

Correzione esame 21/01/19

$$\bullet f(x) = \sqrt{\frac{6x+7x^2}{x+6}}$$

$$\bullet \lim_{x \rightarrow 0^+} = \frac{\sqrt{6}}{6} = \frac{\sqrt{6}}{6}$$

$$\bullet \int \frac{1}{6\cos(x)+8}$$

$$\bullet \lim_{x \rightarrow +\infty} = +\infty$$

① SDF

$$f(x) = \sqrt{\frac{6x+7x^2}{x+6}}$$

$$x \neq -6$$

$$\frac{6x+7x^2}{x+6} > 0$$

$$\text{Dom } f \in]-6, -\frac{6}{7}[\cup [0, +\infty)$$

$$\begin{cases} 6x+7x^2 > 0 \\ x > -6 \end{cases} \quad \begin{cases} x > 0 \\ x > -\frac{6}{7} \end{cases}$$

$$\begin{array}{c|ccccc} & -6 & -\frac{6}{7} & 0 & +\infty \\ \hline & + & + & 0 & + \\ \hline & + & + & 0 & + \\ \hline & -6 < x < -\frac{6}{7} \vee x > 0 & & & \end{array}$$

Intersezioni

$$f(0) = \sqrt{\frac{0+0}{6}} = 0$$

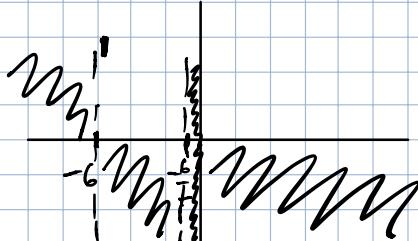
$$f(x) = 0 \in \mathbb{Q}$$

Interezione (0,0)

Segno funzione

$$-6 < x < -\frac{6}{7} \vee x > 0$$

\uparrow
pos



Limiti

$$\lim_{x \rightarrow \infty} \sqrt{\frac{6x+7x^2}{x+6}} = \lim_{x \rightarrow \infty} \sqrt{\frac{x^2(6/x+7)}{x(1+6/x)}} = \sqrt{+x+7} = \sqrt{\infty} = \infty$$

$$\lim_{x \rightarrow -\infty} = \sqrt{\infty} \text{ indefinito}$$

$$\lim_{x \rightarrow -6^+} \sqrt{\frac{6x+7x^2}{x+6}} =$$

perché la funzione non è definita a sinistra di -6 calcola solamente il lim a destra

$$\lim_{x \rightarrow -6^+} \frac{6x+7x^2}{x+6}$$

$$\lim_{x \rightarrow -6^+} (6x+7x^2) \cdot \frac{1}{x+6}$$

$$\lim_{x \rightarrow -6^+} 6x+7x^2$$

↓

$$-6 \cdot 6 + 7 \cdot (-6)^2$$

$$-36 + 7 \cdot 36 = 216$$

$$\lim_{x \rightarrow -6^+} \frac{1}{x+6}$$

↓

$$\frac{1}{0^+} = +\infty$$

Il limite tende a $+\infty$

Derivate

$$\frac{d}{dx} \left(\sqrt{\frac{6x+7x^2}{x+6}} \right) = \frac{1}{2\sqrt{\frac{6x+7x^2}{x+6}}} \cdot \frac{(6+14x^2) \cdot (x+6) - (6x+7x^2) \cdot 1}{(x+6)^2}$$

$$\frac{\sqrt{x+6} (7x^2 + 84x + 36)}{2\sqrt{6x+7x^2} \cdot (x+6)^2} \cdot \frac{\sqrt{x+6}}{\sqrt{x+6}}$$

