Correzione esame 21/01/19

- $f(x) = \sqrt{\dfrac{6x+7x^2}{x+6}}$
- $\displaystyle\int \frac{1}{6\cos(x)+8}$
- $\lim_{x\to 0^+}$ $= \dfrac{\sqrt{6}}{9}$
- $\lim_{x\to +\infty}$ $= +\infty$

$$x^{-9}\int_0^x t^8\sqrt{t^2+6}$$

① SDF

$f(x)=\sqrt{\dfrac{6x+7x^2}{x+6}}$

$x \neq -6$

$\dfrac{6x+7x^2}{x+6} > 0$

$\begin{cases} 6x+7x^2>0 \\ x>-6 \end{cases}$

$\begin{cases} \_\_\_\_ \\ x>-6 \end{cases}$ $\begin{cases} x>0 \\ x>-\frac{6}{7} \end{cases}$

$-\frac{6}{7}$ 0

$+$ $+$

-6 $-\frac{6}{7}$ 0

$+$ $+$

$x<-\frac{6}{7} \vee x>0$

$-6<x<-\frac{6}{7} \vee x>0$

Dom $f \in \left]-6, -\frac{6}{7}\right[\cup \left]0, +\infty\right)$

Intersezioni

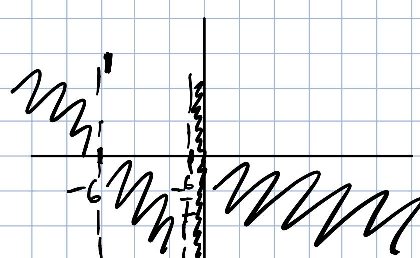
$f(0)=\sqrt{\dfrac{0+0}{6}} = 0$

$f(x)=0 \in Q$ Interseca $(0,0)$

Segno funzione

$-6<x<-\frac{6}{7} \vee x>0$

pos pos



Limiti

$$\lim_{x\to\infty} \sqrt{\frac{6x+7x^2}{x+6}} = \lim_{x\to+\infty} \sqrt{\frac{x^{\cancel{2}}(\frac{6}{x}+7)}{\cancel{x}(1+\frac{6}{x})}} = \sqrt{+x+7} = \sqrt{\infty} = \infty$$

$$\lim_{x\to-\infty} = \sqrt{-\infty} \text{ indefinito}$$

$$\lim_{x\to-6^+} \sqrt{\frac{6x+7x^2}{x+6}} =$$

poiché la funzione non è definita a sinistra di -6 calcolo solamente il lim a destra

$$\lim_{x\to-6^+} \frac{6x+7x^2}{x+6}$$

$$\lim_{x\to-6^+} (6x+7x^2)\cdot\frac{1}{x+6}$$

$$\lim_{x\to-6^+} 6x+7x^2 \qquad \lim_{x\to-6^+} \frac{1}{x+6}$$

$$\downarrow \qquad \downarrow$$

$$-6^+\cdot 6 + 7\cdot -6^2 \qquad \frac{1}{0^+} = +\infty$$

$$-36+7\cdot 36 = 216$$

Il limite tende a $+\infty$

Derivata

$$\frac{d}{dx}\left(\sqrt{\frac{6x+7x^2}{x+6}}\right) = \frac{1}{2\sqrt{\frac{6x+7x^2}{x+6}}} \cdot \frac{(6+14x^2)\cdot(x+6)-(6x+7x^2)}{(x+6)^2}$$

$$\frac{\sqrt{x+6}}{2\sqrt{6x+7x^2}} \cdot \qquad =$$

$$\frac{\sqrt{x+6}\,(7x^2+84x+36)}{2\sqrt{6x+7x^2}\cdot(x+6)^2}$$

Segno derivata

$x+6>0$; $2\sqrt{6x+7x^2}\cdot(x+6)^2>0$

CE $\Downarrow$

$x \in \left(-6,-\frac{6}{7}\right) \cup (0,+\infty)$

$$\frac{\sqrt{x+6}\,(7x^2+84x+36)}{2\sqrt{6x+7x^2}\cdot(x+6)^2}>0$$

$$\begin{cases} \sqrt{x+6}\,(7x^2+84x+36)>0 & ① \\ 2\sqrt{6x+7x^2}\,(x+6)^2>0 & ② \end{cases}$$

