① SDF

$f(x) = \frac{x-1}{x^2-x-6}$ Dom f $x^2-x-6 \neq 0$

$-x = +2x - 3x$

$x^2+2x-3x-6 \neq 0$

$x(x+2) - 3(x+2)$

$(x+2)-(x-3)$

$x+2=0 \longrightarrow x=-2$

$x-3=0 \longrightarrow x=3$

1 - Dom $f \in \mathbb{R} \setminus \{-2,3\}$

2 - P/D

$f(-x) = \frac{-x-1}{-x^2-x-6}$ dispari?

3 - Intersez.

(y) $f(0) \to \frac{-1}{-6} = \frac{1}{6}$ Interseca $(1, \frac{1}{6})$

(x) $f(x)=0$ $\frac{x-1}{x^2-x-6} = 0$ $x-1=0$ $x=1$

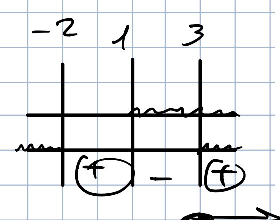
$x \neq -2, x \neq 3$

4 - Segno

$\frac{x-1}{x^2-x-6} > 0$ $x-1>0$ $x>1$

$x^2+x-6>0$ $x<-2 \vee x>3$

$(-\infty,-2) \cup (3,+\infty)$



$S = (-2,1) \cup (3,+\infty)$

5 - limiti

$\lim_{x \to \infty} \frac{x-1}{x^2-x-6} = \frac{\cancel{x}-1}{\cancel{x}^2(1-\frac{1}{x}-\frac{6}{x^2})} = 0$

$\lim_{x \to -\infty} = 0$

$$\lim_{x \to -2^+} \frac{x-1}{x^2 x-6} = \infty$$

$$\lim_{x \to -2^-} \frac{x-1}{x^2-x-6} = -\infty$$

$$\lim_{x \to 3^+} \frac{x-1}{x^2 x-6} = \frac{2}{9-3-6} = \frac{2}{0^+} = +\infty$$

$$\lim_{x \to 3^-} \frac{x-1}{x^2-x-6} = \frac{2^-}{9-3-6} = -\infty$$

$$\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{2g}$$

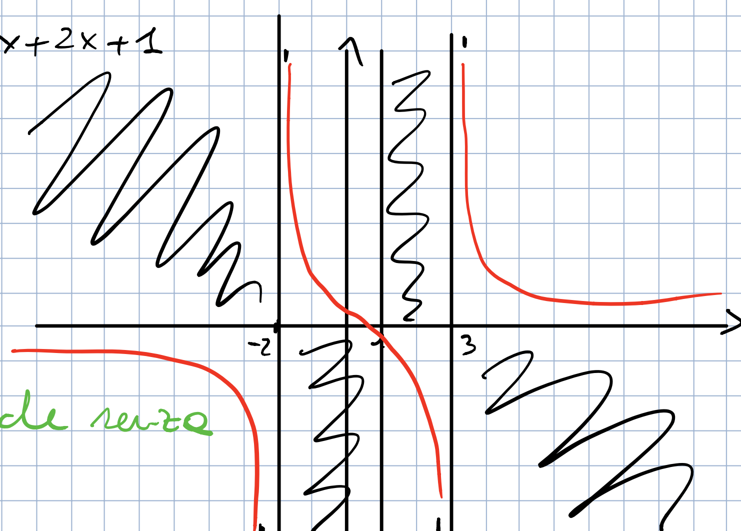
6 - Derivata

$$f(x) = \frac{x-1}{x^2-x-6} \qquad f'(x) = \frac{1\cdot(x^2-x-6)-(x-1)(2x-1)}{(x^2-x-6)^2}$$

$$x^2-x-6-2x^2-1x+2x+1$$

$$\frac{-x^2+2x-7}{(x^2-x-6)^2}$$

Mancano $f'(x)>0$
ma il grafico
era intuibile anche senza



② SDF

$f(x) = e^{-x} - e^{-3x} \rightarrow e^{-x}(1 - e^{-2x})$

$e^{-x} > 0 \quad \forall x \in R$

① Dom $f \in R \setminus \{0\}$ $\quad$ e $f(x) = 0$ se $(1 - e^{-2x}) = 0$

ossia $x = 0$

② Assi

$f(0) \overset{y}{=} 1 - 1 = 0$ $\quad$ (interece 0,0)

$f(x) \overset{x}{=} 0 = e^{-x} - e^{-3x} = 0 \rightarrow 0$

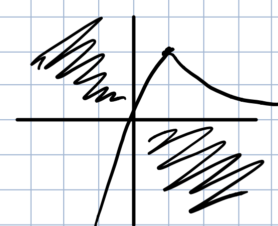
③ Segno

$e^{-x} - e^{-3x} > 0 \qquad e^{-x} > e^{-3x} \qquad -x > -3x$

$-x + 3x > 0 \qquad 2x > 0 \qquad x > \frac{0}{2} \qquad x > 0$

la f è pos da 0 in poi

neg da 0 in dietro



④ lim

$\lim_{x \to \infty} e^{-x} - e^{-3x} = 0$ $\quad$ lim oriz in 0 a ∞

$\lim_{x \to -\infty} e^{-x} - e^{-3x} = -\infty$

⑤ Derivata

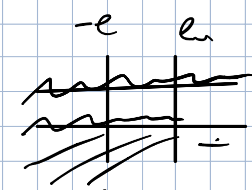
$y = e^{-x} - e^{-3x} = -e^{-x} - (-3e^{-x}) = -e^{-x} + 3e^{-3x}$

$y' > 0 \rightarrow -e^{-x} + 3e^{-3x} > 0 \rightarrow -e^{-x}(1 + 3e^{-2x}) > 0$

$-e^{-x}+3e^{-3x}>0 \rightarrow -e^{-3x}(e^{2x}-3)>0$)

$$\begin{cases} -e^{-3x}>0 & x<0 \\ e^{2x}-3>0 & e^{2x}>3 \rightarrow 2x>\ln(3) \quad x>\dfrac{\ln(3)}{2} \end{cases}$$

$$\begin{cases} -e^{-3x}<0 & \forall x\in\mathbb{R} \\ e^{2x}-3<0 & e^{2x}<3 \quad 2x<\ln(3) \quad x<\dfrac{\ln(3)}{2} \end{cases}$$



$x<\dfrac{\ln(3)}{2}$

quindi cresce da $-\infty$ a $\dfrac{\ln(3)}{2}$

decresce da $\dfrac{\ln(3)}{2}$ a $+\infty$

⑥ Derivata''

$-e^{-x}+3e^{-3x} \Rightarrow e^{-x}-9e^{-3x} = e^{-x}(1-9e^{-2x})$

$e^{-x}>0 \quad \forall x\in\mathbb{R}$

$1-9e^{-2x}>0 \quad 9e^{-2x}>1$

$e^{-2x}>\frac{1}{9}$

$-2x>\ln\frac{1}{9}(3^{-2})$

$x>\dfrac{\ln 3^{-2}}{2}$

$x>\dfrac{\cancel{2}\ln 3}{\cancel{2}}$

$x>\ln 3$

$1-9e^{-2x}=0$

$9e^{-2x}=1$

$e^{-2x}=\frac{1}{9}$

$-2x=\ln\frac{1}{9}$

$-2x=\ln 3^{-2}$

$-2x=-2\ln(3)$

$x=\dfrac{\cancel{2}\ln 3}{\cancel{2}}$

$x=\ln 3$

concava $(-\infty,\ln 3)$ convessa $(\ln,+\infty)$

③ → Esame 24 Novembre 2016 – Parziale

① SDF $f(x) = \frac{x^2 - 5x + 6}{x-4}$ $x-4 \neq 0$
$x \neq 4$

1) Dom $f = x \in \mathbb{R} / \{4\}$
$(-\infty, 4) \cup (4, +\infty)$

2) P/D $f(-x) = \frac{-x^2 + 5x + 6}{-x-4}$ no P/D

3) Intersezione assi

$f(0) = \frac{0 - 0 + 6}{0 - 4} = -\frac{\cancel{6}^{3}}{\cancel{4}_{2}} = -\frac{3}{2}$

