.115      7.967      0.000
    B                  0.908      0.059     15.424      0.000
    INDB               0.832      0.089      9.320      0.000
    INDW               0.202      0.030      6.736      0.000
```

1-1-1 MCCIs

- http://quantpsy.org/medmc/medmc111.htm
- you will need
- a, b, var(a), var(b), cov(a,b), level-2 cov(a,b), var(level-2 cov(a,b))
- From the model results
  - a, b, level-2 cov(a,b)
- From tech3
  - Var(a), var(b), cov(a,b), var(level-2 cov(a,b))
- From tech1
  - Need to find the parameter numbers

1-1-1 MCCI example

- From 111 mediation ex.out

Mean(a) = .523 (from model estimated mean for sa)

Mean(b) = .387 (from model estimated mean for sb)

Tau(a,b) = .000 (from sa with sb in the between portion)

Var(a) = .0013 (from tech3 parameter 13 in the between psi matrix)

Var(b) = .0005 (from tech3 parameter 15 in the between psi matrix)

Cov(a,b) = -.00004 (from tech3 parameter 13 with 15)

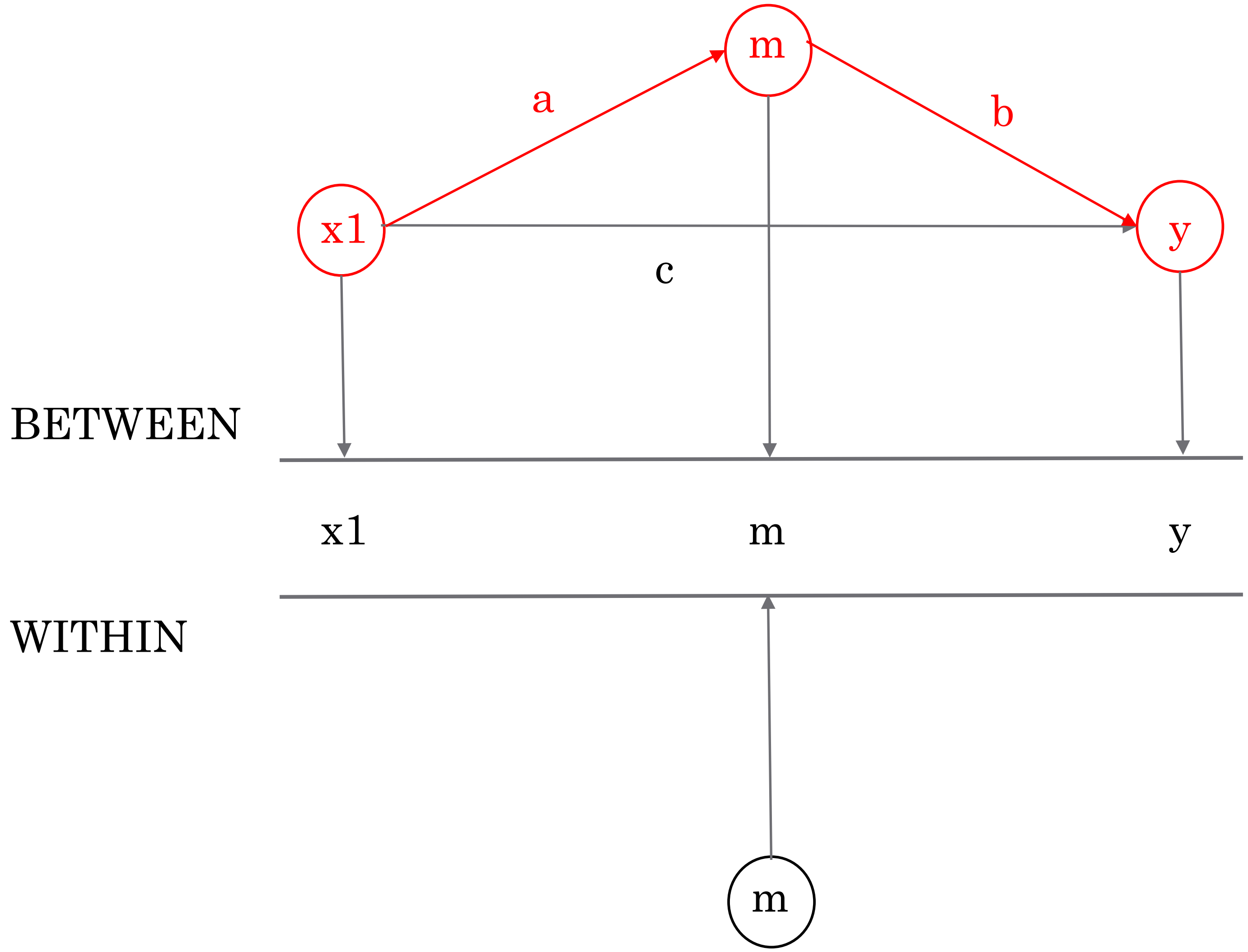
Var(Tau(a,b)) = .0007 (from tech3 parameter 14 - between psi matrix sa with sb)

MCCI [.14, .26]

estimate of indw = .202

*note that it is in the interval is a basic check that you didn't make an error

2-1-2 Mediation
Bayesian Estimation



2-1-2 Mediation example

