Higher-derivative generalization of conformal mechanics

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Higher-derivative analogues of multidimensional conformal particle and many-body conformal mechanics are constructed. Their Newton-Hooke counterparts are derived by applying appropriate coordinate transformations.

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1. Introduction

Conformal algebra in one dimension $so(1,2)$ involves three generators: the generator of time translations H, the generator of dilatations D, and the generator of special conformal transformations K. There are many mechanical systems which exhibit such a symmetry. These include the model of free particle[1], conformal particle[2], harmonic oscillator[3], the system of identical particles which interact with each other by conformal potential (e.g. the Calogero model[4]) and others.

There are several reasons why conformal invariant mechanical systems have attracted interest over the last four decades. First, some of these models are (super)integrable. Second, there are conformal invariant systems which are of physical interest. In particular, some of these models appear in the context of condensed matter physics[5], in physics of black holes[6]. Third, recent proposals on the nonrelativistic version of the AdS/CFT-correspondence stimulate investigations of nonrelativistic conformal algebras and their dynamical realizations.

There are two families of nonrelativistic algebras which contain $so(1,2)$ as a subalgebra. The first family involves conformal extensions of the Galilei algebra which are parameterized by a positive integer or half-integer parameter l[7–11]. This is the reason why the members of this family are called l-conformal Galilei algebras.

The l-conformal extensions of the Newton-Hooke (NH) algebra form the second family of nonrelativistic conformal algebras[9,12]. The l-conformal NH algebra can be viewed as an analogue of the l-conformal Galilei algebra in the presence of universal cosmological repulsion or attraction.

As is well known, the l-conformal extensions of the Galilei algebra and the NH algebra are isomorphic. Indeed, the structure relations of the l-conformal NH algebra can be obtained by a change of the basis $H \to H \pm \frac{1}{R^2}K$ in the l-conformal Galilei algebra, where $\Lambda = \pm\frac{1}{R^2}$ is the nonrelativistic cosmological constant. But when dynamical realizations are considered, this change of the basis leads to a change of the Hamiltonian and consequently alters the dynamics. For example, the Schrödinger group ($l = 1/2$-conformal Galilei group) is the maximal kinematical group of the free particle[1]. At the same time, the NH counterpart of this model is the harmonic oscillator[3].

The free higher-derivative particle[13,14] and the Pais-Uhlenbeck (PU) oscillator[15] can be viewed as higher-derivative analogues of the free particle and the harmonic oscillator, respectively. Recently, symmetries of these higher-derivative models have been extensively studied[13,14,16–23]. In particular it has been shown that the l-conformal Galilei group is the maximal symmetry group of the free $(2l+1)$-order particle[14]. Similarly, the PU oscillator accommodates the l-conformal NH symmetry for a special choice of its oscillation frequencies[16,18,21].

At the same time, higher-derivative analogues of other nonrelativistic conformal invariant mechanical systems are unknown. The purpose of the present work is to construct higher-derivative analogues of the conformal particle and a system of identical particles which interact with each other via a conformal-invariant potential.

The paper is organized as follows. In Sect. 2, we review the free higher-derivative particle and the PU oscillator which exhibits the l-conformal NH symmetry. In Section 3 and Section 4, we construct higher-derivative analogues of the conformal particle and many-body conformal mechanics, respectively. We summarize our results and discuss further possible developments in the conclusion (Section 5). Throughout the work summation over repeated spatial indices is understood. A superscript in braces as well as the number of dots over spatial coordinates designate the number of derivatives with respect to time.