$f(x) = 0$ $\frac{x^2 - 5x + 6}{x-4} = 0$ $x \neq 4$

$\Delta = 25 - 4 \cdot 1 \cdot 6 = 1$ $x_{1,2} = \frac{5 \pm 1}{2} < \begin{matrix} 3 \\ 2 \end{matrix}$ // 0

Interseca in $\left(0, -\frac{3}{2}\right)$

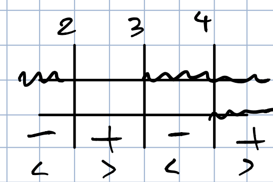
4) Segno

$$\frac{x^2-5x+6}{x-4} > 0$$

$x<2 \ \cup \ x>3$

$x^2-5x+6>0 \to (-\infty,2)\cup(3,+\infty)$

$x-4>0 \to x>4$



negative in in $(-\infty,2)$
$(3,4)$

positive in $(2,3)\cup(4,+\infty)$

5) limiti

$$\lim_{x\to+\infty}\frac{x^2-5x+6}{x-4} = \frac{\cancel{x^2}\left(1-\underbrace{\frac{5}{x}}_{\to 0}+\underbrace{\frac{6}{x^2}}_{\to 0}\right)}{\cancel{x}\left(1-\underbrace{\frac{4}{x}}_{\to 0}\right)} = +\infty$$

$$\lim_{x\to-\infty}\frac{x^2-5x+6}{x-4} = -\infty$$

$$\lim_{x\to4^+}\frac{x^2-5x+6}{x-4} = \frac{16-20+6}{4-4} = \frac{2}{0^+} = +\infty$$

$$\lim_{x\to4^-}\frac{x^2-5x+6}{x-4} = \frac{2}{0^-} = -\infty$$

6) Derivata 1ª

$$y=\frac{x^2-5x+6}{x-4} \qquad y'=\frac{(2x-5)\cdot(x-4)-(x^2-5x+6)}{(x-4)^2}=$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$\frac{2x^2-8x\cancel{-5x}+20-x^2\cancel{+5x}-6}{(x-4)^2} = \boxed{\frac{x^2-8x+14}{(x-4)^2}}$$

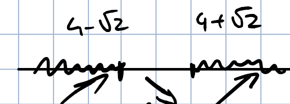
$y' > 0 \quad \frac{x^2 - 8x + 14}{(x-4)^2} > 0 \qquad x^2 - 8x + 14 > 0$

$(x-4)^2 > 0 \quad \forall x \in \mathbb{R}$

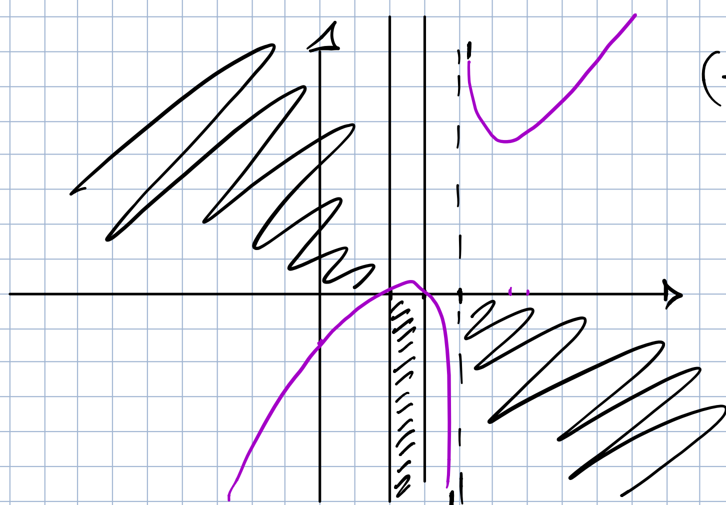
$\Delta = 64 - 4 \cdot 1 \cdot 14 = 8$

$\frac{8 \pm 2\sqrt{2}}{2} < \begin{matrix} \frac{8+2\sqrt{2}}{2} = 4+\sqrt{2} \\ \frac{8-2\sqrt{2}}{2} = 4-\sqrt{2} \end{matrix}$

$x < 4-\sqrt{2} \lor x > 4+\sqrt{2}$



$(-\infty, 4-\sqrt{2})(4+\sqrt{2}, +\infty)$



Int $(0, \frac{3}{2})$

$\sqrt{f(x)} \to$

(a, b, c, d)
2 3 4 5

$\boxed{\log n, n, n^2, x^n, e^n, n!, n^n}$

$$\lim_{n \to +\infty} \frac{2^n + 4n^2}{6^n + 5} = \frac{2^n\left(1 + \frac{4n^2}{2^n} \nearrow 0\right)}{6^n\left(1 + \frac{5}{6^n} \nearrow 0\right)} = \frac{1}{3}\frac{\cancel{2^n}}{\cancel{6^n}} = \frac{1}{3}$$

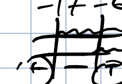
Esame 23/01/17

SDF

$f(x) = 1 - \sqrt{\frac{x+17}{x+6}}$

Dom f: $\frac{x+17}{x+6} \geq 0 \quad \begin{cases} x+17 \geq 0 \\ x+6 > 0 \end{cases} \quad \begin{cases} x \geq -17 \\ x \geq -6 \end{cases}$

$x \leq -17 \lor x > -6$



① Dom $f = (-\infty, -17] \cup]-6, +\infty)$

② P/D $\to f(-x) = 1 - \sqrt{\frac{-x+17}{x+6}}$ la f è dispari(?)

③ Intersez.

$\sqrt{\frac{17}{6}} \to \frac{\sqrt{17}}{\sqrt{6}} \to \frac{\sqrt{17 \cdot 6}}{6} = \frac{\sqrt{102}}{6}$ whatever

$f(0) = 1 - \sqrt{\frac{17}{6}}$ interseca in $\left(0, 1 - \sqrt{\frac{17}{6}}\right)$

$f(x) = 0 \quad 1 - \sqrt{\frac{x+17}{x+6}} = 0 \to \emptyset$

④ Segno

ce: $x+17 \geq 0 \quad x \geq -17$
$x+6 \geq 0 \quad x \geq -6$

$1 - \sqrt{\frac{x+17}{x+6}} > 0 = -\sqrt{\frac{x+17}{x+6}} > 1 = \sqrt{\frac{x+17}{x+6}} < 1 =$

elevo tutto $^2 \to \frac{x+17}{x+6} < 1 = \frac{x+17}{x+6} - 1 < 0 = \frac{\cancel{x}+17-\cancel{x}-6}{x+6} < 0 =$

$\frac{11}{x+6} < 0 = x+6 < 0 = x < -6$

	-17	-6	
$-$	$+$	$+$	
$-$	$-$	$+$	
$+$	$+$	$-$	
$-$	$+$	$+$	

$x \leq -17$

$(-\infty, -17)$ Pos

⑤ $\lim_{x \to +\infty} 1 - \sqrt{\frac{x+17}{x+6}} = \lim_{x \to +\infty} 1 - \lim \sqrt{\frac{x+17}{x+6}}$

$\searrow 1$

regola $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$ quindi $\sqrt{\lim_{x \to \infty} \frac{x+17}{x+6}}$

$\sqrt{\lim_{x \to \infty} \frac{x(1+\frac{17}{x})}{x(1+\frac{6}{x})}} = \sqrt{\lim_{x \to \infty} 1} = \sqrt{1}$ quindi $1 - \sqrt{1} = 0$

$\lim_{x \to -\infty} 1 - \sqrt{\frac{x+17}{x+6}} = 0$ As. orizz a 0.

