ANNO 2017

23/01

$$\int \frac{1}{17\sin x + 6\cos(x) + 18}\,dx$$

uso la sostituzione

$\sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$ dove $t = tg\left(\frac{x}{2}\right)$ $dx = \frac{2}{1+t^2}dt$

$$\int \frac{1}{\frac{34t}{1+t^2} + \frac{6-6t^2}{1+t^2} + \frac{18+18t^2}{\underbrace{1+t^2}_{\text{ho fatto mcm}}}} \cdot \frac{2}{1+t^2}\,dt$$

che diventa

$$\int \frac{12\cancel{(1+t^2)}}{\underbrace{{}_6\cancel{12}t^2 + {}^{17}\cancel{34}t + \cancel{24}^{12}}} \cdot \frac{dt}{\cancel{(1+t^2)}}$$ => Razionale con grado N < grado D => guardo il Δ => $\Delta = 1 > 0$

$t_1 = -\frac{3}{2}$ $t_2 = -\frac{4}{3}$

Riscrivo il tutto come $a(t-t_1)(t-t_2)$ e cerco $A, B \in \mathbb{R}$ tc

$$\frac{1}{6t^2+17t+12} = \frac{A}{t+\frac{3}{2}}{}^{*} + \frac{B}{t+\frac{4}{3}} = \frac{A\left(t+\frac{4}{3}\right) + B\left(t+\frac{3}{2}\right)}{\left(t+\frac{3}{2}\right)\left(t+\frac{4}{3}\right)}$$

Raccolgo A e B che contengono t

$$\frac{(A+B)t + \frac{3}{4}A + \frac{3}{2}B}{6t^2+17t+12}$$

Trova A+B = al coeff del termine che contiene t => 0

e il resto = al coeff del termine di grado 0 => 1

$$\begin{cases} A+B=0 \\ \frac{3}{4}A + \frac{3}{2}B = 1 \end{cases} \quad \begin{cases} A=-B \\ -\frac{3}{4}B + \frac{3}{2}B = 1 \end{cases} \quad \begin{cases} A=-B \\ \frac{3}{2}B = 1 \end{cases} \quad \underbrace{\begin{cases} B = 2/3 \\ A = -2/3 \end{cases}}_{\text{Inserisco il risultato qui}^{*}}$$

$$\int -\frac{2}{3}\cdot\frac{1}{t+\frac{2}{3}} + \frac{2}{3}\cdot\frac{1}{t+\frac{4}{3}} = -\frac{2}{3}\int\frac{1}{t+\frac{2}{3}} + \frac{2}{3}\int\frac{1}{t+\frac{4}{3}}$$

$$-\frac{2}{3}\log\left|t+\frac{2}{3}\right| + \frac{4}{3}\log\left|t+\frac{4}{3}\right|$$

Sostituisco t

$$-\frac{2}{3}\log\left|tg\left(\frac{x}{2}\right)+\frac{2}{3}\right| + \frac{}{3}\log\left|tg\left(\frac{x}{2}\right)+\frac{4}{3}\right.$$

06/02

$y = 8x - 18 \quad x = \frac{1}{8}y - \frac{\cancel{18}^{9}}{\cancel{8}_{4}} \quad dx = \frac{1}{8}dy$

$\int \arctan \sqrt{8x - 18}\, dx$

Sostituisco

$\int \arctan \sqrt{y} \cdot \frac{1}{8} dy = \frac{1}{8} \int \arctan \sqrt{y}\, dy$

Per parti

$\frac{1}{8}\left[\left(\underbrace{y \cdot \arctan \sqrt{y}}_{F(x)\ g(x)}\right) - \int \frac{1}{1+y} \cdot \frac{1}{2\sqrt{y}} \cdot y\right]$

$= \frac{1}{8}\left[y \arctan\sqrt{y} - \int \frac{\sqrt{y}}{2(1+y)}\right]$

Già pronto

$-\frac{1}{2}\int \frac{\sqrt{y}}{1+y}$

Sostituisco $\sqrt{y} = t$

$y = t^2 \quad \frac{1}{2\sqrt{y}} = dt \quad dt\, 2\sqrt{y} = dy$

$-\frac{1}{2}\int \frac{t}{1+t^2} 2t\, dt = \frac{2}{2}\int \frac{t^2}{1+t^2} dt$ → Grado di N = Grado D