Segue derivação

$$x+6 > 0; 2\sqrt{6x+7x^2} \cdot (x+6)^2 > 0$$

\leftarrow

\Downarrow

$$\frac{\sqrt{x+6} (7x^2 + 84x + 36)}{2\sqrt{6x+7x^2} \cdot (x+6)^2} > 0$$

$$x \in (-6, -\frac{6}{7}) \cup (0, +\infty)$$

$$\left\{ \begin{array}{l} \sqrt{x+6} (7x^2 + 84x + 36) > 0 \quad (1) \\ 2\sqrt{6x+7x^2} (x+6)^2 > 0 \quad (2) \end{array} \right.$$

$$(1) \left\{ \begin{array}{l} \sqrt{x+6} > 0 \\ 7x^2 + 84x + 36 > 0 \end{array} \right. \quad \left. \begin{array}{l} x > -6 \\ \Delta = 7056 - 4 \cdot 7 \cdot 36 = 6048 \end{array} \right.$$

$$\Delta = 7056 - 4 \cdot 7 \cdot 36 = 6048$$

$$x_{1,2} = \frac{-84 \pm 6\sqrt{42}}{14} =$$

$$6\sqrt{42} \leftarrow \sqrt{5}$$

$$(1) \left\{ \begin{array}{l} x > -6 \\ \dots \end{array} \right.$$

$$\left(-\infty, \frac{-42 - 6\sqrt{42}}{7} \right) \cup \left(\frac{-42 + 6\sqrt{42}}{7}, +\infty \right)$$

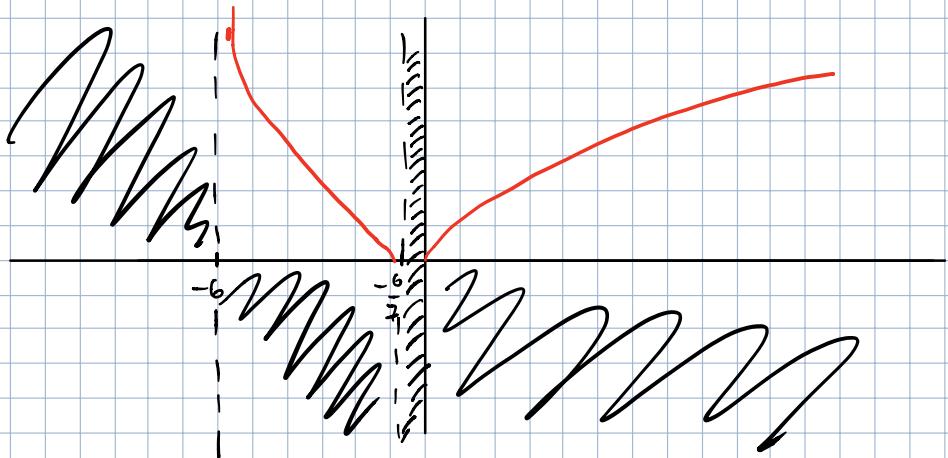
$$(1) x \in \left(\frac{-42 + 6\sqrt{42}}{7}, +\infty \right)$$

$$(2) x \in (-\infty, -6) \cup \left(-6, -\frac{6}{7} \right) \cup (0, +\infty)$$

$x \in (0, +\infty)$ cresce de 0, $+\infty$

decrece de $-\infty, 0$

$\lim f(0, +\infty)$



$$\textcircled{2} \quad \int \frac{l}{6 \cos(x) + 8} dx \quad \text{sost. trigonometrica} \quad \frac{1 - \tan\left(\frac{x}{2}\right)^2}{\cos(x)} = \frac{1 - \tan\left(\frac{x}{2}\right)^2}{1 + \tan\left(\frac{x}{2}\right)^2}$$

$$\int \frac{1}{6 \cdot \frac{1 - \tan\left(\frac{x}{2}\right)^2}{1 + \tan\left(\frac{x}{2}\right)^2} + 8} dx$$

$y = \tan\left(\frac{x}{2}\right) \quad x = 2 \arctan(y) \quad [2 \arctan^{-1}(x) \rightarrow 2 \arctan x]$
 $\cos(x) = \frac{1 - y^2}{1 + y^2} \quad dx = \frac{2}{1 + y^2} dy \quad \frac{\sin}{\cos} = \tan$
 $\sin(y) = \frac{2y}{1 + y^2} \quad \frac{d}{dy} \arctan = \frac{1}{1 + y^2}$

