

Correzione secondo appello

$$f(x) = e^{\left(\frac{x+4}{4x+5x^2}\right)}$$

$$4x+5x^2 \neq 0$$

$$x(4+5x) \neq 0$$

$$x \neq 0$$

$$4+5x \neq 0$$

$$5x \neq -4$$

$$x \neq -\frac{4}{5}$$

$$\text{Dom } f \in \mathbb{R} \setminus \{x=0, x=-\frac{4}{5}\}$$

Intersezioni

$$f(0) = e^0 = 1 \quad \text{intersezione } (0, 1)$$

$$\text{Segno } e^{\left(\frac{x+4}{4x+5x^2}\right)} > 0 \quad \text{C.E. } x \neq 0 \quad x \neq -\frac{4}{5}$$

Se f exp è sempre > 0 quindi $x \in \mathbb{R} \setminus \{-\frac{4}{5}, 0\}$
sempre pos

Limiti

$$\lim_{x \rightarrow +\infty} e^{\left(\frac{x+4}{4x+5x^2}\right)} = \lim_{x \rightarrow +\infty} e^{\frac{x(1+\frac{4}{x})}{x^2(\frac{4}{x}+5)}} = \frac{1}{\frac{1}{\infty}} = 0^+ = e^{0^+} = 1$$

$$\lim_{x \rightarrow -\infty} e^{\left(\frac{x+4}{4x+5x^2}\right)} = 1$$

$$\lim_{x \rightarrow 0^+} e^{\left(\frac{x+4}{4x+5x^2}\right)} = e^{\frac{4}{0^+}} = e^\infty = \infty$$

$$\lim_{x \rightarrow 0^-} e^{\left(\frac{x+4}{4x+5x^2}\right)} = e^{\frac{4}{0^-}} = e^{-\infty} = 0$$

$$\lim_{x \rightarrow -\frac{4}{5}^+} e^{\left(\frac{x+4}{4x+5x^2}\right)} = \exp \left[\begin{array}{l} \lim_{x \rightarrow -\frac{4}{5}^+} (x+4) \\ \lim_{x \rightarrow -\frac{4}{5}^+} 1 \\ \lim_{x \rightarrow -\frac{4}{5}^+} \frac{4}{4(-\frac{4}{5})^+ + 5(-\frac{4}{5})^{+2}} \end{array} \right] \quad \text{①}$$

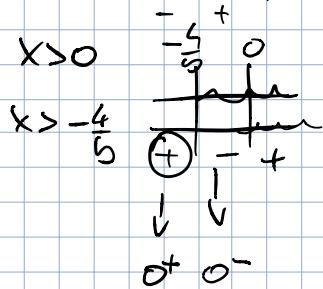
②

$$\textcircled{1} \lim_{x \rightarrow -\frac{4}{5}^+} (x+4) = -\frac{4}{5} + 4 = \frac{-4+20}{5} = \frac{16}{5}$$

$$\textcircled{2} \lim_{x \rightarrow -\frac{4}{5}^+} \frac{1}{4(-\frac{4}{5}) + 5(-\frac{4}{5})^2} = \frac{1}{-\frac{16}{5} + 5(+\frac{16}{25})} = \frac{1}{\frac{80}{25}} = \frac{25}{80} = \frac{1}{16} = \infty$$

$$5x^2 + 4x > 0$$

$$x(5x+4) > 0 \quad x > 0$$



$$e^{\frac{16}{5} \cdot \infty} = 1$$

per $\lim_{x \rightarrow -\frac{4}{5}^-}$ sare $\frac{1}{0^+} \rightarrow +\infty$ quindi $e^{\frac{16}{5} \cdot +\infty} = +\infty$

Derivata

$$\begin{aligned} \frac{d}{dx} e^{\left(\frac{x+4}{4x+5x^2}\right)} &= e^{\left(\frac{x+4}{4x+5x^2}\right)} \cdot \frac{(4x+5x^2)-(x+4) \cdot (4+10x)}{(4x+5x^2)^2} = \\ &= e^{\left(\frac{x+4}{4x+5x^2}\right)} \cdot \frac{4x+5x^2 - (4+4) \cdot (4+10x)}{(4x+5x^2)^2} = \frac{4x+10x^2+16+40x}{44x+10x^2+16} \\ &= e^{\left(\frac{x+4}{4x+5x^2}\right)} \frac{4x+5x^2 - 44x-10x^2-16}{(4x+5x^2)^2} = e^{\left(\frac{x+4}{4x+5x^2}\right)} \frac{-40x-5x^2-16}{(4x+5x^2)^2} \\ &= \frac{e^{\left(\frac{x+4}{4x+5x^2}\right)} \cdot (-40x-5x^2-16)}{(4x+5x^2)^2} \end{aligned}$$

$$= -40e^{\frac{x+4}{4x+5x^2}} x - 5e^{\frac{x+4}{4x+5x^2}} x^2 - 16e^{\frac{x+4}{4x+5x^2}} \frac{(4x+5x^2)^2}{(4x+5x^2)^2}$$

Segno derivate

$$\begin{aligned} & -40e^{\frac{x+4}{4x+5x^2}} x - 5e^{\frac{x+4}{4x+5x^2}} x^2 - 16e^{\frac{x+4}{4x+5x^2}} \frac{(4x+5x^2)^2}{(4x+5x^2)^2} > 0 \quad (\exists) \\ & x \neq 0 \quad x \neq -\frac{4}{5} \\ & = -40e^{\frac{x+4}{4x+5x^2}} x - 21e^{\frac{x+4}{4x+5x^2}} \text{ so } = -e^{\frac{x+4}{4x+5x^2}} \cdot (40x+21) > 0 \end{aligned}$$

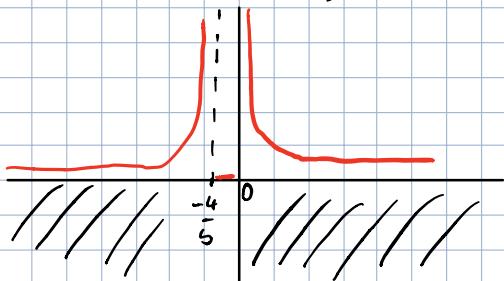
$$\begin{cases} -e^{\frac{x+4}{4x+5x^2}} > 0 \rightarrow x \in \emptyset \\ (40x+21) > 0 \rightarrow x > -\frac{21}{40} \end{cases} \rightarrow x \in \emptyset$$

quindi < 0

$$\begin{cases} -e^{\frac{x+4}{4x+5x^2}} < 0 \rightarrow x \in \mathbb{R} \\ (40x+21) < 0 \rightarrow x < -\frac{21}{40} \end{cases} \rightarrow x \in (-\infty, -\frac{21}{40})$$