① $\begin{cases} \sqrt{x+6}>0 \\ 7x^2+84x+36>0 \end{cases}$ $\Big\}\ x>-6$

$\Downarrow$

$\Delta = 7056-4\cdot 7\cdot 36 = 6048$ $\to 6\sqrt{42}$

$x_{1,2} = \frac{-84 \pm 6\sqrt{42}}{14} =$

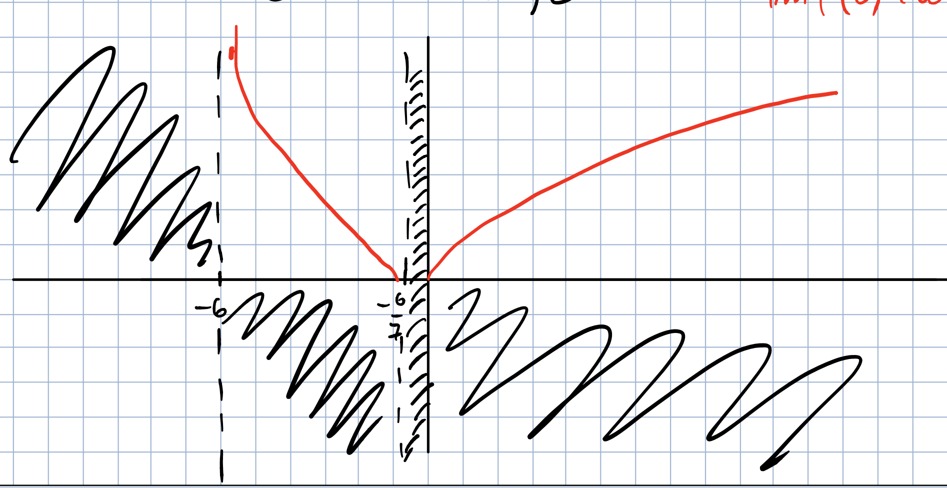
① $\begin{cases} x>-6 \\ \left(-\infty, \frac{-42-6\sqrt{42}}{7}\right) \cup \left(\frac{-42+6\sqrt{42}}{7},+\infty\right) \end{cases}$

$\begin{cases} ① \ x \in \left(\frac{-42+6\sqrt{42}}{7},+\infty\right) \\ ② \ x \in (-\infty,-6) \cup \left(-6,-\frac{6}{7}\right) \cup (0,+\infty) \end{cases}$

$x \in (0,+\infty)$ cresce da $0,+\infty$

decresce da $-\infty,0$

$\mathrm{Im}\, f\, (0,+\infty)$



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$$\int \frac{1}{6\cos(x)+8}\,dx \qquad \text{sost. trigonometrica} \quad \cos(x)=\frac{1-\tan\left(\frac{x}{2}\right)^2}{1+\tan\left(\frac{x}{2}\right)^2}$$

$$\int \frac{1}{6\cdot\frac{1-\tan\left(\frac{x}{2}\right)^2}{1+\tan\left(\frac{x}{2}\right)^2}+8}\,dx$$

$y=\tan\left(\frac{x}{2}\right) \quad x=2\arctan(x) \quad [2\tan^{-1}(x)\to 2\arctan x]$

$\cos(x)=\frac{1-y^2}{1+y^2} \quad dx=\frac{2}{1+y^2}\,dy \quad \frac{\text{sen}}{\cos}=\tan$

$\text{sen}(x)=\frac{2y}{1+y^2} \quad \frac{d}{dx}\arctan=\frac{1}{1+y^2}$

$$\int \frac{1}{6\cdot\frac{1-\tan\left(\frac{x}{2}\right)^2}{1+\tan\left(\frac{x}{2}\right)^2}+8}\cdot\frac{2}{1+y^2}\,dy=$$

$$\int \frac{1}{6\cdot\frac{1-y^2}{1+y^2}+8}\cdot\frac{2}{1+y^2}\,dy=\int\frac{1}{\frac{6-6y^2}{1+y^2}+8}\cdot\frac{2}{1+y^2}\,dy$$

sopra

$$\int \frac{1+y^2}{6-6y^2+8+8y^2}\cdot\frac{2}{1+y^2}\,dy=\int\frac{\cancel{1+y^2}}{14+2y^2}\cdot\frac{2}{\cancel{1+y^2}}\,dy=$$

$$\int\frac{2}{14+2y^2}\,dy=\int\frac{2(1)}{2(7+y^2)}\,dy=\int\frac{1}{7+y^2}\,dy=$$

$$\boxed{\int\frac{1}{x^2+a^2}\,dx=\frac{1}{a}\arctan\left(\frac{x}{a}\right)} \quad \frac{1}{\sqrt{7}}\arctan\left(\frac{y}{\sqrt{7}}\right)=$$

$$=\frac{1}{\sqrt{7}}\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right)=\frac{\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right)}{\sqrt{7}}+c=$$

FAC

razionalizzaz.

$$\frac{\arctan\left(\frac{\sqrt{7}\tan\left(\frac{x}{2}\right)}{7}\right)}{\sqrt{7}}=\frac{\sqrt{7}\arctan\left(\frac{\sqrt{7}\tan\left(\frac{x}{2}\right)}{7}\right)}{7}+c$$

