A NEW STUDY ON THE STRONGLY LACUNARY QUASI CAUCHYNESS

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ABSTRACT. In this paper, the concept of an N_θ^2 quasi-Cauchy sequence is
introduced. We proved interesting theorems related to N_θ^2-quasi-Cauchy se-
quences. A real valued function f defined on a subset A of $\mathbb{R}$, the set of real
numbers, is N_θ^2 ward continuous on A if it preserves N_θ^2 quasi-Cauchy se-
quences of points in A, i.e. $(f(\alpha_k))$ is an N_θ^2 quasi-Cauchy sequence whenever
(α_k) is an N_θ^2 quasi-Cauchy sequences of points in A, where a sequence (α_k)
is called N_θ^2 quasi-Cauchy if $(\Delta^2 \alpha_k)$ is an N_θ quasi-Cauchy sequence where
$\Delta^2 \alpha_k = \alpha_{k+2} - 2\alpha_{k+1} + \alpha_k$ for each positive integer k.

1. INTRODUCTION

The concept of continuity and any concept involving continuity play a very im-
portant role not only in pure mathematics but also in other branches of sciences
involving mathematics especially in computer science, information theory, econom-
ics, and biological science.

Buck ([5]) introduced Cesaro continuity in 1946. Thereafter, a number of au-
thors Posner ([42]), Antoni and Salat ([1]), Spigel and Krupnik ([43]) have studied
A-continuity defined by a regular summability matrix A. Some authors, Das and
Savaş ([34]), Borsik and Salat ([3]) have studied A-continuity for methods of almost
convergence or for related methods. Connor and Grosse-Erdman ([12]) have given
sequential definitions of continuity for real functions calling G-continuity instead

of A-continuity by means of a sequential method, or a method of sequential convergence, and their results cover the earlier works related to A-continuity where a method of sequential convergence, or briefly a method, is a linear function G defined on a linear subspace of all sequences of points in $\mathbb{R}$ denoted by c_G, into $\mathbb{R}$. A sequence $\boldsymbol{\alpha} = (\alpha_k)$ is said to be G-convergent to ℓ if $\boldsymbol{\alpha} \in c_G$,then $G(\boldsymbol{\alpha}) = \ell$. In particular, lim denotes the limit function $\lim \boldsymbol{\alpha} = \lim_k \alpha_k$ on the linear space c, where c denotes the space of convergent sequences. On the other hand, Çakallı has introduced a generalization of compactness ([13]), a generalization of connectedness ([20]), via a method of sequential convergence (see also [26] and [39]).

In recent years, using the same idea, many kinds of continuities were introduced and investigated, not all but some of them we state in the following: slowly oscillating continuity ([14]), [45]), quasi-slowly oscillating continuity ([11]), Δ-quasi-slowly oscillating continuity ([21], and [19]), ward continuity ([16], [6]), δ-ward continuity ([15]), δ^2-ward continuity ([4]), statistical ward continuity ([17], [18]), lacunary statistical ward continuity ([9]), ρ-statistically ward continuity ([8]), λ-statistically ward continuity ([28]), and N_θ−ward continuity ([7], [24], [25] [35], [36] . Investigation of some of these kinds of continuities lead some authors to find conditions on the domain of a function for some theorems related to uniform continuity of a real function in terms of sequences in the above manner ([45, Theorem 8], [16, Theorem 7], and [6, Theorem 1]).

The notion of strongly lacunary or N_θ convergence was introduced, and studied by Freedman, Sember, and M. Raphael in [33] in the sense that a sequence (α_k) of points in $\mathbb{R}$ is N_θ convergent to an $L \in \mathbb{R}$, which is denoted by $N_\theta - \lim \alpha_k = L$, if $\lim_{r \to \infty} \frac{1}{h_r} \sum_{k \in I_r} |\alpha_k - L| = 0$, where $I_r = (k_{r-1}, k_r]$, and $k_0 \neq 0$, $h_r : k_r - k_{r-1} \to \infty$ as $r \to \infty$ and $\theta = (k_r)$ is an increasing sequence of positive integers. Throughout this paper, it is assumed that $\liminf_r \frac{k_r}{k_{r-1}} > 1$. The sums of the form $\sum_{k_{r-1}+1}^{k_r} |\alpha_k|$ frequently occur, and will often be written for convenience as $\sum_{k \in I_r} |\alpha_k|$.