Ora

$$\begin{cases} x \in (-\infty, -\frac{21}{40}) \\ (4x+5x^2)^2 \quad x \in \mathbb{R} \setminus \{0, -\frac{4}{5}\} \end{cases} \rightarrow x \in (-\infty, -\frac{4}{5}) \cup (-\frac{4}{5}, -\frac{21}{40})$$



$$\textcircled{3} \quad \lim_{x \rightarrow 0} \frac{\cos(sx^3) - 1}{\log(1+9x^6)}$$

$$\lim_{x \rightarrow +\infty} \frac{\cos(sx^3) - 1}{\log(1+9x^6)}$$

$$\lim_{x \rightarrow +\infty} \frac{\cos(sx^3) - 1}{\log(1+9x^6)} = \lim_{x \rightarrow +\infty} \frac{1}{\log(1+9x^6)} = 0$$

e $\cos(sx^3) - 1$ é perto de $-3, -1$ quando

$$\lim_{x \rightarrow +\infty} \frac{\cos(sx^3) - 1}{\log(1+9x^6)} = \boxed{0}$$

$$\lim_{x \rightarrow 0} \frac{\cos(sx^3) - 1}{\log(1+9x^6)} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx} \cos(sx^3) - 1}{\frac{d}{dx} \log(1+9x^6)} =$$

$$\lim_{x \rightarrow 0} \frac{-15x^2 \cdot \operatorname{sen}(sx^3)}{9x} = -\frac{15x^2}{5} \cdot \operatorname{sen}(sx^3) \cdot \frac{1+9x^6}{18x^3} =$$

$$\lim_{x \rightarrow 0} -\frac{s(9x^6 + 1) \operatorname{sen}(sx^3)}{18x^3} = -\frac{s}{18} \left(\lim_{x \rightarrow 0} (9x^6 + 1) \cdot \lim_{x \rightarrow 0} \frac{\operatorname{sen}(sx^3)}{x^3} \right)$$

$$\lim_{x \rightarrow 0} -\frac{s \cdot s \cos(sx^3)}{18} = -\frac{s \cdot s}{18} = \boxed{-\frac{25}{18}}$$

$$\frac{s \cdot s \cos(sx^3)}{18}$$

$$\begin{aligned}
 & \textcircled{2} \quad \int \frac{e^x}{e^{2x} + 4e^x + 5} dx \\
 & \qquad \qquad \qquad \begin{aligned} u &= e^x \\ du &= e^x dx \end{aligned} \\
 & \int \frac{1}{u^2 + 4u + 5} du = \int \frac{1}{u^2 + 4u + 4 + 1} du \\
 & \int \frac{1}{(u+2)^2 + 1} du \quad \begin{aligned} t &= u+2 \\ dt &= du \end{aligned} \quad \int \frac{1}{t^2 + 1} \\
 & \int \frac{1}{t^2 + 1} dt = \frac{1}{2} \arctan\left(\frac{x}{2}\right) \\
 & \frac{1}{2} \arctan(t) \rightarrow \arctan(u+2) \rightarrow \boxed{\arctan(e^x+2) + C}
 \end{aligned}$$

19/07/16

$$f(x) = \log(9x^2 + 18x + 17)$$

$$9x^2 + 18x + 17 > 0 \quad \Delta = 324 - 612 = -288$$

no sol R

Dam $f \in R$

intersezioni

$$f(0) = \log(0+0+17)$$

$$f(x) = 0 \in Q \quad \text{intersezione } (0, \ln(17))$$

Segno

escludo log sempre $> 0 \quad \forall x \in \mathbb{R}$

$$\log(9x^2 + 18x + 17) > 0 \quad \forall x \in \mathbb{R}$$

quindi la funzione e' sempre positiva

Limiti

$$\lim_{x \rightarrow +\infty} \log(9x^2 + 18x + 17) = +\infty$$

$$\lim_{x \rightarrow -\infty} \log(9x^2 + 18x + 17) = +\infty \quad \text{il } \lim_{x \rightarrow -\infty} \text{ di un polinomio}$$

di grado pari il cui coefficiente del termine di grado massimo e' positivo, e' uguale a $+\infty$

Derivate

$$y = \log(9x^2 + 18x + 17)$$

$$y' = \frac{d}{dx} (\ln(x)) \cdot \frac{d}{dx} (9x^2 + 18x + 17)$$

$$\frac{1}{9x^2 + 18x + 17} \cdot 18x + 18 = \frac{18x + 18}{9x^2 + 18x + 17}$$

Segno derivate

$$\frac{18x + 18}{9x^2 + 18x + 17} > 0$$

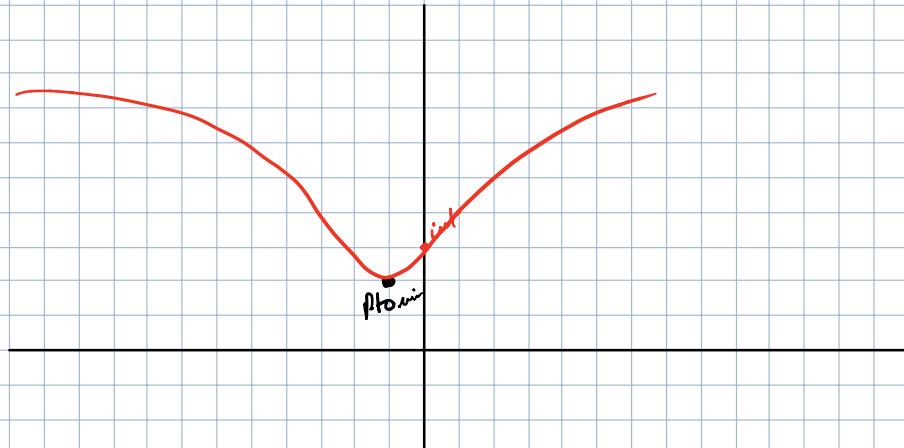
$$\begin{cases} 18x + 18 > 0 & x > -1 \\ 9x^2 + 18x + 17 > 0 & x \in \mathbb{R} \end{cases}$$

