



m | C | A

Automated Property-Based Testing for OCaml Modules

Ernest Ng

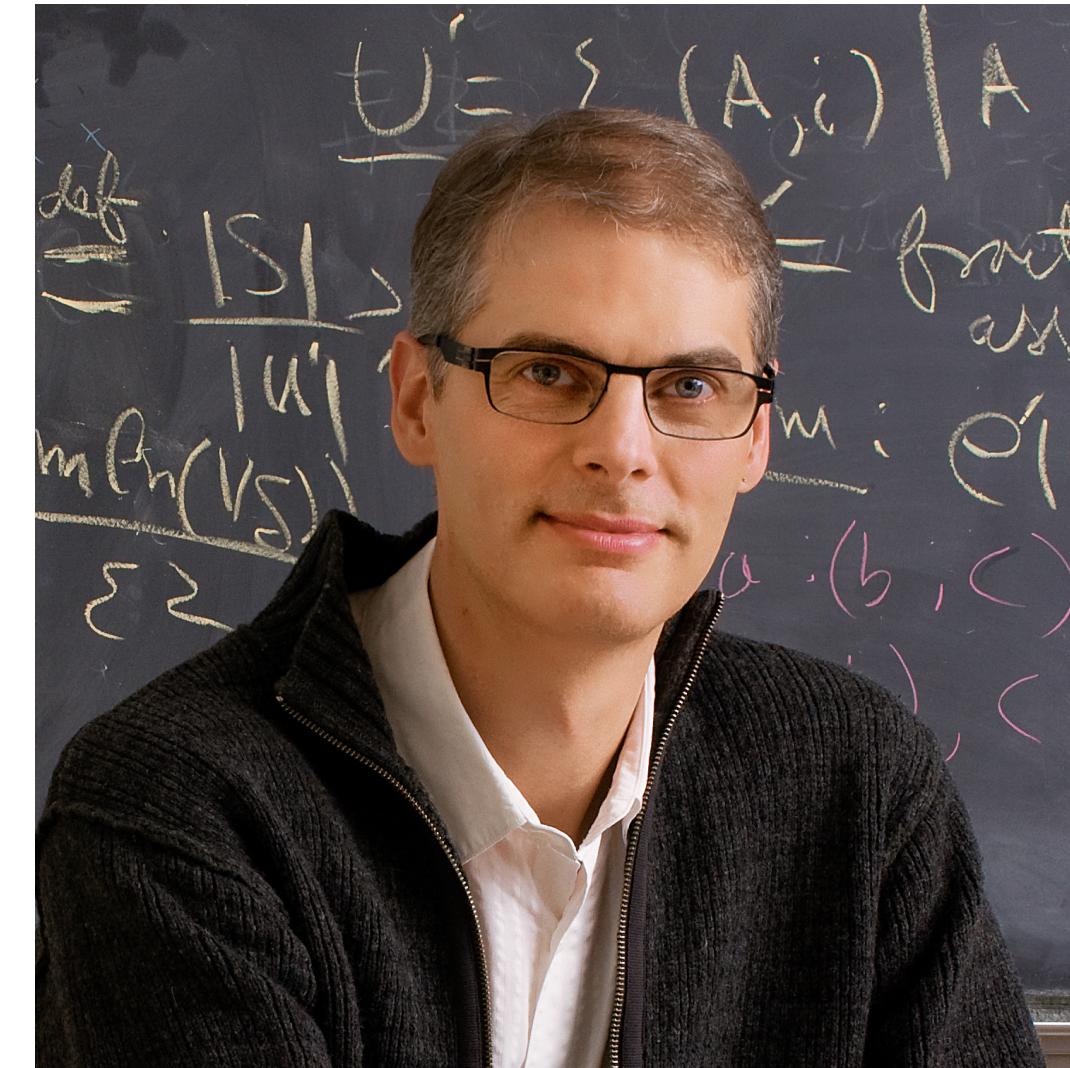
Advised by Harry Goldstein & Benjamin C. Pierce

PLClub, July 28 2023

Acknowledgements



Harry Goldstein



Benjamin C. Pierce

- Carl Eastlund (Jane Street)
- Jan Midgaard (Tarides)

Motivation

**modules
matter
most**

- Bob Harper (2011)



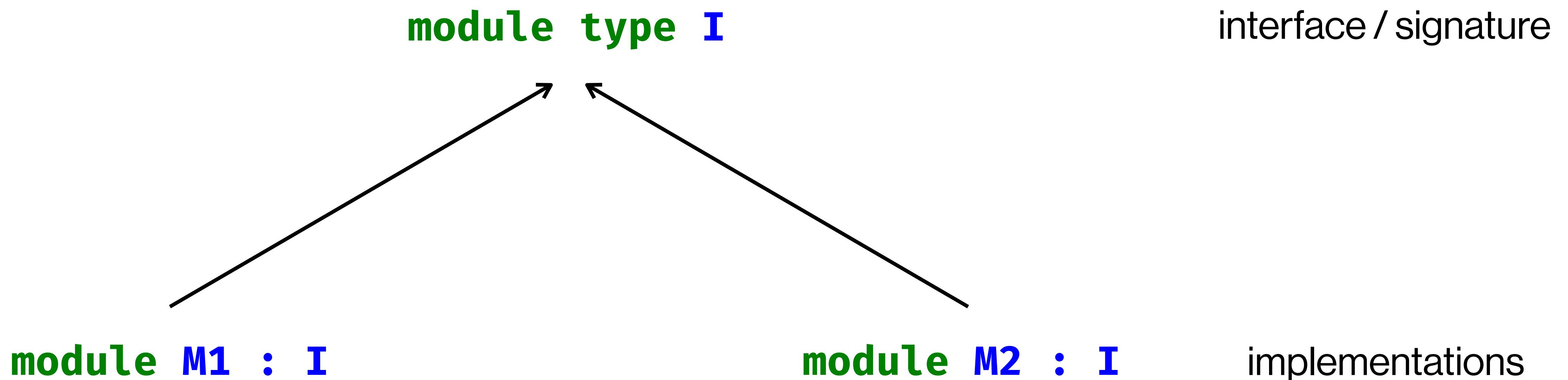
Barbara Liskov,

Programming with Abstract Data Types (1974):

- **Abstract data types** are completely characterised by operations over the ADT
- **Encapsulation**
(implementation details should be hidden from clients)



Representation independence



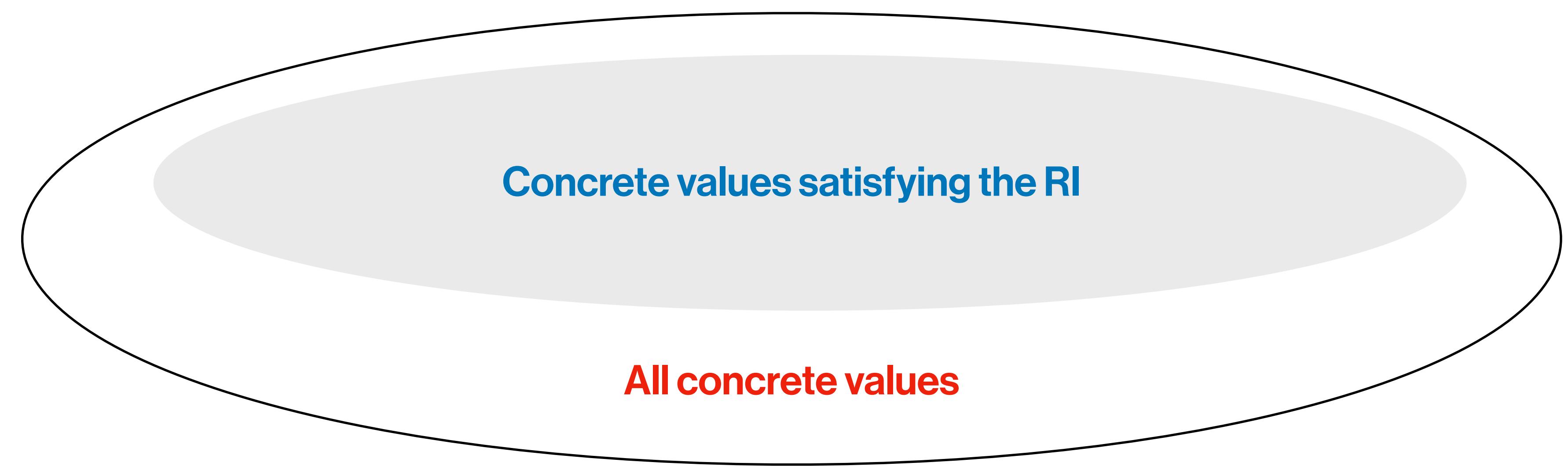
Example: Finite Sets

```
module type SetIntf = sig
  type 'a t
  val empty      : 'a t
  val add        : 'a → 'a t → 'a t
  val intersect : 'a t → 'a t → 'a t
  ...
  val invariant : 'a t → bool
end
```

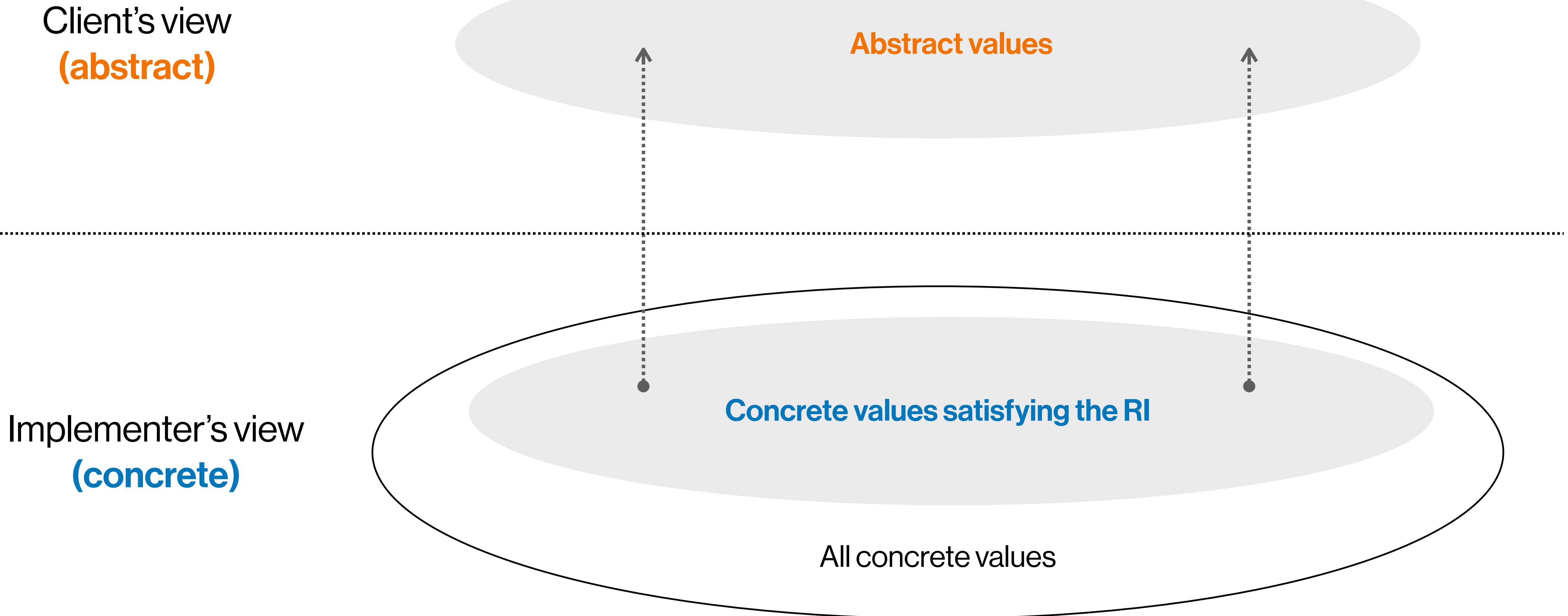
Representation invariants

Distinguish **valid concrete values**
from **invalid concrete values**

Implementer's view
(concrete)



