Statistics of a simple transmission mode on a lossy chaotic background

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Scattering on a resonance state coupled to a complicated background is a typical problem for mesoscopic quantum many-body systems as well as for wave propagation in the presence of a complex environment. On average, such a simple mode acquires an effective damping, the so-called "spreading" width, due to mixing with the background states. Modelling the latter by random matrix theory and employing the strength function formalism, we derive the joint distribution of the reflection and total transmission at arbitrary absorption in the background. The distribution is found to possess a remarkable symmetry between its reflection and transmission sectors, which is controlled by the ratio of the spreading to escape width. This in turn results in a symmetry relation between the marginal densities, despite the absence of the flux conservation law at finite absorption. As an application, we study the statistics of total losses in the system at arbitrary coupling to the background.

Strength function phenomena [1] have a rich history of various applications in atomic and nuclear physics [2–6] as well as in open mesoscopic systems [7–10]. In such problems, one deals with a "simple" excitation (associated with a specific signal) that is coupled to the background of many "complicated" (usually chaotic) states. As a result of this coupling, the simple mode is spread over exact stationary states with a rate determined by the so-called spreading width [1, 2]. Transmission through such a simple mode is therefore characterised by the competition between the two damping mechanisms, escape to the continuum and spreading over the background, and becomes strongly suppressed when the ratio $\eta = \Gamma_\downarrow/\Gamma_0$ of the spreading ($\Gamma_\downarrow$) to escape (Γ_0) width exceeds unity [10].

Under real laboratory conditions, there are also sources of a coherence loss in quantum transport, with finite absorption being one of them [11]. This has dramatic consequences in scattering, since the S matrix becomes no longer unitary. For open quantum or wave chaotic systems, exact analytical results were recently obtained for various scattering characteristics at finite absorption [12–15]. Recent advances in experimental techniques have made it possible to change absorption in a controlled way and to test the theory with high accuracy in microwave cavity experiments [16], including in particular the statistics of reflection and transmission coefficients [17–19], complex impedances [20–22], and decay rates [23].

On the theory side, the resonance scattering formalism [24] is well adopted to treat both dynamical and statistical features of such systems on equal footing [25–27]. When combined with random matrix theory (RMT) to model internal chaotic dynamics [28, 29], it offers a powerful tool to describe universal fluctuations in scattering, see [30, 31] for recent reviews. The approach is also flexible in incorporating system-specific effects. In particular, the simple mode in such a context was recently introduced [10] as a useful model for quantifying fluctuations in an established (deterministic) transmission that are induced by complex environments. For the complete characterisation of the scattering process, however, both transmission and reflection fluctuations need to be treated at the same time. This becomes even more challenging at finite losses,

since the two are no longer related by the flux conservation.

Here, we develop a general approach for scattering on the simple mode coupled to a lossy chaotic background. We derive exact results for the joint distribution of reflection and total transmission at arbitrary absorption and show that the distribution reveals a specific symmetry between its reflection and transmission sectors under the involution $\eta \to \eta^{-1}$. We also study marginal densities and the statistics of total losses.

Simple mode.— Let us consider a simple state with energy ε_0, which is coupled to the continuum by means of the decay amplitudes A_c, where index c runs over all scattering channels open at the given scattering energy E. In the resonance approximation, A_c may be assumed to be energy-independent quantities, leading to a multichannel Breit-Wigner formula [24] $S_{ab}^{(0)}(E) = \delta_{ab} - iA_a^*A_b/(E - \varepsilon_0 + \frac{i}{2}\Gamma_0)$ for the scattering matrix elements. The escape width Γ_0 is then given by the sum of the partial (per channel) widths, $\Gamma_0 = \sum_c |A_c|^2$. This ensures the unitarity of the S matrix (at real E).

