SER for Optimal Combining in Rician Fading with Co-channel Interference

Muralikrishnan Srinivasan S. Kalyani
Department of Electrical Engineering,
Indian Institute of Technology, Madras,
Chennai, India 600036.
{ee14d206,skalyani}@ee.iitm.ac.in

Abstract

Approximate Symbol error rate (SER) expressions are derived for receive diversity system employing optimum combining, when both the desired and the interfering signals are subjected to Rician fading, for the case of a) equal power uncorrelated interferers b) unequal power interferers c) interferer correlation. The derived expressions are applicable for an arbitrary number of receive antennas and interferers and for any QAM constellation. Furthermore, we derive a simple closed form expression for SER in the interference-limited regime, for the special case of Rayleigh faded interferers. A close match is observed between the SER result obtained through the derived analytical expression and the one obtained from Monte-Carlo simulations, for a variety of QAM constellations.

Index Terms

Optimum combining, Rician fading, SER, QAM, Wishart matrices, Hypergeometric functions

I. Introduction

Various diversity combining schemes have been proposed to exploit space diversity offered by adaptive antenna arrays. Optimum combining (OC) proposed in [1] is a scheme that maximizes signal to interference plus noise ratio (SINR). Performance of OC receivers has been extensively studied for various cases, such as a) when both the desired and interfering signals are subjected to Rayleigh fading [2]–[5], b) when the number of interferers is more than the number of receiving antennas [6], c) when dual-antenna diversity reception is used with multiple interferers in M-PSK

[7], d) for arbitrary number of receiving antennas and equal power interferers [8], [9], e) when user signal is Rayleigh faded and interferers are correlated [10], [11]. Outage probability of OC, in the presence of Rayleigh faded interferers, has also been sufficiently studied [12]. Performance of OC in co-operative relay systems like amplify and forward (AF) and decode and forward (DF) relays systems, in the presence of co-channel interference, has also been recently well studied [13]–[16].

Many practical scenarios exist, where both the desired and interfering signals may have line-of-sight (LOS) paths. Such scenarios include indoor propagation, micro-cellular channels, satellite channels, inter-vehicular communications, etc. Symbol error probability expressions (SEP) for OC have been derived, when either the desired signals or the interfering signals undergo Rician fading, while the other undergoes Rayleigh fading for arbitrary number of antennas and interferers [2].

Existing 4G and emerging 5G systems are both interference-limited. Hence, receiver techniques like OC, which mitigate interference, will play a key role in the performance analysis of these systems. In this context, a characterization of OC receivers which takes into account practical scenarios such as unequal interference power and correlation among interferers, is important. Though OC has been well studied in literature, the focus has typically been on equal power uncorrelated interferers. We believe ours is the first work to derive SER expressions for OC considering a) mixture of Rayleigh and Rician faded interferers, b) unequal power interferers (which occurs typically in most wireless systems) and c) correlated interferers (which occurs due to correlated channel fading, shadowing and from spatial distribution of transmitters [17]–[19]).

The SER expressions are derived by determining expressions for the moment generating function (MGF) of the SINR η. The derived SER expressions are functions of a double infinite series. However, for evaluation, we truncate all infinite series to finite series with arbitrarily small truncation error. The series terms are functions of Tricomi hypergeometric functions, which has been used extensively in analyzing throughput and rate of MISO systems over various fading channels [20]–[22]. We also derive expression for the moments of the SINR η. Our results are compared with corresponding Monte-Carlo simulations and a close match is observed.

All our SER expressions and all other SER expressions in [2], [10] involve an explicit evaluation of determinant. Hence, a simple approximation, which avoids determinant evaluation, is also derived when the interferers are subjected to Rayleigh fading and this approximation holds when the noise power $\sigma^2 \approx 0$. We exploit the shifted factorial determinant and Vandermonde determinant properties in [23] to derive this approximation.