As obliquo $= y = mx + q$ $\quad m = \lim_{x \to +\infty} \frac{f(x)}{x}$ $\quad q = \lim_{x \to \infty} f(x) - mx$

$$m = \lim_{x \to \infty} \frac{1 - \sqrt{\frac{x+17}{x+6}}}{x} \begin{cases} \lim_{x \to \infty} 1 - \sqrt{\frac{x+17}{x+6}} = 0 \\ \lim_{x \to \infty} x = \infty \end{cases} 0 \cdot \infty = 0$$

no asintoti obliqui

⑥ Derivata

$$y = 1 - \sqrt{\frac{x+17}{x+6}} \quad \overset{\text{prop rad}}{} \quad y' = 1 - \frac{\sqrt{x+17}}{\sqrt{x+6}} \quad \text{prof oler.} \quad (f+g)' = f' + g'$$

$$= \frac{d}{dx}(1) - \frac{d}{dx}\left(\frac{\sqrt{x+17}}{\sqrt{x+6}}\right) = 0 - \frac{\frac{d}{dx}(\sqrt{x+17}) \cdot \sqrt{x+6} - \sqrt{x+17} \cdot \frac{d}{dx}(\sqrt{x+6})}{(\sqrt{x+6})^2} =$$

$$- \frac{\left(\frac{1}{2\sqrt{x+17}}\right) \cdot (\sqrt{x+6}) - \sqrt{x+17} \cdot \frac{1}{2\sqrt{x+6}}}{x+6} = \frac{\frac{1}{2\sqrt{x+17}} \cdot (\sqrt{x+6}) - \frac{\sqrt{x+17}}{2\sqrt{x+6}}}{x+6} =$$

$$- \frac{\frac{x+6-x-17}{2\sqrt{(x+17) \cdot (x+6)}}}{x+6} = - \frac{\frac{-11}{2\sqrt{x^2+6x+17x+102}}}{x+6} = - \frac{\frac{-11}{2\sqrt{x^2+23x+102}}}{x+6}$$

$$\frac{11}{2\sqrt{x^2+23x+102} \cdot (x+6)}$$

Segno derivata

$$\frac{11}{2\sqrt{x^2+23x+102}\,(x+6)} > 0 \qquad 2\sqrt{x^2+23x+102}\,(x+6) > 0 \quad \overset{\text{tutto fratto}}{=} \quad 2$$

$= x\sqrt{x^2+23x+102}\,(x+6) > 0$

a) $\begin{cases} x+6>0 \\ x\sqrt{x^2+23x+102} > 0 \end{cases}$ $\begin{cases} x>-6 \\ x \le -17 \vee x \ge -6 \end{cases}$

elevo^2

$\cancel{\sqrt{x^2+23x+102}} > 0$

$\Delta = 529 - 4\cdot 102 = 121 \to \sqrt{\ } \to 11$
$\qquad\quad 408$

$x_{1,2} = \frac{-23 \pm 11}{2}$ ⟨ $-\frac{12}{2} = -6$; $-\frac{34}{2} = -17$

$-6 \quad 0$

$(0, +\infty)$

$\begin{cases} x \le -17 \vee x \ge -6 \\ x > 0 \end{cases}$

$-17 \quad -6 \quad 0$

$(0, +\infty)$

b) $\begin{cases} x\sqrt{x^2+23x+102} < 0 \\ x < -6 \end{cases}$

$\begin{cases} x < 0 \\ x^2+23x+102 \end{cases}$

$\shortparallel$

$x \le -17 \vee x \ge -6$

$-17 \quad -6 \quad 0$

$(-\infty, -17)\ (-6, 0)$

$-17 \quad -6 \quad 0$

$(-\infty, -17)$

$\cup$:

$(-\infty,-17) \cup (0,+\infty)$

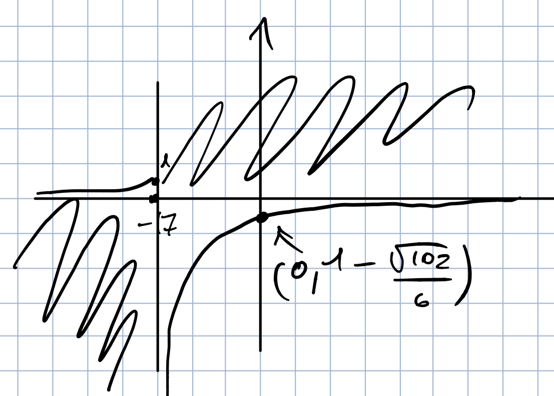
$\nearrow \qquad \nearrow$

$-17 \qquad 0$

$+ \quad - \quad +$

$\nearrow \quad \searrow \quad \nearrow$

$f(-17) = 1$



2) Calcolare $\displaystyle\int \frac{1}{17\operatorname{sen}(x) + 6\cos(x) + 18}\,dx$

$$\int \frac{1}{17\,\text{sen}(x)}\,dx + \int \frac{1}{6\cos(x)}\,dx + \int \frac{1}{18}\,dx$$

$$\frac{1}{17}\int \frac{1}{\text{sen}(x)}\,dx + \boxed{\frac{1}{6}\int \frac{1}{\cos(x)} + \frac{1}{18}x}$$

$$\text{sen}^2(t) = 1 - \cos^2(t)$$

$$\cos^2(t) = 1 - \text{sen}^2(t)$$

$$\frac{1}{17}\int \frac{\text{sen}(x)}{\text{sen}^2(x)}\,dx + \frac{1}{6}\int \frac{\cos(x)}{\cos^2(x)}\,dx$$

$$\frac{1}{17}\int \frac{\text{sen}(x)}{1-\cos^2(x)}$$

$$y = \cos(x) \qquad dy = -\text{sen}(x) \qquad dx = \frac{1}{t'}\,dt$$

$$1 - \text{sen}^2(t) = \cos^2(x)$$

$$1 - \cos^2(x) = \text{sen}^2(x)$$

$$\frac{1}{17}\int \frac{\cancel{\text{sen}(x)}}{1-\cos^2(x)} \cdot -\frac{1}{\cancel{\text{sen}(x)}}\,dy = \frac{1}{17}\int -\frac{1}{1-\cos^2(x)}\,dy = \frac{1}{17}\int -\frac{1}{\text{sen}^2(x)}\,dy$$

$$\frac{1}{17}\int -\frac{1}{1-\cos^2(x)}\,dy = \frac{1}{17}\int -\frac{1}{1-y^2}\,dy \to -\frac{1}{17}\int \frac{1}{1-y^2}\,dy$$

$$\int \frac{1}{x^2-a^2}\,dx \to \frac{1}{2a} \cdot \ln\left(\left|\frac{x-a}{x+a}\right|\right)$$

$$\frac{1}{17} \cdot \frac{1}{2} \cdot \ln\left(\left|\frac{t-1}{t+1}\right|\right)$$

$$\frac{1}{34} \cdot \ln\left(\frac{\cos(x)-1}{\cos(x)+1}\right) + c$$

$$\frac{1}{6}\int \frac{1}{\cos(x)} = \frac{1}{6}\int \frac{\cos(x)}{\cos^2(x)} = \frac{1}{6}\int \frac{\cos(x)}{1-\text{sen}^2(x)}$$

$$t = \text{sen}(x) \qquad t' = \cos(x) \qquad \frac{1}{\cos(x)}\,dy$$

$$= \frac{1}{6}\int \frac{1}{1-t^2}\,dt = -\frac{1}{6} \cdot \frac{1}{2}\ln\left(\left|\frac{t-1}{t+1}\right|\right)$$

$$t^2 \text{ " } 1^2$$

$$= -\frac{1}{12}\ln\left(\left|\frac{\text{sen}(x)-1}{\text{sen}(x)+1}\right|\right) + c$$