Following [1, 2], taking into account the interaction between such a mode and the surrounding background described by a Hamiltonian $H_{\rm bg}$ results in the modified expression

$$S_{ab}(E) = \delta_{ab} - i\frac{A_a^*A_b}{E - \varepsilon_0 + \frac{i}{2}\Gamma_0 - g(E)}, \qquad (1)$$

where $g(E) = V^\dagger(E - H_{\rm bg})^{-1}V$ is the strength function and V stands for a coupling vector to N background states. The latter usually have a very complex structure, fluctuating strongly on the scale of the mean level spacing $\Delta \sim 1/N$. When averaged over this fine structure, the scattering amplitudes acquire an extra damping given by the spreading width $\Gamma_\downarrow \equiv 2\pi{\rm Im}\langle g(\varepsilon_0 - i0)\rangle = 2\pi\|V\|^2/N\Delta$. Introducing a natural control parameter $\eta = \Gamma_\downarrow/\Gamma_0$, we can cast the S matrix at the resonance energy ε_0 in the following convenient form:

$$S = 1 - \frac{1}{1 + i\eta K}(1 - S^{(0)}). \qquad (2)$$

Here $K \equiv 2g(\varepsilon_0)/\Gamma_\downarrow$ has the meaning of the (dimensionless) local Green's function of the complex background [32].

The unitary matrix $S^{(0)}$ stands for the deterministic part of S, $S_{ab}^{(0)} = \delta_{ab} - \frac{2}{\Gamma_0} A_a^* A_b$, accounting for the direct mixing of the channels. Expression (2) provides the multichannel generalisation of two-channel formulae derived recently in [10].

The established connection of S to the background spectrum enables us accommodate its physically relevant properties. Following the RMT paradigm [28, 29], we model $H_{\rm bg}$ by a random $N \times N$ matrix drawn from the Gaussian orthogonal (GOE) or unitary (GUE) ensemble, depending on the presence or absence of time-reversal invariance (TRI), respectively. Universal fluctuations are then expected to occur in the limit $N \gg 1$. Furthermore, homogeneous dissipation can be easily taken into account by uniform broadening $\Gamma_{\rm abs}$ of the background states. Since such a damping is operationally equivalent [12] to the purely imaginary shift $\varepsilon_0 + \frac{i}{2}\Gamma_{\rm abs}$ in the Green's function K, the latter becomes complex,

$$K = (2/\Gamma_\downarrow)g(\varepsilon_0 + i\Gamma_{\rm abs}/2) \equiv u - iv, \qquad (3)$$

with $v > 0$ being the local density of states (normalized as $\langle v \rangle = 1$) [32]. The universal statistics of mutually correlated random variables u and v is solely determined by the (dimensionless) absorption rate $\gamma = 2\pi\Gamma_{\rm abs}/\Delta$. They have the following joint probability density function (JPDF) [32]:

$$P(u,v) = \frac{1}{2\pi v^2}P_0(x), \qquad (4)$$

with $x = (u^2 + v^2 + 1)/2v > 1$. In the present context, the function $P_0(x)$ has the meaning of the distribution of reflection induced by the background [33]. This function is known exactly for both symmetry classes as well as in the crossover regime of gradually broken TRI [14, 34]. We now apply these findings to derive nonperturbative results for the reflection and transmission distributions at arbitrary values of η and γ.

Scattering in a given channel 'a' is commonly studied by means of the coefficients of reflection $R \equiv |S_{aa}|^2$ and total transmission $T \equiv \sum_{b \neq a} |S_{ab}|^2$. Making use of Eq. (2), one finds that these two quantities are expressed as follows

$$R = \frac{(S_{aa}^{(0)} + \eta v)^2 + \eta^2 u^2}{(1 + \eta v)^2 + \eta^2 u^2}, \qquad (5a)$$

$$T = \frac{1}{(1 + \eta v)^2 + \eta^2 u^2}T_0, \qquad (5b)$$

where $T_0 \equiv \sum_{b \neq a} |S_{ab}^{(0)}|^2 = 1 - (S_{aa}^{(0)})^2$ is the total transmission coefficient in a "clean" system. At zero absorption, we have $v \equiv 0$ and thus $R + T = 1$ in agreement with the flux conservation. The later is no longer valid at finite absorption, when S becomes subunitary. Such a unitarity deficit can be naturally described by the following positive quantity:

$$D \equiv 1 - R - T = \frac{2(1 - S_{aa}^{(0)})\eta v}{(1 + \eta v)^2 + \eta^2 u^2} \leq \frac{1 - S_{aa}^{(0)}}{2}, \qquad (6)$$