II. System Model

Let N_R denote the number of receive antennas, N_I denote the number of interferers, $\mathbf{c}$ denote the $N_R \times 1$ channel from the transmitter to the user, $\mathbf{c}_i$ denote the $N_R \times 1$ channel from the i^{th} interferer to the user, x denote the desired user symbol belonging to unit energy QAM constellation and x_i denote the i^{th} interferer symbol also belonging a unit energy QAM constellation. The received vector is given by

$$\mathbf{y} = \mathbf{c}x + \sum_{i=1}^{N_I} \mathbf{c}_i x_i + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is the $N_R \times 1$ additive white complex Gaussian noise vector, with power of σ^2 per dimension i.e., $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{N_R})$. The interferer channels are modeled as i.i.d Rician i.e., $\mathbf{c}_i \sim \mathcal{CN}(\sqrt{a'}\mathbf{m_i'}, b'\mathbf{I}_{N_R})$, where $a' = \frac{\kappa_i}{\kappa_i+1}$, $b' = \frac{1}{\kappa_i+1}$, κ_i the ratio of the power of line of sight component to the scattering component of the interferer signals and $\mathbf{m_i'}$ is an $N_R \times 1$ arbitrary vector with elements of unit magnitude. The user channel is also assumed to be i.i.d Rician i.e., $\mathbf{c} \sim \mathcal{CN}(\sqrt{a}\mathbf{m}, b\mathbf{I}_{N_R})$, where $a = \frac{\kappa_s}{\kappa_s+1}$, $b = \frac{1}{\kappa_s+1}$. Note that, the Rician parameter κ_s, is the ratio of the power of line of sight component to that of the scattering component and $\mathbf{m}$ is an $N_R \times 1$ mean vector with elements of unit magnitude and uniform phase. Let $E_I'' = N_I \times E_I'$ denote the total energy of the interfering signals, where E_I' is the mean energy of each of the interfering signals. The covariance matrix of the interference term plus the noise term is given by

$$\mathbf{R} = E_I'\mathbf{C}'\mathbf{C}'^H + \sigma^2\mathbf{I} = E_I\mathbf{C}\mathbf{C}^H + \sigma^2\mathbf{I}, \tag{2}$$

where $\mathbf{C}' = [\mathbf{c}_1, ..., \mathbf{c}_{N_I}]$, $\mathbf{C}' \sim \mathcal{CN}(\sqrt{a'}\mathbf{M}', b'\mathbf{I}_{N_R} \otimes \mathbf{I}_{N_I})$. $\mathbf{M}'$ is an arbitrary deterministic matrix obtained by stacking $\mathbf{m_i'}$s, such that, $\mathbf{M}' = [\mathbf{m_1'}, \mathbf{m_2'}, \mathbf{m_3'}, ..., \mathbf{m_{N_I}'}]$ and $tr(\mathbf{M}'\mathbf{M}'^H) = N_R N_I$. Here, $\mathbf{C} \sim \mathcal{CN}(\mathbf{M}, \mathbf{I}_{N_R} \otimes \mathbf{I}_{N_I})$, $E_I = E_I' \times b'$ and $\mathbf{M} = \sqrt{a'}/\sqrt{b'}\mathbf{M}' = \sqrt{\kappa_i}\mathbf{M}'$. The received SINR for the OC is given by [8]

$$\eta = E_D \mathbf{c}^H \mathbf{R}^{-1} \mathbf{c}, \tag{3}$$

where E_D is the mean energy of the user signal. Using the standard assumption that the contribution of the interference and the noise at the output of optimal combiner, for a fixed

η, can be well-approximated to be Gaussian as in [24] and [25] and references therein, the probability of symbol error for an M-ary square QAM constellation is given by [26],

$$\mathcal{P}_e \approx k_1 Q(\sqrt{k_2\eta}) - k_3 Q(\sqrt{k_2\eta})^2, \quad (4)$$

where $k_1 = 4\big(1 - \frac{1}{\sqrt{M}}\big)$, $k_2 = \frac{3}{M-1}$, $k_3 = \frac{k_1^2}{4}$ and the Q-function is given by $Q(x) = \frac{1}{2\pi}\int_x^\infty e^{-u^2/2}du$. The assumption that the contribution of the interference and the noise at the output of the optimum combiner, for a fixed η, is Gaussian, is valid even when the number of interferers N_I is small [25] and such a system model assumption is made in a number of papers [8], [27], [28] to derive the SER expression. Using the approximation $Q(x) \approx \frac{1}{12}e^{-\frac{1}{2}x^2} + \frac{1}{4}e^{-\frac{2}{3}x^2}$, one can write $\mathcal{P}_e$ as [29],