$$\int \frac{l}{6 \cdot \frac{1 - \tan\left(\frac{x}{2}\right)^2}{1 + \tan\left(\frac{x}{2}\right)^2} + 8} \cdot \frac{2}{1 + y^2} dy =$$

$$\int \frac{l}{6 \cdot \frac{1 - y^2}{1 + y^2} + 8} \cdot \frac{2}{1 + y^2} dy = \int \frac{l}{\underbrace{6 - 6y^2}_{1 + y^2} + 8} \cdot \frac{2}{1 + y^2} dy$$

$$\int \frac{1 + y^2}{6 - 6y^2 + 8 + 8y^2} \cdot \frac{2}{1 + y^2} dy = \int \frac{1 + y^2}{14 + 2y^2} \cdot \frac{2}{1 + y^2} dy =$$

$$\int \frac{2}{14 + 2y^2} dy = \int \frac{2}{2(7 + y^2)} dy = \int \frac{l}{7 + y^2} dy =$$

$$\boxed{\int \frac{1}{x^2 + Q^2} dx = \frac{1}{Q} \operatorname{arctan}\left(\frac{x}{Q}\right)} \quad ||| \quad \frac{1}{\sqrt{7}} \operatorname{arctan}\left(\frac{y}{\sqrt{7}}\right) =$$

$$= \frac{1}{\sqrt{7}} \operatorname{arctan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right) = \frac{\operatorname{arctan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right)}{\sqrt{7}} + C =$$

razionalizzazione:

$$\frac{\operatorname{arctan}\left(\frac{\sqrt{7} \tan\left(\frac{x}{2}\right)}{7}\right)}{\sqrt{7}} = \frac{\sqrt{7} \operatorname{arctan}\left(\frac{\sqrt{7} \tan\left(\frac{x}{2}\right)}{7}\right)}{7} + C$$

③ $\lim_{x \rightarrow +\infty} x^{-\frac{1}{9}} \int_0^x t^{\frac{8}{9}} \sqrt{t^2 + 6} dt$ é contínua e derivável,
não relopital

$$\lim_{x \rightarrow +\infty} \frac{\int_0^x t^{\frac{8}{9}} \sqrt{t^2 + 6} dt}{x^{\frac{1}{9}}} \stackrel{H}{\rightarrow} \lim_{x \rightarrow +\infty} \frac{x^{\frac{8}{9}} \sqrt{x^2 + 6}}{9x^{\frac{1}{9}}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 6}}{9} = \lim_{x \rightarrow +\infty} \sqrt{x^2 + 6} = +\infty \quad \lim_{x \rightarrow +\infty} 9 = 9 \quad \nearrow = +\infty$$

$\Downarrow x \rightarrow 0^+$

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x^2 + 6}}{9} = \frac{\sqrt{6}}{9}$$

$$\lim_{x \rightarrow +\infty} (x(\ln x)^n)^{-1} \int_1^x (\ln t)^n dt \quad l = \gamma \\ g = \ln x^n$$

$$\lim_{x \rightarrow +\infty} \frac{\int_1^x (\ln t)^n dt}{x(\ln x)^n} \stackrel{H}{\rightarrow} \frac{\cancel{\ln(x)^n}}{\cancel{\ln(x)^n} + n \ln(x)^{n-1}} \underset{\infty}{\cancel{1}} = 0$$

$$f(x) = \sqrt{\frac{x^2 - 4x}{1 - x^2}}$$

$$\frac{x^2 - 4x}{1 - x^2} \geq 0 \quad \text{e} \quad 1 - x^2 \neq 0 \Rightarrow -x^2 \neq 1$$

$$\downarrow \\ x \neq \pm 1$$

$$\begin{cases} x^2 - 4x \geq 0 \\ 1 - x^2 > 0 \end{cases} \Rightarrow \begin{cases} x < -4 \text{ ou } x > 0 \\ -1 < x < 1 \end{cases} \quad \begin{matrix} \text{---} & \text{---} \\ x < -4 & x > 0 \\ \text{---} & \text{---} \end{matrix}$$