2. Free higher-derivative particle and the Pais-Uhlenbeck oscillator

Let us review certain useful facts about the free higher-derivative particle and the PU oscillator which enjoys the l-conformal NH symmetry. The action functional of the former model reads[14]

$$S = \frac{1}{2}\int dt\, \lambda_{ij} x_i x_j^{(2l+1)}, \tag{1}$$

where dimensionless parameter l can take positive integer or half-integer values and

$$\lambda_{ij} = \begin{cases} \delta_{ij}, & i,j = 1,2,..,d, \quad \text{for half-integer } l; \\ \epsilon_{ij} = -\epsilon_{ji}, & i,j = 1,2, \quad \text{for integer } l, \end{cases}$$

with $\epsilon_{12} = 1$. The action functional (1) is invariant under the transformations[14]

$$t' = t + a + bt^2 + ct, \quad x'_i(t') = x_i(t) + \sum_{k=0}^{2l} a_i^{(k)} t^k + lcx_i(t) + 2lbtx_i(t) - \omega_{ij} x_j(t),$$

where a, b, c, $a_i^{(k)}$, and ω_{ij} are infinitesimal parameters. Generators of these transformations form the l-conformal Galilei algebra[8–10].

The NH counterpart of the model (1) can be constructed by applying the Niederer transformation[3]

$$t = R \tan(\tau/R), \quad x_i(t) = \frac{\chi_i(\tau)}{\cos^{2l}(\tau/R)}, \tag{2}$$

where R is a constant which has the dimension of time. For half-integer l, the implementation of this transformation to (1) results in the action functional[16,18]

$$S = \frac{1}{2}\int d\tau\, L_{PU} = \frac{1}{2}\int d\tau\, \chi_i \prod_{k=0}^{l-1/2}\left(\frac{d^2}{d\tau^2} + \frac{(2k+1)^2}{R^2}\right)\chi_i, \tag{3}$$

while for integer l one has[21]

$$S = \frac{1}{2}\int d\tau\, L_{PU} = \frac{1}{2}\int d\tau\, \epsilon_{ij}\, \chi_i \prod_{k=1}^{l}\left(\frac{d^2}{d\tau^2} + \frac{(2k)^2}{R^2}\right)\dot{\chi}_j. \tag{4}$$

The actions functionals (3) and (4) describe the $(2l+1)$-order PU oscillator for a particular choice of its oscillation frequencies. This model is invariant under the transformations [16,18,21]

$$\begin{aligned}
&\tau' = \tau + a + \frac{cR}{2}\sin\frac{2\tau}{R} + bR^2\sin^2\frac{\tau}{R}, \\
&\chi'_i(\tau') = \chi_i(\tau) + lc\cos\frac{2\tau}{R}\chi_i(\tau) + lbR\sin\frac{2\tau}{R}\chi_i(\tau) + \sum_{k=0}^{2l} R^k \sin^k\frac{\tau}{R}\cos^{2l-k}\frac{\tau}{R}a_i^{(k)} - \omega_{ij}\chi_j(\tau),
\end{aligned} \tag{5}$$

whose generators form the l-conformal Newton-Hooke algebra[9,10,12].

The NH counterparts presented in this paper all correspond to the case of negative cosmological constant. The case of positive cosmological constant can be straightforwardly reproduced by a formal change $R \to iR$.

3. Higher-derivative analogue of conformal particle

Let us consider a multidimensional conformal particle whose action functional has the form[2]

$$S = \frac{1}{2}\int dt \left(\dot{x}_i\dot{x}_i - \frac{g}{x_i x_i}\right). \tag{6}$$

This action is invariant under transformations

$$t' = t + a + ct + bt^2, \quad x'_i(t') = x_i(t) + \frac{c}{2}x_i(t) + btx_i(t) - \omega_{ij}x_j(t), \quad \omega_{ij} = -\omega_{ji},$$

whose generators form $so(1,2) \oplus so(d)$ subalgebra of the $l = 1/2$-conformal Galilei algebra. For the model of a free higher-derivative particle (1), the same subalgebra is realized by the generators which produce the following transformations

$$t' = t + a + ct + bt^2, \quad {x'}_i(t') = x_i(t) + lcx_i(t) + 2lbtx_i(t) - \omega_{ij}x_j(t), \quad \omega_{ij} = -\omega_{ji}. \quad (7)$$