→ Divido

$$\begin{array}{l|l} t^2 \quad - \quad - & t^2 + 1 \\ -t^2 \quad -1 & \\ \hline \quad -1 & 1 \end{array}$$

$= \underbrace{\int 1 - \frac{1}{1+t^2}}$

Integrale di una somma = Somma di integrali forma $\dfrac{f'(x)}{1+[f(x)]^2} = arctg(f(x))$

$= \quad + \int_{||} - \int \frac{1}{1+t^2}$

Adesso riprendo gli integrali già risolti e poi sostituisco le 2 variabili temporanee usate

$= \quad + t - arctg(t)$

$$\frac{(8x-18)\, arctg\sqrt{8x-18} - \sqrt{8x-18} + arctg(\sqrt{8x-18})}{8}$$

20/02

$$\int (8x+18)\log(x^2+7)\,dx$$

Espando e scrivo come ① ②

$$\int 8x\cdot\log(x^2+7)dx + \int 18\log(x^2+7)\,dx = 8\int x\log(x^2+7)dx + 18\int \log(x^2+7)dx$$

① Risolvo per parti

$$8\left[\frac{x^2}{2}\cdot\log(x^2+7) - \int\frac{x^2}{2}\cdot\frac{2x}{x^2+7}\right] = 8\left[\frac{x^2}{2}\cdot\log(x^2+7) - \underbrace{\int\frac{x^3}{x^2+7}}\right] =$$

Divido

$$\begin{array}{cccc|c} x^3 & -^2 & -^1 & -^0 & x^2+7 \\ \hline -x^3 & & -7x & & x \\ / & & -7x & & \end{array}$$

Riscrivo come

$$-\int x - \frac{7x}{x^2+7}\,dx$$

$$-\int x\,dx + \frac{7}{2}\int\frac{\frac{2}{\cancel{7}}\cancel{7}x}{x^2+7}\,dx = -\frac{x^2}{2} + \frac{7}{2}\log|x^2+7|$$

$$① = 8\left[\frac{x^2}{2}\cdot\log(x^2+7) - \frac{x^2}{2} + \frac{7}{2}\log|x^2+7|\right)$$

2) Per parti

$$18\left[(x\cdot\log(x^2+7)) - \int x\cdot\frac{2x}{x^2+7}\right] = 18\left[x\cdot\log(x^2+7) - \right.$$

$$-\int \frac{2x^2}{x^2+7} = -2\int \frac{x^2}{x^2+7}$$ Divisione

$$\begin{array}{ccc|c} x^2 & -' & -^0 & x^2+7 \\ \hline -x^2 & & -7 & 1 \\ / & & -7 & \end{array}$$

Riscrivo come

$$-2\int 1 - \frac{7}{x^2+7}$$

$$= -2x + 2\int \frac{7}{x^2+7}$$

$$-2x - 14\int \frac{1}{x^2+7}\,dx$$ chiamo $\frac{x}{\sqrt{7}} = y$ $dx = \sqrt{7}\,dy$

$$= \int \frac{\sqrt{7}}{7y^2+7}\,du = \frac{1}{\sqrt{7}}\int \frac{1}{y^2+7} = \frac{1}{\sqrt{7}}\operatorname{arctg}(y)$$

$$= 2x - 2\sqrt{7}\operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right)$$

SOLUZIONE

$$4x^2\log(x^2+7) - 4x^2 + 28\log(x^2+7) + 18x\log(x^2+7) + 2x - 2\sqrt{7}\operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right)$$

20/06

$y = x^4 \quad 4x^3 dx = dy \quad dx = \frac{dy}{4x^3}$

$\int x^3 \cdot \cos(x^4) dx$ Sostituisco

$\int \cancel{x^3} \cdot \cos(y) \cdot \frac{dy}{\cancel{4x^3}} = \frac{1}{4} \int \cos(y) dy = \frac{1}{4} \sin(x^4) + c$

19/07

$\int \log(9x^2 + 24x + 16) dx$

Per parti

$x \log(9x^2+24x+16) - \int x \cdot \frac{1}{9x^2+24x+16} \cdot 18x+24 \, dx$

$x \cdot \log(9x^2+24x+16) - \frac{6}{9} \int \frac{x \cancel{(3x+4)}}{(3x+4)^{\cancel{2}}}$ $\quad y = 3x+4 \quad dx = 3x$

