Correzione secondo appello

$$f(x) = e^{\left(\frac{x+4}{4x+5x^2}\right)}$$

$4x+5x^2 \neq 0$

$x(4+5x) \neq 0$
$x \neq 0$
$4+5x \neq 0 \quad 5x \neq -4$
$x \neq -\frac{4}{5}$

Dom $f \in \mathbb{R} \setminus \left\{x=0,\ x=-\frac{4}{5}\right\}$

Intersezioni

$f(0) = e^0 = 1$

$f(x) = 0 \in \emptyset$

Interseca $(0,1)$

Segno $e^{\left(\frac{x+4}{4x+5x^2}\right)} > 0$

CE $x \neq 0 \quad x \neq -\frac{4}{5}$

la f exp è sempre $>0$ quindi $x \in \mathbb{R} \setminus \left\{-\frac{4}{6}, 0\right\}$
sempre pos

Limiti

$$\lim_{x \to +\infty} e^{\left(\frac{x+4}{4x+5x^2}\right)} = \lim_{x \to +\infty} e^{\frac{x\left(1+\frac{4}{x}\right)}{x^2\left(\frac{4}{x}+5\right)}} = \frac{1}{\infty} = 0^+ \quad = e^{0^+} = 1$$

$$\lim_{x \to -\infty} e^{\left(\frac{x+4}{4x+5x^2}\right)} = 1$$

$$\lim_{x \to 0^+} e^{\left(\frac{x+4}{4x+5x^2}\right)} = e^{\frac{4}{0^+}} = e^{\infty} = \infty$$

$$\lim_{x \to 0^-} e^{\left(\frac{x+4}{4x+5x^2}\right)} = e^{\frac{4}{0^-}} = e^{-\infty} = 0$$

$$\lim_{x \to -\frac{4}{5}^+} e^{\left(\frac{x+4}{4x+5x^2}\right)} = \exp\left[\begin{array}{ll} \lim_{x \to -\frac{4}{5}^+} (x+4) & (1) \\ \lim_{x \to -\frac{4}{5}^+} \dfrac{1}{4\cdot\left(-\frac{4}{5}\right)^+ + 5\left(-\frac{4}{5}\right)^{+2}} & (2) \end{array}\right]$$

① $\lim_{x \to -\frac{4}{5}^+} (x+4) = -\frac{4}{5} + 4 = \frac{-4+20}{5} = \frac{16}{5}$

② $\lim_{x \to -\frac{4}{5}^+} \frac{1}{4\left(-\frac{4}{5}\right) + 5\left(-\frac{4}{5}\right)^2} = \frac{1}{-\frac{16}{5} + 5\left(+\frac{16}{25}\right)} = 0 = \frac{1}{0} = -\infty$

$\frac{80}{25} \to \frac{16}{5}$

$5x^2+4x > 0$

$x(5x+4) > 0 \qquad x > 0$

$x > -\frac{4}{5}$

$-\quad +$

$-\frac{4}{5} \quad 0$

$(+) \quad - \quad +$

$\downarrow \quad \downarrow$

$0^+ \quad 0^-$

$e^{\frac{16}{5} \cdot -\infty} = 1$

per $\lim_{x \to -\frac{4}{5}^-}$ sarà $\frac{1}{0^+} \to +\infty$ quindi $e^{\frac{16}{0} \cdot +\infty} = +\infty$

## Derivata

$$\frac{d}{dx} e^{\left(\frac{x+4}{4x+5x^2}\right)} = e^{\left(\frac{x+4}{4x+5x^2}\right)} \cdot \frac{(4x+5x^2)-(x+4)\cdot(4+5\cdot 2x)}{(4x+5x^2)^2} =$$

$$e^{\left(\frac{x+4}{4x+5x^2}\right)} \cdot \frac{4x+5x^2-(4+4)\cdot(4+10x)}{(4x+5x^2)^2} =$$

$\to 4x+10x^2+16+40x$

$\downarrow$

$44x+10x^2+16$

$$= e^{\left(\frac{x+4}{4x+5x^2}\right)} \frac{4x+5x^2-44x-10x^2-16}{(4x+5x^2)^2} = e^{\left(\frac{x+4}{4x+5x^2}\right)} \frac{-40x-5x^2-16}{(4x+5x^2)^2}$$

$$\frac{e^{\left(\frac{x+4}{4x+5x^2}\right)} \cdot (40x-5x^2-16)}{(4x+5x^2)^2} =$$

$$= \frac{-40e^{\frac{x+4}{4x+5x^2}}x - 5e^{\frac{x+4}{4x+5x^2}}x^2 - 16e^{\frac{x+4}{4x+5x^2}}}{(4x+5x^2)^2}$$

## Segno derivata

$$\frac{-40e^{\frac{x+4}{4x+5x^2}}x - 5e^{\frac{x+4}{4x+5x^2}}x^2 - 16e^{\frac{x+4}{4x+5x^2}}}{(4x+5x^2)^2} > 0$$

CE

$x \neq 0 \quad x \neq -\frac{4}{5}$

raccogli $e^{\frac{x+4}{4x+5x^2}}$

$$= \frac{-40e^{\frac{x+4}{4x+5x^2}}x - 21e^{\frac{x+4}{4x+5x^2}}}{(4x+5x^2)^2} > 0 = \frac{-e^{\frac{x+4}{4x+5x^2}} \cdot (40x+21)}{(4x+5x^2)^2} > 0 \quad {*}N$$