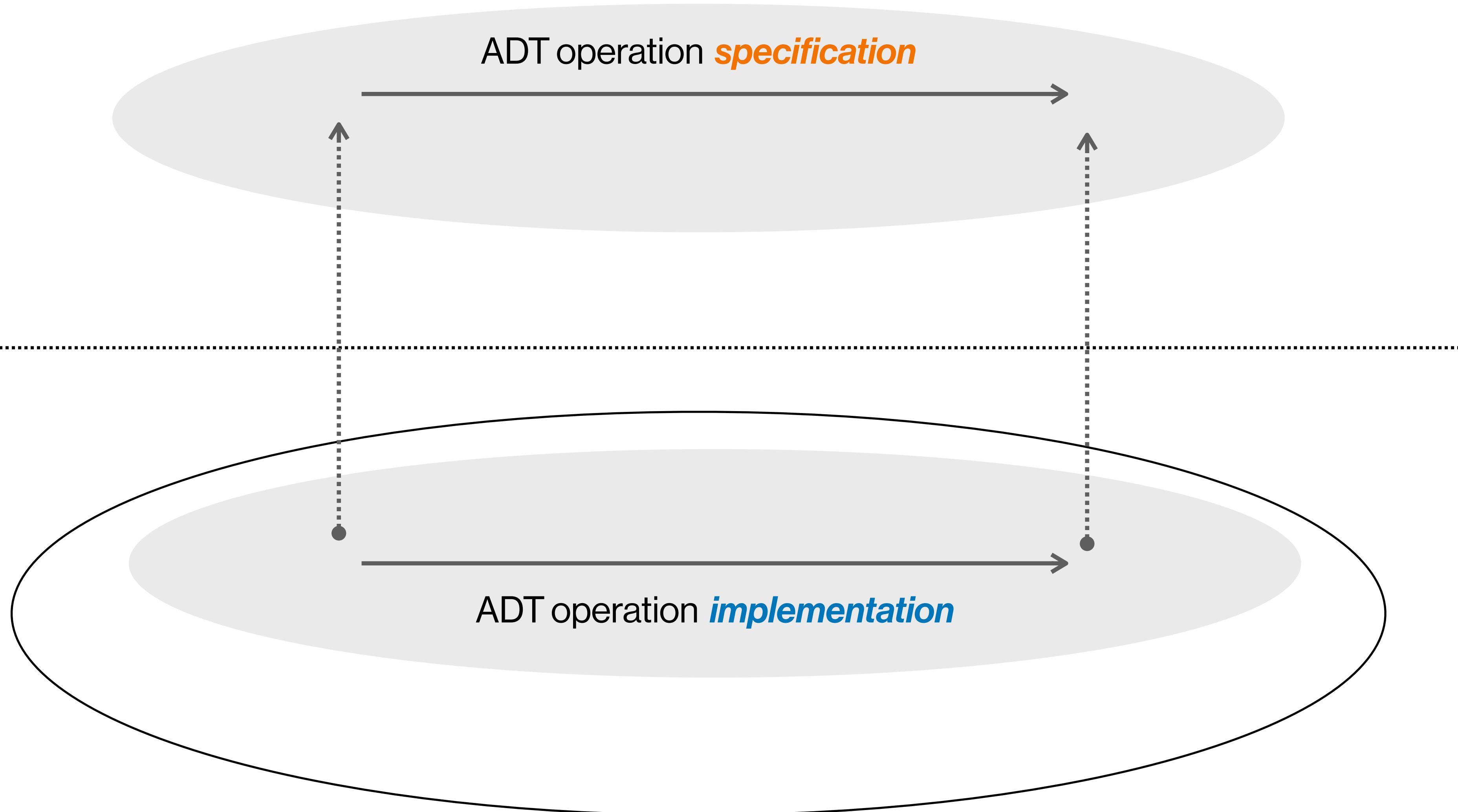
Rep. invariant determines which **concrete** values
are valid representations of **abstract** values



All concrete operations should preserve the RI

Client's view
(abstract)

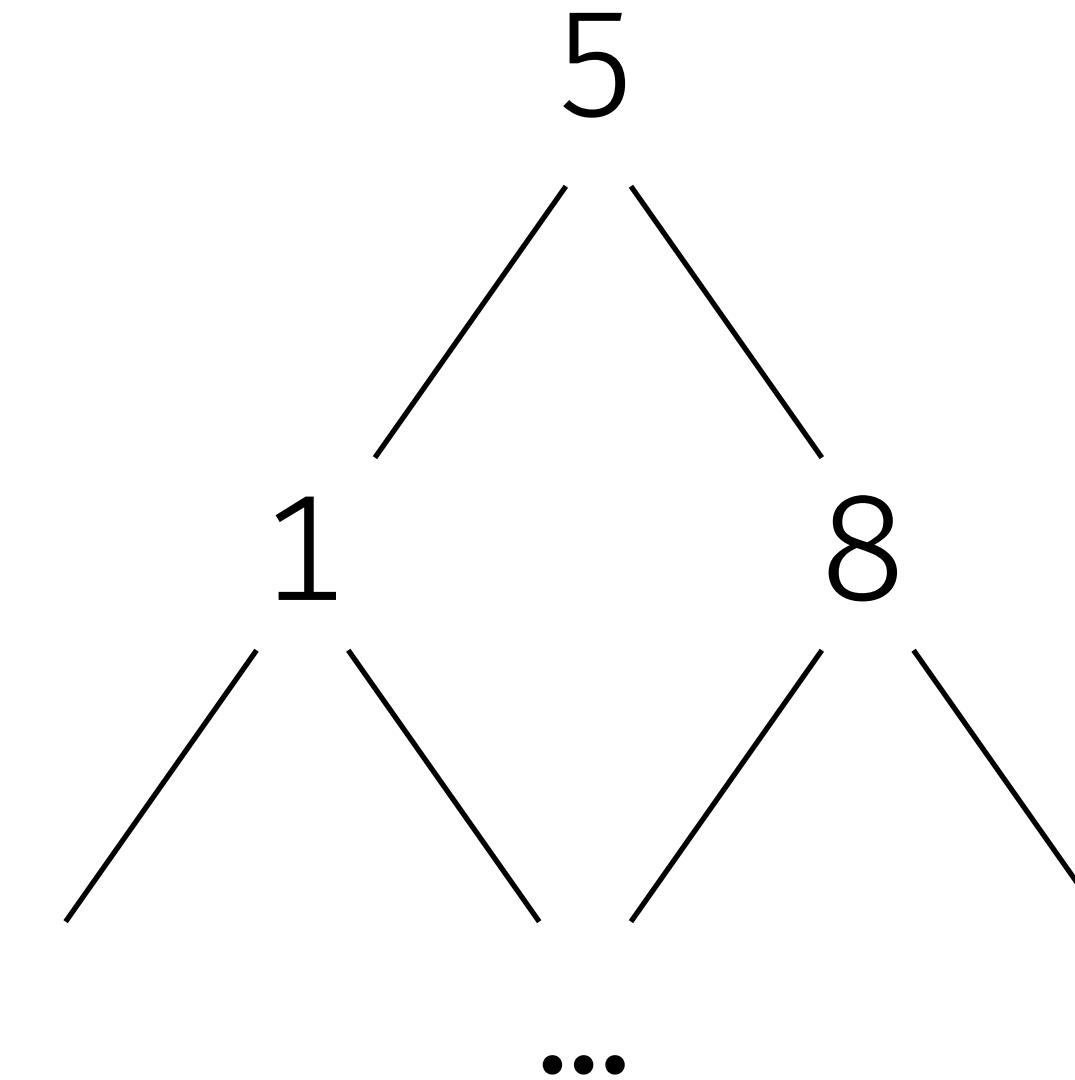
Implementer's view
(concrete)



$\{1, 5, 8, \dots\} \rightsquigarrow [1; 5; 8; \dots]$

```
module ListSet : SetIntf = struct
  type 'a t = 'a list
  (* No duplicates in list *)
  let invariant s = ...
  ...
end
```

{1, 5, 8, ...} \rightsquigarrow



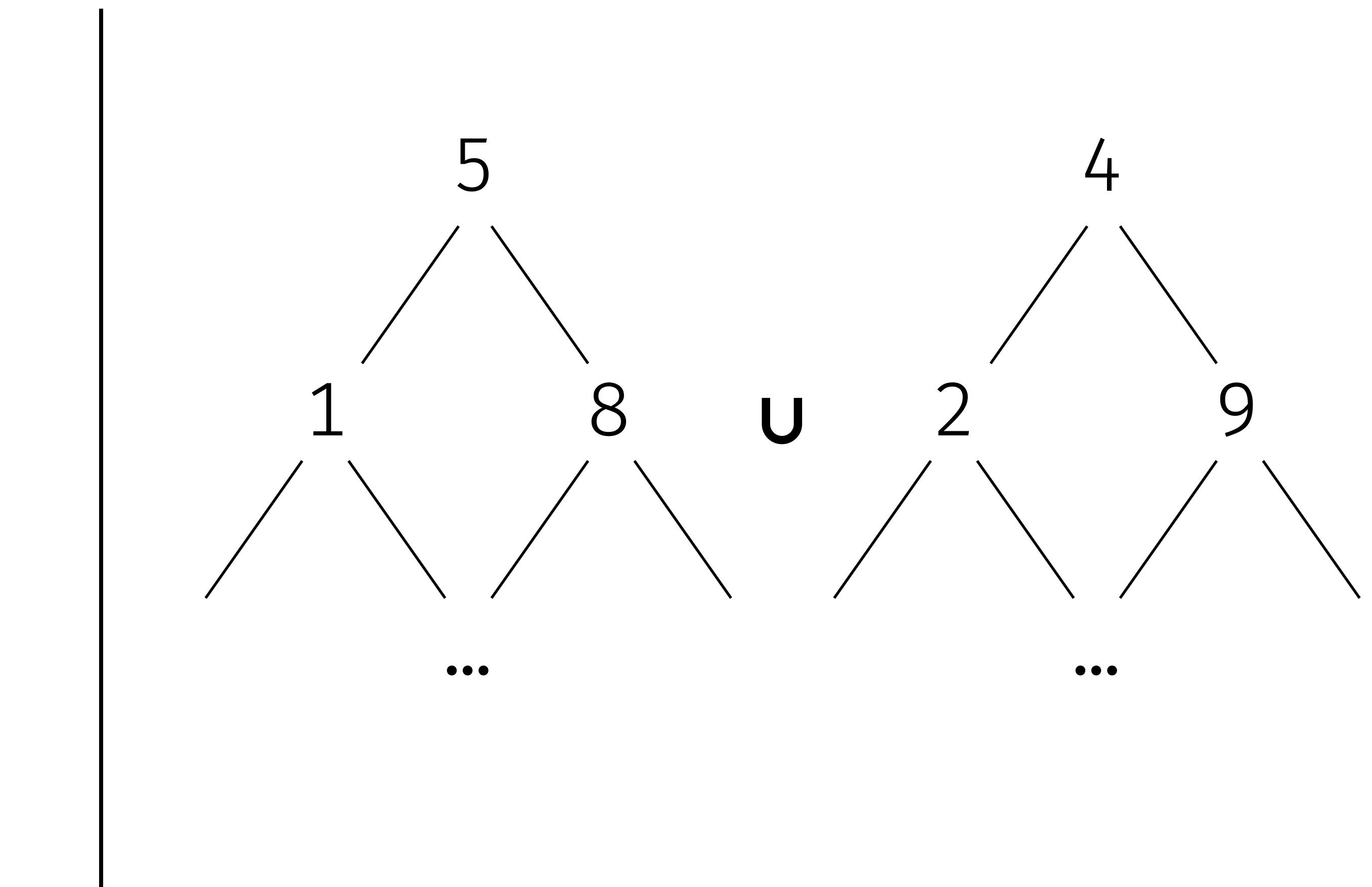
```
type 'a tree =
| Empty
| Node of 'a tree * 'a * 'a tree
```

```
module BSTSet : Set_intf = struct
  type 'a t = 'a tree
  (* BST invariant *)
  let invariant s = ...
  ...
end
```