DA COPIARE

$dx = \frac{dy}{3}$

$+\frac{1}{9} \int \frac{y-4 \cdot \frac{1}{\cancel{3}}}{y} \frac{dy}{\cancel{3}}$ $\quad x = \frac{y-4}{3}$

$= +\frac{1}{9} \int \frac{y-4}{y} dy = +\frac{5}{9} \int \frac{\cancel{y}}{\cancel{y}} 1 \quad +24 \int \frac{1}{y}$

$= \frac{6 \, 3x+4}{9} + \frac{8 \log(3x+4)}{3}$

RISULTATO

$x \log(9x^2+24x+16) + \frac{2}{3}(3x+4) + \frac{8 \log(3x+4)}{3} + c$

27/09

$$\int \frac{5x}{9x^2+24x+\underset{16+1}{17}}$$ Grado N < Grado D $\Delta<0$

Chiamo $\boxed{x=ay+b}$ e sostituisco $*^2$

$9(a^2y^2+b^2+2aby)+24(ay+b)+17$ Svolgo

$9a^2y^2+9b^2+18aby+24ay+24b+17$ Isolo i termini con dentro y

$(9a^2)y^2+(18ab+24a)y+9b^2+24b+17$ *

Pongo il coefficiente del termine con $y^2=1$ e quello con $y'=0$

Quindi

$9a^2=1$ $a^2=\frac{1}{9}$ $\boxed{a=\frac{1}{3}}$

$(18ab+24a)=0$ $6a(3b+4)=0$ $2(3a+4)=6b+8=0$ $b=-\frac{4}{3}$

Vado a sostituire a e b qui *

$$\left(\cancel{9}\frac{1}{\cancel{9}}\right)y^2+\underbrace{\left(18\cdot\frac{1}{3}\cdot\left(\frac{-4}{3}\right)+24\cdot\frac{1}{3}\right)}_{0}y+\cancel{9}\cdot\frac{16}{\cancel{9}}+24\cdot\left(\frac{-4}{3}\right)+17$$

$y^2+\cancel{0y}+1=y^2+1$ Sostituisco a e b qui $*^2$

$x=\frac{1}{3}y-\frac{4}{3}$ $x=\frac{y-4}{3}$ $3x=y-4$

Sostituisco al N e sotto scrivo tutto come ho trovato, ricordando di cambiare anche dx

$y=3x+4$ $dy=3dx$ $dx=\frac{dy}{3}$

da sostituire alla fine

$$5\int\frac{\left(\frac{1}{3}y-\frac{4}{3}\right)}{y^2+1}\,\frac{dy}{3}=$$

$\frac{1}{3} \cdot 5 \int \frac{\frac{1}{3}y - \frac{4}{3}}{y^2+1} \frac{dy}{3} = \frac{5}{9} \int \frac{y}{y^2+1} dy - \frac{5}{9} \int \frac{4}{y^2+1}$

$\frac{5}{18} \int \frac{2y}{y^2+1} = \frac{5}{18} \log(y^2+1)$

$-\frac{20}{9} \int \frac{1}{y^2+1} dy = -\frac{20}{9} \operatorname{arctg}(y)$ Sostituisco y

$\frac{5}{18} \log(9x^2+16+24x+1) - \frac{20}{9} \operatorname{arctg}(3x+4)$

RISULTATO

$= \frac{5}{18} \log(9x^2+24x+17) - \frac{20}{9} \operatorname{arctg}(3x+4)$

ANNO 2016

25/01

Grado N = Grado D

→ Divisione

$$\int \frac{x^2}{(x-3)(x-5)}dx = \int \frac{x^2}{x^2-8x+15}$$

da copiare

$$\begin{array}{r|l} x^2 & x^2-8x+15 \\ \hline -x^2 & 1 \\ / & \end{array}$$

Riscrivo come

$$\int 1 + \frac{8x-15}{x^2-8x+15} = \overbrace{x} + \underbrace{\int \frac{8x-1}{x^2-8}}$$