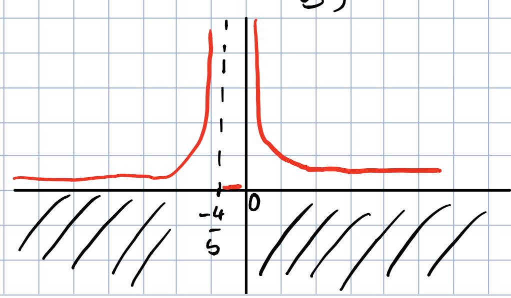
$$N{*}\begin{cases} -e^{\frac{x+4}{4x+5x^2}} > 0 \rightarrow x \in \emptyset \\ (40x-21) > 0 \rightarrow x > -\frac{21}{40} \end{cases} \rightarrow x \in \emptyset$$

quindi $< 0$

$$\begin{cases} -e^{\frac{x+4}{4x+5x^2}} < 0 \rightarrow x \in \mathbb{R} \\ (40x-21) < 0 \rightarrow x < -\frac{21}{40} \end{cases} \rightarrow x \in \left(-\infty, -\frac{21}{40}\right)$$

ora

$$\begin{cases} x \in \left(-\infty, -\frac{21}{40}\right) \\ (4x+5x^2)^2 \quad x \in \mathbb{R} \setminus \left\{0, -\frac{4}{5}\right\} \end{cases} \rightarrow x \in \left(-\infty, -\frac{4}{5}\right) \cup \left(-\frac{4}{5}, -\frac{21}{40}\right)$$



③ $\lim_{x\to 0}$, $\lim_{x\to+\infty}$ $\dfrac{\cos(5x^3)-1}{\log(1+9x^6)}$

$$\lim_{x\to+\infty} \frac{\cos(5x^3)-1}{\log(1+9x^6)} = \lim_{x\to+\infty} \frac{1}{\log+9x^6} = 0$$

e $\cos(5x^3)-1$ è fra $-3, -1$ quindi

$$\lim \frac{\cos(5x^3)-1}{\log(1+9x^6)} = \boxed{0}$$

$$\lim_{x\to 0} \frac{\cos(5x^3)-1}{\log(1+9x^6)} \overset{H}{=} \lim_{x\to 0} \frac{\frac{d}{dx}\cos(5x^3)-1}{\frac{d}{dx}\log(1+9x^6)} =$$

$$\lim_{x\to 0} \frac{-15x^2\cdot \operatorname{sen}(5x^3)}{\frac{54x}{1+9x^6}} = -\cancel{15}^{5}\cancel{x^2}\cdot\operatorname{sen}(5x^3)\cdot\frac{1+9x^6}{\cancel{54}\cancel{x^5}\,18\,x^3} =$$

$$\lim_{x\to 0} -\frac{5(9x^6+1)\operatorname{sen}(5x^3)}{18x^3} = -\frac{5}{18}\left(\lim_{x\to 0}\underset{\uparrow 1}{(9x^6+1)}\cdot\lim_{x\to 0}\frac{\operatorname{sen}5x^3}{x^3}\right)$$

$$\downarrow H \quad \frac{\cancel{15x^2}^{\,5}\cos(5x^3)}{\cancel{3x^2}}$$

$$\lim_{x\to 0} \frac{-5\cdot 5\cos(5x^3)}{18} = -\frac{5\cdot 5}{18} = \boxed{-\frac{25}{18}}$$

② $\int \frac{e^x}{e^{2x}+4e^x+5}\,dx$ $\qquad y = e^x$

$dy = e^x\,dx$

$$\int \frac{1}{y^2+4y+5}\,dy = \int \frac{1}{\underbrace{y^2+4y+4}+1}\,dy$$

$$\int \frac{1}{(y+2)^2+1}\,dy \qquad t = y+2 \qquad dt = t+2\,dy \qquad \int \frac{1}{t^2+1}$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

$$\frac{1}{1}\arctan(t) \to \arctan(y+2) \to \boxed{\arctan(e^x+2)+C}$$

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19/07/16

$f(x) = \log(9x^2+18x+17)$

$9x^2+18x+17>0 \qquad \Delta = 324-612 = -288$

no sol $\mathbb{R}$

Dom $f \in \mathbb{R}$

Intersezioni

$f(0) = \log(0+0+17)$

$f(x) = 0 \in \emptyset$ $\qquad$ Interseca $(0, \ln(17))$

Segno

essendo log sempre >0 $\forall x \in \mathbb{R}$

$\log(9x^2+18x+17)>0 \quad \forall x \in \mathbb{R}$

quindi la funzione e' sempre positiva

Limiti

$$\lim_{x\to+\infty} \log(9x^2+18x+17)=+\infty$$

$\lim_{x\to-\infty} \log(9x^2+18x+17)=+\infty$ il $\lim_{x\to-\infty}$ di un polinomio di grado pari il cui coefficente del termine di grado massimo e' positivo, è uguale a $+\infty$

Derivate

$$y=\log(9x^2+18x+17)$$

$$y'=\frac{d}{dx}(\ln(x))\cdot\frac{d}{dx}(9x^2+18x+17)$$

$$\frac{1}{9x^2+18x+17}\cdot 18x+18 = \frac{18x+18}{9x^2+18x+17}$$

Segno derivate

$$\frac{18x+18}{9x^2+18x+17}>0 \qquad \begin{cases} 18x+18>0 & x>-1 \\ 9x^2+18x+17>0 & x\in\mathbb{R} \end{cases}$$