$$\begin{array}{c} -1 \\ \text{pero} \\ \text{super} \\ + \\ x \in (-1, +\infty) \end{array}$$

p. to min in (-1)

$$\ln((9 \cdot (-1)^2 + 18 \cdot (-1) + 17)) = 9 - 18 + 17 = \ln(8)$$

$$= \ln(2^3) = 3\ln(2) \approx 2,07 \quad \text{p. to min } (-1, 2,07)$$



$$\int \log(9x^2 + 24x + 16) dx$$

$$l = \log(9x^2 + 24x + 16)$$

$$g^1 = 1 \quad g = x$$

$$\int \log(9x^2 + 24x + 16) \cdot 1 dx$$

$$l^1 = \frac{1}{9x^2 + 24x + 16} \cdot 18x + 24 =$$

$$x \log(9x^2 + 24x + 16) - \int \frac{6x}{3x+4} dx \left| \frac{1}{(3x+4)^2} \cdot 6(3x+4) \right.$$

$$x \log(9x^2 + 24x + 16) - 6 \int \frac{x}{3x+4} dx \left| = \frac{6}{3x+4} \right.$$

$$x \log(9x^2 + 24x + 16) - 6 \int \left(\frac{1}{3} - \frac{4}{3(3x+4)} \right) dx \left| \begin{array}{c} x \\ -x + \frac{1}{3} \\ \hline \frac{1}{3} \end{array} \right. > \log(u)$$

$$= x \log(9x^2 + 24x + 16) + 8 \int \frac{1}{u} du - 2 \int 1 dx$$

$$u = 3x + 4$$

$$du = 3dx$$

$$\frac{8 \log(u)}{3} + x \log(9x^2 + 24x + 16) - 2 \int 1 dx$$

$$-2x + x \log((3x+4)^2) + \frac{8}{3} \log(3x+4) + C$$

$$\int \operatorname{arctg}(\sqrt{8x-18}) dx$$

$$y = \sqrt{8x-18} \quad x =$$

$$du = \frac{1}{2\sqrt{8x-18}} \cdot 8^4 dx$$

$$\int \operatorname{arctan}(\sqrt{8x-18}) \cdot \frac{1}{\frac{4}{\sqrt{8x-18}}} dx$$

$$dx = \frac{1}{4} dy$$

$$\int \operatorname{arctan}(\sqrt{8x-18}) \cdot \frac{\sqrt{8x-18}}{4} dx$$

$$\frac{1}{4} \int \operatorname{arctan}(y) \cdot y dy \rightarrow \text{per parti: } \int u dv = uv - \int v du$$

$$u = \operatorname{arctan}(y)$$

$$\frac{1}{4} \int \operatorname{arctan}(y) \cdot \frac{y^2}{2} - \int \frac{y^2}{2} \cdot \frac{1}{1+y^2} dy \quad du = \frac{1}{1+y^2} dy$$

$$dv = y' dy \quad v = \frac{y^2}{2}$$

$$\lim_{x \rightarrow +\infty} x^{-3} \int_0^x t \log(e^{3t} + 5) dt \quad \text{Hop.}$$

$$\lim_{x \rightarrow +\infty} \frac{\int_0^x t \log(e^{3t} + 5) dt}{x^3}$$

$$\frac{d}{dx}$$

$$\frac{d}{dx}$$

$$\lim_{x \rightarrow +\infty} \frac{\log(e^{3x} + 5)}{3x^2} = \lim_{x \rightarrow +\infty} \frac{\log(e^{3x} + 5)}{3x} = \text{Hop ancora}$$

$$\frac{\frac{d}{dx} \log(e^{3x} + 5)}{\frac{d}{dx} 3x} \rightarrow \lim_{x \rightarrow +\infty} \frac{\cancel{e^{3x}}}{\cancel{3}} = \lim_{x \rightarrow +\infty} \frac{e^{3x}}{e^{3x} + 5} = \lim_{x \rightarrow +\infty} \frac{e^{3x}}{e^{3x}}$$

Ancora Hopital $\rightarrow \lim_{x \rightarrow +\infty} \frac{3e^{3x}}{3e^{3x}} = \underline{\underline{1}}$

$$\int \frac{x^2}{(x-3)(x-5)} dx$$

Visto che $\deg D(x) < \deg N(x)$
faccio divisione di polinomi

$$1 + \frac{8}{x} - \frac{120}{x^2} + \frac{1}{x^2-8x+15}$$

$$x^2-8x+15 \rightarrow (x-3)(x-5)$$

$$\begin{array}{r} x^2 \quad 0x \quad 0 \\ -x^2 + 8x - 15 \\ \hline 8x - 15 \\ -8x + 64 \\ \hline 49 - 120 \\ \hline x \end{array} \quad \begin{array}{l} x^2-8x+15 \\ \hline 1+\frac{8}{x} \end{array}$$

$$\frac{A}{(x-3)} + \frac{B}{(x-5)} = \frac{Ax-5A+Bx-3B}{(x-3)(x-5)} =$$

$$\frac{x(B+A)-5A-3B}{(x-3)(x-5)} = \begin{cases} B+A=0 \\ -5A-3B=1 \end{cases} \quad \begin{cases} A=-B \\ 5B-3B=1 \end{cases} \quad \begin{cases} A=-\frac{1}{2} \\ B=\frac{1}{2} \end{cases}$$

$$\frac{1}{(x-3)(x-5)} = -\frac{1}{2} \frac{1}{(x-3)} + \frac{1}{2} \frac{1}{(x-5)}$$

$$\int \frac{x^2}{(x-3)(x-5)} = \int 1 - \frac{112}{x} - \frac{1}{2} \frac{1}{(x-3)} + \frac{1}{2} \frac{1}{(x-5)}$$

$$\int 1 dx - \int \frac{112}{x} dx - \frac{1}{2} \int \frac{1}{(x-3)} dx + \frac{1}{2} \int \frac{1}{(x-5)} dx$$

$$x - 112 \ln(x) - \frac{1}{2} \ln(x-3) + \frac{1}{2} \ln(x-5) + C$$

$$f(x) = \frac{x^2}{(x-3)(x-5)}$$

$$x^2 - 8x + 15 \neq 0$$

$$\Delta = 64 - 4 \cdot 15 = 4$$

$$x_{1,2} = \frac{8 \pm 2}{2}$$

s

3

$$\text{Dom } f \subset \mathbb{R} / \{3, 5\}$$

$$x \neq 5, 3 \quad x < 3 \cup x > 5$$

Intersezione assi:

$$f(0) = \frac{0}{15} = 0$$

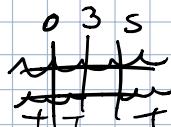
Intersezione (0,0)

$$f(x) = 0 \quad \frac{x^2}{x^2 - 8x + 15} = 0 \in \emptyset$$

Segno f:

$$\frac{x^2}{x^2 - 8x + 15} > 0$$

$$\begin{cases} x^2 > 0 \quad \forall x \in \mathbb{R} \\ x < 3 \vee x > 5 \end{cases}$$



$$x < 3 \vee x > 5$$

$$-\infty \xrightarrow{3} \downarrow 3, 5 \quad \nearrow 5 \rightarrow \infty$$

Limiti

$$\lim_{x \rightarrow \pm\infty} \frac{x^2}{x^2 - 8x + 15} = \frac{x^2}{x^2(1 - \frac{8}{x} + \frac{15}{x^2})} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x^2}{x^2 - 8x + 15} = 1$$

$$\lim_{x \rightarrow 3^+} \frac{x^2}{x^2 - 8x + 15} = \frac{9}{9 - 24 + 15} = \frac{9}{0^+} = +\infty$$



$$\lim_{x \rightarrow 3^-} \frac{x^2}{x^2 - 8x + 15} = \frac{9}{0^-} = -\infty$$

$$\lim_{x \rightarrow 5^+} \frac{x^2}{x^2 - 8x + 15} = \frac{25}{25 - 40 + 15} = \frac{25}{0^+} = +\infty$$

$$\lim_{x \rightarrow 5^-} \frac{x^2}{x^2 - 8x + 15} = -\infty$$

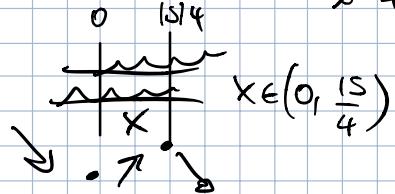
Derivata

$$\frac{\frac{d}{dx}(x^2)}{x^2 - 8x + 15} = \frac{2x \cdot (x^2 - 8x + 15) - x^2 \cdot (2x - 8)}{(x^2 - 8x + 15)^2} =$$

$$\frac{2x^3 - 16x^2 + 30x - 2x^3 - 8x^2}{(x^2 - 8x + 15)^2} = \frac{-8x^2 + 30x}{(x^2 - 8x + 15)^2} > 0 \quad \begin{cases} -8x^2 + 30x > 0 \\ \forall x \in \mathbb{R} \end{cases}$$

$$\begin{cases} x \in (0, \frac{15}{4}) \rightarrow s. \quad x \neq 3 \\ \forall x \in \mathbb{R} \end{cases} \quad \begin{matrix} \nearrow \\ \downarrow \\ (0, 3] \cup [3, \frac{15}{4}) \end{matrix}$$

$$\begin{cases} x > 0 \\ -8x + 30 > 0 \end{cases} \quad \begin{cases} x > 0 \\ x < \frac{30}{8} = \frac{15}{4} \end{cases}$$

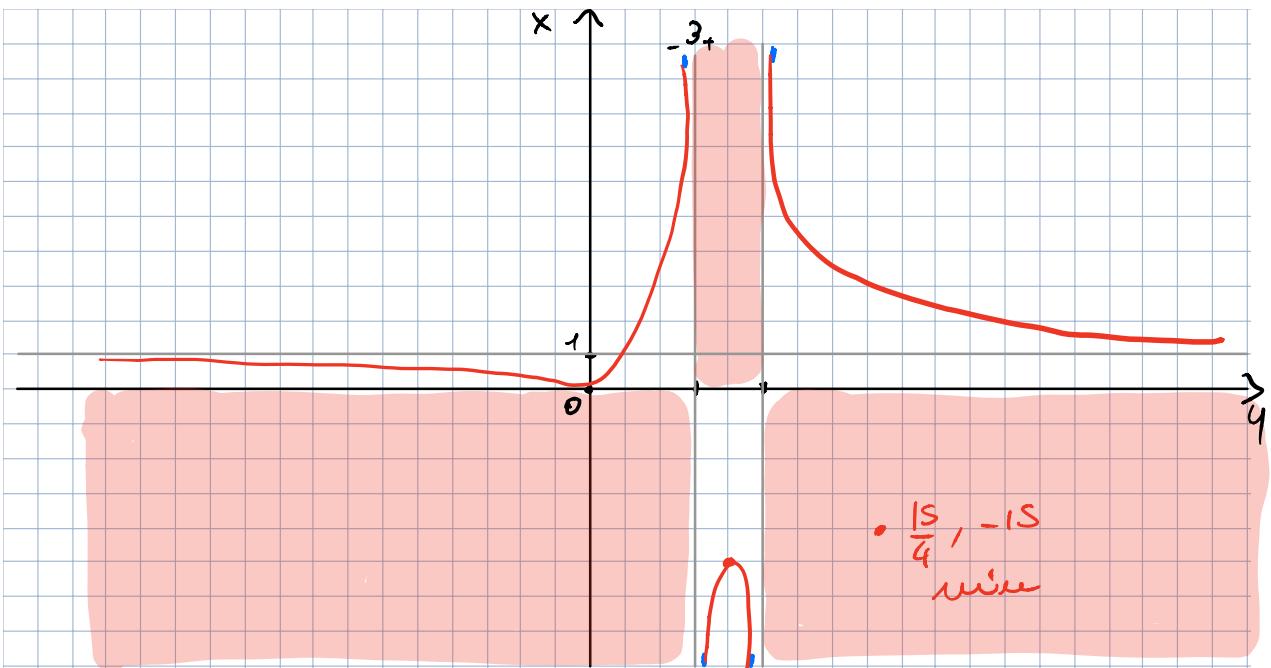


p.t.o min (0) (0,0)

p.t.o max ($\frac{15}{4}$) ($\frac{15}{4}, -15$)

$f(0) \rightarrow 0$ min in $0, 0$ (intersez.)