```
TITLE:     2-1-2 mediation
DATA:      FILE IS ex9.10.dat;

VARIABLE:  NAMES ARE x1 m y x2 m2 w clus;
           !BETWEEN = w;

usevariables are clus m yb xb;

           CLUSTER = clus;

           BETWEEN ARE xb yb;

define:
  xb = cluster_mean(x1);
  yb = cluster_mean(y);

ANALYSIS:  TYPE = TWOLEVEL RANDOM;
           estimator=bayes; !this will provide bayesian credible intervals

MODEL:
           %WITHIN%

           m;

           %BETWEEN%
           xb yb;
           m on xb(a);
           yb on m(b);
           yb on xb;

MODEL CONSTRAINT:
           NEW (indb);

           indb = a*b;

OUTPUT:    sampstat CINTERVAL;
```

I had to force x1 and y to be
Between only because I simulated
The data for the 1-1-1 example

For the write up:
X – skill at baseline
M – effort measured daily
Y – performance at follow-up

Model fit – PPP and DIC

- Bayesian posterior predictive *p* values (PPP)
  - The model test statistic, the chi-square value, is calculated on the basis of the data is compared to the same test statistic, but then defined for the simulated data.
  - Then, the ppp value is defined as the proportion of chi-square values obtained in the simulated data that exceed that of the actual data.
  - The ppp values around .50 indicate a well-fitting model.
- Deviance Information Criterion (DIC)
  - Is a comparative fit index like AIC or BIC
  - Values closer to 0 indicate better fit

```
MODEL FIT INFORMATION

Number of Free Parameters                              10

Bayesian Posterior Predictive Checking using Chi-Square

          95% Confidence Interval for the Difference Between
          the Observed and the Replicated Chi-Square Values