$-1$

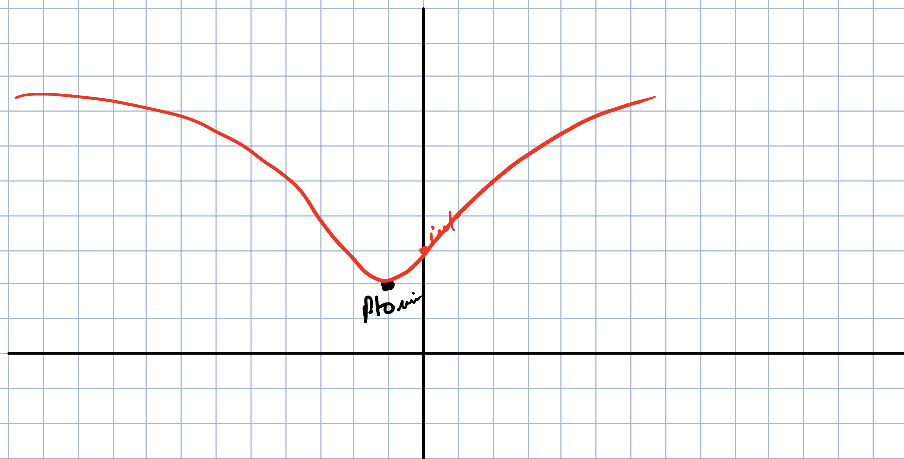
$+$

$x\in(-1,+\infty)$

p.to min in $(-1)$

$\ln(9\cdot(-1)^2+18\cdot(-1)+17) = 9-18+17 = \ln(8)$

$=\ln(2^3) = 3\ln(2) \simeq 2{,}07$ pto min $(-1, 2{,}07)$




$\int \log(9x^2+24x+16)\,dx$ $\quad f = \log(9x^2+24x+16)$

$g' = 1 \quad g = x$

$\int \log(9x^2+24x+16)\cdot 1\,dx$ $\quad f' = \frac{1}{9x^2+24x+16}\cdot 18x+24 =$

$x\log(9x^2+24x+16) - \int \frac{6x}{3x+4}\,dx$ $\quad \frac{1}{(3x+4)^{\cancel{2}}}\cdot 6\cancel{(3x+4)}$

$= \frac{6}{3x+4}$

$x\log(9x^2+24x+16) - 6\int \frac{x}{3x+4}\,dx$

$x\log(9x^2+24x+16) - 6\int \left(\frac{1}{3} - \frac{4}{3(3x+4)}\right)dx$

$$\begin{array}{r|l} x & 3x+4 \\ -x+\frac{1}{3} & \frac{1}{3} \\ \hline \end{array}$$

$> \log(u)$

$= x\log(9x^2+24x+16) + 8\int \frac{1}{u}\,du - 2\int 1\,dx$

$u = 3x+4$

$du = 3dx$

$\frac{8\log(u)}{3} + x\log(9x^2+24x+16) - 2\int 1\,dx$

$-2x + x\log((3x+4)^2) + \frac{8}{3}\log(3x+4) + c$

$$\int \text{arctg}\left(\sqrt{8x-18}\right) dx$$

$y = \sqrt{8x-18}$ $\quad x =$

$$dy = \frac{1}{\cancel{2}\sqrt{8x-18}} \cdot 8^4 dx$$

$$dx = \frac{1}{y'} dy$$

$$\int \arctan(\sqrt{8x-18}) \cdot \frac{1}{\frac{4}{\sqrt{8x+18}}}$$

$$\int \arctan(\sqrt{8x-18}) \cdot \frac{\sqrt{8x+18}}{4} dx$$

$$\frac{1}{4}\int \arctan(y) \cdot y \, dy \rightarrow \text{per parti} \int u \, dv = uv - \int v \, du$$

$u = \arctan(y)$

$dv = y' dy \qquad v = \frac{y^2}{2}$

$du = \frac{1}{1+y^2} dy$

$$\frac{1}{4}\int \arctan(y) \cdot \frac{y^2}{2} - \int \frac{y^2}{2} \cdot \frac{1}{1+y^2} dy$$

---

$$\lim_{x\to+\infty} x^{-3} \int_0^x t \log\left(e^{3t}+5\right) dt \qquad \text{Hop.}$$

$$\lim_{x\to+\infty} \frac{\int_0^x t \log\left(e^{3t}+5\right) dt}{x^3} \quad \begin{matrix} \frac{d}{dx} \\ \frac{d}{dx} \end{matrix}$$

$$\lim_{x\to+\infty} \frac{\cancel{x}\log\left(e^{3x}+5\right)}{3x^{\cancel{2}}} = \lim_{x\to+\infty} \frac{\log\left(e^{3x}+5\right)}{3x} = \text{Hop ancora}$$

$$\frac{\frac{d}{dx} \log\left(e^{3x}+5\right) \rightarrow \ln(g) \cdot g'}{\frac{d}{dx} \quad 3x \quad \rightarrow 3}$$

$$= \lim_{x\to+\infty} \frac{\frac{\cancel{3}e^{3x}}{e^{3x}+5}}{\cancel{3}} = \lim_{x\to+\infty} \frac{e^{3x}}{e^{3x}+5}$$