The purpose of this paper is to investigate the notion of N_θ^2 ward continuity and prove interesting theorems.

2. N_θ^2-WARD CONTINUITY

A function defined on a subset A of $\mathbb{R}$ is called strongly lacunary continuous or N_θ continuous if it preserves N_θ convergent sequences of points in A, i.e. $(f(\alpha_k))$

is N_θ convergent whenever (α_k) is an N_θ convergent sequence of points in A. A function defined on a subset A of $\mathbb{R}$ is strongly lacunary continuous if and only if it is ordinary continuous.

A function defined on a subset A of $\mathbb{R}$ is called strongly lacunary ward continuous or N_θ-ward continuous if it preserves N_θ-quasi-Cauchy sequences of points in A, i.e. $(f(\alpha_k))$ is N_θ-quasi-Cauchy whenever (α_k) is an N_θ-quasi-Cauchy sequence of points in A (see [24]), where a sequence (α_k) of points in $\mathbb{R}$ is called strongly lacunary quasi-Cauchy, or N_θ-quasi-Cauchy if $(\Delta \alpha_k)$ is N_θ-convergent to 0 ([7, 24]). A sequence (α_k) of points in $\mathbb{R}$ is called strongly lacunary δ-quasi-Cauchy, or N_θ-δ-quasi-Cauchy if $(\Delta \alpha_k)$ is an N_θ quasi-Cauchy sequence, i.e.

$$\lim_{r \to \infty} \frac{1}{h_r} \sum_{k \in I_r} |\Delta^2 \alpha_k| = 0$$

where $\Delta^2 \alpha_k = \alpha_{k+2} - 2\alpha_{k+1} + \alpha_k$ for each positive integer k. ΔN_θ and $\Delta^2 N_\theta$ will denote the set of N_θ-quasi-Cauchy sequences of points in $\mathbb{R}$, and the set of N_θ-δ-quasi-Cauchy sequences of points in $\mathbb{R}$, respectively.

Definition 2.1. *A sequence (α_k) of points in $\mathbb{R}$ is called N_θ^2-quasi-Cauchy, or strongly lacunary δ^2-quasi-Cauchy if $(\Delta^2(\alpha_k))$ is an N_θ quasi-Cauchy sequence, i.e.*

$$\lim_{r \to \infty} \frac{1}{h_r} \sum_{k \in I_r} |\Delta^3 \alpha_k| = 0$$

where $\Delta^3 \alpha_k = \alpha_{k+3} - 3\alpha_{k+2} + 3\alpha_{k+1} - \alpha_k$ for each positive integer k.

The sum of two N_θ^2-quasi-Cauchy sequences is N_θ^2-quasi-Cauchy, $(\lambda \alpha_k)$ is N_θ^2-quasi-Cauchy for every constant $\lambda \in \mathbb{R}$. Therefore the set of N_θ^2-quasi-Cauchy sequences becomes a vector space that includes the vector space of convergent sequences, i.e. the vector space of all convergent sequences is a vector subspace of the vector space of all N_θ^2-quasi-Cauchy sequences. On the other hand, the vector space of all convergent sequences is also a vector subspace of the vector space of all N_θ-quasi-Cauchy sequences, and the vector space of all N_θ-quasi-Cauchy sequences is a vector subspace of the vector space of all N_θ^2-quasi-Cauchy sequences.

Now we give some interesting examples that show importance of the interest.