$+8x-15$

Grado N < (

Guardo Δ

$\Delta = 4 > 0 \quad x_1 = 3 \quad x_2 = 5$

$$= \frac{A}{(x-3)} + \frac{B}{(x-5)} = \frac{A(x-5)+B(x-3)}{(x-3)(x-5)}$$

$$= \frac{(A+B)x - 5A - 3B}{(x-3)(x-5)} = \begin{cases} A+B=8 \\ -5A-3B=-15 \end{cases} \quad \underbrace{\begin{cases} A=-9/2 \\ B=25/2 \end{cases}}_{\text{Sostituisco}}$$

$$\int \frac{-9}{2} \cdot \frac{1}{(x-3)} + \frac{25}{2} \cdot \frac{1}{(x-5)} dx = \frac{-9}{2}\int \frac{1}{x-3}dx + \frac{25}{2}\int \frac{1}{x-5}dx$$

RISULTATO

$$x - \frac{9}{2}\log|x-3| + \frac{25}{2}\log|x-5|$$

08/02

$$\int \frac{1}{3\sin x - 5\cos x + 5}$$ Sostituisco tramite parametriche

$\sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t}{1+t^2}$ $t = tg\left(\frac{x}{2}\right)$ $dx = \frac{2}{1+t^2}dt$

$$= \int \frac{1}{\frac{6t}{1+t^2} - \frac{5-5t^2}{1+t^2} + 5(1+t^2)} \cdot \frac{2}{1+t^2}\, dt = \int \frac{\cancel{(1+t^2)}}{6t - \cancel{5} + 5t^2 + \cancel{5} + 5t^2} \cdot \frac{2}{\cancel{(1+t^2)}}\, dt$$

$$= 2\int \frac{1}{10t^2+6t}\, dt = 2\int \frac{1}{2t(5t+3)} = \frac{A}{2t} + \frac{B}{5t+3}$$

$$= \frac{A(5t+3)+B(2t)}{2t(5t+3)} = A5t + 3A + 2tb = t(5A+2B) + 3A$$

$$\begin{cases} 5A+2B=0 \\ 3A=1 \end{cases} \quad \begin{cases} A=1/3 \\ B=-5/6 \end{cases}$$ Sostituisco

$$2\int \frac{1}{3}\cdot\frac{1}{2t} - \frac{5}{6}\cdot\frac{1}{5t+3}\, dt = \frac{2}{3}\int\frac{1}{2t} - \frac{10}{6}\int\frac{1}{5t+3}$$

$$= \frac{2}{6}\int\frac{1}{t} - \frac{2\cancel{5}}{6}\cdot\frac{1}{\cancel{5}}\int\frac{5t}{5t+3}$$

RISULTATO

$$\frac{2}{6}\log|t| - \frac{2}{6}\log|5t+3| = \frac{1}{3}\log\left|tg\left(\frac{x}{2}\right)\right| - \frac{1}{3}\log\left|5tg\left(\frac{x}{2}\right)+3\right|$$

23/02

Grado N = Grado D → Divido

$$\int \frac{9x^2+32x+44}{9x^2+30x+41}$$

$9x^2+32x+44$	$9x^2+30x+41$
$-9x^2-30x-41$	1
$/ \quad 2x+3$	

Riscrivo come

$$\int 1+\frac{2x+3}{9x^2+30x+41}\,dx = \underbrace{x}_{\text{da copiare}} + \int \frac{2x+3}{9x^2+30x+41}$$

Grado N < Grado D
Quando Δ
$\Delta < 0$

$x = ay+b$ → sostituisco

$$9(ay+b)^2+30(ay+b)+41 = 9(a^2y^2+2aby+b^2)+30ay+30b+41$$

$$(9a^2)y^2+18aby+9b^2+30ay+30b+41 = (9a^2)y^2+(18ab+30a)y+9b^2+30b+41$$

$9a^2=1 \qquad a^2=1/9 \qquad a=1/3$

$18ab+30a=0 \qquad 18ab+30\cdot\frac{1}{3}=0 \qquad 18ab+10=0 \qquad \cancel{18}b=-\frac{\cancel{10}^{5}}{\cancel{18}_{9}} \qquad b=-\frac{5}{3}$