Ancora Hopital $\rightarrow \lim_{x \to +\infty} \frac{\cancel{3}e^{\cancel{3x}}}{\cancel{3}e^{3x}} = 1$

---

$$\int \frac{x^2}{(x-3)(x-5)} dx$$

$x^2-8x+15$

Visto che $\deg D(x) < \deg N(x)$ faccio divisione di polinomi

$$\begin{array}{rrr|l} x^2 & 0x & 0 & x^2-8x+15 \\ -x^2 & +8x & -15 & 1+\frac{8}{x} \\ \hline / & 8x & -15 & \\ & -8x & +64 & \\ \hline & / & 49-\frac{120}{x} & \end{array}$$

$$1+\frac{8}{x}-\frac{120}{x}+\frac{1}{x^2-8x+15}$$

$$x^2-8x+15 \rightarrow (x-3)(x-5)$$

$$\frac{A}{(x-3)}+\frac{B}{(x-5)} = \frac{Ax-5A+Bx-3B}{(x-3)(x-5)} =$$

$$\frac{x(B+A)-5A-3B}{(x-3)(x-5)} = \begin{cases} B+A=0 \\ -5A-3B=1 \end{cases} \quad \begin{cases} A=-B \\ 5B-3B=1 \\ 2B=1 \end{cases} \quad \begin{cases} A=-\frac{1}{2} \\ B=\frac{1}{2} \end{cases}$$

$$\frac{1}{(x-3)(x-5)} = -\frac{1}{2}\frac{1}{(x-3)}+\frac{1}{2}\frac{1}{(x-5)}$$

$$\int \frac{x^2}{(x-3)(x-5)} = \int 1-\frac{112}{x}-\frac{1}{2}\frac{1}{(x-3)}+\frac{1}{2}\frac{1}{(x-5)}$$

$$\int 1\,dx - \int \frac{112}{x}dx - \frac{1}{2}\int \frac{1}{(x-3)}dx + \frac{1}{2}\int \frac{1}{(x-5)}dx$$

$$x - 112\ln(x) - \frac{1}{2}\ln(x-3)+\frac{1}{2}\ln(x-5)+c$$

$$f(x) = \frac{x^2}{(x-3)(x-5)}$$
$x^2-8x+15$

$x^2-8+15 \neq 0$

$\Delta = 64 - 4 \cdot 15 = 4$

$x_{1,2} = \frac{8 \pm 2}{2} \begin{cases} 5 \\ 3 \end{cases}$

$x \neq 5, 3 \qquad x<3 \cup x>5$

$\mathrm{Dom} f \in \mathbb{R} / \{3, 5\}$

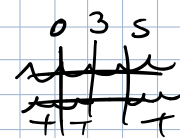
Intersezione assi

$$f(0) = \frac{0}{15} = 0$$

Interseca $(0,0)$

$$f(x)=0 \quad \frac{x^2}{x^2-8x+15} = 0 \in \emptyset$$

Segno $f$:




$$\frac{x^2}{x^2-8x+15} > 0$$

$$\begin{cases} x^2 > 0 \quad \forall x \in \mathbb{R} \\ x<3 \ \cup \ x>5 \end{cases}$$

$x<3 \cup x>5$

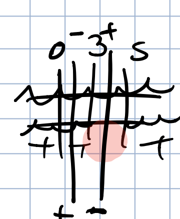
$\nearrow -\infty, 3 \qquad \searrow 3,5 \qquad \nearrow 5, +\infty$

Limiti

$$\lim_{x \to +\infty} \frac{x^2}{x^2-8x+15} = \frac{\cancel{x^2}}{\cancel{x^2}\left(1-\frac{8}{x}+\frac{15}{x^2}\right)} = 1$$

$$\lim_{x \to -\infty} \frac{x^2}{x^2-8x+15} = 1$$

$$\lim_{x \to 3^+} \frac{x^2}{x^2-8x+15} = \frac{9}{9-24+15} = \frac{9}{0^+} = -\infty$$




$$\lim_{x \to 3^-} \frac{x^2}{x^2-8x+15} = \frac{9}{0^-} = +\infty$$

$$\lim_{x\to 5^+} \frac{x^2}{x^2-8x+15} = \frac{25}{25-40+15} = \frac{25}{0^+} = +\infty$$

$$\lim_{x\to 5^-} \frac{x^2}{x^2-8x+15} = -\infty$$

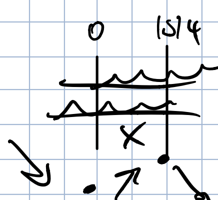
# Derivata

$$f'\cdot g - f\cdot g'$$

$$\frac{x^2}{x^2-8x+15} \begin{matrix} f \\ g \end{matrix} = \frac{2x\cdot(x^2-8x+15) - x^2\cdot(2x-8)}{(x^2-8x+15)^2} =$$

$$\frac{\cancel{2x^3}-16x^2+30x-\cancel{2x^3}-8x^2}{//} = \frac{-8x^2+30x}{(x^2-8x+15)^2} > 0 \quad \begin{cases} -8x^2+30x>0 \\ \forall x \in \mathbb{R} \end{cases}$$

$$\begin{cases} x>0 \\ -8x+30>0 \end{cases} \quad \begin{cases} x>0 \\ x < \cancel{\frac{30}{8}}\ \frac{15}{4} \end{cases}$$




$x \in \left(0, \frac{15}{4}\right)$

$$\begin{cases} x \in \left(0, \frac{15}{4}\right) \\ \forall x \in \mathbb{R} \end{cases} \to S: \left(0, \frac{15}{4}\right) \nearrow$$