Are these equivalent?

$$\{1, 5, 8\} \cup \{2, 4, 9\}$$

$$[1; 5; 8] + [2; 4; 9]$$



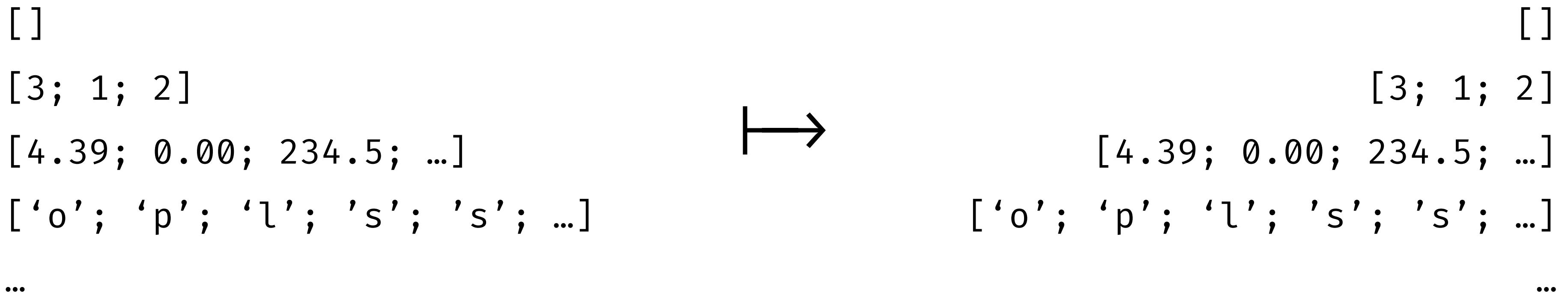
Observational equivalence

equivalent
inputs \mapsto equivalent
outputs

How do we test for
observational equivalence ?

Property-based testing

$\text{rev}(\text{rev lst}) = \text{lst}$

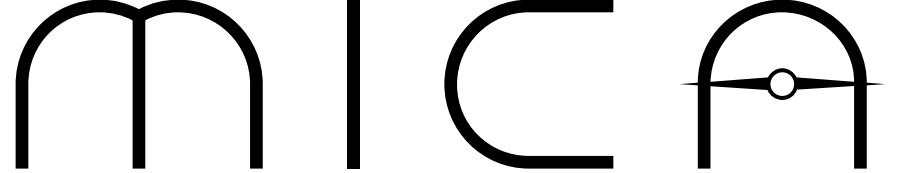


$$\forall \text{stk} : \text{stack}. \forall x : \text{int}.$$
$$\text{pop}(\text{push } x \text{ } \text{stk}) \cong \text{stk}$$

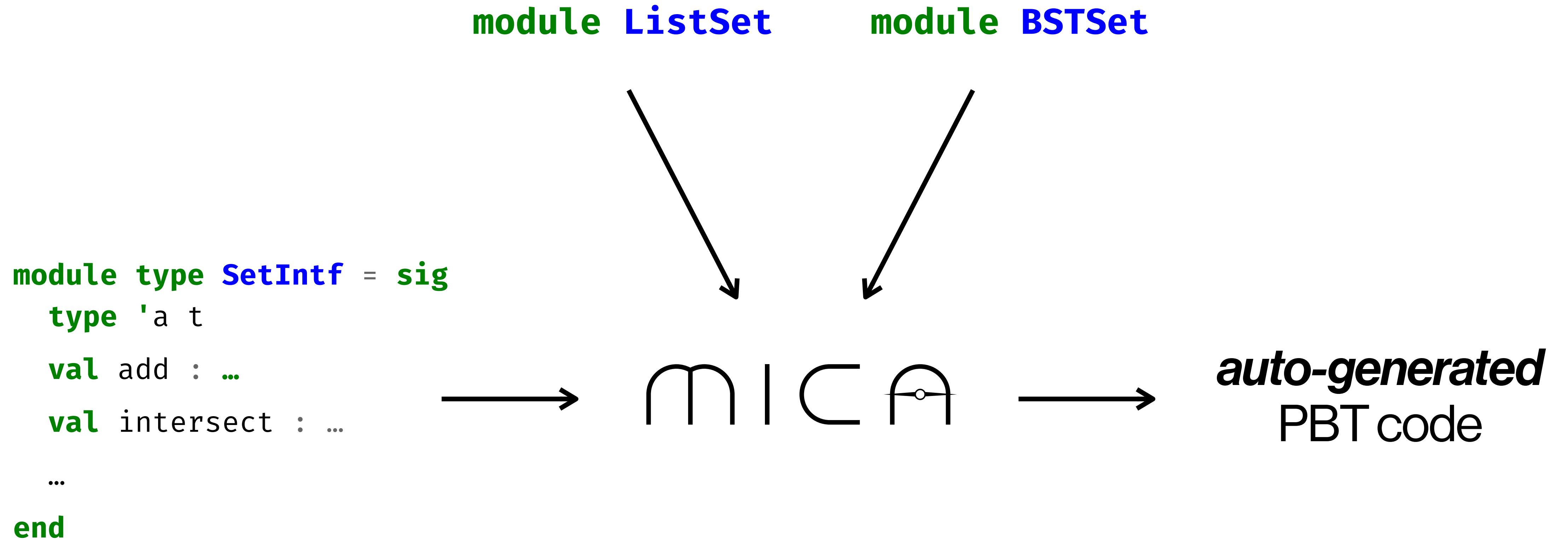
```
QC.forAll genStack $ \stk → do  
    x ← genInt  
    stk' ← pop (push x stk)
```

Problem:

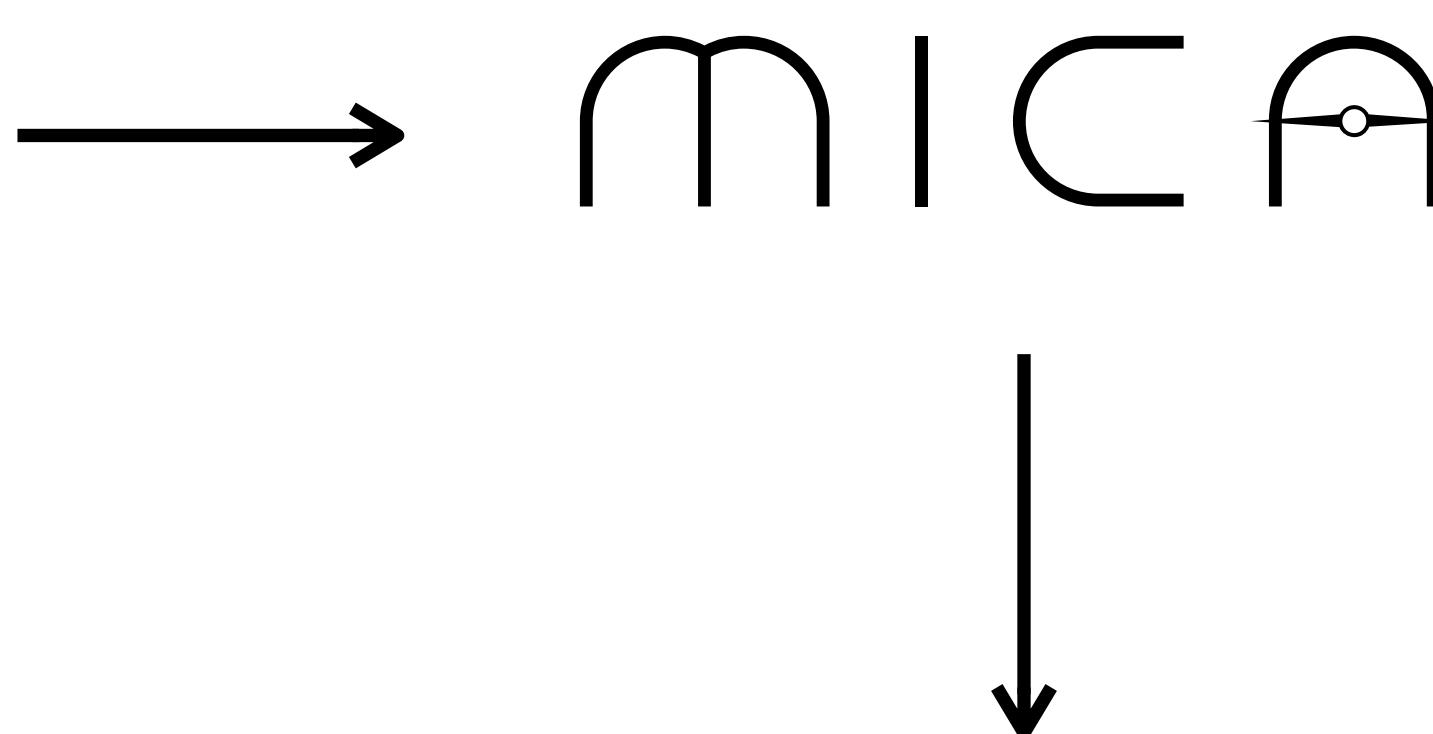
writing PBT code for different modules is time-consuming!

Solution: The logo consists of the lowercase letters 'm', 'i', 'c', and 'a' arranged vertically. A small circle is positioned at the top of the vertical stroke of the letter 'c'.

m | C A overview



```
module type SetIntf = sig
  type 'a t
  val empty      : ...
  val add        : ...
  val intersect  : ...
  ...
end
```

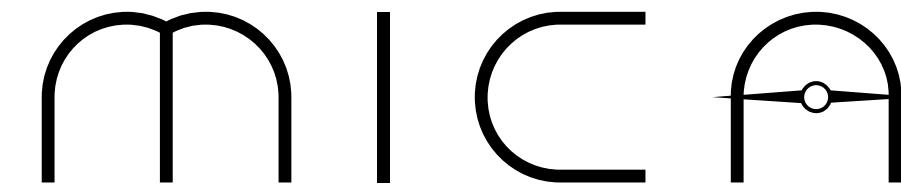


```
type expr =
| Empty
| Add of int * expr
| Intersect of expr * expr
...
type ty     = Bool | Int | T
type value =
| ValBool of bool
| ValInt of int
| ValT of int M.t

val gen_expr : ty → expr Generator.t
val interp   : expr → value
```

*auto-generated
PBT code*

Demo



automatically produces:

```
type expr =
| Empty
| Add of int * expr
| Intersect of expr * expr
...
type ty    = Bool | Int | T
type value =
| ValBool of bool
| ValInt of int
| ValT of int M.t
```

datatype definitions
representing
symbolic commands



MICΑ automatically produces:

Generator for **well-typed** sequences
of symbolic commands

val gen_expr : ty → expr **Generator.t**

`gen_expr` **T**

well-typed command sequences
that return type **T**

`gen_expr T`

well-typed command sequences
that return type **T**

Intersect (Add 2 Empty) Empty



`gen_expr T`

well-typed command sequences
that return type **T**

Intersect (Add 2 Empty) Empty



Is_empty (Size Empty)