Sostituisco

$$\left(\cancel{9}\cdot\frac{1}{\cancel{9}}\right)y^2+\left(\cancel{18\cdot\left(-\frac{5}{9}\right)+30}\left(\frac{1}{3}\right)\right)y+\cancel{9}\left(\frac{25}{\cancel{9}}\right)+41$$

$$y^2+\cancel{0y}+16 = y^2+16$$

$x=\frac{1}{3}y-\frac{5}{3} \qquad x=\frac{y-5}{3}$

$3x=y-5 \qquad y=3x+5$

$dy=3dx \qquad dx=dy/3$

$$\int \frac{2\left(y\cdot\left(-\frac{5}{3}\right)\right)+3}{y^2+16}\,\frac{1}{3}dy$$

$$\int \frac{2y \cdot - \frac{10}{3} + 3}{y^2+16} \cdot \frac{1}{3} dy = \frac{1}{3} \int \frac{\frac{2y-10+9}{3}}{y^2+16} dy$$

$$= \frac{1}{3} \cdot \frac{1}{3} \int \frac{2y-1}{y^2+16} dy = \frac{1}{9} \left[\int \frac{2y}{y^2+16} dy - \int \frac{1}{y^2+16} dy \right]$$

$$= \frac{1}{9} \log|y^2+16| - \frac{1}{9} \int \underbrace{\frac{1}{y^2+16}}_{\text{quadrato perfetto}} = \frac{1}{9} \log|y^2+16| - \frac{1}{9} \int \frac{1}{y^2+16}$$

RISULTATO

$$x + \frac{1}{9} \log(y^2+16) - \frac{1}{9} \cdot \frac{1}{4} \operatorname{arctg}\left(\frac{y}{4}\right) + c$$

Sostituisco

$$x \cancel{y} + \frac{1}{9} \log((3x+5)^2+16) - \frac{1}{36} \operatorname{arctg}\left(\frac{3x+5}{4}\right) + c$$

$$= \frac{x + \log(9x^2+30x+41)}{9} - \frac{\operatorname{arctg}\left(\frac{3x+5}{4}\right)}{36} + c$$

21/06

$$\int x^5 \log(x^2-16)$$

Risolvo per parti

$$\underbrace{\frac{x^6}{6}\cdot \log(x^2-16)}_{\text{da copiare}} - \int \frac{x^6}{6}\cdot\frac{2x}{x^2-16}$$

$$-\frac{1}{3}\int \frac{x^7}{x^2-16}$$

N ≥ D Divisione

$$\begin{array}{l|l} x^7 - {}^6 - {}^5 - {}^4 - {}^3 - {}^2 - {}^1 - {}^0 & x^2-16 \\ \hline -x^7 \quad +16x^5 & x^5+16x^3+256x \\ / \quad -16x^5 & \\ \quad / \quad +256x^3 & \\ \quad\quad -256x^3 & \\ \quad\quad / \quad +4096x & \end{array}$$

Riscrivo come

Riscrivo come

$$-\frac{1}{3}\int x^5+16x^3+256x+\frac{4096x}{x^2-16}\,dx$$

$$-\frac{1}{3}\int x^5dx - \frac{1}{3}\int 16x^3dx - \frac{1}{3}\int 256x\,dx - \frac{1}{3}\int\frac{4096x}{x^2-16}\,dx$$

$$-\frac{1}{3}\frac{x^6}{6} - \frac{\cancel{16}^{4}}{3}\frac{x^4}{\cancel{4}} - \frac{\cancel{256}^{128}}{3}\frac{x^2}{\cancel{2}_1} - \frac{2048}{3}\log|x^2-16|+C$$

RISULTATO

$$\frac{x^6}{6}\log(x^2-16) - \frac{1}{18}x^6 - \frac{4}{3}x^4 - \frac{128}{3}x^2 - \frac{2048}{3}\log|x^2-16|+C$$

Facendo mcm

$$\frac{3x^6\cdot\log(x^2-16) - x^6 - 24x^4 - 768x^2 - 12288\log|x^2-16|}{18} + C$$

19/07

$\int e^x \cdot \sqrt{e^x+4}$

$y = e^x + 4 \quad e^x = y - 4$

$de^x = \frac{dy}{e^x}$

$\int \cancel{e^x} \cdot \sqrt{y} \cdot \frac{dy}{\cancel{e^x}} = \int \sqrt{y}\, dy = \int y^{1/2} dy$