$x \neq 3$

$$\searrow \quad \left(0, 3\right[ \cup \left]3, \frac{15}{4}\right) \quad \searrow$$

$$\nearrow \qquad \nearrow$$

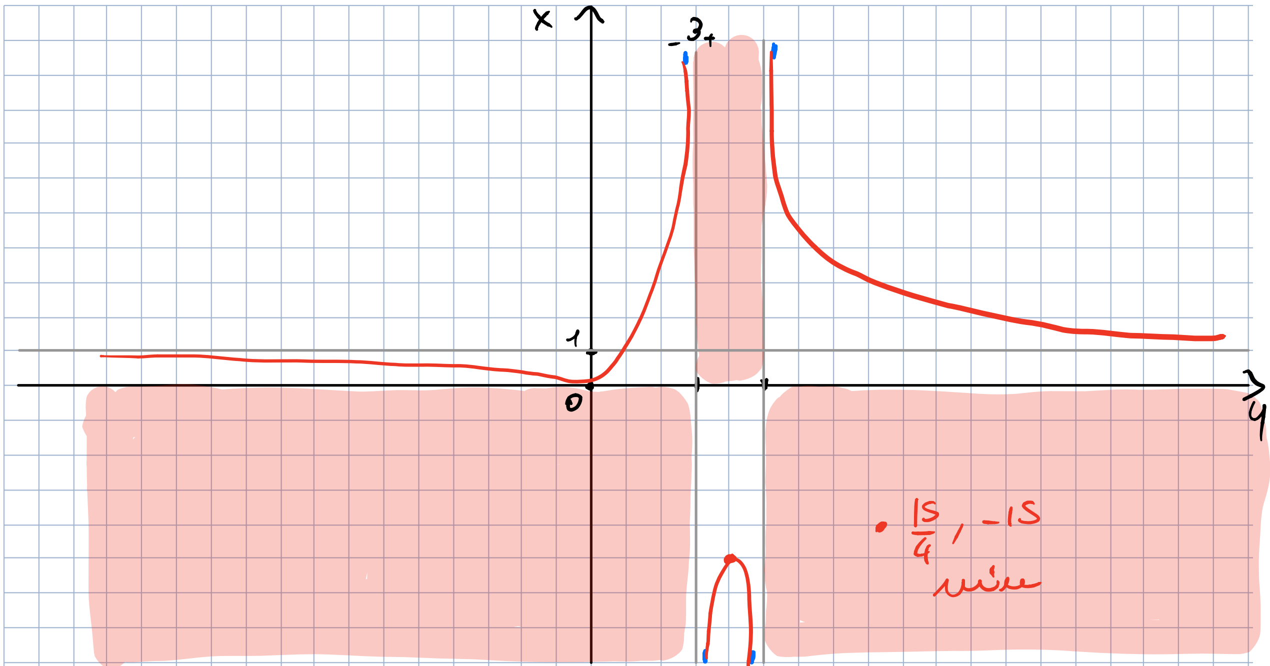
p.to min $(0)$ $(0,0)$

p.to max $\left(\frac{15}{4}\right)$ $\left(\frac{15}{4}, -15\right)$

$f(0) \to 0$ min in $0,0$ (intersez.)

$$f\left(\frac{15}{4}\right) \to \frac{\left(\frac{15}{4}\right)^2}{\left(\frac{15}{4}\right)^2 - \left(\cancel{8}^{\,2}\cdot\frac{15}{\cancel{4}}\right) + 15} = \frac{\frac{225}{16}}{\frac{225}{16} - 30 + 15} = \frac{\frac{225}{16}}{\frac{225}{16} - 15}$$

$$= \frac{\frac{225}{16}}{-\frac{15}{16}} = \frac{225}{\cancel{16}} \cdot -\frac{\cancel{16}}{15} = -15$$




Interseca (0,0)

asintoto orizzontale 1

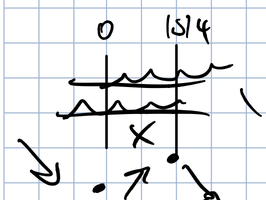
asintoti verticali

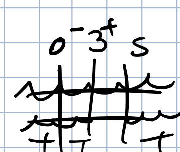
$5^+$ $+\infty$ $\quad$ $5^-$ $-\infty$

$3^+$ $-\infty$ $\quad$ $3^-$ $+\infty$

p.to min (0) (0,0)

p.to max $\left(\frac{15}{4}\right)$ $\left(\frac{15}{4}, -15\right)$

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$$\lim_{x\to 0} \frac{x\,\text{sen}(x)}{\text{sen}(e^{3x}-1)-3x} = \frac{0\,\text{sen}(0)}{\text{sen}(\underset{1}{e^0}-1)-0} = 0$$

---

# Es svolti

**es 1** $$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[x^5\cos(x)+3\cos(2x)\right]dx$$

risulta 0. perché è l'integrale di una f. dispari su un intervallo simmetrico rispetto all'origine

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^5\cos(x)\,dx + 3\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos(2x)\,dx =$$