③ $\lim_{x \to +\infty} x^{-9} \int_0^x t^8 \sqrt{t^2+6}\, dt$ è continua e derivabile, uso Hôpital

$$\lim_{x \to +\infty} \frac{\int_0^x t^8 \sqrt{t^2+6}\, dt}{x^9} \xrightarrow{\hat{H}} \lim_{x \to +\infty} \frac{\cancel{x^8}\sqrt{x^2+6}}{9\cancel{x^8}} =$$

$$= \lim_{x \to +\infty} \frac{\sqrt{x^2+6}}{9} = \left.\begin{array}{l} \lim_{x \to +\infty} \sqrt{x^2+6} = +\infty \\ \lim_{x \to +\infty} 9 = 9 \end{array}\right\rangle = +\infty$$

$\Downarrow x \to 0^+$

$$\lim_{x \to 0^+} \frac{\sqrt{x^2+6}}{9} = \frac{\sqrt{6}}{9}$$

$$\lim_{x \to +\infty} (x(\log x)^n)^{-1} \int_1^x (\log t)^n\, dt$$

$f = x$
$g = \ln x^n$

$$\lim_{x \to +\infty} \frac{\int_1^x (\log t)^n dt}{x(\log x)^n} \xrightarrow{\hat{H}} \frac{\cancel{\log(x)^n}}{\cancel{\ln(x)^n} + n \ln(x)^{n-1}} \; 1 = 0$$

∞

$f(x) = \sqrt{\frac{x^2-4x}{1-x^2}}$ $\qquad \frac{x^2-4x}{1-x^2} \geq 0$ e $1-x^2 \neq 0 \Rightarrow -x^2 \neq 1$

$\Downarrow$

$x \neq \pm 1$

$\begin{cases} x^2-4x \geq 0 \; * \\ 1-x^2 > 0 \; \bullet \end{cases}$ $\qquad \bullet \; -x^2 > -1$

$x^2 < 1$

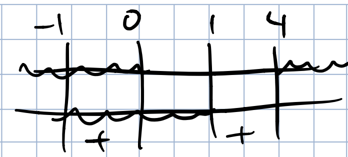
$x < \pm 1$ $\quad -1 \quad 1$ $\quad +$ $\quad +$

$-1 < x < 1$

$* \begin{cases} x(x-4) > 0 \\ -1 < x < 1 \end{cases} \rightarrow \begin{cases} x > 0 \\ x > 4 \end{cases}$ $\quad 0 \quad 4$ $\quad +$ $\quad +$

$x < 0 \cup x > 4$

quindi $\begin{cases} x<0 \cup x>4 \\ -1<x<1 \end{cases}$



$-1<x<0 \cup 1<x<4$

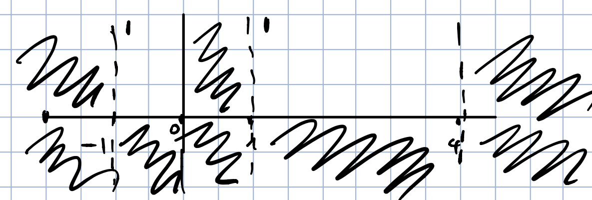
$\boxed{\text{Dom} f \in \,]-1, 0) \cup \,]1, 4)}$

Intersezioni assi:

$f(0) = 0$ $\quad f(x) = 0 \in D \rightarrow$ interseca $(0,0)$

Segno

pos $(-1,0) \cup (1,4)$



limiti

$\lim_{x \to +\infty} \sqrt{\frac{x^2-4x}{1-x^2}} \rightarrow$ indefinito

$\lim_{x \to -\infty} \sqrt{\frac{x^2-4x}{1-x^2}} \rightarrow$ indefinito

$\lim_{x \to -1^+} = +\infty$ grafico $\quad \lim_{x \to +1^+} = +\infty$

$\lim_{x \to -1^-} = i(\infty)$ $\quad \lim_{x \to +1^-} = i(\infty)$

Derivata

$\frac{d}{dx} \sqrt{\frac{x^2-4x}{1-x^2}} \rightarrow \frac{d}{dx} \sqrt{g} \cdot \frac{d}{dx} \left(\frac{x^2-4x}{1-x^2}\right) =$

$$\frac{1}{2\sqrt{\frac{x^2-4}{1-x^2}}} \cdot \frac{(2x-4)(1-x^2)-(x^2-4x)\cdot(-2x)}{(1-x)^2} =$$

$$\frac{1}{2\,\frac{\sqrt{x^2-4x}}{\sqrt{1-x^2}}} \cdot \frac{2x-\cancel{2x^3}-4+4x^2+\cancel{2x^3}-8x^2}{(1-x)^2} =$$