which gives the part of the total flux in the channel that gets dissipated in the background. The deficit $D = 0$ identically at
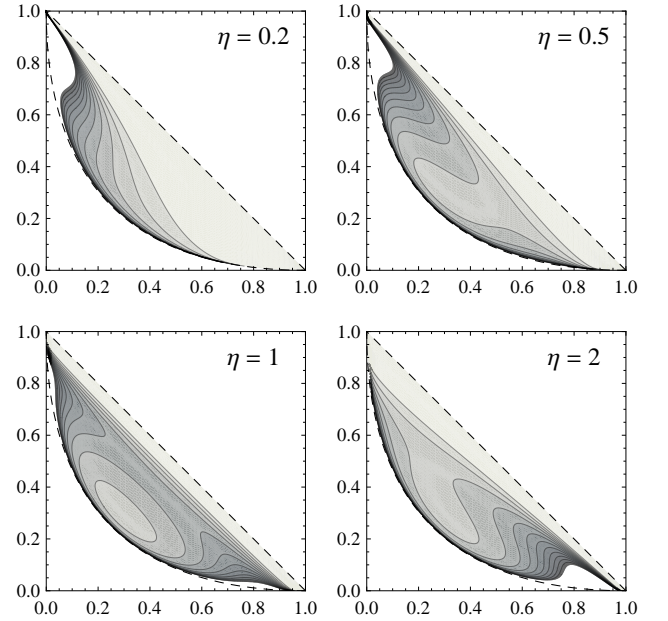

FIG. 1. Contour plots of the joint distribution (7) of reflection (abscissa) and transmission (ordinate) for the GUE case of broken time-reversal invariance at $\gamma = 1$ and various η. Dashed lines indicate the boundaries of the distribution support. Note the symmetry of the distribution at $\eta = 1$ and at the reciprocal values of η.

$S_{aa}^{(0)} = 1$, when the channel is closed. It covers its maximum range $0 \leq D \leq 1$ at $S_{aa}^{(0)} = -1$, when the wave gets reflected in full after the interaction with the background. (Note that both cases correspond to zero transmission.) We will study the probability distribution of D below as well.

Joint and marginal distributions.— It is instructive to consider first the case of perfect coupling, $T_0 = 1$ ($S_{aa}^{(0)} = 0$). We reserve the notation $t = T|_{T_0=1}$ and $r = R|_{T_0=1}$. Making use of relations (5), the JPDF of reflection and transmission $P_\eta(r,t)$ is then found from the known function $P(u,v)$ by applying the calculus of Jacobians. After some algebra, we arrive at the following attractive formula:

$$P_\eta(r,t) = \frac{2}{\pi(1 - r - t)^2\sqrt{y}}P_0\left(\frac{\eta^{-1}r + \eta t}{1 - r - t}\right), \qquad (7)$$

for $1 - r - t > 0$ and $y = 1 + 2rt - (1-r)^2 - (1-t)^2 > 0$, being zero otherwise. Since y is symmetric under the interchange $r \leftrightarrow t$, it can be also cast as follows

$$y = (r_+ - r)(r - r_-) = (t_+ - t)(t - t_-), \qquad (8)$$

with $r_\pm = (1 \pm \sqrt{t})^2$ and $t_\pm = (1 \pm \sqrt{r})^2$. It follows at once that function (7) has the following important symmetry:

$$P_\eta(r,t) = P_{\eta^{-1}}(t,r). \qquad (9)$$

This shows that the background coupling η controls the weight of the total flux distribution between its reflection and transmission sectors. In particular, distribution (7) becomes
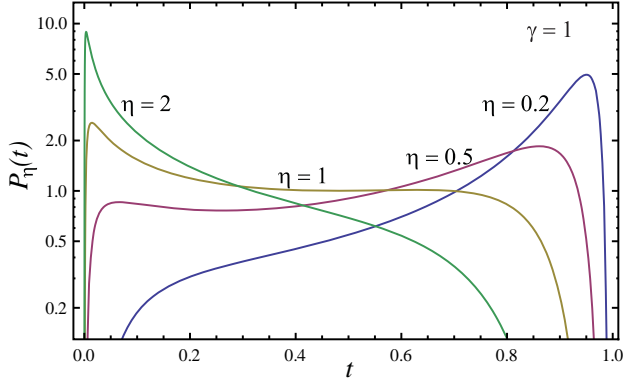
FIG. 2. Transmission distribution (10) for the GUE case of broken time-reversal invariance at $\gamma = 1$ and various η. By the symmetry property (11), the corresponding reflection distributions would be given by the same curves at the reciprocal values of η.