Taking into account (1), let us consider the action functional of the form

$$S = \frac{1}{2}\int dt \left(\lambda_{ij}x_i x_j^{(2l+1)} - V(x)\right)$$

with an arbitrary function $V = V(x)$. To obtain higher-derivative generalization of the model (6), let us require this action functional to be invariant under the transformations (7). This restriction is satisfied when the function $V = V(x)$ has the form

$$V(x) = \frac{g}{(x_i x_i)^{1/2l}}.$$

So, the model

$$S = \frac{1}{2}\int dt \left(\lambda_{ij}x_i x_j^{(2l+1)} - \frac{g}{(x_i x_i)^{1/2l}}\right) \quad (8)$$

can be viewed as a higher-derivative generalization of conformal particle (6). The dynamics of this system is governed by the following equations of motion

$$\lambda_{ij}x_j^{(2l+1)} = -\frac{g}{2l}\frac{x_i}{(x_j x_j)^{(2l+1)/2l}}. \quad (9)$$

It is interesting to note that a one-dimensional analogue of this dynamical equation can be obtained via the method of nonlinear realization[24–26]. Indeed, let us consider the exponential parametrization of the group $SO(1,2)$[27]

$$G = G(t,z,u) = e^{itH}e^{izK}e^{iuD}, \quad [H,D] = iH,\ [H,K] = 2iD,\ [D,K] = iK.$$

Left multiplication by a group element $G(a,b,c)$ produces the following infinitesimal coordinate transformations

$$\delta t = a + bt^2 + ct, \quad \delta z = b(1 - 2tz) - cz, \quad \delta u = c + 2bt, \quad (10)$$

where a, b, and c are infinitesimal parameters. Then one constructs the left-invariant Maurer-Cartan one-forms[27]

$$G^{-1}dG = i(\omega_H H + \omega_D D + \omega_K K),$$

where we denoted

$$\omega_H = e^{-u}dt, \quad \omega_D = du - 2zdt, \quad \omega_K = e^{u}(dz + z^2 dt).$$

To obtain higher-derivative dynamical realization of $SO(1,2)$ group, firstly let us introduce the new variable

$$\rho = e^{lu}.$$

During next step, we discard the variable z from our consideration with the aid of the constraint

$$\omega_D = 0 \ \Rightarrow \ z = \frac{1}{2l}\frac{\dot\rho}{\rho}.$$

Considering this relation, the transformations (10) may be rewritten as

$$\delta t = a + bt^2 + ct, \quad \delta\rho = lc\rho + 2lbt\rho. \tag{11}$$

One may obtain $SO(1,2)$-invariant higher-derivative equations by using both the function

$$\Omega = \frac{\omega_K}{\omega_H} = \frac{1}{2l}\rho^{\frac{2}{l}}\left(\frac{\ddot\rho}{\rho} - \frac{2l-1}{2l}\frac{\dot\rho^2}{\rho^2}\right)$$

and the differential operator

$$D = \rho^{\frac{1}{l}}\frac{d}{dt}, \tag{12}$$

which are invariant under the transformations (11). In particular, one-dimensional analogue of (9) for $l = 1, \frac{3}{2}, 2, \frac{5}{2}$ can be reproduced as follows

$$\begin{aligned}
l = 1: &\quad D\Omega = -\frac{g}{4} \ \Rightarrow \ \dddot\rho = -\frac{g}{2\rho^2};\\
l = \frac{3}{2}: &\quad D^2\Omega + 3\Omega^2 = -\frac{g}{9} \ \Rightarrow \ \rho^{(4)} = -\frac{g}{3\rho^{5/3}};\\
l = 2: &\quad D^3\Omega + 16\Omega D\Omega = -\frac{g}{16} \ \Rightarrow \ \rho^{(5)} = -\frac{g}{4\rho^{3/2}};\\
l = \frac{5}{2}: &\quad D^4\Omega + 31\Omega D^2\Omega + 26(D\Omega)^2 + 45\Omega^3 = -\frac{g}{25} \ \Rightarrow \ \rho^{(6)} = -\frac{g}{5\rho^{7/5}}.
\end{aligned}$$