Quindi $\frac{1}{34}\ln\left(\left|\frac{\cos(x)-1}{\cos(x)+1}\right|\right) - \frac{1}{12}\ln\left(\left|\frac{\text{sen}(x)-1}{\text{sen}(x)+1}\right|\right) + \frac{1}{18}x + c$

① SDF $\quad f(x) = \dfrac{x^2}{(x-3)(x-5)}$

$(x-3)(x-5) \neq 0$

$x \neq 3$

$x \neq 5$

Dom $f \in \mathbb{R} \setminus \{3, 5\}$

Dom $f = (-\infty, 3[\cup]3, 5[\cup]5, +\infty)$

P/D $= f(-x) = \dfrac{-x^2}{(-x-3)(-x-5)} \rightarrow$ dispari

Intersezioni

$y\; f(0) = \underline{0} \rightarrow 0$

$f(x) = 0 \rightarrow \dfrac{x^2}{(x-3)(x-5)} =$ $\quad x^2 = 0 \in Q \quad$ (base $= 0$)

$x - 3 = 0 \quad x \neq +3$

$x - 5 = 0 \quad x \neq +5$

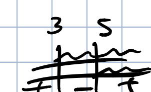
Interseca in $(0,0)$

Segno

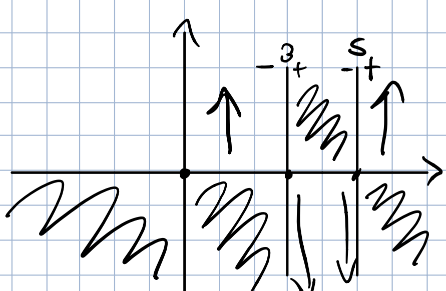
$\dfrac{x^2}{(x-3)(x-5)} > 0$

$\begin{cases} x^2 > 0 & \forall x \in \mathbb{R} \setminus \{0\} \\ (x-3)(x-5) > 0 \end{cases}$

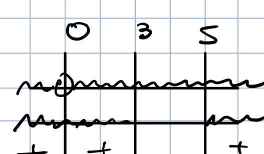
$x > 3 \cup x > 5$



$(-\infty, 3) \cup (5, +\infty)$



$\begin{cases} \forall x \in \mathbb{R} / \{0\} \\ (-\infty, 3)(5 + \infty) \end{cases}$



pos in $(-\infty, 0)(0, 3)(5, +\infty)$

limiti

$$\lim_{x\to\infty} \frac{x^2}{(x-3)(x-5)} = \frac{x^2}{\underset{\displaystyle x^2-8x+15}{\overset{\shortparallel}{x^2-3x-5x+15}}} = \frac{\cancel{x^2}}{\cancel{x^2}\left(1-\frac{8}{x}+\frac{15}{x^2}\right)} = 1$$

$\lim_{x\to-\infty} = 1$ os or $+\infty$ a 1, $-\infty$ a 1

$\lim_{x\to 3^+} = -\infty$ os vert 3^+ a $-\infty$

$\lim_{x\to 3^-} = +\infty$ os vert 3^- a $+\infty$

$\lim_{x\to 5^+} = +\infty$ os vert 5^+ a $+\infty$

$\lim_{x\to 5^-} = -\infty$ os vert 5^- a $-\infty$

derivata

$$y = \frac{x^2}{(x-3)(x-5)} \to \frac{x^2}{x^2-8x+15} \qquad \frac{f' g - f \cdot g'}{g^2}$$

$$y' = \frac{2x\cdot(x^2-8x+15) - x^2\cdot(2x-8)}{(x^2-8x+15)^2}$$

$$y' = \frac{\cancel{2x^3}-16x^2+30x-\cancel{2x^3}+8x^2}{(x^2-8x+15)^2} = \frac{-8x^2+30x}{(x^2-8x+15)^2}$$

Segno derivata

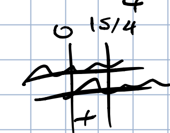
$$\frac{-8x^2+30x}{(x^2-8x+15)^2}>0 \qquad ce = x\neq 5,\ x\neq 3$$

Fattorizzo raccogliendo $-2x$

$$\frac{-2x(4x-15)}{(x^2-8x+15)^2}>0$$

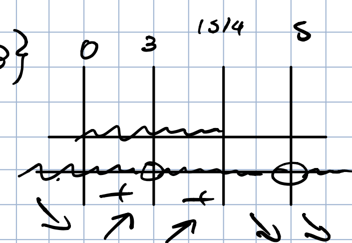
$$\begin{cases}-2x(4x-15)>0\\(x^2-8x+5)^2>0\end{cases}$$

$$\begin{cases}-2x>0\\4x-15>0\end{cases} \qquad \begin{cases}x>0\\x<\frac{15}{4}\end{cases}$$



$\left(0, \frac{15}{4}\right)$

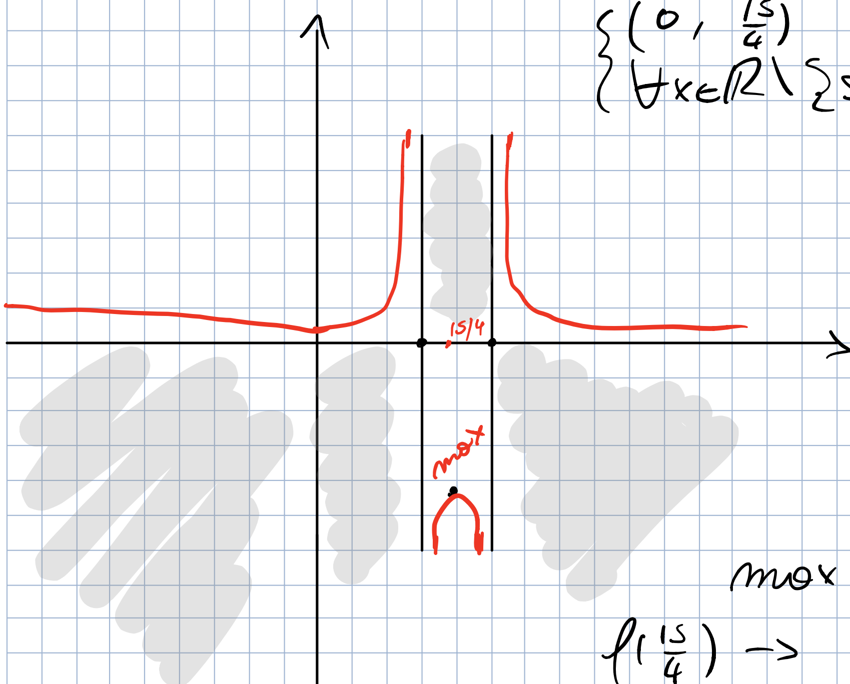
$$\begin{cases}\left(0, \frac{15}{4}\right)\\ \forall x\in\mathbb{R}\setminus\{5,3\}\end{cases}$$



$(0,3)\left(3,\frac{15}{4}\right)$

$\|$

$\left(0,\frac{15}{4}\right)\setminus\{3\}$



max in $15/4$

$$f\left(\frac{15}{4}\right) \to \frac{\left(\frac{15}{4}\right)^2}{\left(\frac{15}{4}-3\right)\left(\frac{15}{4}-5\right)} = \frac{225}{16}$$

$$\frac{3}{4}\cdot -\frac{5}{4} = -\frac{15}{16}$$

$$\frac{225}{\cancel{16}}\cdot -\frac{\cancel{16}}{15} = -15$$

max$\left(\frac{15}{4},-15\right)$

$(-\infty,-15)(f(0),+\infty)$

$$f(x) = \frac{x}{x^2 - 24x + 128}$$

1SDF 16/17

$D: b^2 - 4ac = 64$

$$x_{1,2} = \frac{24 \pm 8}{2} \begin{cases} \cancel{\frac{32}{2}} \; 16 \\ \cancel{\frac{16}{2}} \; 8 \end{cases}$$