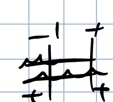
$$\frac{1}{2\,\frac{\sqrt{x^2-4x}}{\sqrt{1-x^2}}} \cdot \frac{2x-4-4x^2}{(1-x)^2} = \frac{1}{\cancel{2}\,\frac{\sqrt{x^2-4x}}{\sqrt{1-x^2}}} \cdot \frac{\cancel{2}(x-2-2x^2)}{(1-x)^2} =$$

$$\frac{\sqrt{1-x^2}}{\sqrt{x^2-4x}} \cdot \frac{x-2-2x^2}{(1-x)^2} = \frac{\sqrt{1-x^2}\,(x-2-2x^2)}{\sqrt{x^2-4x}\,(1-x)^2}$$

Segno derivata

$$\frac{\sqrt{1-x^2}\,(x-2-2x^2)}{\sqrt{x^2-4x}\,(1-x)^2}$$

CE: • $1-x^2<0$ $\quad x<\pm 1$



• $x^2-4x<0$

• $\sqrt{x^2-4x}\,(1-x)^2>0$

• $(-\infty,-1)\cup(1,+\infty)$

• $(0,4)$

• $x=0$
$x=4$
$x=1$ NO

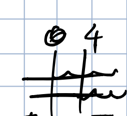
$CE\ [-\infty,-1]\cup[0,+\infty]$

$$\star\begin{cases}\sqrt{1-x^2}\,(x-2-2x^2)\geq 0\\ \bullet\ \sqrt{x^2-4x}\,(1-x)^2>0\end{cases}$$

$$\star\begin{cases}\sqrt{1-x^2}\geq 0\\ (x-2-2x^2)>0\end{cases} \quad \begin{cases}[-1,1]\\ x\in\emptyset\end{cases} \Rightarrow (x\in\emptyset)$$

$x^2-4x>0$

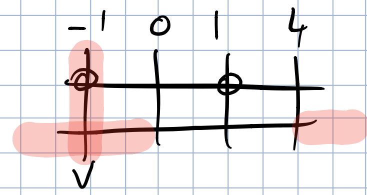
$$\begin{cases}x>0\\ x-4>0 \quad x>4\end{cases}$$



$$\bullet\begin{cases}\sqrt{x^2-4x}\geq 0\\ (1-x)^2>0\end{cases} \quad \begin{cases}(-\infty,0)\cup(4,+\infty)\\ \forall x\in\mathbb{R}\setminus\{1\}\end{cases} \Rightarrow (-\infty,0)\cup(4+\infty)$$

$(-\infty,0)(4,+\infty)$

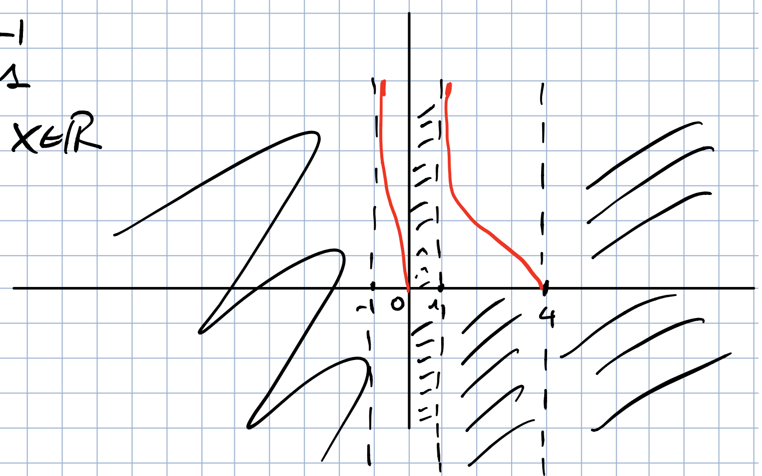
$$S\begin{cases} 1 \\ -1 \\ (-\infty,0)\cup(4,+\infty) \end{cases}$$



$S = -1$

Sempre decrescente

$$\begin{cases} \sqrt{1-x^2} \le 0 \quad < \begin{matrix} -1 \\ 1 \end{matrix} \\ (x-2-2x^2) < 0 \quad x \in \mathbb{R} \end{cases}$$



$$\lim_{x \to +\infty} x^2 \log\left(\frac{3+2x^2}{2x^2}\right) = \lim_{x \to \infty} \log\left(\frac{3+2x^2}{2x^2}\right)^{x^2} =$$

$$\lim_{x \to +\infty} \log\left(\frac{3}{2x^2}+1\right)^{x^2} = \lim_{x \to \infty} \log\left(\frac{1}{\frac{2x}{3}^2}+1\right)^{\frac{2}{3}x^2 \cdot \frac{3}{2}}$$

$$\lim_{x \to +\infty} \frac{3}{2} \log e = \lim_{x \to \infty} \log e^{\frac{3}{2}} = e^x = x = \frac{3}{2}$$

$$\lim_{x \to +\infty} \frac{3}{2} = \frac{3}{2}$$