$$\begin{aligned} f\left(\frac{15}{4}\right) &\rightarrow \frac{\left(\frac{15}{4}\right)^2}{\left(\frac{15}{4}\right)^2 - \left(8 \cdot \frac{15}{4}\right) + 15} = \frac{\frac{225}{16}}{\frac{225}{16} - 30 + 15} = \frac{\frac{225}{16}}{\frac{225}{16} - 15} \\ &= \frac{\frac{225}{16}}{-\frac{15}{16}} = \frac{225}{16} \cdot \frac{-16}{15} = -15 \end{aligned}$$



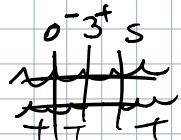
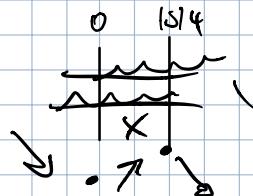
intersezione $(0, 0)$

p.t.o min (0) $(0, 0)$

p.t.o max $(\frac{15}{4})$ $(\frac{15}{4}, -15)$

oscurtozzi orizzontali 1
oscurtozzi verticali

$$\begin{array}{ll} s^+ + \infty & s^- - \infty \\ 3^+ - \infty & 3^- + \infty \end{array}$$



limite $\lim_{x \rightarrow 0} \frac{x \operatorname{sen}(x)}{\operatorname{sen}(e^x - 1) - 3x} = \frac{0 \operatorname{sen}(0)}{\operatorname{sen}(e^0 - 1) - 0} = 0$

Esercizi

$$(Esercizio 1) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} [x^5 \cos(x) + 3 \cos(2x)] dx$$

risulta 0. perché è l'integrale di una
f. disposta su un intervallo simmetrico
rispetto all'origine

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^5 \cos(x) dx + 3 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(2x) dx =$$

\Downarrow per sostituzione $u = 2x$

$$3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(u) du = \frac{3}{2} [\sin(\frac{\pi}{2}) - \sin(-\frac{\pi}{2})] =$$

$$\frac{3}{2} (1 - (-1)) = \frac{3}{2} \cdot 2 = 3$$

$du = 2dx$
 $\frac{\pi}{4} \rightarrow \frac{\pi}{2}$
 $-\frac{\pi}{4} \rightarrow -\frac{\pi}{2}$

$$(Esercizio 2) \int_0^{15} \frac{72}{\sqrt{x+49} + \sqrt{x+1}} dx =$$

$$\frac{72}{\sqrt{x+49} + \sqrt{x+1}} \cdot \frac{\sqrt{x+49} - \sqrt{x+1}}{\sqrt{x+49} - \sqrt{x+1}} = \frac{72 \cdot \sqrt{x+49} - \sqrt{x+1}}{x+49 - (x+1)} =$$

$$\int_0^{15} \frac{3}{2} (\sqrt{x+49} - \sqrt{x+1}) dx = \frac{3}{2} \int_0^{15} 1 \cdot (x+49)^{1/2} dx - \frac{3}{2} \int_0^{15} 1 \cdot (x+1)^{1/2} dx$$

$$= \frac{3}{2} \cdot \frac{(x+49)^{3/2}}{\frac{3}{2}} - \frac{3}{2} \cdot \frac{(x+1)^{3/2}}{\frac{3}{2}} + C =$$

$$= \sqrt{(x+49)^3} - \sqrt{(x+1)^3} + C \rightarrow \left[\sqrt{(x+49)^3} - \sqrt{(x+1)^3} \right]_0^{15} =$$

$$= \sqrt{64^3} - \sqrt{16^3} - (\sqrt{49^3} - \sqrt{1^3}) = 512 - 64 - (343 - 1) = 106$$

(es 3) Trova una primitiva di $f(x) = \frac{\sqrt{x+8}}{\sqrt{x+8} + x+8} + x^{2019}$

$$\int \frac{\sqrt{x+8}}{\sqrt{x+8} + x+8} dx = \int \frac{t}{t+t^2} 2t dt =$$

$$t = \sqrt{x+8} \quad dx$$

$$dt = \frac{1}{2\sqrt{x+8}} dx = \frac{1}{2t} dx$$

$$= \int \frac{2t^2}{t(t+1)} dt = 2 \int \frac{t}{t+1} dt$$

$$dx = 2t dt$$

$$= 2 \int \frac{t+1}{t+1} dt - 2 \int \frac{1}{t+1} dt =$$

$$= 2t - 2 \ln|t+1| + C = 2\sqrt{x+8} - 2 \ln(\sqrt{x+8} + 1) + C$$

$$= 2\sqrt{x+8} - 2 \ln(\sqrt{x+8} + 1) + \frac{x^{2020}}{2020} e^{-} \text{primitiva di } f(x)$$

$$\lim_{x \rightarrow +\infty} x^3 \log \left(\frac{x^3+3}{x^3+5} \right) \quad x^3 = t$$

$$\lim_{t \rightarrow +\infty} t \log \left(\frac{t+3}{t+5} \right) \rightarrow \frac{\log \left(\frac{t+3}{t+5} \right)}{\frac{1}{t}} \rightarrow \text{Höp}$$

$$\frac{(t+5) \left(\frac{1}{t+5} - \frac{t+3}{(t+5)^2} \right)}{t+3} = - \frac{2t^2}{(t+3)(t+5)} =$$

$$-\frac{1}{t^2}$$

$$-2 \lim_{t \rightarrow +\infty} \left(\frac{t^2}{(t+3)(t+5)} \right) = \boxed{-2}$$

$$\int \frac{1}{3\sin(x) - 5\cos(x) + 5} dx = \quad t = \tan\left(\frac{x}{2}\right)$$

$$\sin(x) = \frac{2t}{1+t^2}$$

$$\int \frac{1}{3 \cdot \frac{2t}{1+t^2} - 5 \cdot \frac{1-t^2}{1+t^2} + 5} \cdot \frac{2}{1+t^2} dt = \quad \cos(x) = \frac{1-t^2}{1+t^2}$$

$$t = 2 \arctan(x)$$

$$dx = \frac{2}{1+t^2} dt$$

$$\int \frac{2}{6t - 5(1-t^2) + 5(1+t^2)} dt = \quad \text{lien qui devrait être}$$

$$\int \frac{2}{6t - 8t + st^2 + 8 + st^2} dt = \quad \begin{cases} \frac{1}{3t + st^2} dt = \frac{A}{t} + \frac{B}{3+st} \\ t(3+st) \end{cases}$$