Example 2.2. *Let n be a positive integer. In a group of n people, each person selects at random and simultaneously another person of the group. All of the selected*

persons are then removed from the group, leaving a random number $n_1 < n$ of people which form a new group. The new group then repeats independently the selection and removal thus described, leaving $n_2 < n_1$ persons, and so forth until either one person remains, or no persons remain. Denote by α_n the probability that, at the end of this iteration initiated with a group of n persons, one person remains. Then the sequence $(\alpha_1, \alpha_2, , \alpha_n, ...)$ is an N_θ^2 quasi-Cauchy sequence (see also [46]).

Example 2.3. *In a group of k people, k is a positive integer, each person selects independently and at random one of three subgroups to which to belong, resulting in three groups with random numbers k_1, k_2, k_3 of members; $k_1 + k_2 + k_3 = k$. Each of the subgroups is then partitioned independently in the same manner to form three sub subgroups, and so forth. Subgroups having no members or having only one member are removed from the process. Denote by α_k the expected value of the number of iterations up to complete removal, starting initially with a group of k people. Then the sequence $(\alpha_1, \frac{\alpha_2}{2}, \frac{\alpha_3}{3}, ..., \frac{\alpha_n}{n}, ...)$ is a bounded non-convergent N_θ^2 quasi-Cauchy sequence ([37]).*

We note that any N_θ-quasi Cauchy sequence is also $N_\theta - \delta^2$-quasi Cauchy, but the converse is not always true. $\Delta^3 N_\theta$ will denote the set of $N_\theta - \delta^2$-quasi-Cauchy sequences of points in $\mathbb{R}$.

Now we give the definition of $N_\theta - \delta^2$-ward compactness of a subset of $\mathbb{R}$.

Definition 2.4. *A subset A of $\mathbb{R}$ is called N_θ^2 ward (or $N_\theta - \delta^2$-ward) compact if any sequence of points in A has an $N_\theta - \delta^2$ quasi-Cauchy subsequence, i.e. if whenever $\boldsymbol{\alpha} = (\alpha_k)$ is a sequence of points in A there is a subsequence $\boldsymbol{\beta} = (\beta_k) = (\alpha_{k_k})$ of $\boldsymbol{\alpha}$ with $N_\theta - \lim_{k \to \infty} \Delta^3 \boldsymbol{\beta}_k = 0$.*

Firstly, we note that any finite subset of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact, the union of finite number of $N_\theta - \delta^2$-ward compact subsets of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact, and the intersection of any family of $N_\theta - \delta^2$-ward compact subsets of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact. The sum of finite number of $N_\theta - \delta^2$-ward compact subsets of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact, the product λA of an $N_\theta - \delta^2$-ward compact subset of $\mathbb{R}$ and a constant real number λ is $N_\theta - \delta^2$-ward compact. Furthermore any subset of an $N_\theta - \delta^2$-ward compact set is $N_\theta - \delta^2$-ward compact and any bounded subset of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact. Any compact subset of $\mathbb{R}$ is also $N_\theta - \delta^2$-ward compact,

and the converse is not always true. For example, any open bounded interval is $N_\theta - \delta^2$-ward compact, which is not compact. On the other hand, the set $\mathbf{N}$ is not $N_\theta - \delta^2$-ward compact. We note that any slowly oscillating compact subset of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact (see also [14], and [21] for the results on slowly oscillating compactness), and any quasi-slowly oscillating compact subset of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact where a subset A of $\mathbb{R}$ is called quasi-slowly oscillating compact (see [11]) if whenever $\boldsymbol{\alpha} = (\alpha_n)$ is a sequence of points in A, there is a quasi-slowly oscillating subsequence $\boldsymbol{\beta} = (\beta_{n_k})$ of $\boldsymbol{\alpha}$.

We note that if a closed subset E of $\mathbb{R}$ is N_θ-ward compact, then it is also $N_\theta - \delta^2$-ward compact, and any sequence of points in E has a (P_n, s)-absolutely almost convergent subsequence (see [30], [40], [47], and [48]).

Now we give the definition of N_θ^2-ward continuity in the following.