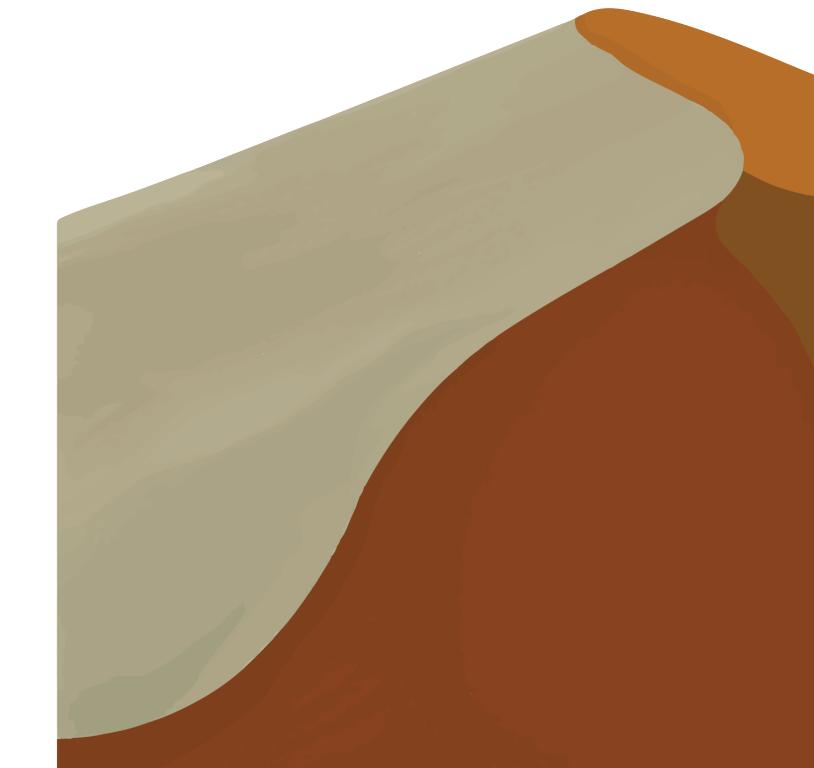
MICAH automatically produces:

Interpreter for symbolic commands

val interp : expr → value

MICA automatically produces:

Executable for testing observational equivalence



DUNE

Generator

generate **random**
command sequences

```
(Is_empty  
  (Intersect (Rem 8 (Add 7 Empty))  
            (Union (Add 2 Empty)  
                  (Union Empty Empty)))
```

Generator

generate **random**
command sequences

```
(Is_empty  
  (Intersect (Rem 8 (Add 7 Empty))  
            (Union (Add 2 Empty)  
                  (Union Empty Empty)))
```

Interpreter

interpret commands
over modules

... → **module BSTSet** → ...
... → **module ListSet** → ...

Generator

generate **random**
command sequences

```
(Is_empty  
  (Intersect (Rem 8 (Add 7 Empty))  
            (Union (Add 2 Empty)  
                  (Union Empty Empty)))
```

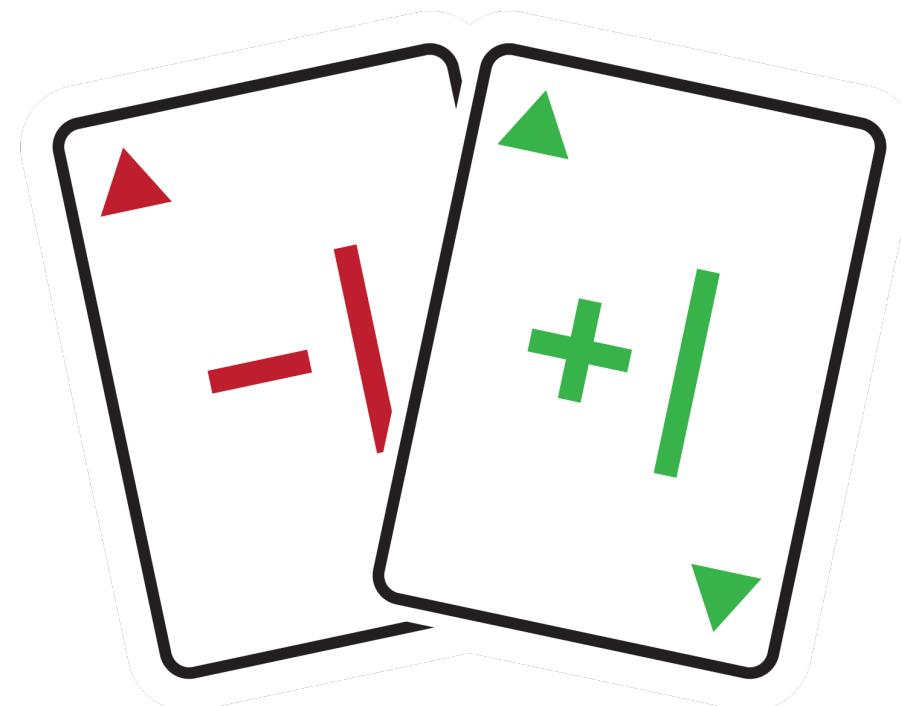
Interpreter

interpret commands
over modules

... → **module BSTSet** → ...
... → **module ListSet** → ...

Executable

test for
observational
equivalence





Jane Street



BASE_QUICKCHECK



CORE

Parser

inhabitedtype/ angstrom



Parser combinators built for speed and memory efficiency

Code generator

fpottier/pprint

A pretty-printing combinator library for OCaml



Examining the *automatically generated* PBT code

The **expr** type

expressions = sequences of symbolic commands

module type SetIntf = sig		
type 'a t	↔	type expr =
val empty : 'a t	↔	Empty
val add : 'a t → 'a t → 'a t	↔	Add of int * expr
val union: 'a t → 'a t → 'a t	↔	Union of expr * expr
val size: 'a t → int	↔	Size of expr
val is_empty : 'a t → bool	↔	Is_empty of expr
...		...
end		

Possible **types** & **values** that can be returned by **exprs**

```
type ty =  
| Bool  
| Int  
| T
```

```
type value =  
| ValBool of bool  
| ValInt of int  
| ValT of int M.t
```

Generator for expr's

```
let rec gen_expr (ty : ty) : expr Generator.t =
  let%bind k = QC.size in
  match ty, k with
  | (T, 0) → return Empty
  | (T, _) →
    let intersect =
      let%bind e1 = QC.with_size ~size:(k / 2) (gen_expr T) in
      and e2 = QC.with_size ~size:(k / 2) (gen_expr T) in
      QC.return @@ Intersect(e1, e2) in
    ...
    QC.union [ intersect; ... ]
  ...
  ...
```

Invoking QC generators for opaque types

```
module type Map_intf = sig
  type t
  val from_list : AssocList.t → t
  ...
end

let rec gen_expr (ty : ty) : expr Generator.t =
  match ty, QC.size with
  | (T, _) → ...
    let from_list =
      let%bind xs = [%quickcheck.generator: AssocList.t] in
      G.return @@ From_list xs
    in G.union [ from_list; ... ]
  ...

```

Interpreter for expr's

```
module ExprToImpl (M : SetInterface) = struct

let rec interp (expr : expr) : value =
  match expr with
  | Empty → ValT (M.empty)
  | Add(x1, e2) → match interp e2 with
    | ValT e' → ValT (M.add x1 e')
    ...
  | Union(e1, e2) →
    match (interp e1, interp e2) with
    | (ValT e1', ValT e2') → ValT (M.union e1' e2')
    ...
  ...
end
```