$$A(3+st) + Bt$$

$$3A + stA + Bt \rightarrow t(stA + B) + 3A$$

$$\begin{cases} SA + B = 0 \\ 3A = 1 \end{cases} \quad \begin{cases} \frac{S}{3} = -B \\ A = \frac{1}{3} \end{cases} \quad B = -\frac{S}{3}$$

$$\int \frac{1}{3+st^2} = \int \frac{\frac{1}{3}}{t} dt + \int \frac{-\frac{S}{3}}{3+st} dt$$

$$\frac{1}{3} \ln(t) + \frac{1}{3} \ln(3+st)$$

$$\frac{1}{3} \ln(\tan\frac{x}{2}) + \frac{1}{3} \ln(3+s \tan\frac{x}{2})$$

$$\int \frac{9x^2 + 32x + 44}{9x^2 + 30x + 41} dx \quad \int \frac{2x+3}{9x^2 + 30x + 41} dx$$

$$\int \frac{2x+3}{9x^2+30x+41} dx + \int 1 dx$$

$9x^2+32x+44$	$9x^2+30x+41$
$-9x-30x-41$	
<hr/>	
	$2x+3$

$$\int \left(\frac{18x+30}{9(9x^2+30x+41)} - \frac{1}{3(9x^2+30x+41)} \right) dx + \int 1 dx$$

$$\frac{1}{9} \int \frac{18x+30}{9(9x^2+30x+41)} dx - \frac{1}{3} \int \frac{1}{3(9x^2+30x+41)} dx + \int 1 dx$$

$$y = 9x^2+30x+41 \quad \frac{1}{9} \int \frac{1}{4} dy - \frac{1}{3} \int \frac{1}{3(9x^2+30x+41)} dx + \int 1 dx$$

$$dy = 18x+30 dx$$

$$\frac{\log(4)}{9} - \frac{1}{3} \int \frac{1}{(3x+s)^2+16} ds + \int 1 dx$$

↓

$$s = 3x+s \quad \frac{\log(4)}{9} - \frac{1}{9} \int \frac{1}{s^2+16} ds + x$$

$$ds = 3dx$$

$$\frac{\log(4)}{9} - \frac{1}{9} \int \frac{1}{16(\frac{s^2}{16}+1)} ds + x$$

$$\frac{\log(4)}{9} - \frac{1}{144} \int \frac{1}{\frac{s^2}{16}+1} ds + x$$

$$p = \frac{s}{4} \quad dp = \frac{1}{4} ds \quad \frac{\log(4)}{9} - \frac{1}{36} \int \frac{1}{p^2+1} dp + x$$

$$\frac{\log(4)}{9} - \frac{1}{36} \operatorname{arctan}(p) + x + C$$

$$\frac{\log(9x^2+30x+41)}{9} - \frac{1}{36} \operatorname{arctan}\left(\frac{3x+s}{4}\right) + x + C$$

$$\lim_{x \rightarrow 0} x^{-4} \int_0^x \sin(4t^3) e^{-t} dt$$

$$\lim_{x \rightarrow 0} \frac{\int_0^x \sin(4t^3) e^{-t} dt}{x^4}$$

Hop

$$\lim_{x \rightarrow 0} \frac{\sin(4x^3) e^{-x}}{x^4} = \text{non esiste} \quad \begin{matrix} \downarrow \\ \text{perché} \end{matrix}$$

$$\begin{matrix} 0^- \rightarrow -\infty \\ 0^+ \rightarrow +\infty \end{matrix} \quad \cancel{\frac{\sin(1) - (e^{-x})^0}{x^4}} = \frac{\sin(1)}{0} = 0$$

$$f(x) = \frac{x+4}{x^2+7} \quad x^2+7 \neq 0 \quad \forall x \in \mathbb{R}$$

Dau $f \in \mathbb{R}$

Intersezioni

$$f(0) = 0$$

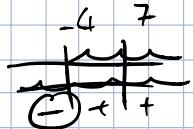
$$\text{Intersezione } (0, \frac{4}{7})$$

$$f(x) = 0 \Leftrightarrow \frac{4}{7}$$

Segni

$$\begin{cases} x+4 > 0 \\ x^2+7 > 0 \end{cases}$$

$$\begin{cases} x > -4 \\ \forall x \in \mathbb{R} \end{cases}$$



$$\text{pos } (-4, +\infty)$$

Limiti

$$\lim_{x \rightarrow \infty} \frac{x+4}{x^2+7} = \frac{x^2(1+\frac{4}{x^2})}{x^2(1+\frac{7}{x^2})} = 0$$

$$\lim_{x \rightarrow -\infty} = 0$$

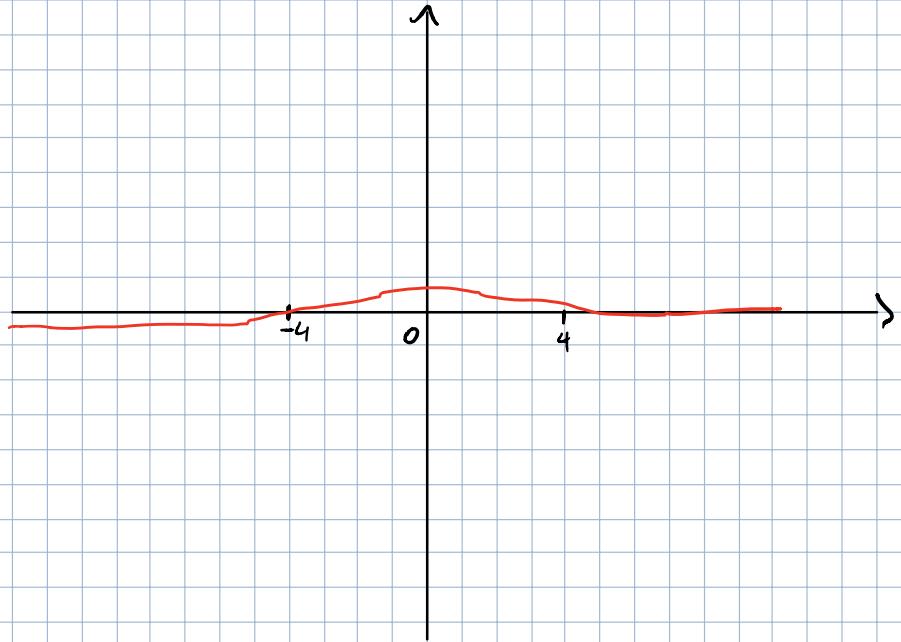
Derivata

$$\frac{-x^2-8x-7}{(x^2+7)^2} > 0$$

$$\Delta = 64 - 4 \cdot (-1) \cdot 7 \rightarrow 36$$

$$\begin{cases} -x^2-8x-7 > 0 \\ \forall x \in \mathbb{R} \end{cases} \quad \begin{cases} x < 1 \vee x > 7 \\ \forall x \in \mathbb{R} \end{cases}$$

