

Find the One for Me

This problem was worth 500 points and was ranked easy. The author of this problem is Dhruv Chawla.

Note: GitHub does not support LaTeX in Markdown. If you want a more readable version of the problem, download the PDF file instead.

Statement

Dhruv really likes the digit 1. So, Aryan gave Dhruv two numbers x and y for his birthday. The numbers only contain the digit 1 because otherwise Dhruv will be disappointed.

Dhruv will be happy if Aryan could also answer the question "What is the middle digit in the product $x \cdot y$ ".

Since Aryan doesn't know how to solve the problem, he asks for your help. You are required to answer t testcases. Help Aryan make Dhruv happy.

Aryan does not tell you the values of x and y because the numbers could be very large to take as input. He instead gives you integers k and l - the number of digits in x and y , respectively.

Note: k and l have the same parity, i.e. k and l are either both even or both odd.

Input Format

The first line of the input contains an integer t - the number of testcases.

Each of the following t lines contain an integer k and l - the number of digits in the numbers x and y that Aryan gave to Dhruv.

Output Format

The output should contain t lines.

The i^{th} line should contain the answer to the i^{th} testcase - the middle digit in the product $x \cdot y$.

Constraints

$$1 \leq t \leq 1000$$

$$1 \leq k \leq 10^9$$

$$1 \leq l \leq 10^9$$

Note: x and y only contain 1 in their digits and no other digit.

Sample Tests

Sample Test 1

Input

```
1 | 2
2 | 1 3
3 | 2 4
```

Output

```
1 | 1
2 | 2
```

Explanation

In the first test case, $k = 1$ and $l = 3$, so $x = 1$ and $y = 111$.

Their product is $x \cdot y = 111$. The middle digit is 1.

In the second test case, $k = 2$ and $l = 4$, $x = 11$ and $y = 1111$.

Their product is $x \cdot y = 12221$. The middle digit is 2.

Sample Test 2

Input

```
1 | 3
2 | 999999998 999999998
3 | 2 1000000000
4 | 690 42
```

Output

```
1 | 8
2 | 2
3 | 6
```

Solution

Time Complexity: $O(t)$

Space Complexity: $O(1)$

To solve this problem, we need to take care of a few cases. Problems with a lot of edge case handling is usually discouraged in competitive programming, but they do exist. This problem is good for practice on the same, we think.

Let's take a look at the cases:

- $k = l$: when x and y have the same number of 1's
 - $k \leq 9$: answer is simply k

- $k \geq 10$:
 - $k \% 9 = 0$: answer is 9
 - $k \% 9 = 1$: answer is 0
 - otherwise answer is $k \% 9 = 1$
- $k \neq l$: when x and y have different number of 1's
 - $\min(k, l) \% 9 = 0$: answer is 9
 - otherwise answer is $\min(k, l) \% 9$

We leave the proof as an exercise for the readers :)

$$\begin{array}{r}
 111,111,111 \\
 \times 111,111,111 \\
 \hline
 111,111,111 \\
 1,111,111,110 \\
 11,111,111,100 \\
 111,111,111,000 \\
 1,111,111,110,000 \\
 11,111,111,100,000 \\
 111,111,111,000,000 \\
 1,111,111,110,000,000 \\
 + 11,111,111,100,000,000 \\
 \hline
 12,345,678,987,654,321
 \end{array}$$

Solution in C++:

```

1  #include <bits/stdc++.h>
2
3  constexpr bool test_cases = true;
4
5  void solve () {
6      int k, l;
7      std::cin >> k >> l;
8
9      if (k == l) {
10         if (k <= 9)
11             std::cout << k;
12         else {
13             if (k % 9 == 0)
14                 std::cout << 9;
15             else if (k % 9 == 1)
16                 std::cout << 0;
17             else

```

```
18         std::cout << k % 9;
19     }
20 }
21 else {
22     int m = std::min(k, l);
23     if (m % 9 == 0)
24         std::cout << 9;
25     else
26         std::cout << m % 9;
27 }
28 std::cout << '\n';
29 }
30
31 int main () {
32     std::ios::sync_with_stdio(false);
33     std::cin.tie(nullptr);
34
35     int cases = 1;
36     if (test_cases)
37         std::cin >> cases;
38     while (cases--)
39         solve();
40
41     return 0;
42 }
```