FIG. 3. Reflection distribution (16) for the GUE case at nonperfect coupling $T_0 = 0.75$, corresponding to $S_{aa}^{(0)} = \pm\sqrt{1 - T_0}$. Note a hard gap $R > 1 - T_0$ of the distribution when $S_{aa}^{(0)} > 0$ and a finite value of $\mathcal{P}_\eta(0)$ when $S_{aa}^{(0)} < 0$. The dashed line shows the distribution at perfect coupling for comparison.

symmetric with respect to the line $r = t$ at the special coupling $\eta = 1$. This discussion is further illustrated on Fig. 1.

The marginal distributions can now be obtained from JPDF (7) by integrating it over r or t. One readily finds the following expression for the transmission distribution:

$$\mathcal{P}_\eta^{(\mathrm{tr})}(t) = \int_{r_-}^{1-t} \frac{dr}{\pi(1-r-t)^2} \frac{2P_0\left(\frac{\eta^{-1}r+\eta t}{1-r-t}\right)}{\sqrt{(r_+ - r)(r - r_-)}}, \quad (10)$$

for $0 \leq t \leq 1$ and zero otherwise. Choosing a new integration variable $p = (r - r_-)/(1 - r - t) \geq 0$, one can further bring $\mathcal{P}_\eta^{(\mathrm{tr})}(t)$ to the form derived recently in [10]. The advantage of representation (10) is that it utilizes the symmetry property (9) explicitly. It becomes then obvious that the distribution of reflection is simply related to that of transmission as follows

$$\mathcal{P}_\eta^{(\mathrm{ref})}(r) = \mathcal{P}_{\eta^{-1}}^{(\mathrm{tr})}(r). \quad (11)$$

This is a remarkable relation showing that despite lacking any apparent connection between the reflection and transmission coefficients at finite absorption, their distribution functions turn out to be linked by symmetry (11). With explicit formulae for P_0 found in [14, 34], Eqs. (7), (10) and (11) provide the exact solution to the problem at arbitrary η and γ.

Further analysis is possible in the physically interesting limiting cases of weak and strong absorption, when the function P_0 is known to take simpler asymptotic forms [32]. At $\gamma \ll 1$, one has $P_0(x) \approx \frac{(\beta\gamma)^{\beta/2+1}}{4\Gamma(\beta/2+1)}\left(\frac{x+1}{4}\right)^{\beta/2} e^{-\frac{\beta\gamma}{4}(x+1)}$, where $\beta = 1$ ($\beta = 2$) stands for the GOE (GUE) case. After the integration in (10), this results in the leading-order result

$$\mathcal{P}_{\eta,\gamma\ll1}^{(\mathrm{tr})}(t) \approx \mathcal{P}_\eta^{(0)}(t) \exp\left[-\frac{\beta\gamma(1 + (\eta-1)\sqrt{t})^2}{8\eta\sqrt{t}(1-\sqrt{t})}\right], \quad (12)$$

where $\mathcal{P}_\eta^{(0)}(t)$ denotes the transmission distribution at zero absorption (i.e. for a stable background) [10],

$$\mathcal{P}_\eta^{(0)}(t) = \frac{1}{\pi\sqrt{t(1-t)}} \frac{1}{\eta^{-1}(1-t) + \eta t}. \quad (13)$$
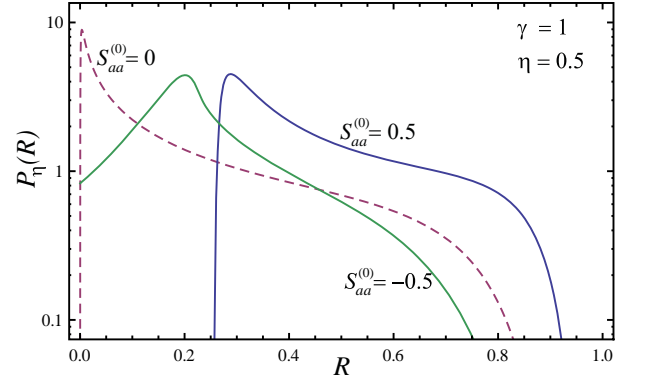
Distribution (13) is insensitive to the presence of TRI. It has a typical bimodal profile with square-root singularities near its edges, which get exponentially suppressed by finite absorption. In the opposite case of $\gamma \gg 1$, making use of $P_0(x) \approx \frac{\beta\gamma}{4}e^{-\frac{\beta\gamma}{4}(x-1)}$ yields the following approximation:

$$\mathcal{P}_{\eta,\gamma\gg1}^{(\mathrm{tr})}(t) \approx \frac{\sqrt{\beta\gamma\eta}(1+\sqrt{t})\exp\left[-\frac{\beta\gamma(1-(\eta+1)\sqrt{t})^2}{8\eta\sqrt{t}(1-t)}\right]}{4\sqrt{\pi}(1-t)t^{3/4}\sqrt{1+(\eta^2-1)t}}. \quad (14)$$

Figure 2 shows $\mathcal{P}_\eta^{(\mathrm{tr})}(t)$ at moderate absorption $\gamma = 1$.

Nonperfect coupling.— In the general case of $S_{aa}^{(0)} \neq 0$, it is also convenient to express the reflection and transmission coefficients (5) in terms of r and t studied above. One finds

$$T = T_0 t, \quad R = S_{aa}^{(0)} + (1 - S_{aa}^{(0)})(r - S_{aa}^{(0)}t). \quad (15)$$

Now only a part (given by T_0) of the incoming flux contributes to the transmission. Thus the distribution of T is obtained by a simple rescaling of expression (10). The reflection coefficient takes a more elaborate form because of the interference between the two reflected waves, the one backscattered directly at the channel interface and the one originating from the background. The corresponding distribution can be found in a closed form using Eqs. (7) and (15) and reads

$$\mathcal{P}_\eta(R) = \int_{T_-}^{T_*} \frac{dT}{\pi(1-R-T)^2} \frac{2P_0(x)}{\sqrt{(T_+ - T)(T - T_-)}}, \quad (16)$$

where $T_* = \min(1 - R, T_+)$, $T_\pm = \frac{1+S_{aa}^{(0)}}{1-S_{aa}^{(0)}}(1 \pm \sqrt{R})^2$, and

$$x = \frac{(1+S_{aa}^{(0)})(R - S_{aa}^{(0)}) + T(\eta^2 + S_{aa}^{(0)})}{\eta(1 + S_{aa}^{(0)})(1 - R - T)}. \quad (17)$$

It reduces to Eq. (11) at perfect coupling, $S_{aa}^{(0)} = 0$.

A particular feature of the reflection distribution (16) is the dependence of its support on the sign of $S_{aa}^{(0)}$ (see Fig. 3 and
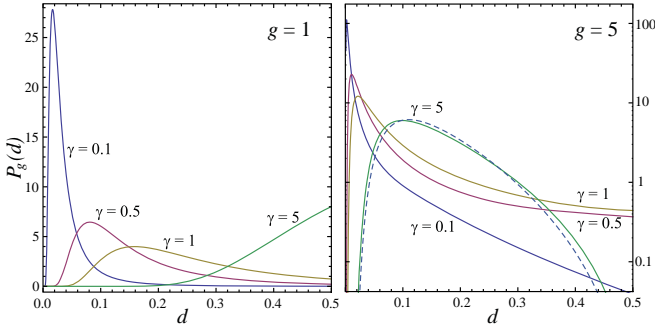
FIG. 4. Distribution (18) of the loss parameter $d = 1 - r - t$ for the GUE case at the increased absorption γ. The background coupling $g = 1$ (left) and $g = 5$ (right, note also the semilog scale in this case). The dashed line on the right panel corresponds to the asymptotic expression (19) at strong absorption.

[10]). The distribution vanishes identically for $R \leq 1 - T_0$, when $S_{aa}^{(0)} > 0$, and covers the whole range $0 \leq R \leq 1$, when $S_{aa}^{(0)} < 0$. This follows from the compatibility requirement $T_- < T_*$ and is, of course, in agreement with definition (5).