① $\text{Dom} \in \mathbb{R} \setminus \{8, 16\}$

② Intersezioni

$f(0) = 0$ (interseca 0,0)

$f(x) = 0 \rightarrow 0$

③ Segno

$$\frac{x^3}{x^2 - 24x + 128} > 0$$

$x^3 > 0 \quad x > 0$

$x < 8 \vee x > 16$

	0	8	16	
−	+	+	+	
+	+	−	+	
−	+	−	+	

$(0, 8) \cup (16, +\infty)$

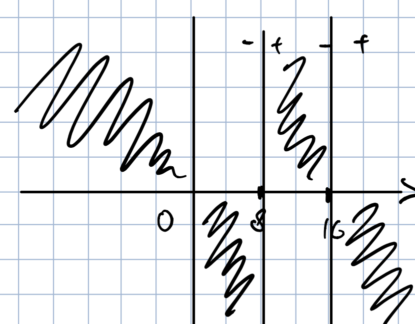
④ limiti

$$\lim_{x \to \infty} \frac{x^3}{x^2\left(1 - \frac{24}{x} + \frac{128}{x^2}\right)} = \frac{x}{1} = +\infty$$

$$\lim_{x \to -\infty} = -\infty$$

$$\lim_{x \to 8^+} = -\infty \qquad \lim_{x \to 8^-} = +\infty$$

$$\lim_{x \to 16^+} = +\infty \qquad \lim_{x \to 16^-} = -\infty$$



$y = mx + q \qquad m = \lim_{x \to \infty} \frac{f(x)}{x} \qquad q = f(x) - mx$

$$m = \lim_{x \to \infty} \frac{x^3}{x^2 - 24x + 128} \cdot \frac{1}{x} = \frac{x^3}{x^3 - 24x^2 + 128x} = 1$$

$$q = \lim_{x \to \infty} \frac{x^3}{x^2-24x+128} - x = \frac{x^3 - x(x^2-24x+128)}{x^2-24x+128} =$$

$$= \frac{\cancel{x^3}-\cancel{x^3}+24x^2-128x}{x^2-24x+128} = \frac{\cancel{x^2}\left(24-\frac{128}{x}\right)}{\cancel{x^2}\left(1-\frac{24}{x}+\frac{128}{x^2}\right)} = 24$$

$$y = x+24$$

⑤ Derivate

$$f = \frac{x^3}{x^2-24x+128} \qquad f' = \frac{3x^2\cdot(x^2-24x+128) - x^3\cdot(2x-24)}{(x^2-24x+128)^2}$$

$$= \frac{3x^4-72x^3+384x^2-2x^4+24x^3}{(x^2-24x+128)^2} = \frac{x^4-48x^3+384x^2}{(x^2-24x+128)^2}$$

$$\frac{\overbrace{x^4-48x^3+384x^2}^{>0}}{\underbrace{(x^2-24x+128)^2}_{>0}} \longrightarrow \forall x \in \mathbb{R}$$

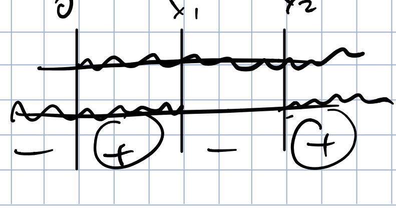
$$x^4-48x^3+384x^2>0 \qquad x^2(x^2-48x+384)>0$$

$$\begin{cases} x^2>0 & \forall x \in \mathbb{R} \\ x^2-48x+384>0 \end{cases}$$

$$\Delta = 2304-\underbrace{4\cdot 384}_{1536} = 768$$

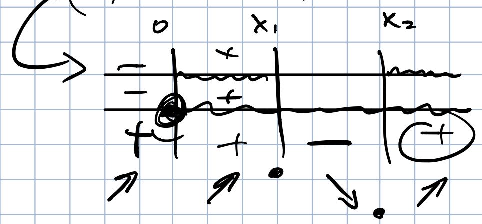
$$x_{1,2} = \frac{48 \pm \sqrt{768}}{2} = \frac{48 \pm 16\sqrt{3}}{2} = \qquad \sqrt{768} = \sqrt{16^2\cdot 3} = \sqrt{16^2}\cdot\sqrt{3} = 16\sqrt{3}$$

$$\frac{\cancel{2}(24 \pm 16\sqrt{3})}{\cancel{2}} \left\langle \begin{array}{l} 24+16\sqrt{3} \quad x_2 \\ 24-16\sqrt{3} \quad x_1 \end{array} \right.$$



$$x < 24-16\sqrt{3} \;\cup\; x > 24+16\sqrt{3}$$

$(0, 24-16\sqrt{3}) \cup (24+16\sqrt{3}, +\infty)$



$(0, x_1) \cup (x_2, +\infty)$

$\emptyset$

$(-\infty, 0)(0, \underline{24-16\sqrt{3}})$

$(-\infty,0)(0,8)(8, 24-16\sqrt{3})(24+16\sqrt{3}, +\infty)$

$x+8$

$x \neq 16$

$max = 24-16\sqrt{3}\ ,\ y?$

$min = 24+16\sqrt{3}\ ,\ y?$

$$f(max) = \qquad \frac{x^3}{x^2-24+128}$$
$$f(min) =$$

$$\frac{(24-16\sqrt{3})^3}{4-16\sqrt{3})^2 - 24(24-16\sqrt{3})+128} = \cancel{x} - 51$$

2012-2013

SDF

$$y = \frac{e^{2x}+8}{e^x+1}$$

Dom $e^x+1 \neq 0$

$e^x \neq -1 \quad \forall x \in R$

Dom $\in R$

Intersez assi

$f(0) = \frac{9}{2}$

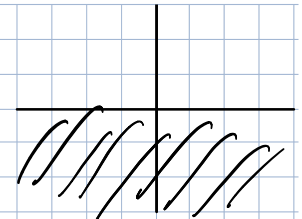
Interseca in $(0, \frac{9}{2})$

$f(x) = 0 \qquad \frac{e^{2x}+8}{e^x+1} = 0 \in \emptyset$

$\uparrow$

Segno

$$\frac{e^{2x}+8}{e^x+1} > 0 \qquad \begin{cases} e^{2x}+8>0 & \forall x\in\mathbb{R} \\ e^{x}+1>0 & \forall x\in\mathbb{R} \end{cases}$$



Sempre pos

Asintoti

$$\lim_{x\to\infty} \frac{e^{2x}+8}{e^x+1} = \frac{e^{\cancel{2}x}\left(1+\frac{8}{e^{2x}}\right)}{\cancel{e^x}\left(1+\frac{1}{\cancel{e^x}}\right)} = \infty$$

$$\lim_{x\to-\infty} = \frac{e^{2x}+8}{e^x+1} = \frac{0+8}{0+1} = 8$$ As orizz a 8

$$y=mx+q \qquad m=\lim_{x\to\infty}\frac{f(x)}{x} \qquad q=\lim_{x\to\infty} f(x)-mx$$

$$m=\lim_{x\to\infty}\frac{e^{2x}+8}{e^x+1}\cdot x = \frac{x e^{2x}+8x}{e^x+1} = \frac{\overset{\infty}{e^{2x}}\left(\overset{\infty}{x}+\frac{8x}{e^{2x}}\right)}{\cancel{e^x}\left(1+\frac{1}{e^x}\right)} = +\infty$$