Definition 2.5. *A function defined on a subset A of $\mathbb{R}$ is called N_θ^2-ward continuous or $N_\theta - \delta^2$-ward continuous if it preserves $N_\theta - \delta^2$-quasi-Cauchy sequences of points in A, i.e. $(f(\alpha_k))$ is $N_\theta - \delta^2$-quasi-Cauchy whenever (α_k) is an $N_\theta - \delta^2$-quasi-Cauchy sequence of points in A.*

We note that if f and g are $N_\theta - \delta^2$-ward continuous functions on a subset A of $\mathbb{R}$, then $f + g$ is $N_\theta - \delta^2$-ward continuous on A. On the other hand, the product λf of an $N_\theta - \delta^2$-ward continuous function and a constant real number λ is $N_\theta - \delta^2$-ward compact, i.e. λf is $N_\theta - \delta^2$-ward continuous whenever f is an $N_\theta - \delta^2$-ward continuous function, and λ is a constant real number, therefore the set of $N_\theta - \delta^2$-ward continuous functions becomes a vector space. We note that the product of $N_\theta - \delta^2$-ward continuous functions need not be $N_\theta - \delta^2$-ward continuous. The function defined by $f(x) = x$ is clearly $N_\theta - \delta^2$-ward continuous whereas the product $f(x).f(x) = x^2$ is not $N_\theta - \delta^2$-ward continuous.

Theorem 2.6. *If f is $N_\theta - \delta^2$-ward continuous on a subset A of $\mathbb{R}$, then it is N_θ-ward continuous on A.*

Proof 2.7. *Assume that f is an $N_\theta - \delta^2$-ward continuous function on A. Let (α_n) be any N_θ-quasi-Cauchy sequence of points in A. Then the sequence*

$$(\alpha_1, \alpha_1, \alpha_1, \alpha_2, \alpha_2, \alpha_2, ..., \alpha_{n-1}, \alpha_{n-1}, \alpha_{n-1}, \alpha_n, \alpha_n, \alpha_n, ...)$$

is also N_θ-quasi-Cauchy. Hence it is $N_\theta - \delta^2$-quasi-Cauchy. As f is $N_\theta - \delta^2$-ward continuous, the sequence

$$(f(\alpha_1), (f(\alpha_1), f(\alpha_2), f(\alpha_2), ..., f(\alpha_{n-1}), f(\alpha_{n-1}), f(\alpha_n), f(\alpha_n), ...)$$

is $N_\theta - \delta^2$-quasi-Cauchy. It follows from this that the sequence $(f(\alpha_n))$ is N_θ-quasi-Cauchy. This completes the proof of the theorem.

Corollary 2.8. *If f is $N_\theta - \delta^2$-ward continuous on a subset A of $\mathbb{R}$, then it is continuous on A.*

Proof 2.9. *The proof easily follows from Theorem 2.6 and* [25, Corollary 3], *so is omitted.*

Theorem 2.10. *The composite of two $N_\theta - \delta^2$-ward continuous functions is $N_\theta - \delta^2$-ward continuous, i.e. if f and g are $N_\theta - \delta^2$-ward continuous functions on $\mathbb{R}$, then the composite gof of f and g is $N_\theta - \delta^2$-ward continuous.*

Proof 2.11. *Let f and g be $N_\theta - \delta^2$-ward continuous functions on $\mathbb{R}$, and (α_n) be an $N_\theta - \delta^2$-quasi Cauchy sequence of points in $\mathbb{R}$. As f is $N_\theta - \delta^2$-ward continuous, the transformed sequence $(f(\alpha_n))$ is an $N_\theta - \delta^2$-quasi Cauchy sequence. Since g is $N_\theta - \delta^2$-ward continuous, the transformed sequence $g(f(\alpha_n))$ of the sequence $(f(\alpha_n))$ is an $N_\theta - \delta^2$-quasi Cauchy sequence. This completes the proof of the theorem.*

It should be noted that any $N_\theta - \delta^2$-ward continuous function is statistically continuous, N_θ-continuous, I-sequentially continuous for any non-trivial admissible ideal I, and G-sequentially continuous for any regular subsequential sequential method G.