It should be noted that the NH counterpart of the model (8) can be obtained via a Niederer transformation (2). The action functional of this counterpart has the form

$$S = \frac{1}{2}\int d\tau\left(L_{PU} - \frac{g}{(\chi_i\chi_i)^{1/2l}}\right), \tag{13}$$

where L_{PU} is defined in (3) and (4). This model holds invariant under $SO(1,2)\oplus SO(d)$-subgroup of transformations (5).

The system (13) can be also viewed as a higher-derivative analogue of the conformal particle in a harmonic trap

$$S=\frac{1}{2}\int d\tau\left(\dot{\chi}_i\dot{\chi}_i-\frac{1}{R^2}\chi_i\chi_i-\frac{g}{\chi_i\chi_i}\right). \tag{14}$$

This action functional can be obtained by applying (2) to (6).

4. Higher-derivative analogue of many-body conformal mechanics

A system of N identical particle whose dynamics is governed by the action functional

$$S=\frac{1}{2}\int dt\left(\sum_{\alpha=1}^{N}\dot{x}_{\alpha,i}\dot{x}_{\alpha,i}-V_{\frac{1}{2}}(x_{1,i},x_{2,i},..,x_{N,i})\right)$$

may exhibit invariance under transformations

$$t'=t+a+bt^2+ct,\quad {x'}_{\alpha,i}(t')=x_{\alpha,i}(t)+a_i^{(0)}+a_i^{(1)}t+\frac{c}{2}x_{\alpha,i}(t)+btx_{\alpha,i}(t)-\omega_{ij}x_{\alpha,j}(t),$$

whose generators form the Schrödinger algebra when the function $V_{\frac{1}{2}}=V_{\frac{1}{2}}(x_{1,i},x_{2,i},..,x_{N,i})$ satisfies the following equations

$$\sum_{\alpha=1}^{N}\frac{\partial V_{\frac{1}{2}}}{\partial x_{\alpha,i}}=0,\quad \sum_{\alpha=1}^{N}x_{\alpha,i}\frac{\partial V_{\frac{1}{2}}}{\partial x_{\alpha,i}}+2V_{\frac{1}{2}}=0,\quad \sum_{\alpha=1}^{N}\left(x_{\alpha,i}\frac{\partial V_{\frac{1}{2}}}{\partial x_{\alpha,j}}-x_{\alpha,j}\frac{\partial V_{\frac{1}{2}}}{\partial x_{\alpha,i}}\right)=0. \tag{15}$$

To construct the higher-derivative analogue of this model, let us consider the following action functional:

$$S=\frac{1}{2}\int dt\left(\sum_{\alpha=1}^{N}\lambda_{ij}x_{\alpha,i}x_{\alpha,j}^{(2l+1)}-V_l(x_{1,i},x_{2,i},..,x_{N,i})\right). \tag{16}$$

This action is invariant under the transformations

$$t'=a+bt^2+ct,\quad {x'}_{\alpha,i}(t')=x_{\alpha,i}(t)+\sum_{k=0}^{2l}a_i^{(k)}t^k+lcx_{\alpha,i}(t)+2lbtx_{\alpha,i}(t)-\omega_{ij}x_{\alpha,j}(t).$$

when the function $V_l=V_l(x_{1,i},x_{2,i},..,x_{N,i})$ obeys the following equations

$$\sum_{\alpha=1}^{N}\frac{\partial V_l}{\partial x_{\alpha,i}}=0,\quad l\sum_{\alpha=1}^{N}x_{\alpha,i}\frac{\partial V_l}{\partial x_{\alpha,i}}+V_l=0,\quad \sum_{\alpha=1}^{N}\left(x_{\alpha,i}\frac{\partial V_l}{\partial x_{\alpha,j}}-x_{\alpha,j}\frac{\partial V_l}{\partial x_{\alpha,i}}\right)=0. \tag{17}$$