Derivata

$$y=\frac{e^{2x}+8}{e^x+1} \qquad y'=\frac{2e^{2x}\cdot(e^x+1)-(e^{2x}+8)\cdot e^x}{(e^x+1)^2}=$$

$$=\frac{2e^{3x}+2e^{2x}-e^{3x}-8e^x}{(e^x+1)^2}=\frac{e^{3x}+2e^{2x}-8e^x}{(e^x+1)^2}$$

Segno derivata

$$\begin{cases} e^{3x}+2e^{2x}-8e^x>0 \\ \forall x\in\mathbb{R} \end{cases}$$

$$e^x(e^{2x}+2e^x-8)>0 = e^x(e^{2x}+4e^x-2e^x-8)>0$$

$2e^x = 4e^x - 2e^x$

$$e^x(e^x(e^x+4)-2(e^x+4))>0$$

$$e^x((e^x+4)(e^x-2))>0 \to \begin{cases} e^x>0 & x\in\mathbb{R} \\ (e^x+4)(e^x-2)>0 \end{cases}$$

$e^x>-4 \quad \forall x\in\mathbb{R}$ sempre vera perché $\forall x \in \mathbb{R}$

$\boxed{e^x>2} \to \ln(2)$

$-4 \quad 2$

$\boxed{(\ln(2), +\infty)}$

$$f(\ln(2)) = \quad y=\frac{e^{2x}+8}{e^x+1}$$

$$\frac{e^{2\ln(2)^2}+8}{e^{\ln(2)}+1} =$$

$$\frac{e^{\ln(2)^2}+8}{e^{\ln(2)}+1} =$$

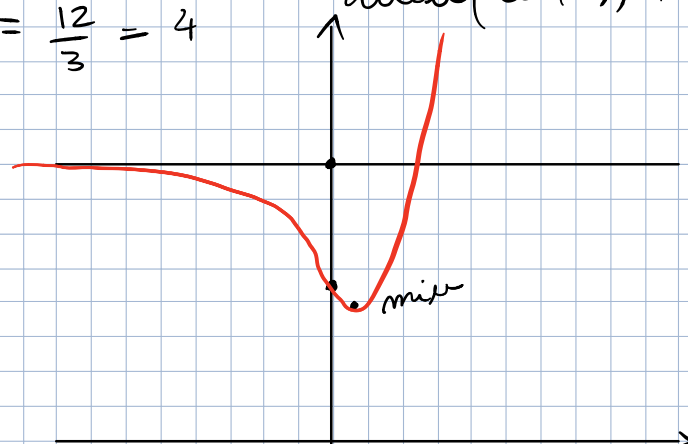
$$x\cdot\ln(a)=\ln(a^x)$$

$$2\ln(2)=\ln(2^2)$$

$$e^{\ln(x)}=x$$

$$\frac{e^{\ln(2)}+8}{3} = \frac{2^2+8}{3} = \frac{12}{3} = 4$$

$\min(\ln(2), 4)$



$$\int \underline{\cos(x)}\cdot\frac{\sin^3(x)-5\sin(x)}{\sin^2(x)-\sin(x)-6}\,\underline{dx}$$

$y=\sin(x)$

$dy=\underline{\cos(x)}\,\underline{dx}$

$$\int\frac{y^3-5y}{y^2-y-6}\,dy$$

$y^3 \quad -5y \quad | y^2-y-$

$\int y+1\,dy + \int \frac{2y+6}{y^2-y-6}\,dy$

$$\begin{array}{l|l} -y^3+y^2+6y & y+1 \\ \hline y^2+1y & \\ -y^2+y+6 & \\ \hline 2y+6 & \end{array}$$

$\int y\,dy + \int 1\,dy +$

$t = y^2-y-6$

$dt = 2y-1\,dy$

$\int \frac{2y+7-1}{t}\,dt \qquad \int \frac{7}{t}\,dt \qquad 7\int \frac{1}{t}\,dt$

$\int y\,dy + \int 1\,dy + 7\ln|t|$

$\frac{y^2}{2} + y + 7\ln|y^2-y-6|$

$\frac{\operatorname{sen}^2 x}{2} + \operatorname{sen} x + 7\ln|\operatorname{sen}^2 x - \operatorname{sen} x - 6| + c$

16-17 ②

$f(x) = 1-\sqrt{\frac{x+17}{x+6}}$

$x+6 \neq 0 \quad x \neq -6$

Dom f $(-\infty,-17)\cup]-6,+\infty)$

$\frac{x+17}{x+6} \geq 0 \qquad x \geq -17$

$x > -6$

Intersez

$f(0) = 1-\sqrt{\frac{17}{6}} = \approx 0,25$ Interseco $(0,\ 0,25)$

$f(x)=0 \quad +\sqrt{\frac{x+17}{x+6}} = +1 \quad \frac{x+17}{x+6} = +1 \rightarrow 1-\left(\frac{x+17}{x+6}\right) =$

$$\frac{x+6-x-17}{x+6} = \frac{-11}{x+6} = 0 \in \emptyset$$

Segno

$$x \neq -6$$

$$1-\sqrt{\frac{x+17}{x+6}} > 0 \to \sqrt{\frac{x+17}{x+6}} < 1 \to \frac{x+17}{x+6} < 1$$

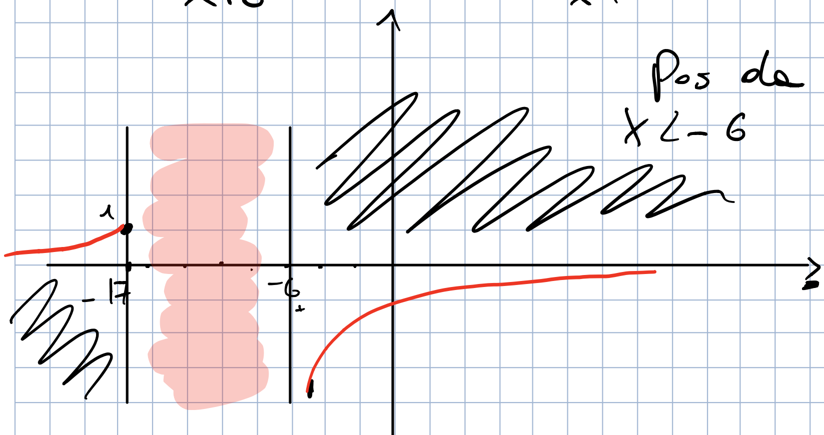
$$-1+\frac{x+17}{x+6} < 0 \to \frac{-\cancel{x}-6+\cancel{x}+17}{x+6} = \frac{11}{x+6} < 0$$

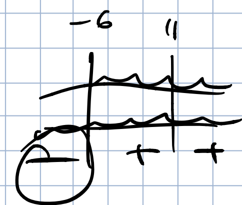
$11 > 0 \ \forall x \in \mathbb{R}$

$x+6 > 0$

$x > -6$

Pos da $x < -6$





$$\lim_{x\to\infty} 1-\sqrt{\frac{x+17}{x+6}} = 1-\sqrt{\frac{x+17}{x+6}} \cdot \frac{1-\sqrt{\frac{x+17}{x+6}}}{1+\sqrt{\frac{x+17}{x+6}}}$$

$$\frac{1-\frac{x+17}{x+6}}{1+\sqrt{\frac{x+17}{x+6}}} \qquad \frac{\frac{x+6-x-17}{x+6}}{1+\sqrt{\frac{x+17}{x+6}}} =$$

$$\underbrace{\frac{\cancel{x}+6-\cancel{x}-17}{x+6}}_{\to -\frac{11}{x+6}} \cdot \frac{1}{1+\sqrt{\frac{x+17}{x+6}}} = \left(-\frac{11}{x+6}\right) \cdot \frac{1}{1+\frac{\sqrt{x+17}}{\sqrt{x+6}}}$$

$$\frac{1}{\frac{\sqrt{x+6}+\sqrt{x+17}}{\sqrt{x+6}}} = \frac{\left(\sqrt{x+6}\right)^{-\frac{1}{2}}}{\sqrt{x+6}+\sqrt{x+17}} \cdot \frac{-11}{(x+6)^{-1}}$$

$$= \frac{-11}{(\sqrt{x+6})(\sqrt{x+6}+\sqrt{x+17})} = \frac{-11}{(x+6)(\sqrt{x^2+23x+112})} = \frac{-11}{+\infty} = 0^-$$