Theorem 2.12. *$N_\theta - \delta^2$-ward continuous image of any $N_\theta - \delta^2$-ward compact subset of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact.*

Proof 2.13. *Assume that f is an $N_\theta - \delta^2$-ward continuous function on a subset A of $\mathbb{R}$, and A is an $N_\theta - \delta^2$-ward compact subset of $\mathbb{R}$. Let (β_n) be any sequence of points in $f(A)$. Write $\beta_n = f(\alpha_n)$ where $\alpha_n \in A$ for each positive integer n. $N_\theta - \delta^2$-ward compactness of A implies that there is a subsequence $(\gamma_k) = (\alpha_{n_k})$ of (α_n) with $N_\theta - \lim_{k\to\infty} \Delta^3 \gamma_k = 0$. Write $(t_k) = (f(\gamma_k))$. As f is $N_\theta - \delta^2$-ward continuous, $(f(\gamma_k))$ is $N_\theta - \delta^2$-quasi-Cauchy. Thus we have obtained a subsequence*

(t_k) of the sequence $(f(\alpha_n))$ with $N_\theta - \lim_{k \to \infty} \Delta^3 t_k = 0$. Thus $f(A)$ is $N_\theta - \delta^2$-ward compact. This completes the proof of the theorem.

Corollary 2.14. $N_\theta - \delta^2$-ward continuous image of any G-sequentially connected subset of $\mathbb{R}$ is G-sequentially connected.

The proof follows from the preceding theorem, and [20, Theorem 1], so is omitted.

Corollary 2.15. $N_\theta - \delta^2$-ward continuous image of any bounded subset of $\mathbb{R}$ is bounded.

The proof follows from [7, Theorem 3.3].

Corollary 2.16. $N_\theta - \delta^2$-ward continuous image of a G-sequentially compact subset of $\mathbb{R}$ is $N_\theta - \delta^2$-ward compact for any regular subsequential method G.

As far as ideal continuity is considered, any $N_\theta - \delta^2$-ward continuous function is I-sequentially continuous for any admissible ideal I ([23]). It follows from [24, Theorem 1] that $(f(\alpha_k))$ is $N_\theta - \delta^2$-quasi Cauchy whenever (α_k) is a quasi-Cauchy sequence of points in a subset A of $\mathbb{R}$ if f is uniformly continuous on A.

It is a well known result that the uniform limit of a sequence of continuous functions is continuous. This is also true in case of $N_\theta - \delta^2$-ward continuity, i.e. uniform limit of a sequence of $N_\theta - \delta^2$-ward continuous functions is $N_\theta - \delta^2$-ward continuous.

Theorem 2.17. If (f_n) is a sequence of $N_\theta - \delta^2$-ward continuous functions on a subset A of $\mathbb{R}$, and (f_n) is uniformly convergent to a function f, then f is $N_\theta - \delta^2$-ward continuous on A.

Proof 2.18. Let (α_k) be any $N_\theta - \delta^2$-quasi-Cauchy sequence of points in A, and let ε be any positive real number. By the uniform convergence of (f_n), there exists an $n_1 \in \mathbf{N}$ such that $|f(x) - f_n(x)| < \frac{\varepsilon}{9}$ for $n \geq n_1$ and for every $x \in E$. As f_{n_1} is $N_\theta - \delta^2$-ward continuous on A, there exists an $n_2 \in \mathbf{N}$ such that for $r \geq n_2$

$$\frac{1}{h_r} \sum_{k \in I_r} |f_{n_1}(\alpha_{k+3}) - 3f_{n_1}(\alpha_{k+2}) + 3f_{n_1}(\alpha_{k+1}) - f_{n_1}(\alpha_k)| < \frac{\varepsilon}{9}.$$

Now write $n_0 = max\{n_1, n_2\}$. Thus for $r \geq n_0$ we have

$\frac{1}{h_r} \sum_{k \in I_r} |f(\alpha_{k+3}) - 3f(\alpha_{k+2}) + 3f(\alpha_{k+1}) - f(\alpha_k)| = \frac{1}{h_r} \sum_{k \in I_r} |f(\alpha_{k+3}) - 3f(\alpha_{k+2}) +$