Loss statistics.— We now apply the obtained results to discuss the distribution of the unitary deficit (6), which is a useful measure of total losses in the system. By the construction $D = (1 - S_{aa}^{(0)})d$, where $d = D|_{T_0=1} = 1 - r - t$ is the deficit at perfect coupling. We note that property (9) enforces the deficit distribution to depend on η and η^{-1} in a symmetric way. It is therefore convenient to introduce $g \equiv \frac{1}{2}(\eta + \frac{1}{\eta}) \geq 1$ as the effective coupling constant to the background. After some algebra, we arrive at the following exact result for the distribution of the loss parameter d ($0 \leq 2d \leq 1$):

$$\mathcal{P}_g(d) = \int_0^\pi \frac{d\theta}{\pi d^2} P_0 \left[\frac{g - \sqrt{(g^2-1)(1-2d)}\cos\theta}{d} - g \right]. \tag{18}$$

At the special coupling $g = 1$ ($\eta = 1$), this expression simplifies further to $\mathcal{P}_{g=1}(d) = \frac{1}{d^2} P_0 \left(\frac{1-d}{d} \right)$. The asymptotic forms of $\mathcal{P}_g(d)$ can also be obtained in the limits of weak and strong absorption. In particular, in the latter case it reads

$$\mathcal{P}_{\gamma \gg 1}(d) \approx \frac{\beta\gamma e^{\frac{\beta\gamma}{4}(g-1-\frac{g}{d})}}{4d^2} I_0 \left[\frac{\beta\gamma\sqrt{(g^2-1)(1-2d)}}{4d} \right], \tag{19}$$

where $I_0(x)$ is a modified Bessel function. For arbitrary absorption, expression (18) can be evaluated further only in the GUE case, when P_0 takes the following simple form [32, 35]: $P_0(x) = \frac{1}{2}[\frac{\gamma}{2}(x+1)A + B]e^{-\gamma(x+1)/2}$ with the γ-dependent constants $A = e^\gamma - 1$ and $B = 1 + \gamma - e^\gamma$. Performing the subsequent integration results in

$$\mathcal{P}_\eta^{(\text{gue})}(d) = \frac{1}{2d^2} \left(B - A\gamma \frac{\partial}{\partial \gamma} \right) F, \tag{20}$$

where $F = e^{\frac{\gamma}{2}(g-1-\frac{g}{d})} I_0 \left(\frac{\gamma\sqrt{(g^2-1)(1-2d)}}{2d} \right)$. The distribution for various values of γ and g is shown on Fig. 4. The be-

haviour of $\mathcal{P}_g(d)$ in the GOE case is similar and can be qualitatively described by rescaling $\gamma \to \frac{\gamma}{2}$ in (20).

It is worth noting that the unitary deficit is closely related to the time-delay matrix Q at finite absorption [12] as well as to the so-called probability of no return $\tau \equiv 1 - R$ [13]. The former is defined by $Q = \Gamma_{\text{abs}}^{-1}(1 - S^\dagger S)$, yielding $D = \Gamma_{\text{abs}} Q_{aa}$, whereas the later is given by $\tau = D + T$. Refs. [12, 13] provide the exact multi-channel formulae for the mean eigenvalue density of Q and for the distribution of τ in a different setting of fully chaotic scattering without a direct mixing between the channels. The two distributions are distinct in general, but reduce to the same expression in the special case of the single channel, since $T = 0$ then identically. It turns out that the deficit distribution (18) is also given by the very same expression, provided that $d = \tau/2$ and η is identified with the degree of system openness. This can be substantiated by noting that zero transmission is realized in the present model at $S_{aa}^{(0)} = -1$, resulting in $D = 2d$ and $S_{aa} = \frac{1-i\eta K}{1+i\eta K}$. The latter is the usual form for the elastic (single-channel) scattering [25], with η now playing the role of the channel coupling. This proves the connection observed.

Discussion.— The approach developed shows that scattering on the simple mode coupled to the complex background serves as a sensitive probe of its internal structure. Fluctuations in scattering originate from those of the background states and are found to depend only on the interplay between the spreading width and the losses in the environment. In particular, the joint distribution of reflection and transmission reveals the remarkable symmetry (9). This can be traced to the symmetry properties of the local density of states, first found for ergodic states in [32, 34] and then generalised to multifractal spectra at Anderson transition [36] and at critical points of other disordered systems [37]. Studying the symmetry of relevant multifractal exponents has recently become accessible experimentally [38]. The formalism presented here offers the promising way to study manifestations of such symmetries at the level of scattering characteristics. We also note the flexibility of the approach in incorporating other real-world effects. In particular, inhomogeneous losses can be included following [39]. Therefore, we expect our results to find further exciting applications within a broader context of wave chaotic systems with complex environments.

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