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^5\cos(x)\,dx = 0$

per sostituzione $y=2x$

$dy = 2dx$

$\frac{\pi}{4} \to \frac{\pi}{2}$

$-\frac{\pi}{4} \to -\frac{\pi}{2}$

$$\frac{3}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos(y)\,dy = \frac{3}{2}\left[\operatorname{sen}\left(\frac{\pi}{2}\right)-\operatorname{sen}\left(-\frac{\pi}{2}\right)\right] =$$

$$\frac{3}{2}\left(1-(-1)\right) = \frac{3}{\cancel{2}}\cdot\cancel{2} = 3$$

**es 2** $$\int_0^{15}\frac{72}{\sqrt{x+49}+\sqrt{x+1}}\,dx =$$

$$\frac{72}{\sqrt{x+49}+\sqrt{x+1}}\cdot\frac{\sqrt{x+49}-\sqrt{x+1}}{\sqrt{x+49}-\sqrt{x+1}} = \frac{\cancel{72}\cdot\sqrt{x+49}-\sqrt{x+1}}{x+\cancel{49}-(x+1)} =$$

$$\int_0^{15}\frac{3}{2}\left(\sqrt{x+49}-\sqrt{x+1}\right)dx = \frac{3}{2}\int_0^{15}1\cdot(x+49)^{1/2}dx - \frac{3}{2}\int_0^{15}1\cdot(x+1)^{\frac{1}{2}}dx$$

$f'\cdot f^{\alpha}$ $\qquad \int f'(x)\cdot[f(x)]^n = \frac{f(x)^{n+1}}{n+1}$

$$= \frac{3}{2}\cdot\frac{(x+49)^{3/2}}{\frac{3}{2}} - \frac{3}{2}\cdot\frac{(x+1)^{3/2}}{\frac{3}{2}} + c =$$

$$= \sqrt{(x+49)^3}-\sqrt{(x+1)^3}+c \to \left[\sqrt{(x+49)^3}-\sqrt{(x+1)^3}\right]_0^{15} =$$

$$= \sqrt{64}^3-\sqrt{16}^3-\left(\sqrt{49}^3-\sqrt{1}^3\right) = 512-64-(343-1) = 106$$

(es 3) Trova una primitiva di $f(x) = \frac{\sqrt{x+8}}{\sqrt{x+8}+x+8} + x^{2019}$

$$\int \frac{\sqrt{x+8}}{\sqrt{x+8}+x+8}\,dx = \int \frac{t}{t+t^2}\,2t\,dt =$$

$t = \sqrt{x+8}\,dx$

$dt = \frac{1}{2\sqrt{x+8}}\,dx = \frac{1}{2t}\,dx$

$dx = 2t\,dt$

$$= \int \frac{2t^{\cancel{2}}}{\cancel{t}(t+1)}\,dt = 2\int \frac{t}{t+1}\,dt$$

$$= 2\int \frac{t+1}{t+1}\,dt - 2\int \frac{1}{t+1}\,dt =$$

$$= 2t - 2\ln|t+1| + c = 2\sqrt{x+8} - 2\ln(\sqrt{x+8}+1) + C$$

$$= 2\sqrt{x+8} - 2\ln(\sqrt{x+8}+1) + \frac{x^{2020}}{2020}$$ è primitiva di $f(x)$

---

$$\lim_{x\to+\infty} x^3 \log\left(\frac{x^3+3}{x^3+5}\right) \qquad x^3 = t$$

$$\lim_{t\to+\infty} t\log\left(\frac{t+3}{t+5}\right) \to \frac{\log\left(\frac{t+3}{t+5}\right)}{\frac{1}{t}} \to \text{Hôp}$$

$$\frac{\frac{(t+5)\left(\frac{1}{t+5} - \frac{t+3}{(t+5)^2}\right)}{t+3}}{-\frac{1}{t^2}} = -\frac{2t^2}{(t+3)(t+5)} =$$

$$-2\lim_{t\to+\infty}\left(\frac{t^2}{(t+3)(t+5)}\right) = \boxed{-2}$$

$=0$

$$\int \frac{1}{3\sin(x) - 5\cos(x) + 5}\, dx = \qquad t = \tan\left(\frac{x}{2}\right)$$

$$\sin(x) = \frac{2t}{1+t^2}$$

$$\int \frac{1}{3\cdot\frac{2t}{1+t^2} - 5\,\frac{1-t^2}{1+t^2} + 5} \cdot \frac{2}{1+t^2}\, dt = \qquad \cos(x) = \frac{1-t^2}{1+t^2}$$

$$t = 2\arctan(x)$$

$$dx = \frac{2}{1+t^2}\, dt$$

$$\int \frac{2}{6t - 5(1-t^2) + 5(1+t^2)}\, dt =$$

fino qui okay

$$\int \frac{2}{6t - \cancel{5} + 5t^2 + \cancel{5} + 5t^2}\, dt = \boxed{\int \frac{1}{3t + 5t^2}\, dt = \quad t(3+5t)} \qquad \frac{A}{t} \qquad \frac{B}{3+5t}$$

$$A(3+5t) + Bt$$

$$3A + 5tA + Bt \to t(5A+B) + 3A$$

$$\begin{cases} 5A + B = 0 \\ 3A = 1 \end{cases} \qquad \begin{cases} \frac{5}{3} = -B \\ A = \frac{1}{3} \end{cases} \qquad B = -\frac{5}{3}$$

$$\int \frac{1}{3+5t^2} = \int \frac{\frac{1}{3}}{t}\, dt + \int \frac{-\frac{5}{3}}{3+5t}\, dt$$

$$\frac{1}{3}\ln(t) + \frac{1}{3}\ln(3+5t)$$

$$\frac{1}{3}\ln\left(\tan\frac{x}{2}\right) + \frac{1}{3}\ln\left(3 + 5\tan\left(\frac{x}{2}\right)\right)$$

?!