$\lim_{x \to -\infty} f(x) = 0^+$ asintoti orizzontali

$\lim_{x \to -6^+} = -\infty$ $\quad 1-\sqrt{\frac{-6+17}{-6^+ +6}} = 1-\frac{11^+}{0^+} = -\infty$

$\searrow 1-(+\infty)$

$\lim_{x \to -6^-} =$ non c'é

$$\lim_{x \to -17^-} = 1-\sqrt{\frac{-17^- +17}{-17^- +6}} = 1-\sqrt{\frac{0^-}{-11^-}} = 1-\sqrt{\frac{0^+}{11}} = 1$$

Derivata prima

$$y' = -\left[\frac{1}{2\sqrt{\frac{x+17}{x+6}}}\left(\frac{1\cdot(x+6)-(x+17)\cdot 1}{(x+6)^2}\right)\right]$$

$$= -\left[\frac{1}{2\sqrt{\frac{x+17}{x+6}}}\left(\frac{-11}{(x+6)^2}\right)\right] = \frac{(\sqrt{x+6})\cdot -11}{2\cdot\sqrt{x+17}\cdot(x+6)^2} =$$

$$\frac{11}{2}\,\frac{1}{(x+6)^2(\sqrt{x+6})(\sqrt{x+7})}$$

$-\ \ 6$

$+ \mid +$

$\nearrow \mid \nearrow$

Limite concorso

$$\lim_{x \to \infty} x^{-3}\int_0^x \frac{17t^3+6x+18}{t+1}$$

Sono continue e derivabili e $t+1 \neq 0 \to t \neq -1$

$\downarrow$

Hopital

$$\lim_{x \to \infty} \frac{\int_0^x \frac{17t^3+6x+18}{t+1}}{x^3}$$

$$\lim_{x\to\infty} \frac{17x^3+6x+8}{\frac{x+1}{3x^2}} = \frac{17x^3+6x+8}{x+1} \cdot \frac{1}{3x^2} =$$

$$\lim_{x\to\infty} \frac{17x^3+6x+8}{3x^3+3x^2} = \frac{x^3\left(17+\frac{6}{x^2}+\frac{8}{x^3}\right)}{x^3\left(3+\frac{3}{x}\right)} = \frac{17}{3}$$

Seconda prova 16/17

$$f(x) = \sqrt{8x^3-18x^2+3x}$$

$$8x^3-18x^2+3x \geq 0$$

$$x(8x^2-18x+3) \geq 0$$

$$\begin{cases} x \geq 0 \\ 8x^2-18x+3>0 \end{cases}$$

$$\Delta = 324 - \underbrace{4\cdot 3\cdot 8}_{96} = 228 \quad \sqrt{228} \to 2\sqrt{57}$$

$$\frac{+18 \pm 2\sqrt{57}}{16}$$

$$x_1 = \frac{\cancel{2}(9+\sqrt{57})}{\cancel{16}} = \frac{9+\sqrt{57}}{8}$$

$$x_2 = \frac{9-\sqrt{57}}{8}$$

$$x < \frac{9-\sqrt{57}}{8} \lor x > \frac{9+\sqrt{57}}{8}$$

$$x < 15 \cup x > 15$$

$$(0, 15) \cup (15, +\infty)$$

Intersezione

$$f(0) = \sqrt{0} = 0$$

$f(x)=0$ $\quad \sqrt{8x^3-18x^2+3x} = 0$ una radice può essere $=0$ solo $\sqrt{x}$ $\quad x=0$ $\quad 8x^3-18x^2+3x=0$

$x=0$

$$x_1 = \frac{9+\sqrt{57}}{8} \qquad x_2 = \frac{9-\sqrt{57}}{8} \qquad (0,0)$$

Segno

$$\sqrt{8x^3-18x^2+3x} > 0$$

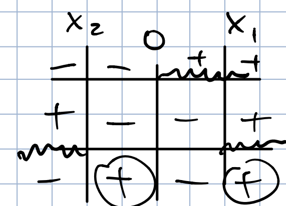
$$8x^3-18x^2+3x>0$$

$$x(8x^2-18x+3)>0$$

$$\begin{cases} x>0 \\ 8x^2-18x+3>0 \end{cases}$$

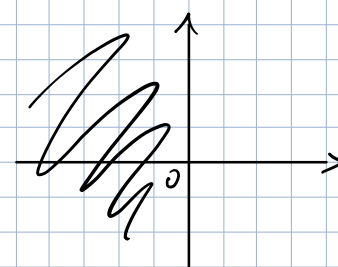
$$\downarrow$$

$$x<x_2 \cup x>x_1$$



$$x_2<x<0 \ \cup \ x>x_1$$

Limiti



$$\lim_{x\to\infty} \sqrt{8x^3-18x^2+3x} = +\infty$$

$$\lim_{x\to-\infty} \sqrt{8x^3-18x^2+3x} = \text{indefinito}$$

$$y=mx+q \qquad mx = \lim_{x\to\infty} \sqrt{8x^3-18x^2+3x} \cdot \frac{1}{x} = \frac{\sqrt{8x^3-18x^2+3x}}{x}$$

Derivata

$$\frac{d}{dx}\sqrt{\underset{\uparrow g}{8x^3-18x^2+3x}} = \qquad = \frac{1}{2\sqrt{g}}\cdot(24x^2-24x+3) =$$

$$\frac{d}{dg}\sqrt{g}\cdot\frac{dx}{dx}(8x^3-18x^2+3x) =$$

$$= \frac{1}{2\sqrt{8x^3-18x^2+3x}}\cdot(24x^2-24x+3) = \frac{24x^2-24x+3}{2\sqrt{8x^3-18x^2+3x}}$$

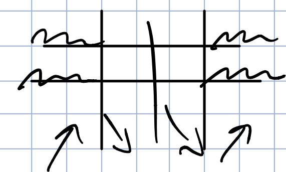
Segno derivata

$$\frac{24x^3-24x+3}{2\sqrt{8x^3-18x^2+3x}} \geq 0$$

$$24x^2-24x+3>0$$

$$\begin{cases} x>0 \\ 8x^2-18x+3>0 \end{cases} = x<x_1 \cup x>x_2$$

$= 576 - \underbrace{4\cdot 24\cdot 3}_{288} = 288 \to 2\sqrt{57} =$



$f\left(\frac{9-\sqrt{57}}{8}\right)$

$\downarrow$

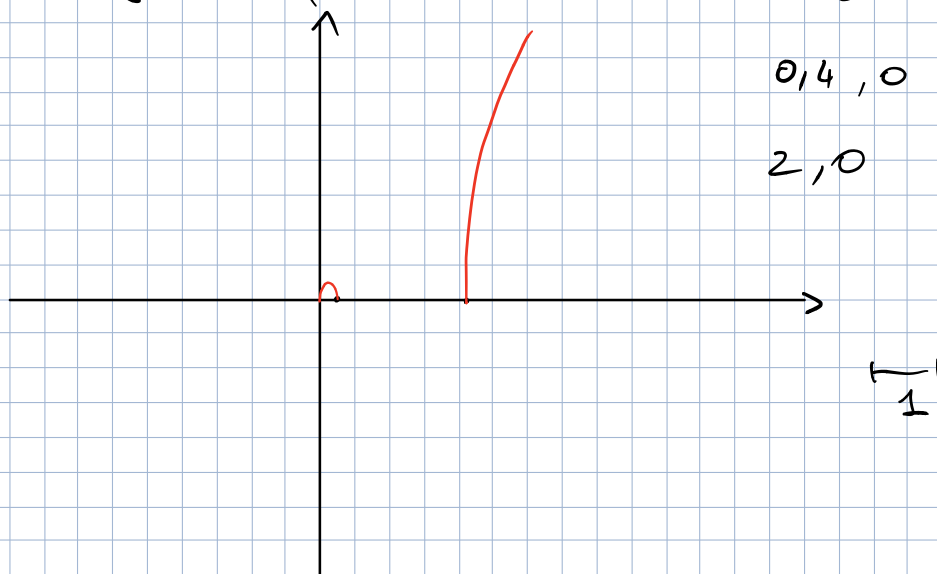
$\sqrt{8\left(\frac{9-\sqrt{67}}{8}\right)^3 - 18\left(\frac{9-\sqrt{57}}{8}\right)^2 + 3\left(\frac{9-\sqrt{57}}{8}\right)}$

