OpenNodal Theory Manual

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Revision Log

Revision	Date	Affected Pages	Revision Description
0	August 11, 2022	All	Initial Release

Acronyms

Contents

1. Background

OpenNodal is an open source nodal solver for nuclear reactors. OpenNodal uses the polynomial nodal method with additive correction factors for the CMFD form solved.

The polynomial solution is achieved via a nonlinear iteration scheme described in [3] and [1]. The correction factors that couple the CMFD solve with the polynomial form of the nodal equations is described in [2].

2. Additional Theory

While the listed references provide a complete description of the polynomial nodal equations and the nonlinear iteration procedure, some correlations are implied but not explicitly stated. One of these implied values is $\bar{S}$, so a more explicit description is given here.

Fundamental equations given or implied in [1], [2], and [3]:

$$l,m,n = \begin{cases} i = 1,2,\ldots,I & u = x,y,z \\ j = 1,2,\ldots,J & v \neq u \\ k = 1,2,\ldots,K & w \neq u,v \end{cases}$$

$$J_{gu}^{mn}(u_l) = -\left[\frac{h_u^l}{2D_g^{lmn}} + \frac{h_u^{l-1}}{2D_g^{l-1,mn}}\right]^{-1} \left(\bar{\phi}_g^{lmn} - \bar{\phi}_g^{l-1,mn}\right) + \tilde{D}_{gu}^{l,l-1,mn}\left(\bar{\phi}_g^{lmn} + \bar{\phi}_g^{l-1,mn}\right)$$

$$J_{gu}^{mn}(u) = \frac{1}{h_w^n h_v^m}\int_{v_m}^{v_{m+1}}\int_{w_n}^{w_{n+1}} J_{gu}(u,v,w)\,dv\,dw$$

$$L_{gv}^{mn}(u) = \frac{1}{h_w^n}\int_{w_n}^{w_{n+1}} \left(J_{gv}(u,v_{m+1},w) - J_{gv}(u,v_m,w)\right)\,dw$$

$$S_{gu}^{mn}(u) = \frac{1}{h_v^m}L_{gv}^{mn}(u) + \frac{1}{h_w^n}L_{gw}^{mn}(u)$$

$$\bar{S}_{gu}^{lmn} = \frac{1}{h_u^l}\int_{u_l}^{u_{l+1}} S_{gu}^{mn}(u)\,du$$

To get a clear form for how to compute $\bar{S}$, we first get $\bar{L}$ in terms of J:

$$\bar{L}_{gv}^{lmn} = \frac{1}{h_w^n h_u^l}\int_{u_l}^{u_{l+1}}\int_{w_n}^{w_{n+1}} \left(J_{gv}(u,v_{m+1},w) - J_{gv}(u,v_m,w)\right)\,dw\,du = J_{gv}^{ln}(v_{m+1}) - J_{gv}^{ln}(v_m)$$

So then $\bar{S}$ can be expressed as:

$$\bar{S}_{gu}^{lmn} = \frac{1}{h_u^l}\int_{u_l}^{u_{l+1}} \left(\frac{1}{h_v^m}L_{gv}^{mn}(u) + \frac{1}{h_w^n}L_{gw}^{mn}(u)\right)\,du = \frac{1}{h_v^m}\bar{L}_{gv}^{lmn} + \frac{1}{h_w^n}\bar{L}_{gw}^{lmn}$$

Which gives $\bar{S}$ in a form that can now be explicitly computed using flux, ϕ, for a current, J, expressed as a function of ϕ.

Bibliography

[1] Jess C. Gehin. *A Quasi-Static Polynomial Nodal Method for Nuclear Reactor Analysis*. PhD thesis, Massachusetts Institute of Technology, Department of Nuclear Engineering, 1992.

[2] Kord S. Smith. An analytic nodal method for solving the two-group, multidimensional, static and transient neutron diffusion equations, 1979.

[3] Kord S. Smith. Nodal method storage reduction by nonlinear iteration. *Transactions of the American Nuclear Society*, 44:265–266, 1983.