

Say window length = N

Linear Model:  $\hat{y}_i = ax_i + b$

$$\mathcal{L} = \sum_{n=1}^N (\hat{y}_i - y_i)^2$$

$$\mathcal{L} = \sum_{n=1}^N (ax_i + b - y_i)^2 \text{ where } y_i \text{ is ECG data from this window}$$

$$\frac{\partial \mathcal{L}}{\partial a} = \sum_{n=1}^N 2(ax_i + b - y_i)(x_i)$$

$$\frac{\partial \mathcal{L}}{\partial b} = \sum_{n=1}^N 2(ax_i + b - y_i)$$

$$\text{Setting } \frac{\partial \mathcal{L}}{\partial a} = 0, a \sum_{n=1}^N x_i^2 + b \sum_{n=1}^N x_i = a \sum_{n=1}^N x_i y_i$$

$$\text{Setting } \frac{\partial \mathcal{L}}{\partial b} = 0, a \sum_{n=1}^N x_i + Nb = \sum_{n=1}^N y_i$$

As a matrix,

$$\begin{matrix} & A & & \\ & & B & \\ \begin{pmatrix} \sum_{n=1}^N x_i^2 & \sum_{n=1}^N x_i \\ \sum_{n=1}^N x_i & N \end{pmatrix} & \begin{pmatrix} a \\ b \end{pmatrix} & = & \begin{matrix} Y \\ \begin{pmatrix} \sum_{n=1}^N x_i y_i \\ \sum_{n=1}^N y_i \end{pmatrix} \end{matrix} \end{matrix}$$

Then solving for B,

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\text{Constant}}{\text{Det}(A)} \begin{matrix} \text{Constant} \\ \begin{pmatrix} N & -\sum_{n=1}^N x_i \\ -\sum_{n=1}^N x_i & \sum_{n=1}^N x_i^2 \end{pmatrix} \end{matrix} \begin{pmatrix} \sum_{n=1}^N x_i y_i \\ \sum_{n=1}^N y_i \end{pmatrix}$$