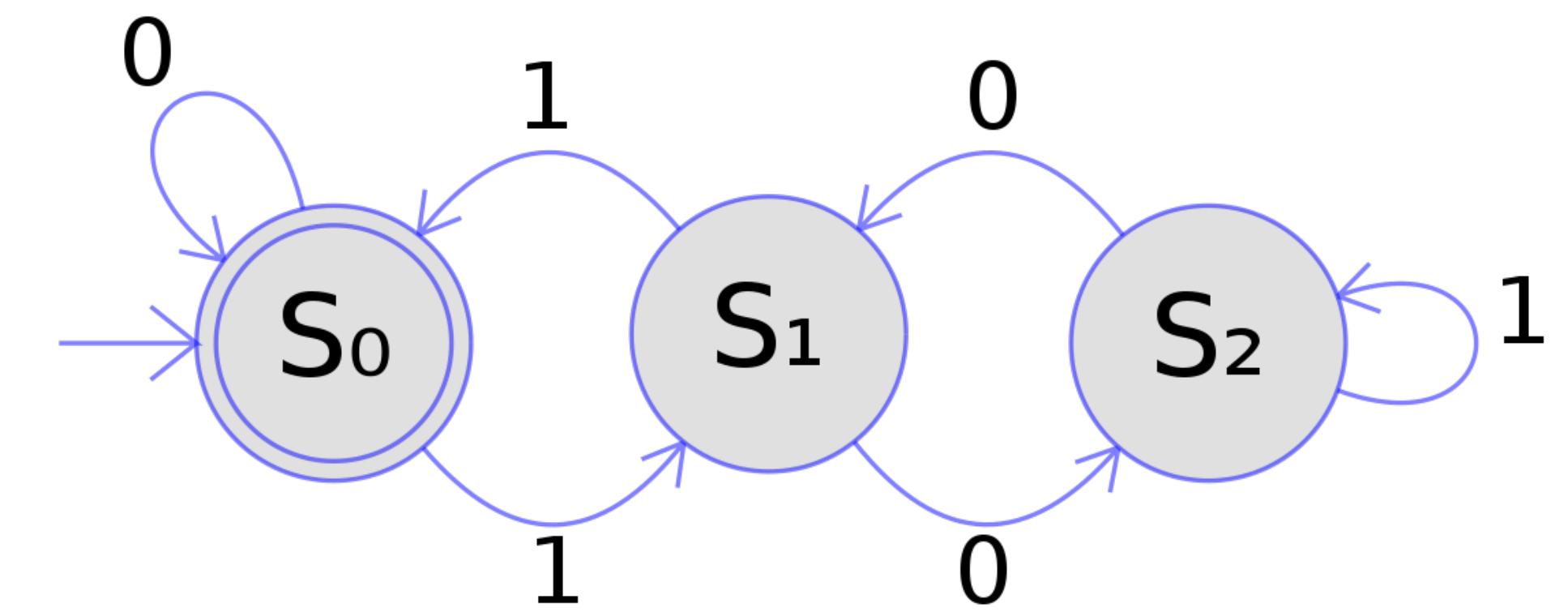
Case studies

Regex matching

Brzozowski derivatives

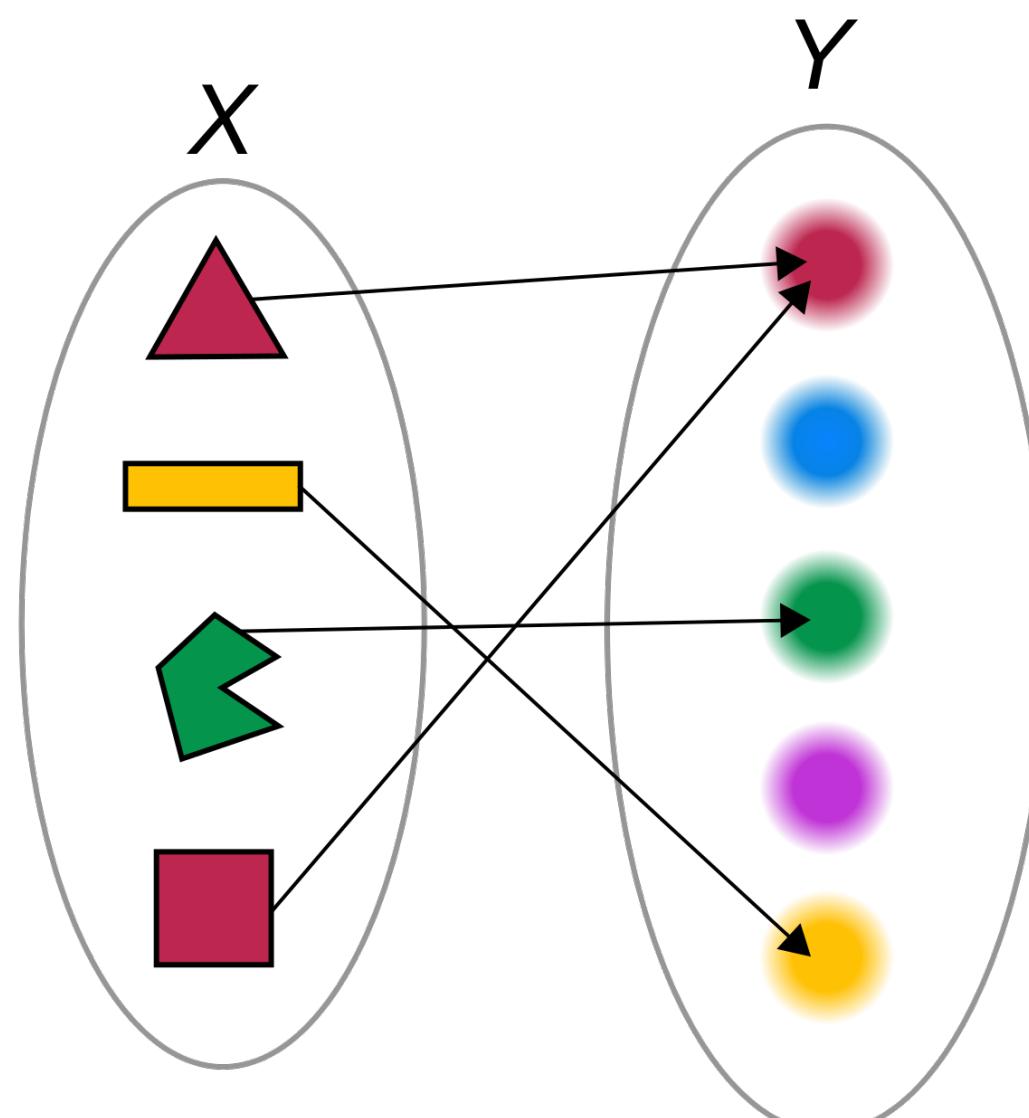
$$u^{-1}S = \{v \in \Sigma^* \mid uv \in S\}$$

DFA

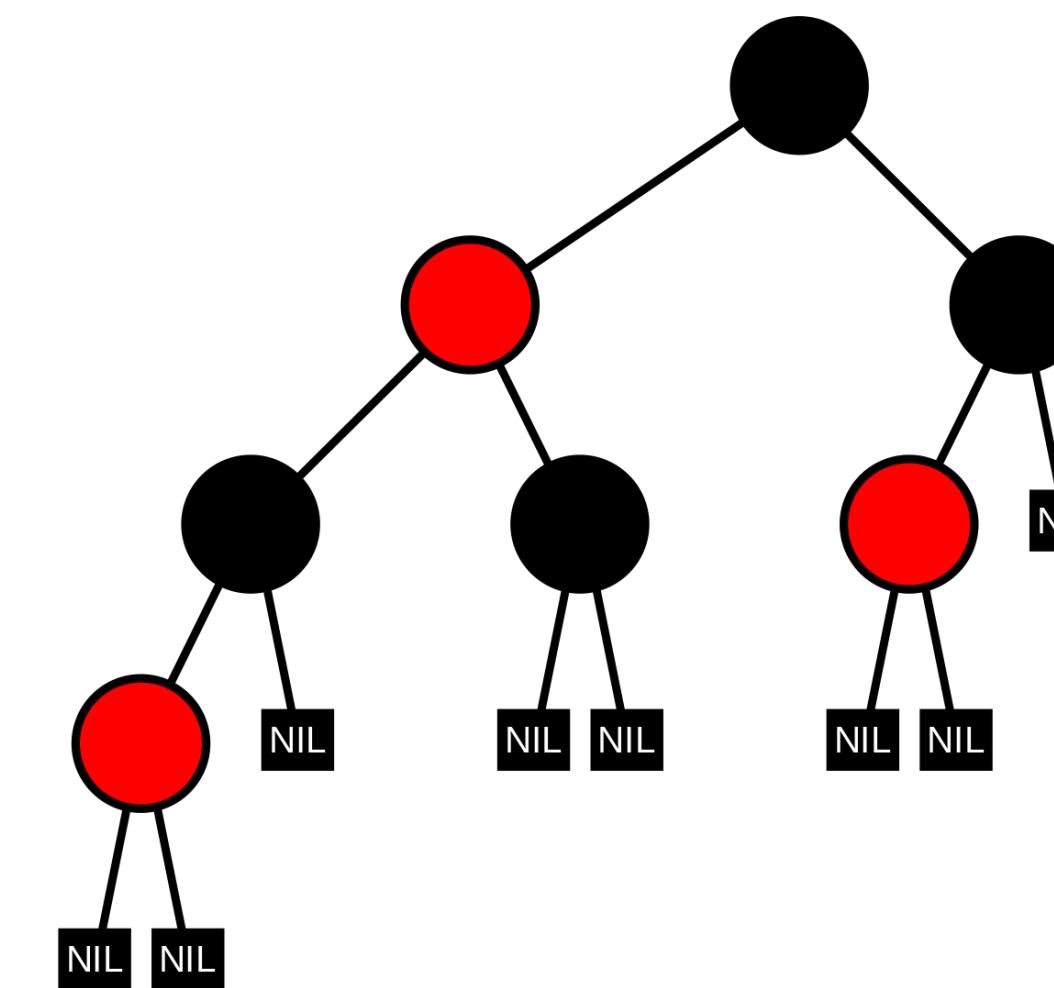


Functional maps

Association lists



Red-Black Trees

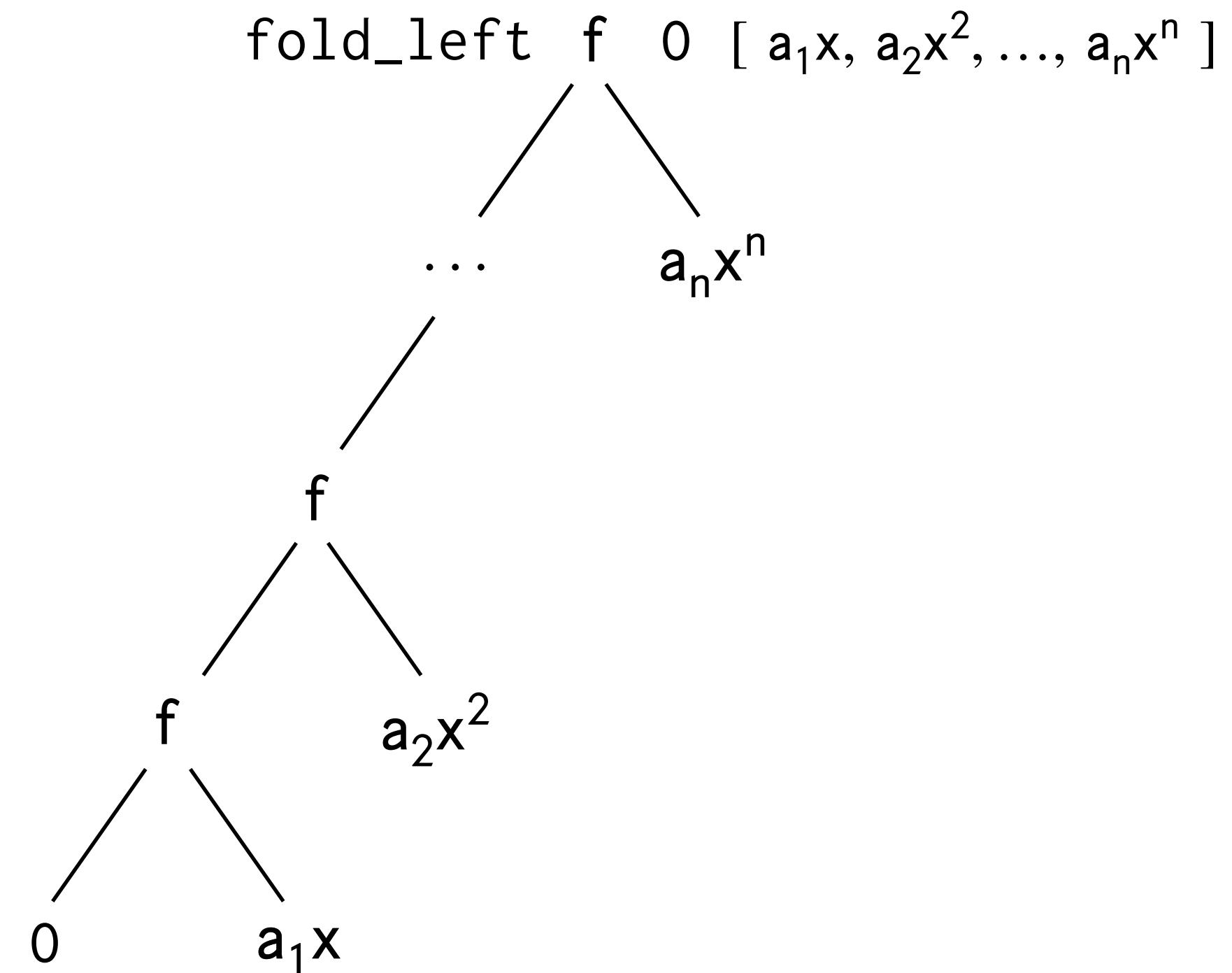


Polynomials

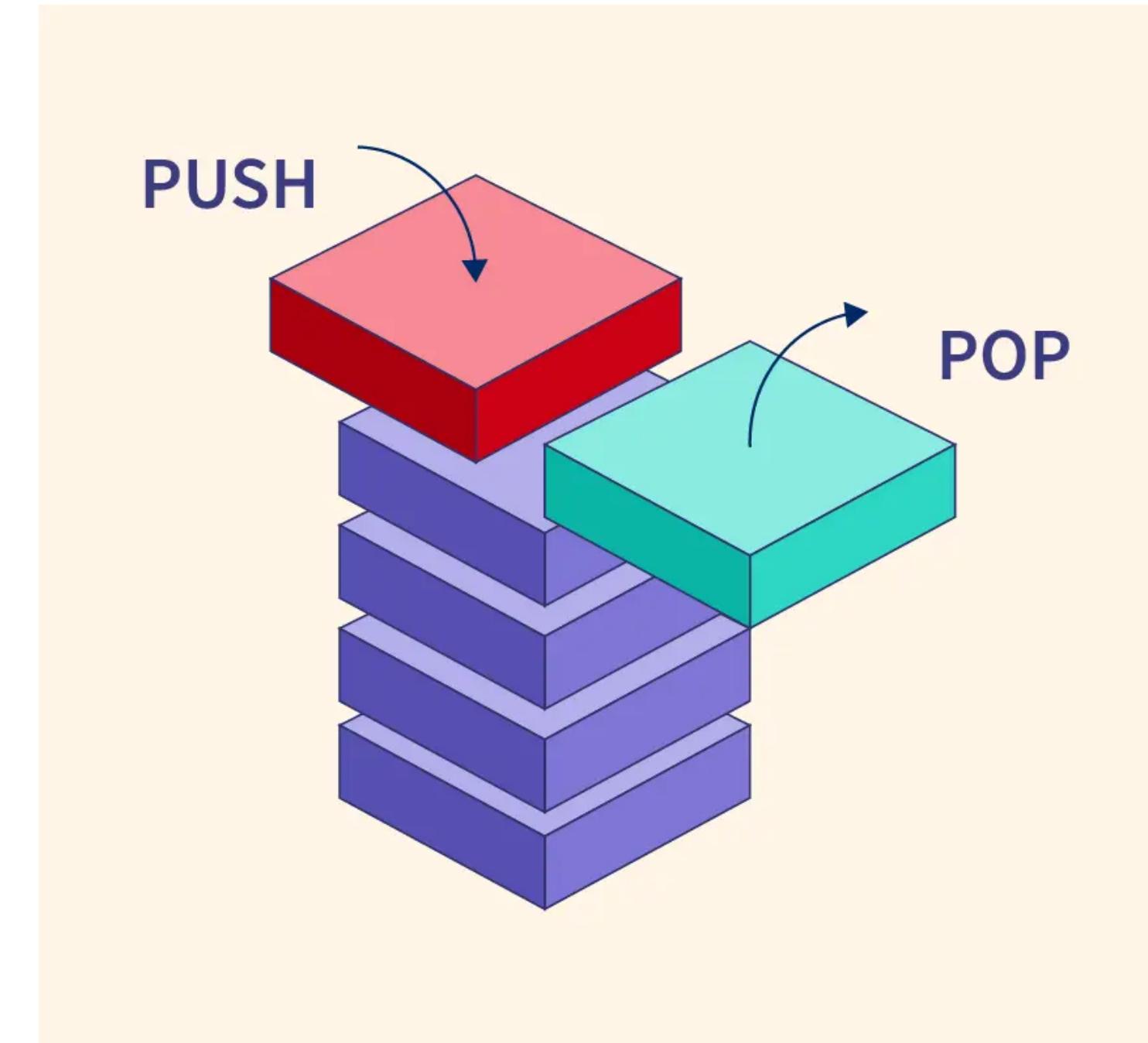
Horner's algorithm

$$\begin{aligned} p(x_0) &= a_0 + x_0 \left(a_1 + x_0 \left(a_2 + \cdots + x_0 (a_{n-1} + b_n x_0) \cdots \right) \right) \\ &= a_0 + x_0 \left(a_1 + x_0 \left(a_2 + \cdots + x_0 b_{n-1} \right) \right) \\ &\vdots \\ &= a_0 + x_0 b_1 \\ &= b_0. \end{aligned}$$