$3f(\alpha_{k+1}) - f(\alpha_k) - [f_{n_1}(\alpha_{k+3}) - 3f_{n_1}(\alpha_{k+2}) + 3f_{n_1}(\alpha_{k+1}) - f_{n_1}(\alpha_k)] + [f_{n_1}(\alpha_{k+3}) -$

$3f_{n_1}(\alpha_{k+2}) + 3f_{n_1}(\alpha_{k+1}) - f_{n_1}(\alpha_k)] \leq \frac{1}{h_r}\sum_{k\in I_r}|f(\alpha_{k+3}) - f_{n_1}(\alpha_{k+3})| + \frac{1}{h_r}\sum_{k\in I_r}3|f(\alpha_{k+2}) -$

$f_{n_1}(\alpha_{k+2})| + \frac{1}{h_r}\sum_{k\in I_r}3|f(\alpha_{k+1}) - f_{n_1}(\alpha_{k+1})| + \frac{1}{h_r}\sum_{k\in I_r}|f(\alpha_k) - f_{n_1}(\alpha_k)| + \frac{1}{h_r}\sum_{k\in I_r}|f_{n_1}(\alpha_{k+3}) -$

$3f_{n_1}(\alpha_{k+2}) + 3f_{n_1}(\alpha_{k+1}) - f_{n_1}(\alpha_k)|$

$< \frac{\varepsilon}{9} + \frac{\varepsilon}{9} + \frac{\varepsilon}{9} + \frac{\varepsilon}{9} < \varepsilon$. *Hence*

$$\lim_{r\to\infty}\frac{1}{h_r}\sum_{k\in I_r}|f(\alpha_{k+3}) - 3f(\alpha_{k+2}) + 3f_{n_1}(\alpha_{k+1}) - f(\alpha_k)| = 0.$$

This completes the proof of the theorem.

Theorem 2.19. *The set of N_θ^2-ward continuous functions on a subset A of $\mathbb{R}$ is a closed subset of the set of continuous functions on A. i.e. $\overline{\Delta^3 N_\theta(A)} = \Delta^3 N_\theta(A)$ where $\overline{\Delta^3 N_\theta(A)}$ denotes the set of all cluster points of $\Delta^3 N_\theta(A)$.*

Proof 2.20. *Let f be an element in $\overline{\Delta^3 N_\theta(A)}$. Then there exists a sequence (f_n) of points in $\Delta^3 N_\theta(A)$ such that $\lim_{n\to\infty} f_n = f$. It follows from Theorem 2.17 that $f \in \Delta^3 N_\theta(A)$, which completes the proof of the theorem.*

Corollary 2.21. *The set of N_θ^2-ward continuous functions on a subset A of $\mathbb{R}$ is a complete subset of the set of continuous functions on A.*

3. CONCLUSION

In this paper, the concept of an N_θ^2-quasi-Cauchy sequence is introduced, and investigated. In this investigation, we proved interesting theorems related to N_θ^2-ward continuity, and some other kinds of continuities. One may expect this investigation to be a useful tool in the field of analysis in modeling various problems occurring in many areas of science, dynamical systems, computer science, information theory, and biological science. For a further study, we suggest to investigate N_θ^2-quasi-Cauchy sequences of fuzzy point, or soft points (see [22], [2], and [32] for the definitions and related concepts), and we suggest to investigate N_θ^2-quasi-Cauchy double sequences (see [27], [31], [41], and [38] for the concepts in double case). Another suggestion for another further study is to introduce and give investigations of N_θ^2-quasi-Cauchy sequences of points in cone normed spaces (see [29], and [44], for basic concepts in topological vector space valued cone metric, and cone normed spaces). However due to the change in settings, the definitions and methods of proofs will not always be analogous to those of the present work.

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References

[1] J. Antoni, and T. Salat, On the *A*-continuity of real functions, Acta Math. Univ. Comenian, **39** (1980) 159-164.