---

$$\int \frac{9x^2 + 32x + 44}{9x^2 + 30x + 41}\, dx \qquad \int \frac{2x+3}{9x^2+30x+41}\, dx$$

$$\int \frac{2x+3}{9x^2+30x+41}\,dx + \int 1\,dx \qquad \begin{array}{l|l} 9x^2+32x+44 & 9x^2+30x+41 \\ -9x^2-30x-41 & 1 \\ \hline / \quad 2x+3 & \end{array}$$

$$\int\left(\frac{18x+30}{9(9x^2+30x+41)} - \frac{1}{3(9x^2+30x+41)}\right)dx + \int 1\,dx$$

$$\frac{1}{9}\int\frac{18x+30}{9(9x^2+30+41}\,dx - \frac{1}{3}\int\frac{1}{3(9x^2+30x+41)}\,dx + \int 1\,dx$$

$$y = 9x^2+30+41 \qquad \frac{1}{9}\int\frac{1}{4}\,dy - \frac{1}{3}\int\frac{1}{3(9x^2+30x+41)}\,dx + \int 1\,dx$$
$$dy = 18x+30\,dx$$

$$\frac{\log(4)}{9} - \frac{1}{3}\int\frac{1}{(3x+5)^2+16}\,dx + \int 1\,dx$$

$$s = 3x+5 \qquad \frac{\log(4)}{9} - \frac{1}{9}\int\frac{1}{s^2+16}\,ds + x$$
$$ds = 3dx$$

$$\frac{\log(4)}{9} - \frac{1}{9}\int\frac{1}{16\left(\frac{s^2}{16}+1\right)}\,ds + x$$

$$\frac{\log(4)}{9} - \frac{1}{144}\int\frac{1}{\frac{s^2}{16}+1}\,ds + x$$

$$p = \frac{s}{4} \quad dp = \frac{1}{4}\,ds \qquad \frac{\log(4)}{9} - \frac{1}{36}\int\frac{1}{p^2+1}\,dp + x$$

$$\frac{\log(4)}{9} - \frac{1}{36}\,\operatorname{arctan}(p) + x + c$$

$$\frac{\log\left(9x^2+30x+41\right)}{9} - \frac{1}{36}\,\operatorname{arctan}\left(\frac{3x+5}{4}\right) + x + c$$

$$\lim_{x \to 0} x^{-4} \int_0^x \mathrm{sen}(4t^3)\, e^{-t}\, dt$$

$$\lim_{x \to 0} \frac{\int_0^x \mathrm{sen}(4t^3)\, e^{-t}\, dt}{x^4}$$ Hop

$$\lim_{x \to 0} \frac{\mathrm{sen}(4x^3)\, e^{-x}}{x^4} = \text{non esiste}$$ perché

$0^- \to -\infty$

$0^+ \to +\infty$

$$\frac{\cancel{x^4}\left(\mathrm{sen}(1) - \overset{0}{\frac{e^{-x}}{x^2}}\right)}{\cancel{x^4}} = \frac{\mathrm{sen}(1)}{0} = 0$$

$$f(x)=\frac{x+4}{x^2+7}$$ $x^2+7\neq 0$ $x^2\neq 7$ $\forall x\in\mathbb{R}$

Dom $f \in \mathbb{R}$

Intersezioni

$f(0)= 0$

$f(x)=0$ e $\frac{4}{7}$

Interseca $(0, \frac{4}{7})$

Segno

$$\begin{cases} x+4>0 \\ x^2+7>0 \end{cases} \quad \begin{cases} x>-4 \\ \forall \in\mathbb{R} \end{cases}$$

$-4$ $7$

$-$ $+$ $+$

pos $(-4,+\infty)$

Limiti

$$\lim_{x\to\infty} \frac{x+4}{x^2+7} = \frac{\cancel{x}^2(1+\frac{4}{x_2})}{\cancel{x}^2(1+\frac{7}{x^2})} = 0$$