$= y^{3/2} \cdot \frac{2}{3} = (e^x+4)^{3/2} \cdot \frac{2}{3} + c = \frac{2}{3}\sqrt{(e^x+4)^3} + C$

14/09

$\int (x+7)\log(x^2+16)$

Svolgo

$\int x\log(x^2+16) + 7\log(x^2+16)dx =$

1

1 $\int x\log(x^2+16)$ Per parti

$\underbrace{\frac{x^2}{2}\cdot\log(x^2+16)}_{\text{da copiare}} - \int \frac{x^2}{\cancel{2}}\cdot\frac{\cancel{2}x}{x^2+16} = -\int\frac{x^3}{x^2+16}$

Divido

$$\begin{array}{l|l} x^3 \; -^2 \; -^1 \; -^0 & x^2+16 \\ \hline -x^3 \quad -16x & x \\ / \quad -16x & \end{array}$$

Riscrivo come $-\int x - \frac{16x}{x^2+16} =$

$= -\int x + 16\int\frac{x}{x^2+16} = \underbrace{-\frac{x^2}{2} + 8\log|x^2+16|}$ da copiare

2 $7\int\log(x^2+16)\,dx =$ Per parti

$7\left[x\cdot\log(x^2+16) - \int x\cdot\frac{2x}{x^2+16}\right]$ Divido

$= \underbrace{7x\log|x^2+16|}_{\text{da copiare}} - 14\int\frac{x^2}{x^2+16}$

$$\begin{array}{l|l} x^2 \; -^1 \; -^0 & x^2+16 \\ \hline -x^2 & 1 \\ \quad -16 & \end{array}$$

Riscrivo come

$$-14\left(\int 1 \quad \frac{-16}{x^2+16}\right) = -14\int 1 \quad +14\int \frac{16}{x^2+16}$$

$$= -14x + 224 \cdot \frac{1}{4} arctg \frac{x}{4} + c = -14x + 56 arctg \frac{x}{4}$$

Risultato

$$\frac{x^2}{2} \log(x^2+16) - \frac{x^2}{2} + 8\log(x^2+16) + 7x\log(x^2+16) - 14x + 56 arctg \frac{x}{4} + c$$

ANNO 2015

26/01

$$\int x \arctan(2x+3)$$

Per parti

$$\underbrace{\frac{x^2}{2}\cdot \arctan(2x+3)}_{\text{da copiare}} - \int \frac{x^2}{2}\cdot\frac{2}{1+(2x+3)^2} =$$

$$-\int \frac{x^2}{4x^2+12x+10}$$

Divido

$$\begin{array}{l|l} x^2 \quad -1 \quad -0 & 4x^2+12x+10 \\ -x^2 & \frac{1}{4} \\ \quad -3x - \frac{5}{2} & \end{array}$$

Riscrivo come

$$-\int \frac{1}{4} + \frac{-3x}{4x^2+12x+10} - \frac{5}{2}\cdot\frac{1}{4x^2+12x+10}\,dx =$$

$$= -\int \frac{1}{4}\,dx - \int \frac{3x}{4x^2+12x+10}\,dx + \frac{5}{2}\int \frac{1}{4x^2+12x+10}\,dx$$

① ② ③

$$= \boxed{-\frac{1}{4}x}$$

② $\Delta < 0$

$x = ay + b$

$$4(ay+b)^2 + 12(ay+b) + 10 = 4a^2y^2 + 4b^2 + 8aby + 12ay + 12b + 10$$

$$(4a^2)y^2 + (8ab+12a)y + 4b^2 + 12b + 10$$

$4a^2 = 1 \quad a^2 = \frac{1}{4} \quad a = \frac{1}{2}$

$8ab + 12a = 0 \quad 4b + 6 = 0 \quad b = -\frac{6}{4} = -\frac{3}{2}$

$$\left(\cancel{4}\cdot\frac{1}{\cancel{4}}\right)y^2+\left(\underbrace{8\cdot\frac{1}{2}\cdot\left(-\frac{3}{2}\right)+12\,\frac{1}{2}}_{\substack{-6\,+6\\0}}\right)y+\cancel{4}\cdot\frac{9}{\cancel{4}}+\cancel{12}^{6}\left(-\frac{3}{\cancel{2}_1}\right)+10$$