                              -11.819             9.920

          Posterior Predictive P-Value                0.528

Information Criteria

          Deviance (DIC)                           1905.678
          Estimated Number of Parameters (pD)        67.588
```

Frequentist Confidence Intervals VS. Bayesian Credible Intervals

- Frequentist confidence interval
  - The parameter is fixed, but unknown – data is random
  - The CI will include the true value of the parameter 95% of the time over repeated experiments

- Bayesian Credible Interval
  - Parameters are random, and described by a distribution – data is fixed
  - A Bayesian CI will include the true value of the parameter with 95% probability

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Within Level						
Variances						
M	2.019	0.127	0.000	1.796	2.280	*
Between Level						
M ON						
XB	0.725	0.094	0.000	0.536	0.909	*
YB ON						
M	1.557	0.353	0.000	1.228	2.628	*
XB	-0.315	0.290	0.033	-1.175	0.024	
Means						
XB	-0.079	0.099	0.189	-0.280	0.098	
Intercepts						
YB	0.061	0.101	0.291	-0.137	0.270	
M	-0.003	0.070	0.486	-0.130	0.147	
Variances						
XB	0.904	0.144	0.000	0.685	1.198	*
Residual Variances						
YB	0.045	0.044	0.000	0.006	0.168	*
M	0.131	0.051	0.000	0.041	0.240	*
New/Additional Parameters						
INDB	1.150	0.292	0.000	0.808	2.020	*

CREDIBILITY INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
Within Level							
Variances							
M	1.726	1.796	1.828	2.019	2.239	2.280	2.363
Between Level							
M ON							
XB	0.511	0.536	0.563	0.725	0.878	0.909	0.962
YB ON							
M	1.165	1.228	1.257	1.557	2.288	2.628	2.938
XB	-1.597	-1.175	-0.996	-0.315	-0.025	0.024	0.108
Means							
XB	-0.341	-0.280	-0.249	-0.079	0.072	0.098	0.177
Intercepts							
YB	-0.177	-0.137	-0.111	0.061	0.239	0.270	0.293
M	-0.187	-0.130	-0.112	-0.003	0.117	0.147	0.171
Variances							
XB	0.613	0.685	0.708	0.904	1.149	1.198	1.450
Residual Variances							
YB	0.004	0.006	0.007	0.045	0.152	0.168	0.198
M	0.028	0.041	0.051	0.131	0.220	0.240	0.268
New/Additional Parameters							
INDB	0.689	0.808	0.853	1.150	1.795	2.020	2.413

MULTILEVEL MODERATION

Multilevel Moderation

- Moderation: when the effect of Y on X varies as a function of W
- Traditional methods conflate lower- and higher-order effects by not separating them into their orthogonal components, which causes model misspecification
- We need to examine level-specific moderation effects
  - Decomposing effects is necessary
  - Level-1 variables have both within and between components