[2] C.G. Aras, A. Sonmez, H. Çakallı, An approach to soft functions, J. Math. Anal., **8** 2 129-138.

[3] J. Borsik, and T. Salat, On *F*-continuity of real functions, Tatra Mt. Math. Publ. **2** (1993) 37-42.

[4] Naim L. Braha, H. Cakalli, A new type continuity for real functions, J. Math. Anal. **7** 6 (2016) 68-76.

[5] R.C. Buck, Solution of problem 4216, Amer. Math. Monthly, **55** (1948) 36.

[6] D. Burton, and J. Coleman, Quasi-Cauchy Sequences, Amer. Math. Monthly, **117** 4 (2010) 328-333.

[7] H. Cakalli, N_θ-ward continuity, Abstr. Appl. Anal. **2012** Article ID 680456, (2012) 8 pages. doi:10.1155/2012/680456.

[8] H. Cakalli, A Variation on Statistical Ward Continuity, Bull. Malays. Math. Sci. Soc. DOI 10.1007/s40840-015-0195-0

[9] H. Çakalli, C.G. Aras, and A. Sonmez, Lacunary statistical ward continuity , AIP Conf. Proc. **1676** (2015) Article Number: 020042 doi: 10.1063/1.4930468

[10] H. Cakalli, and H. Kaplan, Strongly lacunary delta ward continuity, AIP Conf. Proc. **1676**, (2015) Article Number: 020063 http://dx.doi.org/10.1063/1.4930489

[11] I. Canak, and M. Dik, New Types of Continuities, Abstr. Appl. Anal. (2010) Article ID 258980, 6 pages. doi:10.1155/2010/258980

[12] J. Connor, and K.-G. Grosse-Erdmann, Sequential definitions of continuity for real functions, Rocky Mountain J. Math. **33** 1 (2003) 93-121.

[13] H. Çakallı, Sequential definitions of compactness, Appl. Math. Lett. **21** 6 (2008) 594-598.

[14] H. Çakallı, Slowly oscillating continuity, Abstr. Appl. Anal. **2008** (2008) Article ID 485706, 5 pages. doi:10.1155/2008/485706 .

[15] H. Çakallı, δ-quasi-Cauchy sequences, Math. Comput. Modelling, **53** 1-2 (2011) 397-401.

[16] H. Çakallı, Forward continuity, J. Comput. Anal. Appl. **13** 2 (2011) 225-230.

[17] H. Çakallı, Statistical ward continuity, Appl. Math. Lett. **24** 10 (2011) 1724-1728.

[18] H. Çakallı, Statistical-quasi-Cauchy sequences, Math. Comput. Modelling, **54** 5-6 (2011) 1620-1624.

[19] H. Çakallı, On Δ-quasi-slowly oscillating sequences, Comput. Math. Appl. **62** 9 (2011) 3567-3574.

[20] H. Çakallı, Sequential definitions of connectedness, Appl. Math. Lett. **25** 3 (2012) 461-465.

[21] H. Çakallı, I. Canak, M. Dik, Δ-quasi-slowly oscillating continuity, Appl. Math. Comput. **216** 10 (2010) 2865-2868.

[22] H. Çakallı, and Pratulananda Das, Fuzzy compactness via summability, Appl. Math. Lett. **22** 11 (2009) 1665-1669.

[23] H. Çakallı, and B. Hazarika, Ideal Quasi-Cauchy sequences, J. Inequal. Appl. (2012) 2012: 234. doi:10.1186/1029-242X-2012-234

[24] H. Çakallı, and H. Kaplan, A study on N_θ-quasi-Cauchy sequences, Abstr. Appl. Anal. (2013) Article ID 836970, 4 pages http://dx.doi.org/10.1155/2013/836970

[25] H. Cakalli and H. Kaplan, A variation on strongly lacunary ward continuity, J. Math. Anal. **7** 3 (2016) 13-20.

[26] H. Çakallı, and O. Mucuk, On connectedness via a sequential method, Rev. Un. Mat. Argentina, **54** 2 (2013) 101-109.