$$\mathcal{P}_e \approx \sum_{l=1}^{5} a_l e^{-b_l \eta}, \quad (5)$$

where $a_1 = \frac{k_1}{12}$, $a_2 = \frac{k_1}{4}$, $a_3 = \frac{-k_3}{144}$, $a_4 = \frac{-k_3}{16}$, $a_5 = \frac{-k_3}{24}$, $b_1 = \frac{k_2}{2}$, $b_2 = \frac{2k_2}{3}$, $b_3 = k_2$, $b_4 = \frac{4k_2}{3}$ and $b_5 = \frac{7k_2}{6}$. The average SER obtained by averaging $\mathcal{P}_e$ over all channel realizations is,

$$\begin{aligned} SER \approx \mathbb{E}_\eta[\mathcal{P}_e] = \mathbb{E}_\eta[\sum_{l=1}^{5} a_l e^{-b_l\eta}] &= \sum_{l=1}^{5} a_l \mathbb{E}_\eta[e^{-b_l\eta}] \\ &= \sum_{l=1}^{5} a_l M_\eta(s)|_{s=-b_l}, \end{aligned} \quad (6)$$

where $M_\eta(s)$ is the MGF of η and $\mathbb{E}_\eta$ denotes expectation over SINR η. Finding an expression for SER in (6) requires determining an expression for the MGF of SINR η. This is derived in the following section.

III. SER EXPRESSIONS FOR EQUAL POWER UNCORRELATED INTERFERERS

A general expression for the MGF $M_\eta(s)$ of SINR η, can now be obtained from Theorem 1 of [2]. We now further simplify for the specific case of Rician distribution. Let $n_2 = max(N_R, N_I)$ and $n_1 = min(N_R, N_I)$.

$$M_\eta(s) = (-1)^{N_R}(\sigma^2/E_I)^{(N_R-n_1)}\mathbb{E}_{\mathbf{\Lambda}_R}\left[\left(\prod_{i=1}^{n_1}\frac{\sigma^2/E_I + \lambda_i}{\lambda_i^{(N_R-n_1)}}\right)\frac{det(\mathbf{J})}{V_{n_1}(\mathbf{\Lambda}_R)}\right], \quad (7)$$

where $V_{n_1}(\mathbf{\Lambda}_R)$ is the determinant of the Vandermonde matrix formed by eigen values of non-central Wishart matrix $\mathbf{CC}^H$. $\mathbf{J}$ is an $n_1 \times n_1$ matrix with elements,

$$\mathbf{J}_{i,j} = \begin{cases} h_1(s,\lambda_i) - \sum_{t=1}^{N_R-n_1} h_t(s,0)\lambda_i^{t-1}, & j = 1, \\ \lambda_i^{N_R-j}, & j = 2, ..., n_1, \end{cases}$$

and

$$h_t(s,x) = \frac{{}_1F_1(t; N_R; \frac{aN_R s}{xE_I/E_D + \sigma^2/E_D - bs})}{(bsE_D/E_I - \sigma^2/E_I - x)^t}, \tag{8}$$

where ${}_1F_1(.)$ is the confluent hypergeometric function [30] with the series expansion of ${}_1F_1(.)$ given by ${}_1F_1(a;b;z) = \sum_{z=0}^{\infty} \frac{(a)_k z^k}{(b)_k k!}$.

SER is derived in [2] for two cases: a) Rician signal with Rayleigh interferers b) Rayleigh signal with Rician interferers. For the Rician-Rayleigh case, (7) is used along with the eigen value distribution of central Wishart matrix to arrive at a closed form expression for the MGF. In the later case, the fact that user signal $\mathbf{c}$ exhibits Rayleigh fading and hence invariant under unitary transformation is exploited to derive a closed form expression for the MGF.

To the best of our knowledge, there is no open literature that proves that Rician distribution is invariant under unitary transformation. Therefore, for the case of Rician signals with Rician interferers, we propose to evaluate the expectation in (7), by using the eigen value distribution of non-central Wishart matrix and subsequently simplify it by using properties of hyper-geometric functions.