    - Between –variation between units
      - e.g., between classrooms
    - Within – variations around cluster means (within cluster variation)
      - e.g., between students in a classroom
  - Level-2 variables have no within cluster variation
    - e.g., what treatment condition a participant is in
  - *Level 1 only is possible, but rare and requires independence of observations
    - e.g., pregaming days nested within individuals' repeated assessments

Multilevel Moderation Notation

- 1 X (1→1)
  - First 1 is the level at which the moderator is measured
  - Second 1 is the level at which the focal predictor is measured
  - Last 1 is the level at which the outcome is measured
- In this example all three are at level 1
  - We can think of it as the 1's in parentheses is the primary relationship and the 1 outside is the variable moderating that relationship
  - We can rewrite the notation as:
    - W * (X → Y)
- If for example we had treatment condition moderating the relationship between daily mood and daily substance use
  - Tx * (mood → substance use)
  - The notation would be: 2 X (1 → 1)

2 types of moderation

- Random Coefficient Prediction (RCP) Method
  - A slope is predicted by a moderator
  - Very useful for modeling cross-level interactions
  - Cannot treat interaction terms as random slopes
- Latent Moderated Structural Equation (LMS) Method
  - Computing a product involving at least one random Between part of an Level-1 variable as a latent predictor
    - Requires latent interactions among random coefficients by directly representing latent interactions as part of the structural model
  - Can be used for cross- and same-level interactions
  - Can treat interactions as random slopes
  - Produces biased results for non-normal data
    - However, due to central limit theorem, cluster means may be normal even if level-1 variables are not
- *NOTE: RCP and LMS can be applied in the same model

Predicting a slope (RCP method)

- The simplest form of multilevel moderation is
- S on w;
- Because the slope will have been previously specified in the within statement.
  - S | y on x;
- If W predicts S then the relationship between y and x varies as a function of W.

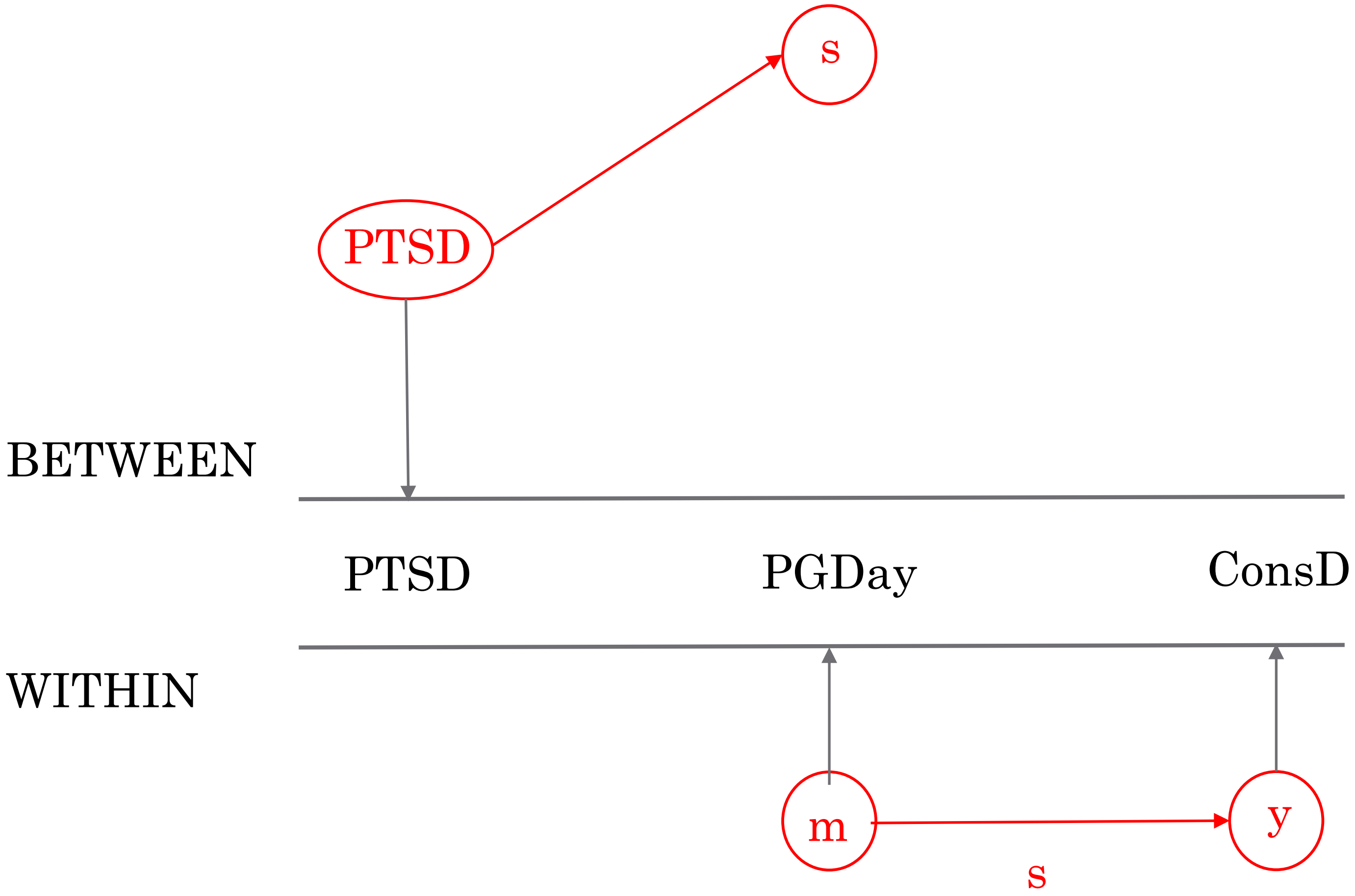
Computing a product (LMS method)

- Computing a product involving at least one random Between part of an Level-1 variable as a latent predictor
- Excerpt from Preacher et al 2016 code for Moderation 2X(1-1) LMS