y^2+1 $\quad x=\frac{1}{2}y-\frac{3}{2}\quad x=\frac{y-3}{2}\quad 2x=y-3$

$y=2x+3 \qquad dx=\frac{dy}{2}$

$$\frac{3}{2}\int\frac{\cancel{2}\left(\frac{y-3}{2}\right)\cdot}{y^2+1}\,dy=-\frac{3}{4}\int\frac{y-3}{y^2+1}$$

$$=\underbrace{-\frac{3}{4}\int\frac{y}{y^2+1}}-\frac{3}{4}\int\frac{-3}{y^2+1}$$

$$-\frac{3}{8}\log(y^2+1)+\frac{9}{4}\operatorname{arctg}(y)+c$$

da copiare

③ $\frac{5}{4}\int\frac{1}{y^2+1}\,dy=\frac{5}{4}\operatorname{arctg}|y|$

$$\frac{x^2}{2}\operatorname{arctg}(2x+3)+\frac{3}{8}\log(4x^2+12x+10)+\frac{14}{4}\operatorname{arctg}(2x+3)-\frac{1}{4}x$$

$$=\frac{3\log(4x^2+12x+10)+(4x^2+28)\operatorname{arctg}(2x+3)-2x}{8}+C$$

09/02

$$\int x^2 \log(x^2-4x+3)$$

Per parti

$$\underbrace{\frac{x^3}{3}\cdot \log(x^2-4x+3)}_{\text{da copiare}} - \int \frac{x^3}{3}\cdot\frac{2x-4}{x^2-4x+3} = -\frac{1}{3}\int\frac{2x^4-4x^3}{x^2-4x+3}$$

$$\begin{array}{l|l} 2x^4-4x^3 \; -\,^2 \; -\,^1 \; -\,^0 & x^2-4x+3 \\ -2x^4+8x^3-6x^2 & \\ \quad 4x^3-6x^2 & 2x^2+4x+10 \\ \quad -4x^3+16x^2-12x & \\ \quad / \; 10x^2-12x & \\ \quad\quad -10x^2+40x-30 & \\ \quad\quad / \; 28x-30 & \end{array}$$

Riscrivo come

$$-\frac{1}{3}\int 2x^2+4x+10 + \frac{28x-30}{x^2-4x+3}\,dx =$$

$$-\frac{2}{3}\int x^2 - \frac{4}{3}\int x - \frac{10}{3}\int dx \quad \underbrace{-\frac{2}{3}\int\frac{14x-15}{x^2-4x+3}}$$

$$\underbrace{-\frac{2}{3}\frac{x^3}{3} - \frac{4}{3}\frac{x^2}{2} - \frac{10}{3}x}_{\text{Da copiare}}$$

Grado minore
cerco A, B
$\Delta<0$

$x=ay+b \quad x=y+4 \quad y=x-4$

$(ay+b)^2-4(ay+b)+30=\underline{a^2y^2}+\underline{aby}+b^2-\underline{4ay}-4b+30=$

$(a^2)y^2+(ab-4a)y+b^2-4b+30=$

$\downarrow$

$=1 \quad a^2=1 \quad a=1 \quad ab-4a=0 \quad b-4=0 \quad b=4$

$y^2+\cancel{(4-4)y}+\cancel{16-16}+30=y^2+30$

$-\frac{2}{3}\int\frac{14(y+4)-15}{y^2+30}=-\frac{2}{3}\int\frac{14y-41}{y^2+30}$

$=\underbrace{-\frac{2}{3}\int\frac{14y}{y^2+30}}-\frac{82}{3}\int\frac{1}{y^2+30}$

$-\frac{14}{3}\log(y^2+30)-\frac{82}{3}\cdot\frac{1}{\sqrt{30}}\operatorname{arctg}\left(\frac{y}{\sqrt{30}}\right)$

NON ESCE

30/06/15

$$\int \frac{4x^3+16x^2+23x-40}{4x^2+20x+41}$$

Grado N > Grado D
Divido

$$\begin{array}{rrrr|l}
4x^3 & +16x^2 & +23x & -40 & 4x^2+20x+41 \\
-4x^3 & -20x^2 & -41x & & \\
\hline
/ & -4x^2 & -18x & -40 & x-1 \\
 & +4x^2 & +20x & +41 & \\
\hline
 & / & 2x & +1 &
\end{array}$$