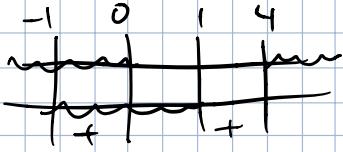
$$-1 < x < 1$$

$$* \begin{cases} x(x-4) > 0 \\ -1 < x < 1 \end{cases} \quad \begin{cases} x > 0 \\ x > 4 \end{cases} \quad \begin{cases} x > 0 \\ x < 4 \end{cases}$$

$$x < 0 \cup x > 4$$

quindi

$$\begin{cases} x < 0 \cup x > 4 \\ -1 < x < 1 \end{cases}$$



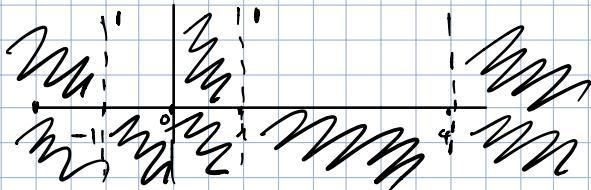
$$[-1 < x < 0 \cup 1 < x < 4] \\ \boxed{\text{Dom}(f) \in]-1, 0) \cup]1, 4)}]$$

Intersezione così:

$$f(0) = 0 \quad f(x) = 0 \text{ e.d.} \rightarrow \text{intersezione } (0, 0)$$

Segno

$$\text{pos } (-1, 0) \cup (1, 4)$$



limiti

$$\lim_{x \rightarrow +\infty} \sqrt{\frac{x^2 - 4x}{1 - x^2}} \rightarrow \text{indeterminato}$$

$$\lim_{x \rightarrow -\infty} \sqrt{\frac{x^2 - 4x}{1 - x^2}} \rightarrow \text{indeterminato}$$

$$\lim_{x \rightarrow -1^+} = +\infty \text{ grafico} \quad \lim_{x \rightarrow +1^-} = +\infty$$

$$\lim_{x \rightarrow -1^-} = i(\infty) \quad \lim_{x \rightarrow +1^+} = i(\infty)$$

Derivate

$$\frac{d}{dx} \sqrt{\frac{x^2 - 4x}{1 - x^2}} \rightarrow \frac{d}{dx} \sqrt{g} \cdot \frac{d}{dx} \left(\frac{x^2 - 4x}{1 - x^2} \right) =$$

$$\frac{1}{2\sqrt{\frac{x^2-4}{1-x^2}}} \cdot \frac{(2x-4)(1-x^2) - (x^2-4x) \cdot (-2x)}{(1-x)^2} =$$

$$\frac{1}{2\sqrt{\frac{x^2-4x}{1-x^2}}} \cdot \frac{2x-2x^3-4+4x^2+2x^3-8x^2}{(1-x)^2} =$$

$$\frac{1}{2\sqrt{\frac{x^2-4x}{1-x^2}}} \cdot \frac{2x-4-4x^2}{(1-x)^2} = \frac{1}{2\sqrt{\frac{x^2-4x}{1-x^2}}} \cdot \frac{2(x-2-2x^2)}{(1-x)^2} =$$

$$\frac{\sqrt{1-x^2}}{\sqrt{x^2-4x}} \cdot \frac{x-2-2x^2}{(1-x)^2} = \frac{\sqrt{1-x^2}(x-2-2x^2)}{\sqrt{x^2-4x}(1-x)^2}$$

Se gør derivate

$$\frac{\sqrt{1-x^2}(x-2-2x^2)}{\sqrt{x^2-4x}(1-x)^2}$$

$$CE: \begin{aligned} & 1-x^2 < 0 \quad x < \pm 1 \\ & x^2-4x < 0 \end{aligned}$$

$$\bullet \sqrt{x^2-4x}(1-x)^2 > 0$$

$$\bullet (-\infty, -1) \cup (1, +\infty)$$

$$\bullet (0, 4)$$

$$\bullet x = 0 \quad CE \in [-\infty, -1] \cup [0, +\infty]$$

$$\begin{array}{l} x = 4 \\ x = 1 \text{ no} \end{array}$$

$$\star \left\{ \sqrt{1-x^2}(x-2-2x^2) \geq 0 \right.$$

$$\bullet \left\{ \sqrt{x^2-4x}(1-x)^2 > 0 \right.$$

$$\star \left\{ \sqrt{1-x^2} \geq 0 \quad \left\{ \begin{array}{l} [-1, 1] \\ (x-2-2x^2) > 0 \end{array} \right. \right. \quad \left. \left. \begin{array}{l} x \in \mathbb{Q} \Rightarrow (x \in \mathbb{Q}) \text{ red} \\ x^2-4x > 0 \end{array} \right. \right. \quad \underline{\quad}$$