```
ANALYSIS: TYPE IS TWOLEVEL RANDOM; ESTIMATOR IS MLR;
ALGORITHM IS INTEGRATION; INTEGRATION IS 4;
MODEL:
%WITHIN%
xw BY x@1; xw*.7; x@.01;
yw BY y@1; yw*.7; y@.01;
s | yw ON xw;
xz | xw XWITH zb; yw ON xz*.2;
%BETWEEN%
xb BY x@1; xb*.7; x@.01;
zb BY z@1; zb*.7; z@.01;
yb BY y@1; yb*.7; y@.01;
yb ON xb*.2 zb*.2; xb WITH zb*.1 s*0; zb WITH s*0;
[x@0 z@0 y@0 xb*0 zb*0 yb*.1 s*.1]; s*.2;
```

Note: zb is part of the latent Interaction term on the within Level but is defined on the between level

2X(1-1) Moderation RCP



Moderation 2 X (1-1) RCP method

```
USEVARIABLES ARE
    pgday
    CONSd
    ptsd1
  ;


CLUSTER IS IDNUM;
within is pgday;
between is ptsd1 ;
ANALYSIS:
  TYPE IS TWOLEVEL RANDOM;
MODEL:
  %WITHIN%
  s | consd on pgday;
  %BETWEEN%
  consd;
  s;
  consd with s;
  consd s on ptsd1 ;
```

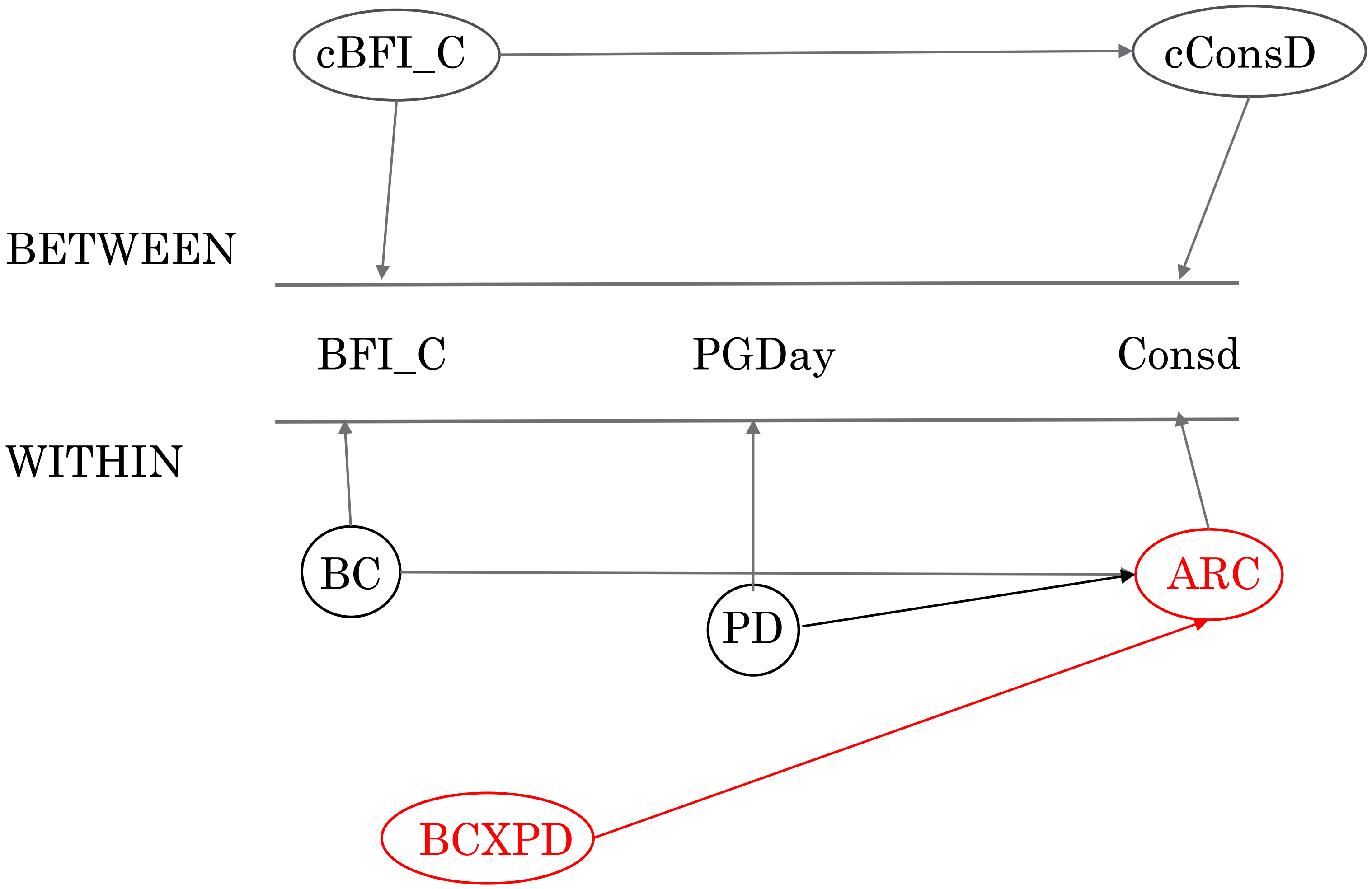
Note that here is the moderation.

By definition, if the relationship between x and y varies as a function of a third variable you have moderation. So a slope being predicted by a third variable fits that definition.

2 X (1-1) RCP OUTPUT

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Residual Variances
    CONSD              0.832      0.070     11.930      0.000

Between Level

 S           ON
    PTSD1             -0.522      0.230     -2.271      0.023

 CONSD       ON
    PTSD1             -0.106      0.020     -5.411      0.000

 CONSD     WITH
    S                  0.139      0.037      3.780      0.000

 Intercepts
    CONSD              0.226      0.017     13.558      0.000
    S                  2.436      0.145     16.783      0.000

 Residual Variances
    CONSD              0.041      0.012      3.478      0.001
    S                  4.400      0.772      5.699      0.000
```