$\frac{-4-\sqrt{23}}{2}, \frac{4\sqrt{23}}{2}$



$$f(x) = \log(9x^2 + 24x + 17)$$

$$9x^2 + 24x + 17 > 0$$

① Dati $f \in \mathbb{R}$

$$\Delta = 576 - 4 \cdot 9 \cdot 17 = -36$$

② Intersezioni

$$f(0) = \log(17)$$

Intersezione $(-\frac{4}{3}, \ln(17))$

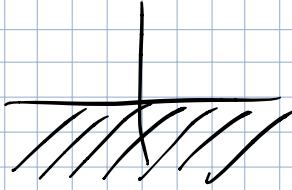
$f(x) = 0$ log può essere = 0 solo quando l'esponente è 1

$$9x^2 + 24x + 17 = 1 \rightarrow 9x^2 + 24x + 17 - 1 = 0 \rightarrow 9x^2 + 24x + 16 = 0$$

$$(3x+4)^2 = 0 \quad 3x+4=0 \quad x = -\frac{4}{3}$$

③ Segno

$$9x^2 + 24x + 17 \quad x \in \mathbb{R}$$



④ limiti

$$\lim_{x \rightarrow +\infty} 9x^2 + 24x + 17 = +\infty$$

$\lim_{x \rightarrow -\infty} = +\infty$ il limite di un polinomio di grado pari il cui coeff. del termine di grado max è positivo è $+\infty$

⑤ Derivata

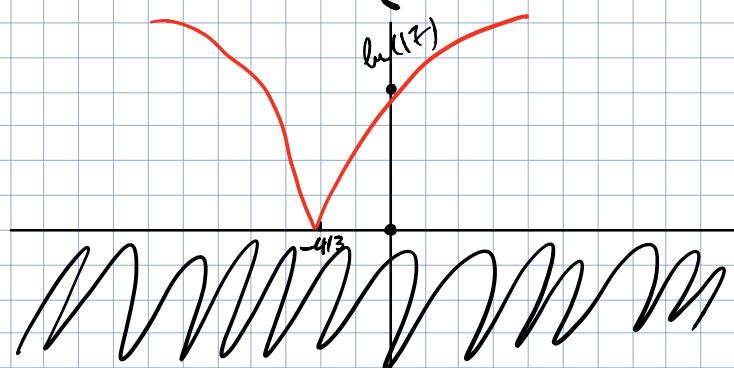
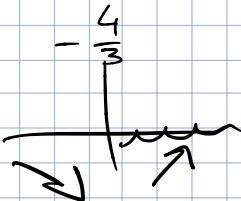
$$\frac{d}{dx} \ln(9x^2 + 24x + 17) = \frac{1}{9x^2 + 24x + 17} \cdot (18x + 24)$$

$$\frac{1}{9x^2 + 24x + 17} \cdot 18x + 24 = \frac{18x + 24}{9x^2 + 24x + 17}$$

⑥ Segno derivate

$$\frac{18x + 24}{9x^2 + 24x + 17} > 0$$

$$\left\{ \begin{array}{l} x > -\frac{24}{18} = -\frac{4}{3} \\ x \in \mathbb{R} \end{array} \right.$$



$$\lim_{x \rightarrow 0^+} x^{-\frac{3}{7}} \int_0^x (\sin(3t))^6 dt = \frac{\sin(3x)^6}{7x^6}$$

$$\frac{3}{7} \lim_{x \rightarrow 0^+} \left(\frac{\sin(x)^6}{x^6} \right) = 1 = \frac{3}{7}$$

livet not

$$\int \frac{\log \log(x)}{x} dx =$$

$$u = \log(x)$$

$$du = \frac{1}{x} dx$$

$$\int \log(u) du = \int \log(u) \cdot 1 du$$

$$f = \log(u) \quad f' = \frac{1}{u}$$

$$g = 1 \quad g' = 1$$

$$= u \log(u) - \int 1 du =$$

$$= \log(x)(\log(\log(x)) - 1) + C$$

$$\lim_{x \rightarrow +\infty} (x(\log x)^4)^{-1} \int_1^x (\log t)^4 dt \rightarrow 1$$

$$\lim_{x \rightarrow +\infty} \frac{(\log(x))^4}{x(\log(x))^4} = \frac{1}{x} = 0$$