It is easy to see that there is a correspondence between potentials $V_{\frac{1}{2}}$ and solutions of the system (17). Indeed, let us suppose that we have a solution $V_{\frac{1}{2}}$ of the system (15). Then we can produce a solution

$$V_l = \sqrt[2l]{V_{\frac{1}{2}}}$$

of the system (17) for any possible value of l. It should be noted that the potential $V_{\frac{1}{2}}$ should be positive-definite in order to obtain a solution V_l for integer l. For example, the potential

$$V_l = g^2 \sqrt[2l]{\sum_{\alpha<\beta} \frac{1}{(x_{\alpha,i} - x_{\beta,i})^2}}$$

is related to the celebrated Calogero model[4].

To obtain NH counterpart of the model (16), it follows to apply Niederer's coordinate transformation of the form

$$t = R \tan{(\tau/R)}, \quad x_{\alpha,i}(t) = \frac{\chi_{\alpha,i}(\tau)}{\cos^{2l}{(\tau/R)}}, \tag{18}$$

to the action functional (16). For half-integer l one finds

$$S = \frac{1}{2}\int d\tau \left[\sum_{\alpha=1}^{N} \chi_{\alpha,i} \prod_{k=0}^{l-1/2} \left(\frac{d^2}{d\tau^2} + \frac{(2k+1)^2}{R^2} \right) \chi_{\alpha,i} - V_l(\chi_{1,i}, \chi_{2,i}, .., \chi_{N,i}) \right], \tag{19}$$

while for integer l one obtains

$$S = \frac{1}{2}\int d\tau \left[\sum_{\alpha=1}^{N} \epsilon_{ij}\, \chi_{\alpha,i} \prod_{k=1}^{l} \left(\frac{d^2}{d\tau^2} + \frac{(2k)^2}{R^2} \right) \dot{\chi}_{\alpha,j} - V_l(\chi_{1,i}, \chi_{2,i}, .., \chi_{N,i}) \right], \tag{20}$$

where the function $V_l = V_l(\chi_{1,i}, \chi_{2,i}, .., \chi_{N,i})$ obeys the same conditions as in (17)

$$\sum_{\alpha=1}^{N} \frac{\partial V_l}{\partial \chi_{\alpha,i}} = 0, \quad l \sum_{\alpha=1}^{N} \chi_{\alpha,i} \frac{\partial V_l}{\partial \chi_{\alpha,i}} + V_l = 0, \quad \sum_{\alpha=1}^{N} \left(\chi_{\alpha,i} \frac{\partial V_l}{\partial \chi_{\alpha,j}} - \chi_{\alpha,j} \frac{\partial V_l}{\partial \chi_{\alpha,i}} \right) = 0.$$

The actions (19) and (20) describe a set of identical Pais-Uhlenbeck oscillators which interact with each other via a conformal-invariant potential. This system can be viewed as a higher-derivative generalization of many-body conformal mechanics considered in[28,29].

5. Conclusion

To summarize, in this work higher-derivative analogues of the conformal particle and a system of identical particles which interact with each other via a conformal-invariant potential were constructed. An appropriate Niederer's coordinate transformation was applied to these higher-derivative systems so as to obtain their NH counterparts.

Turning to further possible developments, the most interesting questions are related to integrability and stability. A construction of supersymmetric extensions of higher-derivative models (8), (13), (16), (19), and (20) is worth studying as well. It would be also interesting to extend the analysis in Refs.[29–32], which is related to nonlocal conformal transformations, to the case of higher-derivative mechanical systems introduced in this paper.

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