$$\lim_{x\to-\infty} = 0$$

Derivata

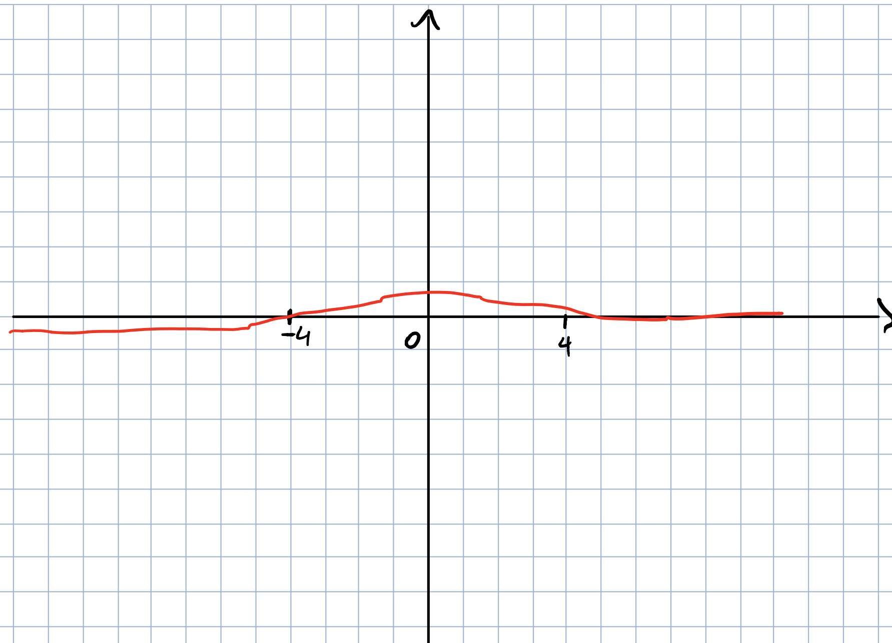
$$\frac{-x^2-8x+7}{(x^2+7)^2}>0$$

$\Delta = 64-4\cdot(-1)\cdot 7 \to 36$

$$\begin{cases} -x^2-8x+7>0 \\ \forall x\in\mathbb{R} \end{cases} \quad \begin{cases} x<1 \lor x>7 \\ \forall x\in\mathbb{R} \end{cases}$$

$-4-\sqrt{23}$ $4\sqrt{23}$

↗ ↘ ↗




$f(x) = \log(9x^2 + 24x + 17)$ $\qquad$ $9x^2 + 24x + 17 > 0$

$\Delta = 576 - 4 \cdot 9 \cdot 17 = -36$

$\qquad\qquad 612$

① Dom $f \in \mathbb{R}$

② Intersezioni

$f(0)$ $\log(17)$ $\qquad$ Interseca $\left(-\frac{4}{3}, \ln(17)\right)$

$f(x)=0$ log può essere $=0$ solo quando l'argomento è 1

$9x^2 + 24x + 17 = 1 \rightarrow 9x^2 + 24x + 17 - 1 = 0 \rightarrow 9x^2 + 24x + 16 = 0$

$(3x+4)^2 = 0 \qquad 3x + 4 = 0 \qquad x = -\frac{4}{3}$

③ Segno

$9x^2 + 24x + 17 \quad x \in \mathbb{R}$

④ limiti

$$\lim_{x\to+\infty} 9x^2+24+17=+\infty$$

$\lim_{x\to-\infty} = +\infty$ il $\lim_{x\to-\infty}$ di un polinomio di grado pari il cui coeff. del termine di grado max è positivo è $+\infty$

⑤ Derivata

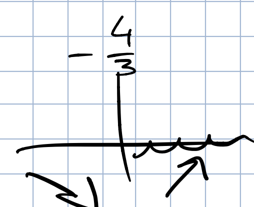
$$\frac{d}{dx}\ln\left(9x^2+24x+17\right) \qquad \frac{d}{dx}\ln(g)\cdot\frac{d}{dx}g$$

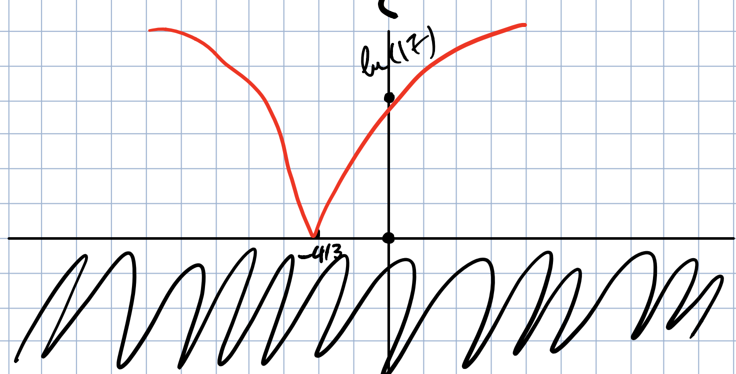
$$\frac{1}{g}\cdot g'$$

$$\frac{1}{9x^2+24x+17}\cdot 18x+24=\frac{18x+24}{9x^2+24x+17}$$

⑥ Segno derivata

$$\frac{18x+24}{9x^2+24x+17}>0 \quad \begin{cases} x>-\cancel{\frac{24}{18}} \; -\frac{4}{3} \\ \forall x\in\mathbb{R} \end{cases}$$



$$\lim_{x\to 0^+} x^{-7}\int_0^x (\operatorname{sen}(3t))^6\,dt \underset{\hat{H}}{=} \frac{\operatorname{sen}(3x)^6}{7x^6}$$

$$\frac{3}{7}\lim \frac{\operatorname{sen}(x)^6}{x^6} \qquad \underbrace{\frac{\operatorname{sen}(x)}{x}=1}_{\text{lim not}} = \frac{3}{7}$$

---

$$\int \frac{\log\log(x)}{x}\,dx = \qquad y=\log(x) \qquad dy=\frac{1}{x}\,dx$$

$$\int \log(y)\,dy = \int \underset{f}{\log(y)}\cdot \underset{g'}{1}\,dy \qquad f=\log(y) \quad f'=\frac{1}{y} \qquad g'=1 \quad g=y$$

$$= y\log(y) - \int 1\,dy =$$

$$y\log(y) - y = \log(x)\log(\log(x)) - \log(x) + c$$

$$= \log(x)(\log(\log(x)) - 1) + c$$

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$$\lim_{x\to+\infty} (x(\log x)^4)^{-1}\int_1^x (\log t)^4\,dt \to \hat{H}$$

$$\lim_{x\to+\infty} \frac{\cancel{(\log(x))^4}}{x\cancel{(\log(x))^4}} = \frac{1}{x} = 0$$