The joint pdf of ordered eigen values $(\lambda_1 > \lambda_2 > ... > \lambda_{n_1})$ of non-central Wishart matrix is given by [31],

$$f(\lambda_1, ..., \lambda_s) = c_1 |\mathbf{\Upsilon}| \prod_{i<j}^{n_1} (\lambda_i - \lambda_j) \prod_{k=1}^{n_1} \lambda_k^{n_2 - n_1} e^{-\lambda_k}, \tag{9}$$

where $\mathbf{\Upsilon}$ is a $n_1 \times n_1$ matrix whose $(i,j)^{th}$ entry $\forall i = 1, ..., n_1$ is given by,

$$\mathbf{\Upsilon}_{i,j} = \begin{cases} {}_0F_1(n_2 - n_1 + 1; w_j \lambda_i), & j = 1, ..., L, \\ \lambda_i^{n_1 - j} \frac{(n_2 - n_1)!}{(n_2 - j)!}, & j = L+1, ..., n_1, \end{cases}$$

and

$$c_1 = \frac{e^{-tr(\mathbf{\Omega})}((n_2 - n_1)!)^{-n_1}}{\prod_{i=1}^{n_1 - L}(n_1 - L - i)! \prod_{i=1}^{L} w_i^{n_1 - L} \prod_{i<j}^{L}(w_i - w_j)}.$$

Note that w_is are the ordered L non-zero eigen-values of the non-centrality matrix, $\mathbf{\Omega} = \mathbf{M}^H \mathbf{M}$ and the series expansion of hypergeometric function ${}_0F_1(.)$ is given by ${}_0F_1(b;z) = \sum_{k=0}^{\infty} \frac{z^k}{(b)_k k!}$.

Substituting (9) in (7),

$$
\begin{aligned}
M_\eta(s) &= c_1(-1)^{N_R}(\sigma^2/E_I)^{(N_R-n_1)}\int_0^\infty \left[\left(\prod_{i=1}^{n_1}\frac{\sigma^2/E_I+\lambda_i}{\lambda_i^{(N_R-n_1)}}\right)\frac{det(\mathbf{J})}{V_{n_1}(\mathbf{\Lambda}_R)}\right]|\mathbf{\Upsilon}| \\
&\qquad\times\prod_{i<j}^{n_1}(\lambda_i-\lambda_j)\prod_{k=1}^{n_1}\lambda_k^{n_2-n_1}e^{-\lambda_k}d\lambda_1...d\lambda_{n_1} \\
&= (-1)^{N_R}(\sigma^2/E_I)^{(N_R-n_1)}\int_0^\infty c_1\left(\prod_{i=1}^{n_1}(\sigma^2/E_I+\lambda_i)\times e^{-\lambda_i}\right)|\mathbf{J}||\mathbf{\Upsilon}|d\lambda_1...d\lambda_{n_1}. \qquad (10)
\end{aligned}
$$

From Theorem 2 in Appendix of [32], it can be observed that, for two arbitrary $m\times m$ matrices $\boldsymbol{\phi}(\boldsymbol{y})$ and $\boldsymbol{\psi}(\boldsymbol{y})$ with ij^{th} element $\phi_i(y_j)$ and $\psi_i(y_j)$ and arbitrary function g(.), where $\mathbf{y}=[y_1,y_2,...y_m]^T$, the following identity holds:

$$
\int\cdots\int_{b\geq y_i\geq a}|\boldsymbol{\phi}(\boldsymbol{y})||\boldsymbol{\psi}(\boldsymbol{y})|\prod_{k=1}^{m}g(y_k)dy_1dy_2...dy_m = det\left(\int_a^b\{\phi_i(y)\psi_j(y)g(y)dy\}_{i,j=1,..,m}\right).
$$