$\boxed{\frac{x^2}{2} - x} \rightarrow$ Da copiare

Riscrivo come

$$\int x-1+\frac{2x+1}{4x^2+20x+41} = \int x\,dx - \int dx + \int \frac{2x+1}{4x^2+20x+41} \quad \Delta<0$$

$x=ay+b \qquad x=\frac{1}{2}y-\frac{5}{2} \qquad x=\frac{y-5}{2} \qquad 2x=y-5, \; y=2x+5 \qquad dx=\frac{dy}{2}$

$4(ay+b)^2+20(ay+b)+41 = \underline{4a^2y^2}+4b^2+\underline{8aby}+\underline{20ay}+20b+4$

$4(a^2)y^2+\underline{(8ab+20a)}y+4b^2+20b+41$

$4a^2=1 \quad a=\frac{1}{2} \qquad 8ab+20a=0 \quad 8\cdot\frac{1}{2}\cdot b+10=0 \quad 4b=-10$

$b=-\frac{5}{2}$

$y^2+\cancel{4}\frac{25}{\cancel{4}}+\cancel{20}\left(-\frac{5}{\cancel{2}}\right)+41 = y^2-9$

Riscrivo come

$$\int \frac{y-5+1}{y^2-9}\,dy = \frac{1}{2}\int \frac{y-4}{y^2-9} \qquad \Delta = 36$$

$$= \frac{A}{(y-3)} + \frac{B}{(y+3)} = \frac{A(y+3)+B(y-3)}{y^2-9} = \frac{(A+B)y+3A-3B}{y^2-9}$$

$$\begin{cases} A+B=1 \\ 3A-3B=-4 \end{cases} \begin{cases} A=1-B \\ 3-3B-3B=-4 \end{cases} \begin{cases} \\ -6B=-7 \end{cases} \begin{cases} A=-\frac{1}{6} \\ B=\frac{7}{6} \end{cases}$$

$$\frac{1}{2}\cdot -\frac{1}{6}\int \frac{1}{y-3} + \frac{1}{2}\cdot\frac{7}{6}\int \frac{1}{y+3}$$

$$-\frac{1}{12}\log|y-3| + \frac{7}{12}\log|y+3| =$$

RISULTATO

$$-\frac{1}{12}\log|2x-2| + \frac{7}{12}\log|2x+8| + \frac{x^2}{2} - x$$

21/01/14

$$\int \frac{\cos x + 3\sin(x)}{\cos x + 1} = \int \frac{\cos x}{\cos x + 1}\,dx + 3\int \frac{\sin x}{\cos x + 1}$$

$$\int \frac{\cos x}{\cos x + 1}\,dx \underbrace{-3\int \frac{-\sin x}{\cos x + 1}}_{= -3\log(\cos x + 1)}$$

$\cos x = \frac{1-t^2}{1+t^2}$ $\quad d\cos x = \frac{2dt}{1+t^2}$ $\quad t = tg\left(\frac{x}{2}\right)$

$$\int \frac{1-t^2}{1+t^2} \cdot \frac{1}{\frac{1-\cancel{t^2}}{1+t^2} + \frac{1+\cancel{t^2}}{1+t^2}} \cdot \frac{2dt}{1+t^2} =$$

$$\int \frac{1-t^2}{1+t^2} \cdot \frac{\cancel{(1+t^2)}}{\cancel{2}} \cdot \frac{\cancel{2}dt}{\cancel{(1+t^2)}} = \int \frac{1-t^2}{1+t^2}\,dt$$

$t^2 - 1 \quad \cancel{1+t}$ $\qquad \frac{(1-t)\cancel{(1+t)}}{(1+t)^{\cancel{2}}}$

$$= \int \frac{1-t}{1+t}\,dt = \underbrace{\int \frac{1}{1+t}}_{\log|1+t|} - \int \frac{t}{1+t}$$

$$\begin{array}{r|l} t & 1+t \\ \hline -t & 1 \\ / \quad -1 & \end{array}$$

$$= -\int 1 - \frac{1}{1+t}$$

$$-\int \frac{-1}{1+t} = +\int \frac{1}{1+t} = +\log|1+t|$$

$$\int 1 \, dt = t$$

$$-3\log(\cos x + 1) + 2\log(tg\frac{x}{2}+1) + tg(\frac{x}{2})$$