-

Infinitesimi

$$\lim_{x \rightarrow \pm\infty} \frac{c}{x} =$$

caso 1) $c > 0 \rightarrow \lim_{x \rightarrow +\infty} \frac{c}{x} = 0^+$

caso 2) $c = 0 \rightarrow \lim_{x \rightarrow \pm\infty} \frac{c}{x} = 0$

caso 3) $c < 0 \rightarrow \lim_{x \rightarrow -\infty} \frac{c}{x} = 0^-$

$$\lim_{x \rightarrow +\infty} \frac{c}{x} = 0^+$$

$$\lim_{x \rightarrow -\infty} \frac{c}{x} = 0^+$$

Altro

$$\lim_{x \rightarrow +\infty} \frac{0^+}{x} = 0^+$$

$$\lim_{x \rightarrow -\infty} \frac{0^-}{x} = 0^-$$

$$\lim_{x \rightarrow +\infty} \frac{0^-}{x} = 0^-$$

$$\lim_{x \rightarrow -\infty} \frac{0^-}{x} = 0^+$$

Casi i logoritmi $f(x) = \log(x)$

$$\lim_{x \rightarrow +\infty} \log(x) = +\infty$$

$$\lim_{x \rightarrow 0^+} \log(x) = -\infty$$

Tabelle

$$\frac{+\infty}{0^+} = +\infty$$

$$\frac{-\infty}{0^+} = -\infty$$

$$(+\infty)^n = +\infty \quad \text{tutti } n > 0$$

$$\frac{+\infty}{0^-} = -\infty$$

$$\frac{-\infty}{0^-} = +\infty$$

$$(+\infty)^n = 0 \quad \text{tutti } n < 0$$

$$(+\infty)^{(+\infty)} = +\infty$$

$$(+\infty)^{(-\infty)} = 0^+$$

$$\sqrt[n]{(+\infty)} = +\infty \quad \text{tutti interi}$$

$$\sqrt[n]{(-\infty)} = -\infty \quad \text{tutti interi, dispari}$$

$$\frac{c > 0}{0^+} = +\infty$$

$$\frac{c > 0}{0^-} = -\infty$$

$$\frac{c < 0}{0^+} = -\infty$$

$$\frac{c < 0}{0^-} = +\infty$$

Metodo di integrazione generale #1

$$y = \frac{N(x)}{D(x)}$$

① Divisione fra $N < D$ se $gr(N) \geq gr(D)$

② Rettorizzazione il den

③ decomporre la frazione in fratti semplici

④ Integrazione dei pezzi

Esempio 1

$$\int \frac{x^3 - 3x - 1}{x^2 - x - 2} dx$$

$$\frac{x^3 - 3x - 1}{x^2 - x - 2} = x+1 + \frac{l}{x^2 - x - 2}$$

$$\begin{array}{r} x^3 - 3x^2 - 3x - 1 \\ -x^3 + x^2 + 2x \\ \hline \end{array} \left| \begin{array}{c} x^2 - x - 2 \\ x+1 \end{array} \right.$$

$$\begin{array}{r} x^2 - x - 1 \\ -x^2 + x + 2 \\ \hline 1 \end{array} \left| \begin{array}{c} 1 \\ 1 \end{array} \right.$$

$$② x^2 - x - 2 = (x-2)(x+1)$$

$$③ \frac{l}{x^2 - x - 2} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{Ax + A + Bx - 2B}{(x-2)(x+1)} =$$

$$\frac{x(A+B) + A - 2B}{(x-2)(x+1)}$$

$$\begin{cases} A+B=0 \\ A-2B=1 \end{cases} \quad \begin{cases} A=-B \\ -B-2B=1 \end{cases} \quad \begin{cases} A=\frac{1}{3} \\ B=-\frac{1}{3} \end{cases}$$

$$-3B=1$$

$$B=-\frac{1}{3}$$

$$\frac{1}{(x-2)(x+1)} = \frac{1}{3} \cdot \frac{1}{x-2} - \frac{1}{3} \cdot \frac{1}{x+1}$$

$$④ \int \frac{x^3 - 3x - 1}{x^2 - x - 2} dx = \int \frac{1}{3(x-2)} - \frac{1}{3(x+1)} dx$$

$$\int x dx + \int dx + \frac{1}{3} \int \frac{1}{(x-2)} dx - \frac{1}{3} \int \frac{1}{(x+1)} dx$$

$$\begin{array}{ccc}
 \downarrow & \downarrow & \\
 y = x-2 & dy = 1 dx & z = x+1 \\
 \frac{x^2}{2} + x + \frac{1}{3} \ln(x-2) - \frac{1}{3} \ln(x+1) + C & dz = 1 dx \\
 \frac{x^2}{2} + x + \frac{1}{3} \ln \frac{(x-2)}{(x+1)} + C
 \end{array}$$

Esempio 2

$$\int \frac{1}{x^2-1} dx \quad \textcircled{1} \deg(D) > \deg(N) \\
 \textcircled{2} \quad x^2-1 = (x-1)(x+1)$$

$$\textcircled{3} \quad \frac{1}{(x-1)(x+1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1) + B(x+1)}{(x+1)(x-1)} =$$

$$\frac{Ax-A+Bx+B}{(x+1)(x-1)} = \frac{x(B+A)-A+B}{(x+1)(x-1)} = \begin{cases} A+B=0 \\ B-A=1 \end{cases} \quad \begin{cases} A=-B \\ B=B=1 \end{cases}$$

$$\begin{cases} A=-\frac{1}{2} \\ B=\frac{1}{2} \end{cases} \quad \textcircled{4} \quad \int \frac{1}{x^2-1} dx = \int -\frac{1}{2} \cdot \frac{1}{x+1} + \frac{1}{2} \cdot \frac{1}{x-1} dx$$

$$\begin{aligned}
 -\frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx &= -\frac{1}{2} \ln(x+1) + \frac{1}{2} \ln(x-1) + C \\
 &= \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + C
 \end{aligned}$$

Metodo Generale #2

- Denar 2° g non scampabile } problemi !
- Potenze di un monomio

Esempio 1

$$\int \frac{1}{x^3+x} dx$$

① non serve $\deg(D) < \deg(N)$

$$② x^3+x = x(x^2+1)$$

$$③ \frac{1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)} =$$

$$\frac{x^2(A+B)+Cx+A}{x(x^2+1)}$$

$$\begin{cases} A+B=0 \\ C=0 \\ A=1 \end{cases} \quad \begin{cases} A=1 \\ B=-1 \\ C=0 \end{cases}$$

$$④ \int \frac{1}{x^3+x} dx = \int \frac{1}{x} dx - \frac{1}{2} \int \frac{2x}{x^2+1} dx = \ln(|x|) - \frac{1}{2} \ln|x^2+1| + C$$

$$= \ln \left| \frac{x}{\sqrt{x^2+1}} \right| + C$$

possano dare come risultati
 ln e arctan

Esempio 2

$$\int \frac{x+1}{x^3-2x^2+x} dx$$

① non serve

$$② x^3-2x^2+x = x(x^2-2x+1) = x(x-1)^2$$

$$③ \frac{x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} =$$

$$\frac{A(x-1)^2 + Bx(x-1) + Cx}{x(x-1)^2} = \frac{x^2(A+B) + x(C-2A-B) + A}{x(x-1)^2}$$

$$\begin{cases} A+B=0 \\ C-2A-B=1 \\ A=1 \end{cases}$$

$$\begin{cases} A=1 \\ B=-1 \\ C=2 \end{cases}$$

$$④ \int \frac{x+1}{x(x-1)^2} dx = \int \frac{1}{x} dx - \int \frac{1}{x-1} dx + 2 \int \frac{1}{(x-1)^2} dx$$

$$\ln|x| - \ln|x-1| + 2 \frac{(x-1)^{-1}}{-1} + C$$

$$\ln\left|\frac{x}{x-1}\right| - \frac{2}{x-1} + C$$

Se per caso

$$\frac{l}{x(x-3)^3} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} + \frac{D}{(x-3)^3}$$