Lecture 9 - Set, TFVS

Friday, 10 October 2025 11:00 AM

Recall: $\{x_n\}$ $\xrightarrow{\text{w.r.t. closed, prob set with}}$ x $\Leftrightarrow$ $\{x_n\}$ $\xrightarrow{\text{w.r.t. closed, prob set with}}$ x $\Leftrightarrow$ $\{x_n\}$ $\xrightarrow{\text{w.r.t. cvx}}$ x

Q: How is $\xrightarrow{\text{w.r.t. cvx}}$ defining topology?

Ex: $\mathcal{B} = \left\{ \left(\frac{1}{n+1}, \frac{1}{n} \right) \cup \left(\frac{1}{n+1}, \frac{1}{n} \right) + \left(\frac{1}{n+1}, \frac{1}{n} \right) \right\}$

$\{x_n\} \xrightarrow{\text{w.r.t. cvx}} 0$ iff $\{x_n\} \xrightarrow{\text{w.r.t. closed, prob set with}} 0$

SOTS: $\mathcal{B}(0) = \{0\}$, all pts affine with every pt in $\mathcal{B}(0)$ $\mathcal{B}(0) = \mathbb{R}^2$

CAN WE DO BETTER THAN ABOVE?

Def: $\mathcal{B} \subseteq \mathbb{R}^n$ is:

1. $\mathcal{B}$ is cvx, bounded, & symmetric, & contains 0 in its interior
2. $\mathcal{B}$ is cvx, bounded, & symmetric, & contains 0 in its relative interior
3. $\mathcal{B}$ is cvx, bounded, & contains 0 in its relative interior

Lemma: Let $\mathcal{B}$ be some probability $\mathcal{B}$ to be:

1. $\mathcal{B}$ is cvx, bounded, & contains 0 in its relative interior

Then $\mathcal{B}$ is cvx, bounded, & contains 0 in its relative interior

Proof: Suppose $\mathcal{B}$ is cvx, bounded, & contains 0 in its relative interior

Let $\mathcal{B}(0)$ be the affine hull of $\mathcal{B}$ & $\mathcal{B}(0)$ be the affine hull of $\mathcal{B}$ & $\mathcal{B}(0)$ be the affine hull of $\mathcal{B}$ & $\mathcal{B}(0)$ be the affine hull of $\mathcal{B}$