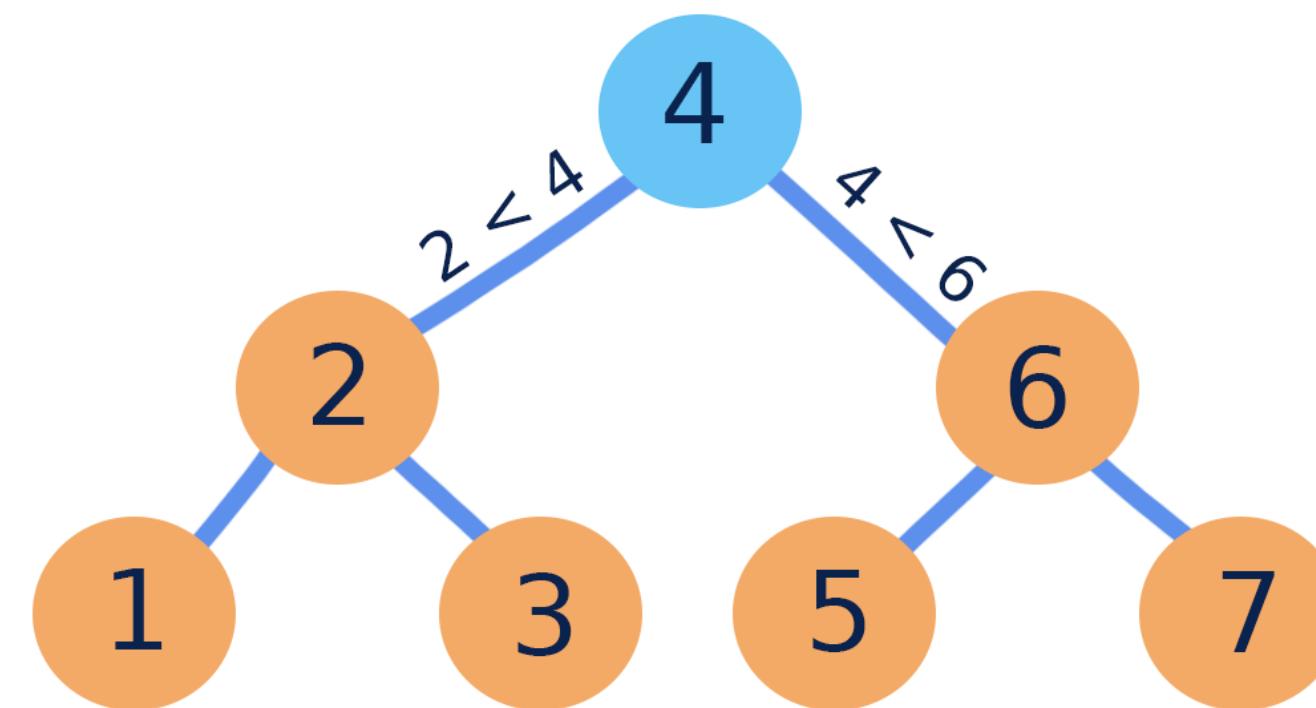
Fold over list of monomials



Stacks

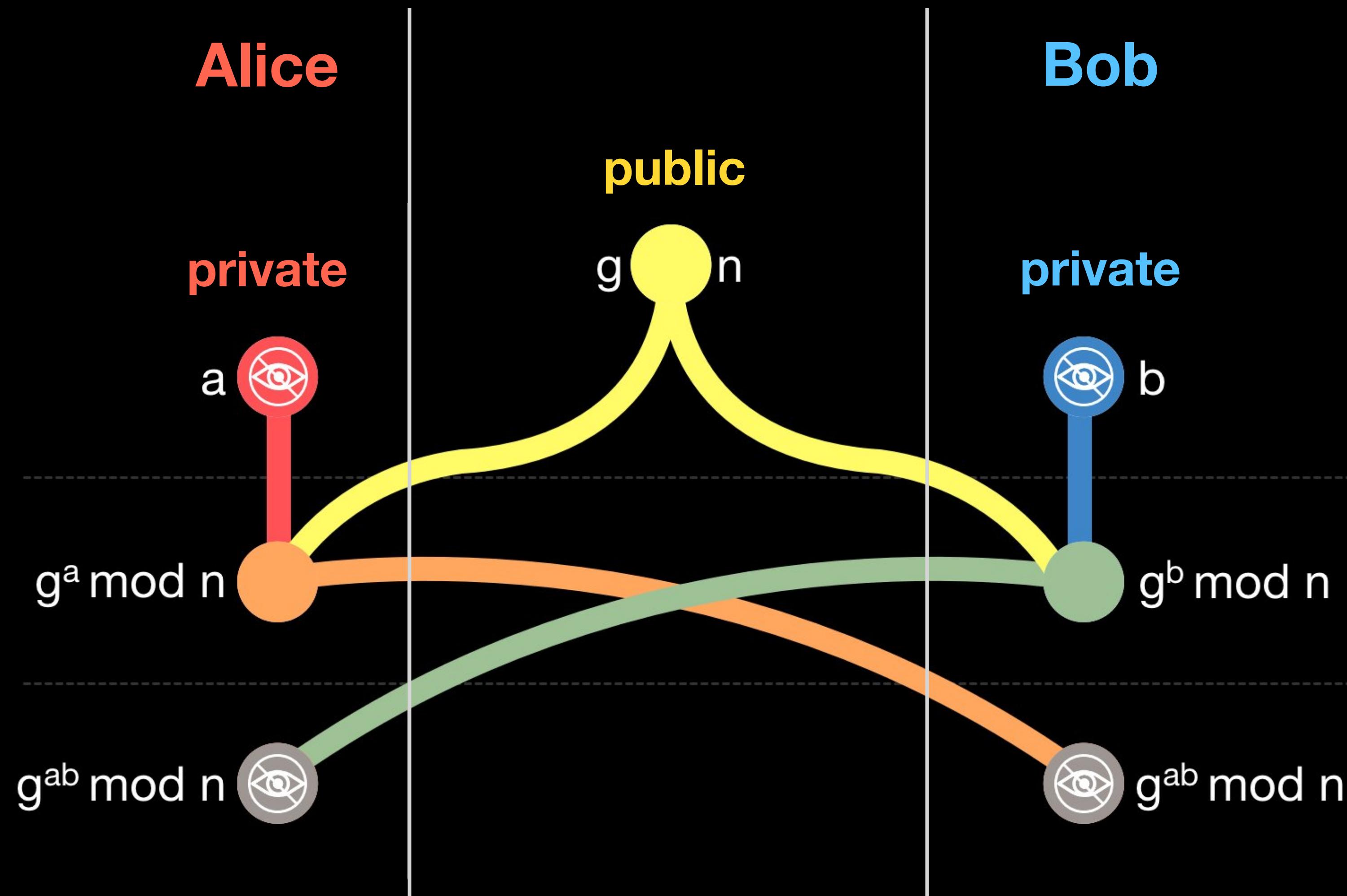


Sets (BSTs, lists)



Diffie-Hellman key exchange

(work in progress)



Challenges with testing Diffie-Hellman

Challenges with testing Diffie-Hellman

```
module type Set = sig
  type 'a t
  ...
end
```

Other examples

one abstract type

DH interface

two abstract types

```
module type DH = sig
  type public_key
  type private_key
  ...
end
```

Challenges with testing Diffie-Hellman

```
module type Set = sig
  type 'a t
  ...
end
```



Other examples

one abstract type

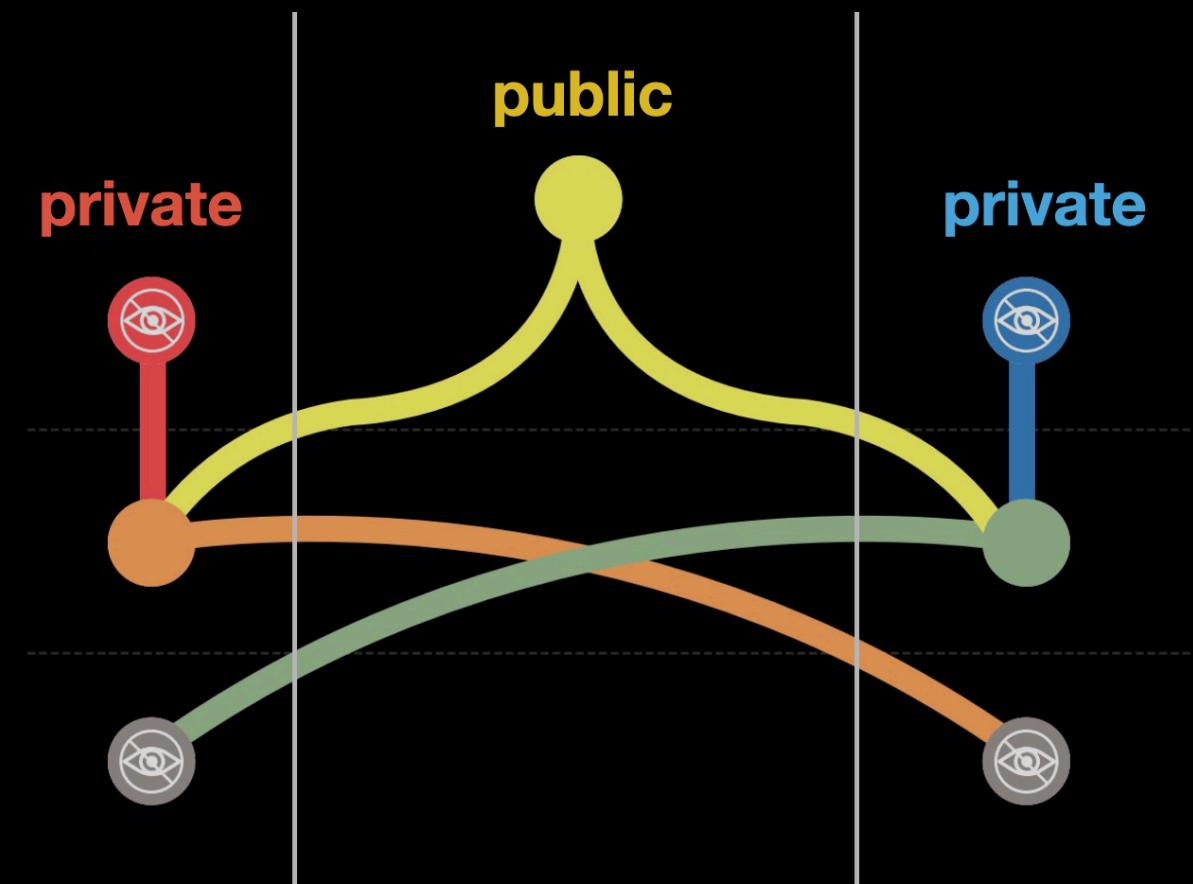
no prescribed order
for calling functions

DH interface

two abstract types

prescribed order
for calling functions
(specified by DH)

```
module type DH = sig
  type public_key
  type private_key
  ...
end
```



Challenges with testing Diffie-Hellman

```
module type Set = sig
  type 'a t
  ...
end
```

Add
Union
Size
...
~~~~~ Set ~~~~ Add  
Union  
Size  
...