1 X (1-1) Moderation LMS



Moderation 1X(1-1) LMS method

```
USEVARIABLES ARE
     pgday
     CONSd
     bfi_c
     cbfi_c
     cconsd
  ;

CLUSTER IS IDNUM;
within is pgday bfi_c consd;
between is cbfi_c cconsd;

define:
cbfi_c = cluster_mean(bfi_c);
cconsd = cluster_mean(consd);

ANALYSIS:
  TYPE IS TWOLEVEL RANDOM;
  algorithm=integration;

MODEL:
  %WITHIN%
  PD by pgday@1; pd@.7; pgday@.1;
  BC by bfi_c@1; bc@.7; bfi_c@.1;
  ARC by consd@1; arc@.7; consd@.1;

 pdXbc | PD xwith BC ;
  ARC on PD BC pdXbc;

  %BETWEEN%
  cconsd cbfi_c;
 cconsd on cbfi_c;
 [cconsd cbfi_c];

OUTPUT: TECH1 TECH3;
```

These are single indicator Latent variables.
*You could use the observed variables and make simple interaction terms.

Xwith is used to create Latent variable interactions

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
PD BY				
PGDAY	1.000	0.000	999.000	999.000
BC BY				
BFI_C	1.000	0.000	999.000	999.000
ARC BY				
CONSD	1.000	0.000	999.000	999.000
ARC ON				
PD	1.270	0.081	15.665	0.000
BC	-0.154	0.025	-6.071	0.000
PDXBC	-3.140	0.220	-14.254	0.000
BC WITH				
PD	-0.515	0.018	-29.039	0.000
Intercepts				
PGDAY	0.059	0.003	19.734	0.000
CONSD	0.100	0.017	5.868	0.000
BFI_C	3.476	0.027	128.403	0.000
Variances				
PD	0.700	0.000	999.000	999.000
BC	0.700	0.000	999.000	999.000
Residual Variances				
PGDAY	0.100	0.000	999.000	999.000
CONSD	0.100	0.000	999.000	999.000
BFI_C	0.100	0.000	999.000	999.000
ARC	0.700	0.000	999.000	999.000
Between Level				
CCONSD ON				
CBFI_C	-0.060	0.020	-2.955	0.003
Means				
CBFI_C	3.474	0.027	128.242	0.000
Intercepts				
CCONSD	0.529	0.077	6.838	0.000
Variances				
CBFI_C	0.440	0.024	18.607	0.000
Residual Variances				
CCONSD	0.162	0.035	4.694	0.000

How to write up the results…

- Analysis plan
  - Link hypotheses to chosen method
  - Describe any data decisions or model building
  - Describe the type of mediation or moderation used (e.g., 1-1-1, 1X(1-1) – RCP, LMS)
  - Describe any additional steps you took, MCCIs, Bayesian CIs
- Results
  - For mediation
    - Report like a typical mediation write up, with special emphasis given to the language used depending on the level of each variable
  - For moderation
    - Emphasize the higher order effects, i.e., interaction terms are more important than main effects