[27] H. Çakallı, and E. Savas, Statistical convergence of double sequences in topological groups, J. Comput. Anal. Appl. **12** 2 (2010) 421-426.

[28] H. Çakallı, A. Sonmez, and C.G. Aras, λ-statistically ward continuity, An. Stiint. Univ. Al. I. Cuza Iasi. Mat. **63** 2 (2017) 313-322. (N.S.) DOI: 10.1515/aicu-2015-0016

[29] H. Çakallı, A. Sonmez, and Ç. Genç, On an equivalence of topological vector space valued cone metric spaces and metric spaces, Appl. Math. Lett. **25** 3 (2012) 429-433.

[30] H. Çakalli, and E.I. Taylan, On Absolutely Almost Convergence, An. Stiint. Univ. Al. I. Cuza Iasi. Mat. (N.S.), DOI: https://doi.org/10.2478/aicu-2014-0032 .

[31] A. Esi, Asymptotically double lacunary equivalent sequences defined by Orlicz functions, Acta Scientiarum-Technology, **36** 2 (2014) 323-329.

[32] A. Esi, M. Acikgoz, On almost lambda-statistical convergence of fuzzy numbers, Acta Scientiarum-Technology, **36** 1 (2014) 129-133.

[33] A.R. Freedman, J.J. Sember, and M. Raphael, Some Cesaro-type summability spaces, Proc. London Math. Soc. **3** 37 (1978) 508-520.

[34] G. Das, and E. Savaş, On the *A*-continuity of real functions, Istanbul Univ. Fen Fak. Mat Derg. **53** (1994) 61-66.

[35] H. Kaplan, H. Cakalli, Variations on strongly lacunary quasi Cauchy sequences, AIP Conf. Proc. **1759** (2016) Article Number: 020051 doi: 10.1063/1.4959665

[36] H. Kaplan, H. Cakalli, Variations on strong lacunary quasi-Cauchy sequences, J. Nonlinear Sci. Appl. **9** (2016) 4371-4380.

[37] M. Keane, Understanding Ergodicity, Integers 11B (2011) 1-11.

[38] D. Djurcic, Ljubia D.R. Kocinac, M.R. Zizovic, Double Sequences and Selections, Abstr. Appl. Anal. (2012) Article ID:497594, 6 pp. DOI: 10.1155/2012/497594, 2012.

[39] O. Mucuk, T. Şahan, On *G*-Sequential Continuity, Filomat, **28** 6 (2014) 1181-1189.

[40] H. Seyhan Ozarslan, and Ş. Yıldız, A new study on the absolute summability factors of Fourier series, J. Math. Anal, 7, 1 (2016) 31-36.

[41] R.F. Patterson and H. Cakalli, Quasi Cauchy double sequences, Tbilisi Mathematical Journal **8** (2) (2015), 211-219.

[42] E.C. Posner, Summability preserving functions, Proc.Amer.Math.Soc. **12** (1961) 73-76.

[43] E. Spigel, and N. Krupnik, On the *A*-continuity of real functions, J. Anal. **2** (1994) 145-155.

[44] A. Sonmez, and H. Çakalli, Cone normed spaces and weighted means, Math. Comput. Modelling, **52** 9-10 (2010) 1660-1666.

[45] R.W. Vallin, Creating slowly oscillating sequences and slowly oscillating continuous functions, With an appendix by Vallin and H. Cakalli, Acta Math. Univ. Comenianae, **25** 1 (2011) 71-78.

[46] P. Winkler, Mathematical Puzzles: A Connoisseurs Collection, A.K.Peters LTD, (2004) ISBN 1-56881-201-9.

[47] Ş. Yıldız, A new theorem on local properties of factored Fourier series, Bull. Math. Anal. Appl. **8** 2 (2016) 1-8.

[48] Ş. Yıldız, On Absolute Matrix Summability Factors of Infinite Series and Fourier Series, Gazi University Journal of Science, **30** 1 (2017) 363-370.

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