```
emptySet : 'a t
getAbsType : ? → 'a t
```

## Other examples

**one abstract type**

**no prescribed order**  
for calling functions

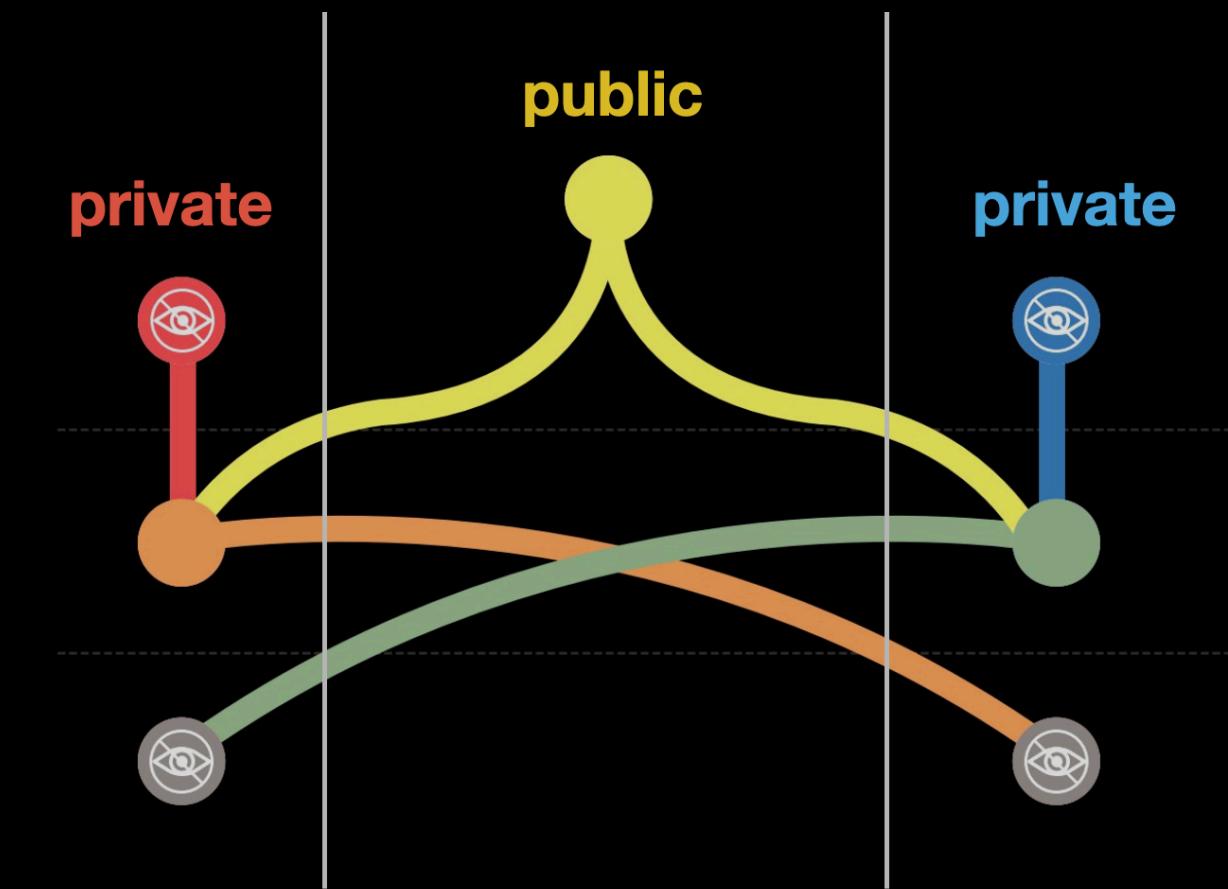
interface tells us how to  
to obtain an '**a t**'

## DH interface

**two abstract types**

**prescribed order**  
for calling functions  
(specified by DH)

```
module type DH = sig
  type public_key
  type private_key
  ...
end
```



no way to get a **private\_key**  
using only the interface

# **Related work**

# History of model-based PBT



Monadic QuickCheck

[Claessen & Hughes 2002]



QuviQ QuickCheck

[Hughes 2016]

# Model-based PBT



QCSTM

[Midgaard 2020]

Model\_quickcheck

[Dumont 2020]

- Algebraic data types for representing symbolic commands
- Mica adds support for invariants + binary operations on abstract types

# Random testing of ML modules



Monolith

[Pottier 2021]

Articheck

[Braibant et al. 2014]

- GADT-based DSLs for testing ML modules
- Mutation-based fuzzing
- Mica automatically derives the requisite PBT code

# Automatic generation of PBT code



Hypothesis  
Ghostwriter

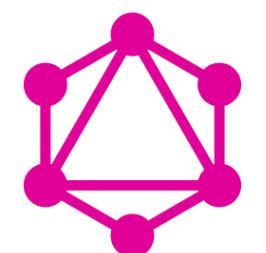
[Hatfield-Dodds et al. 2020]



Clojure

QuickREST

[Karlsson et al. 2020, 2019]



GraphQL

# **Future work**

# Future work

Handle imperative code



# Future work

Encode dependencies in  
generated command sequences

Rem **2** (Add **2** Empty)

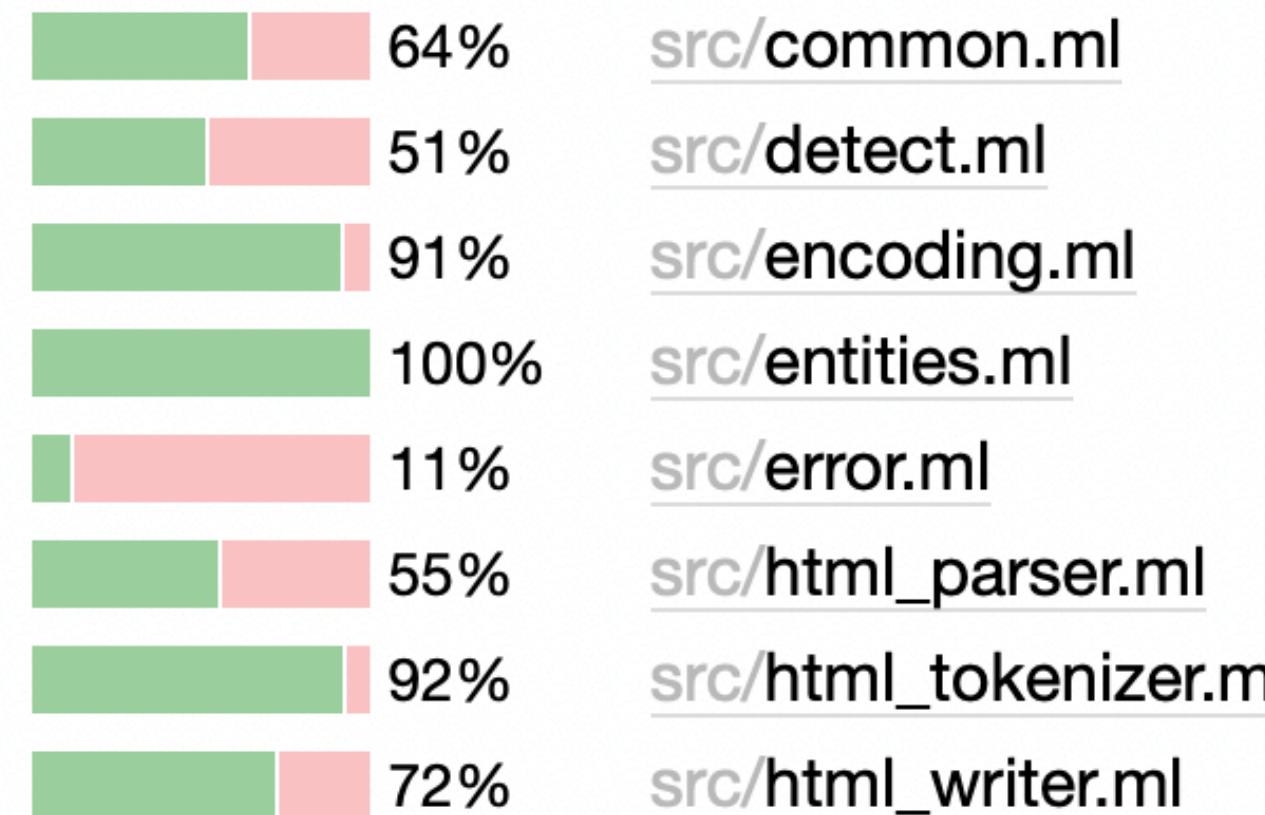


# Future work

## Empirical evaluation

ppx\_bisect

**Coverage report** 72.15%



Haskell QuickCheck

