Critical Statistics for Measurement

PSY 600K
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Agenda

- Critical statistics (seriously):
  - Types of variables.
  - The mean (and what it means in measurement).
  - Variance.
  - Covariance & correlation.

Views of Variables

- **Random**
  - Unknown before a study is carried out.
  - We can model the *expected value* of random variables but we do not know the actual values until we have the data.
- **Observed** vs. **latent** (unobservable - inferred).
  - A variable can be latent without being a construct.
- **Discrete** vs. **continuous**
  - Implications for the way we set up and interpret our statistical models.
  - R&M: If a variable takes on fewer than 15 different values, model it as discrete.
  - This means most item responses are viewed as discrete…

The Mean and What it Means

- You all know how to calculate the mean.
- When you have a binary variable (0 or 1), the mean is (exactly) equivalent to the probability of scoring 1.
  - Getting the item correct, choosing the keyed answer.
- With binary items, it is easy to think of this as the **difficulty** of the item.
  - The probability of getting it right.
  - Higher numbers = lower difficulty.
- We extend this logic to continuous variables as well: the item mean = item difficulty.
  - Difficulty = extremeness.
  - **Not** about cognitive effort!

Variance

- Conceptually, what is the variance of a variable?
- Calculated by:
  - $\hat{\sigma}_X{}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$
- Variance is **good** in psychological tests.
  - What we really want is lots of **true** variance.
  - Need lots of overall variance to make that happen.
- The standard deviation is just another way to write the variance – puts it in interpretable units.
- We will mostly work with variances (not SDs).

Covariance

- Indicator of the relatedness of two variables:
  - $c_{X,Y} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})$
  - $(X_i - \bar{X})$ = distance of the X value from the mean of X
  - $(Y_i - \bar{Y})$ = distance of the Y value from the mean of Y
  - Covariance gets bigger when both $(X_i - \bar{X})$ and $(Y_i - \bar{Y})$ are positive, OR both are negative.
    - When X & Y deviate from the mean in the same direction.
  - Covariance gets smaller when one is positive and the other is negative.

Good Things to Know About Covariances

- The bigger $(X_i - \bar{X})$ and $(Y_i - \bar{Y})$ are, the larger the covariance can be.
  - In other words… having more variance (variability) gives you more potential to see sizable covariances.
  - Having very little variability – everything clustered around the mean – makes it hard to have a high covariance with anything else.
  - Related to *restriction of range.*
- The variance of a variable is its covariance with itself:
- $c_{X,Y} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(X_i - \bar{X}) = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$
- So we can find a variance-covariance matrix by finding the covariances among every possible pair of variables.

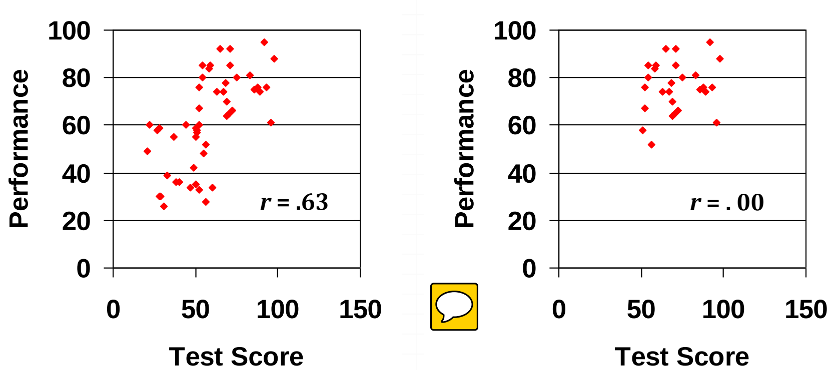
Covariance and Correlation

- Covariances don't have meaningful units – hard to compare and interpret.
- So we often transform them into correlations.
  - $\hat{\rho}_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$ (Equation 2.6)
- This is equivalent to finding the variance using z-scores instead of raw differences from the mean (Equation 2.7).
- What this means is that the correlation is the *standardized* version of the covariance.
  - We can use this later in factor analysis to get standardized estimates of item parameters.

Restriction of Range

- We've already said that you can't have covariance if you don't have variance.
- Sometimes, our measures or our samples prevent us from seeing the full variance of a latent variable.
  - Can you think of examples?
- When this happens, even if a true relationship between the variables exists, we will see only a weak correlation.

Restriction of Range



Implications of Range Restriction

- Item writing:
  - If you write an item that has very little variance, it will not correlate well with other items.
  - If you are surprised by the poor performance of an item, look at its variance – lack of variance is often the reason.
  - This is (one reason) why your pilot/development sample needs to be reasonably representative of your population!
- Using tests:
  - Same principle applies. If there is not much variance in your outcome, you can't predict it. If variance is artificially restricted, you may miss a true relationship.
- Restriction of range makes correlations smaller; not bigger.

Correlations and Statistical Significance

- You can test the likelihood that a particular observed correlation would occur even if the true population correlation is zero.
- However, this is not much use. Why?
- Significance tests for correlations are hugely dependent on sample size.
  - This is true for all sig tests, but tests for correlations are particularly sensitive.
  - A correlation of .10 is significant if you have 272 participants. Not if you have 271. A correlation of .01 is significant if you have 30,000 participants.
- In measurement, we are much more interested in the effect size.

Correlations and Effect Sizes

- Even a small correlation can have a big practical effect.
  - Example: a new cancer treatment
  - With no treatment, 60% of patients die and 40% live.
  - With the treatment, 40% of patients die and 60% live.
  - Is that worthwhile?
- Correlation: $r = .20$, $r^2 = .04$
- In measurement we are much more concerned with *patterns* of correlations than with the "significance" of any one of them.
  - So long as you have a large enough sample to give you some confidence in those patterns…

Linear Combinations

- A linear combination is a mathematical process that involves:
  - Adding
  - Subtracting
  - Multiplying
  - Dividing
  - And NO other math (no square roots, logs, etc.).
- Linear combinations do not change the shape of the underlying distributions of our variables.

Variances of Linear Combinations

- If you have a set of variables X_1, X_2, etc., and you want to add them all together.
  - For example, a set of items you want to add into a total test score.
- The variance of the sum is equal to the sum of the item variances plus the covariances among all the items (counted twice).
  - Assuming you are weighting all the items equally.
  - It's only slightly more complicated for weighted items (see Equation 2.8).

Item and Test Variance

- Imagine an item covariance matrix:

$$\begin{bmatrix} \sigma_{11} & \sigma_{21} & \cdots & \sigma_{j1} \\ \sigma_{12} & \sigma_{22} & \cdots & \vdots \\ \cdots & \cdots & \ddots & \vdots \\ \sigma_{1j} & \sigma_{2j} & \cdots & \sigma_{jj} \end{bmatrix}$$

- The variances of linear combinations formula means that we can find total test variance by adding up all the elements of this matrix.
- We want large item variances *and* large item covariances.

Questions?

For next time:
Classical Test Theory
Read: DeVellis Ch. 2
4th Reading Response