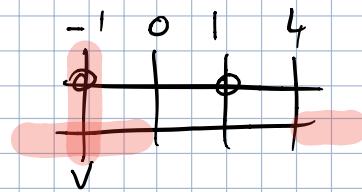
$$\left. \left. \begin{array}{l} (-\infty, 0) \cup (4, +\infty) \\ (\forall x \in \mathbb{R} \setminus \{1\}) \end{array} \right. \right. \quad \left. \left. \begin{array}{l} x > 0 \\ x-4 > 0 \end{array} \right. \right. \quad \begin{array}{l} \text{red} \\ \text{no} \end{array}$$

$$\bullet \left\{ \sqrt{x^2-4x} \geq 0 \quad \left\{ \begin{array}{l} (-\infty, 0) \cup (4, +\infty) \\ (1-x)^2 > 0 \end{array} \right. \right. \quad \left. \left. \begin{array}{l} x > 0 \\ x-4 > 0 \end{array} \right. \right. \quad \begin{array}{l} \text{red} \\ \text{no} \end{array}$$

$$\Rightarrow (-\infty, 0) \cup (4, +\infty)$$

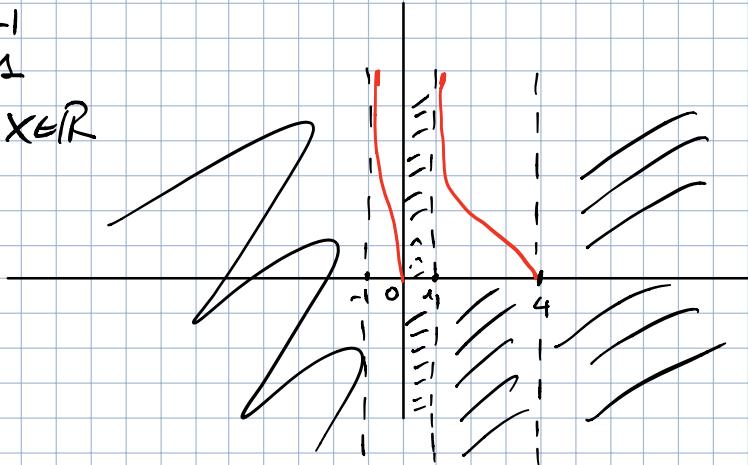
$$(-\infty, 0) \cup (4, +\infty)$$

$$S \begin{cases} 1 \\ -1 \\ (-\infty, 0) \cup (4, +\infty) \end{cases}$$



$S = -1$
Siempre
decreciente

• $\begin{cases} \sqrt{1-x^2} \leq 0 < -1 \\ (x-2-2x^2) < 0 \quad x \in \mathbb{R} \end{cases}$



$$\lim_{x \rightarrow +\infty} x^2 \log \left(\frac{3+2x^2}{2x^2} \right) = \lim_{x \rightarrow \infty} \log \left(\frac{3+2x^2}{2x^2} \right)^{x^2} =$$

$$\lim_{x \rightarrow +\infty} \log \left(\frac{3}{2x^2} + \frac{1}{x^2} \right)^{x^2} = \lim_{x \rightarrow \infty} \log \left(\frac{1}{\frac{2x^2}{3}} + 1 \right)^{\frac{2x^2}{3} \cdot \frac{2}{2}}$$

$$\lim_{x \rightarrow \infty} \frac{3}{2} \log e = \lim_{x \rightarrow \infty} \log e^{\frac{3}{2}} = e^{\frac{3}{2}} = x = \frac{3}{2}$$

$$\lim_{x \rightarrow +\infty} \frac{3}{2} = \frac{3}{2}$$