max $\frac{9-\sqrt{67}}{8}$, 0

min $\frac{9+\sqrt{67}}{8}$, 0

$0{,}4$, 0

$2{,}0$



$\int \operatorname{arctg}(8x-18)\,dx$ sost $\to$ $y = 8x-18$

$dy = 8\,dx$

aggiungi $\cdot 1$

$\frac{1}{8}\int \operatorname{arctg}(y)\,dy = \frac{1}{8}\int \underbrace{\operatorname{arctg}(y)}_{f}\cdot\underbrace{1}_{g'}\,dy$

$f = \operatorname{arctg}(y)$ $\quad f' = \frac{1}{1+y^2}$

$g' = 1$ $\quad g = y$

$\frac{1}{8}\left(\operatorname{arctg}(y)\cdot y - \int y\cdot\frac{1}{1+y^2}\,dy\right)$

$\frac{1}{8}\left(\operatorname{arctg}(y)\cdot y - \int \frac{y}{1+y^2}\,dy\right)$ $\xrightarrow{\text{sost}}$ $t = 1+y^2$

$dt = 2y\,dy$

$$\frac{1}{8}\left(\text{arctg}(4)\cdot 4-\frac{1}{2}\int \frac{1}{t}\,dt\right) \qquad \int \frac{1}{x}\,dx=\ln(|x|)$$

$$\frac{1}{8}\left(\text{arctg}(4)\cdot 4-\frac{1}{2}\ln(|t|)\right)=\frac{1}{8}\left(\text{arctg}(4)\cdot 4-\frac{1}{2}\ln(|1+4^2|)\right)$$

$$=\frac{1}{8}\left(\text{arctg}(8x-18)\cdot(8x-18)\right)-\frac{1}{2}\ln\left(|1+(8x-18)^2|\right)$$

moltiplico

$$=\frac{1}{8}\left(8\,\text{arctg}(8x-18)\cdot x-18\,\text{arctan}(8x-18)-\frac{1}{2}\ln\left(1+(8x-18)^2\right)\right)$$

rimuovo $\left(\frac{1}{8}\right)$

$$=\text{arctg}(8x-18)\cdot x-\frac{9\,\text{arctg}(8x-18)}{4}-\frac{1}{16}\ln\left(1+(8x-18)^2\right)$$

espando

$$=\text{arctg}(8x-18)\cdot x-\frac{9\,\text{arctg}(8x-18)}{4}-\frac{1}{16}\ln\left(\underbrace{1+64x^2-288x+324}_{\text{sommo}}\right)$$

$$=\text{arctg}(8x-18)\cdot x-\frac{9\,\text{arctg}(8x-18)}{4}-\frac{1}{16}\ln\left(64x^2-288x+325\right)+C$$

$$\lim_{x\to 0}\frac{\text{sen}(8x^3-18x^2)}{1-\cos(3x)}=\left[\frac{0}{1}\right]=\text{Hôpital}$$

$$\lim_{x\to 0}\frac{\frac{d}{dx}\,\text{sen}(8x^3-18x^2)}{\frac{d}{dx}\,1-\cos(3x)} \quad \to \text{composte } (f(g))'=f'(g)\cdot g'$$

$$\lim_{x\to 0}\frac{\cos(8x^3-18x^2)\cdot(24x^2-36x)}{3\,\text{sen}(3x)}=$$ raccolgo un 3 per semplificarlo

$$\lim_{x\to 0}\frac{\cos(8x^3-18x^2)\cdot\cancel{3}(8x^2-12x)}{\cancel{3}\,\text{sen}(3x)}=$$ poiché ottengo una forma ind. uso Hôp ancora

$$\lim_{x\to 0} \frac{\frac{d}{dx}\left(\cos(8x^3-18x^2)\cdot(8x^2-12x)\right)}{\frac{d}{dx}\left(\operatorname{sen}(3x)\right)} \qquad f\cdot g = f'\cdot g + f\cdot g'$$

$$\lim_{x\to 0} \frac{\overbrace{-\operatorname{sen}(8x^3-18x^2)(24x^2-36x)}^{f'}\cdot\overbrace{(8x^2-12x)}^{g} + \cos(8x^3-18x^2)\cdot(16x-12)}{3\cos(3x)}$$

$$\lim_{x\to 0} \frac{-3(64x^4-192x^3+144x^2)\cdot\operatorname{sen}(8x^3-18x^2)+\cos(8x^3-18x^2)\cdot(16x-12)}{3\cos(3x)}$$

$\cos(0)=1$

$$\frac{-3(0-0+0)\cdot\operatorname{sen}(0-0)+\cos(0-0)\cdot(-12)}{3}$$

$$\frac{3\cdot 0\operatorname{sen}(0)+\cos(0)\cdot(-12)}{3} = \frac{0-1\cdot 12}{3} = \frac{-\cancel{12}}{\cancel{3}} = -4$$

SDF

$$f(x) = \log(9x^2+12x+17) \qquad 9x^2+12x+17 \neq 0$$

$$\Delta = 144 - \underbrace{4\cdot 9\cdot 17}_{612} = -468 \quad \emptyset$$

Dom $f \in \mathbb{R}$

Intersezioni:

$f(0) = \log(17)$ $\qquad$ Int $(0, \underbrace{\log(17)}_{2{,}8\to 3})$

$f(x)=0 \in \emptyset \to 0$

Segno

$\log(9x^2+12x+17) = x \in \mathbb{R}$ sempre positiva

Limiti

$$\lim_{x\to\infty} \log(9x^2+12x+17) = \infty$$

$\lim\limits_{x\to-\infty} = +\infty$ perché $\log > 0$

$m = \frac{f(x)}{x}$ $\quad \lim\limits_{x\to\infty} \frac{\log(9x^2+12x+17)}{x} = +\infty$

Derivata

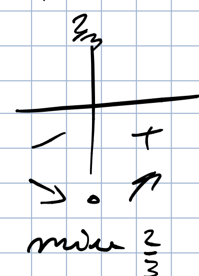
$f(g(x)) = f'(g)\cdot g'$

$\log(9x^2+12x+17) = \frac{1}{9x^2+12x+17} \cdot 18x+12 = \frac{18x+12}{9x^2+12x+17}$

Segno derivata

$\frac{18x+12}{9x^2+12x+17} \geq 0$

$\begin{cases} 18x+12 \geq 0 & x \geq -\frac{12}{18} \quad x \geq \frac{2}{3} \\ 9x^2+12+17 > 0 & \forall x \in \mathbb{R} \end{cases}$



$f(-\frac{2}{3})$ $\quad 9\left(-\frac{2}{3}\right)^2 + \overset{3}{\cancel{12}}\left(-\frac{2}{3}\right) + 17$

$4 - 6 + 17 = 15 \to \ln 15$

$\left(-\frac{2}{3},\ \ln(15)\right)$