```
>>> quickCheck prop_sorted_sort
+++ OK, passed 100 tests; 1684 discarded.
```

List elements (109 in total):

|      |    |
|------|----|
| 3.7% | 0  |
| 3.7% | 17 |
| 3.7% | 2  |
| 3.7% | 6  |
| 2.8% | -6 |
| 2.8% | -7 |

# Future engineering work

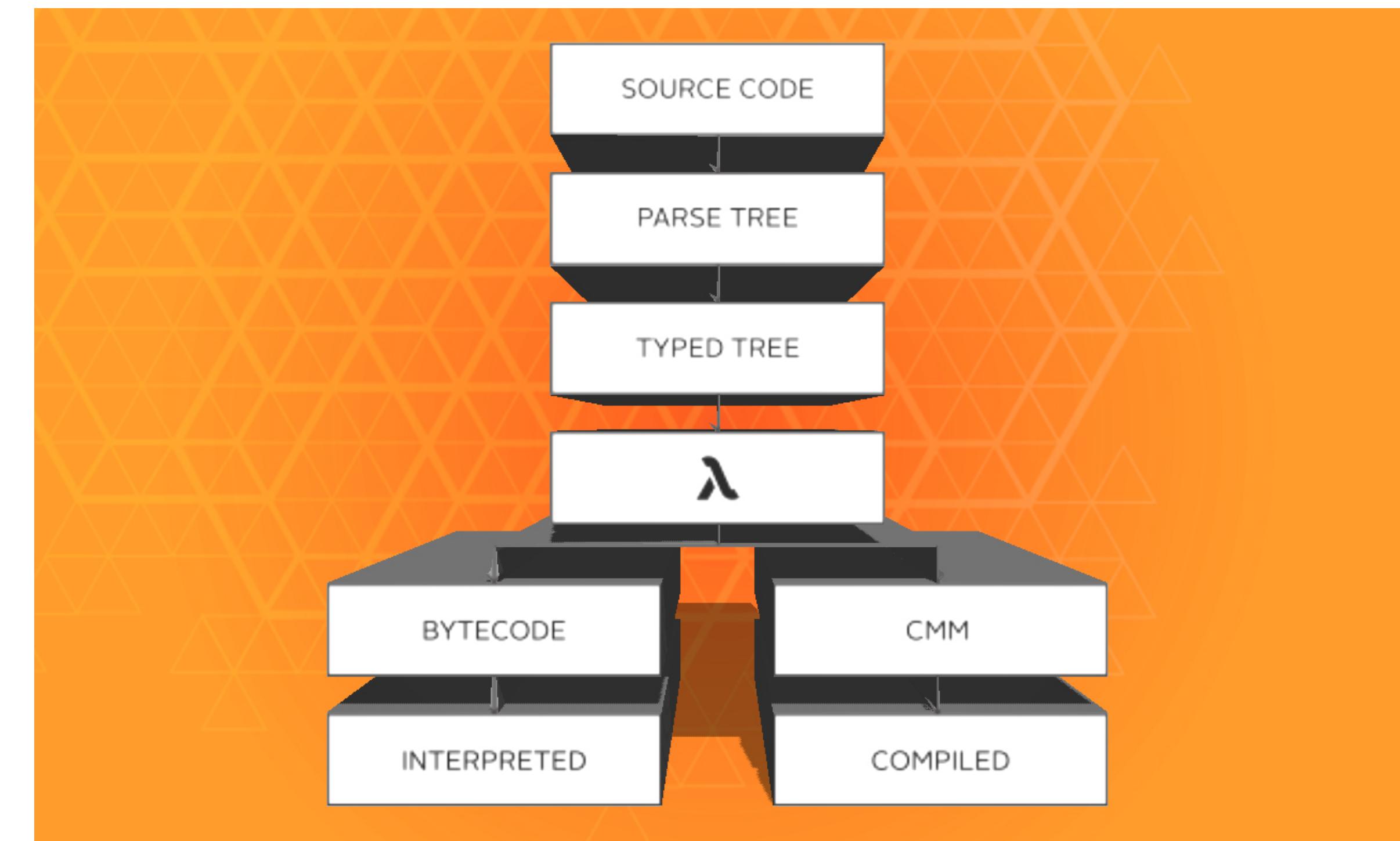
Shrinking

Add **5** (Rem **2** (Add **2** Empty))



Add **5** Empty

Integrate with OCaml compiler



# Takeaways



Graphic from *Real World OCaml*

# Takeaways

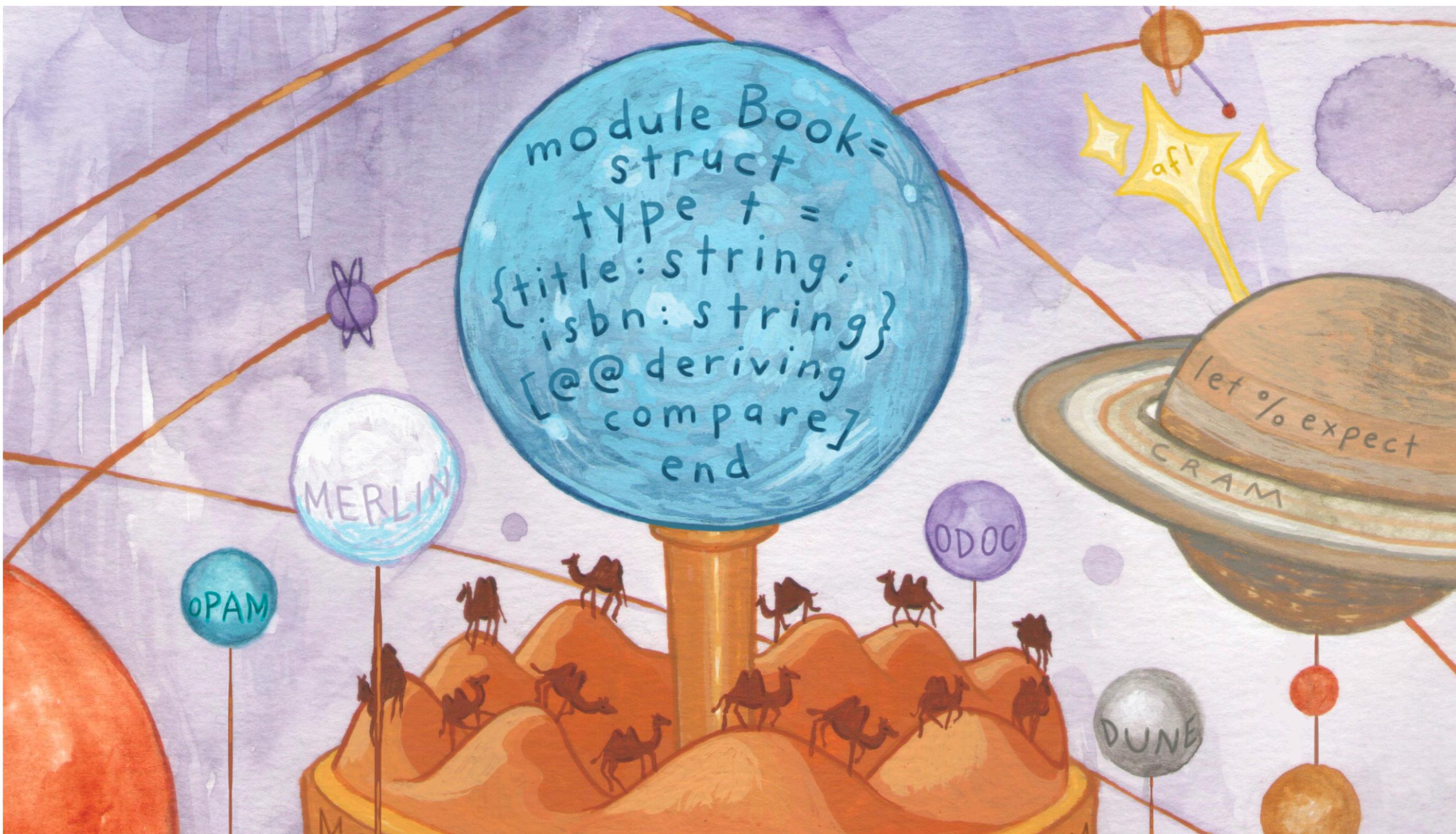
1. Checking observational equivalence requires significant programmer effort



Graphic from *Real World OCaml*

# Takeaways

1. Checking observational equivalence requires significant programmer effort
2.  $M \vdash A$  can automate this process via PBT!



Graphic from *Real World OCaml*

# Thank you!

(Questions?)

[nbernest@seas.upenn.edu](mailto:nbernest@seas.upenn.edu)

[github.com/nbernest/module\\_pbt](https://github.com/nbernest/module_pbt)

# **Appendix**

# **Appendix 1: Bugs caught**

# “Bug Bounty” Overview

- **Total of 15 bugs artificially inserted across all three pairs of modules**
  - (the bugs are detailed in the following slides)
- **14 bugs caught**
- **Experimental limitations:**
  - **Bugs were introduced one at a time (no cascading bugs)**
  - **In each trial, the bug was only introduced in one module, with the other module remaining correct**

## Artificial bugs (Stack & Set examples)

| Artificially introduced bug                                                              | Failing test case                                                                                                                                      | Correct result                    | Erroneous result                      |
|------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------|---------------------------------------|
| Is_empty empty = false                                                                   | Is_empty Empty                                                                                                                                         | TRUE                              | FALSE                                 |
| Peek s = None (forall s)                                                                 | Peek (Push -1 Empty)                                                                                                                                   | -1                                | None                                  |
| push x stack = stack @ [x]                                                               | Peek (Push -8 (Push 3 Empty))                                                                                                                          | -8                                | 3                                     |
| length (h :: t) = length t                                                               | Length (Push -1 Empty)                                                                                                                                 | 1                                 | 0                                     |
| <b>Wrong invariant (List implementation of Sets)</b>                                     | Invariant Empty                                                                                                                                        | TRUE                              | FALSE                                 |
| let invariant (s : 'a list) : bool =<br>(* not @@ *)<br>List.contains_dup ~compare:(=) s | Invariant<br>(Union (Add 5<br>(Union (Union Empty Empty)<br>(Add -3 Empty)))<br>(Rem -10 (Add 5 (Add 9 Empty))))<br>= ...<br>= Invariant [-3; 5; 5; 9] | Invariant<br>[-3; 5; 9]<br>= TRUE | Invariant<br>[-3; 5; 5; 9]<br>= FALSE |
| <b>Not enforcing Set invariant (Union for ListSet)</b>                                   |                                                                                                                                                        |                                   |                                       |
| let union s1 s2 =<br>s1 @ s2 (* ▷ dedup *)                                               |                                                                                                                                                        |                                   |                                       |

```

type 'a tree =
| Empty
| Node of 'a tree * 'a * 'a tree

```

| Artificially introduced bug                                                                                                                                                                                                                                                      | Failing test case                                                                                                                                                                                                                                                                                                                                                                              | Correct | Erroneous |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|-----------|
| <b>Spurious recursive call (BST insertion)</b><br><pre> let rec add x t = match t with ...   Node (lt, n, rt) -&gt;   if x &lt; n   then Node (add x lt, n, <b>add x rt</b>) ... </pre>                                                                                          | <b>Fails BST invariant</b><br>Invariant<br>$\begin{aligned} & (\text{Rem } 2 \text{ (Add } -7 \\ & \quad (\text{Add } -3 \text{ (Intersect Empty Empty)))) \\ & = \dots \\ & = \text{Invariant } (\dots \text{ (Node } (-7, 3, -7))) \end{aligned}$                                                                                                                                            | TRUE    | FALSE     |
| <b>Spurious negation (Intersection of two BSTs)</b><br><pre> let intersect t1 t2 =   let commonElts =     filter ~f:(fun x -&gt; <b>not</b> <del>mem</del> mem x t1)       (inorderTraversal t2) in   List.fold commonElts ~init:empty     ~f:(fun acc x -&gt; add x acc) </pre> | $\begin{aligned} & (\text{Is\_empty} \\ & \quad (\text{Intersect} \text{ (Rem } 8 \text{ (Rem } -7 \text{ (Rem } 7 \text{ Empty)))) \\ & \quad (\text{Intersect} \text{ (Union} (\text{Add } 2 \text{ Empty}) \\ & \quad \quad (\text{Union} \text{ Empty Empty})) \\ & \quad (\text{Add } 8 \text{ (Add } 4 \text{ Empty})) \\ & = \dots \\ & = \text{Is\_empty } \text{Empty} \end{aligned}$ | TRUE    | FALSE     |
| Changing the base case<br>of the BST invariant function<br>to <b>False</b> instead of True<br>(details omitted)                                                                                                                                                                  | Invariant<br>$\begin{aligned} & (\text{Union} \text{ (Add } 9 \text{ (Add } -4 \text{ (Rem } 6 \text{ Empty)))) \\ & \quad (\text{Intersect} \text{ (Add } -10 \text{ (Intersect Empty Empty)))} \\ & \quad (\text{Union} \text{ (Add } -6 \text{ Empty}) \\ & \quad \quad (\text{Union} \text{ Empty Empty)))) \\ & = \text{Invariant } \{-700, -6, -4, 9\} \end{aligned}$                    | TRUE    | FALSE     |

```

type 'a tree =
| Empty
| Node of 'a tree * 'a * 'a tree

```

## BST implementation of Sets: removing an element

```

let rec rem (x : 'a) (t : 'a tree) : 'a tree =
  match t with ...
| Node (lt, n, rt) → ...

```

```

if x > n then Node (rem x lt, n, rem x rt)

```

**Spurious recursive call**

```

if x > n then Node (lt, n, rt)

```

**Missing recursive call**

**Bug not caught**

Technically, [rem x lt] doesn't do anything here

**Bug caught**

| Failing test case                                    | Correct result | Erroneous result |
|------------------------------------------------------|----------------|------------------|
| <b>Size (Rem -2 (Add -2 (Add -3 (Add 4 Empty))))</b> | 2              | 3                |