Using the above property to simplify MGF in (10), we get,

$$
M_\eta(s)=c|\mathbf{N}|, \qquad (11)
$$

where

$$
c=\frac{e^{-tr(\mathbf{\Omega})}((n_2-n_1)!)^{-n_1}(-1)^{N_R}(\sigma^2/E_I)^{(N_R-n_1)}}{\prod_{i=1}^{n_1-L}(n_1-L-i)!\prod_{i=1}^{L}w_i^{n_1-L}\prod_{i<j}^{L}(w_i-w_j)},
$$

$$
\mathbf{N}_{i,j}=\begin{cases}
\int_0^\infty(\frac{\sigma^2}{E_I}+x)e^{-x}x^{n_2-N_R}\,{}_0F_1(n_2-n_1+1;w_ix)[h_1(s,x)-\sum_{t=1}^{N_R-n_1}h_t(s,0)x^{t-1}]dx, \\
\qquad\qquad j=1,\ i=1,...,L, \\
\int_0^\infty(\frac{\sigma^2}{E_I}+x)e^{-x}x^{n_2-N_R}x^{n_1-i}\frac{(n_2-n_1)!}{(n_2-i)!}[h_1(s,x)-\sum_{t=1}^{N_R-n_1}h_t(s,0)x^{t-1}]dx, \\
\qquad\qquad j=1,\ i=L+1,...,n_1, \\
\int_0^\infty(\frac{\sigma^2}{E_I}+x)e^{-x}x^{n_2-N_R}\,{}_0F_1(n_2-n_1+1;w_ix)x^{N_R-j}dx,\ j=2,...,n_1\, i=1,...,L, \\
\int_0^\infty(\frac{\sigma^2}{E_I}+x)e^{-x}x^{n_2-N_R}x^{n_1-i}\frac{(n_2-n_1)!}{(n_2-i)!}x^{N_R-j}dx, \quad j=2,...,n_1\, i=L+1,...,n_1.
\end{cases}
$$

Further simplification of $N_{i,j}$ is given in Appendix A. The final expression for the entries $N_{i,j}$ is given in (12) and this can be substituted in (11) to obtain the final MGF of SINR and the SER can then be obtained using (6). In (12) $p=\sigma^2/E_I$, $q=n_2-n_1+1$, $u=aN_RsE_D/E_I$, $v=bsE_D/E_I-\sigma^2/E_I$. Note that the expression for $N_{i,j}$, for $L=n_1$ or $L=0$, gets significantly simplified.

a) $L=n_1$: If the number of eigen values is $L=n_1$ corresponding to all the interferers being Rician faded, we get $M_\eta(s)=c|\mathbf{N}_{L=n_1}|$, where $c=\frac{e^{-tr(\mathbf{\Omega})}((n_2-n_1)!)^{-n_1}}{\prod_{i<j}^{n_1}(w_i-w_j)}(-1)^{N_R}(\sigma^2/E_I)^{(N_R-n_1)}$

and the value of $|\mathbf{N}_{L=n_1}|$ is evaluated using (12) at $L = n_1$.

b) $L = 0$: If the number of non-zero eigen values is zero i.e., $L = 0$ corresponding to all the interferers being Rayleigh faded, we get $M_\eta(s) = c|\mathbf{N}_{L=0}|$, where

$$c = \frac{1}{\prod_{i=1}^{n_1}(n_1-i)!\prod_{i=1}^{n_1}(n_2-i)!}(-1)^{N_R}(\sigma^2/E_I)^{(N_R-n_1)}$$

and the value of $|\mathbf{N}_{L=0}|$ is evaluated using (13). Also, when we do a Laplace expansion of the determinant $|\mathbf{N}_{L=0}|$ given in (13) along the first column and substitute $\zeta_t(k) = \frac{\sigma^2}{E_I}\Gamma(n_1+n_2-N_R+t-k+1)+\Gamma(n_1+n_2-N_R+t-k+2)$, we observe that the expression for $M_\eta(s)$ for the Rayleigh interferers case is the same as the one obtained in [2, Eq. 13].

A. *SER for interference limited scenario*

Recently, there has been a lot of interest in characterizing the performance of cellular networks/ wireless system in an interference limited scenario. Throughput and rate have been studied in [33]–[37] and references therein, assuming noise can be neglected (i.e., $\sigma^2 = 0$), in an interference limited scenario. Motivated by these works, we now derive SER for the interference limited scenario and show that the SER expression can be substantially simplified in the case of Rayleigh interferers. Note that an interference-limited scenario is possible only for $N_I > N_R$. For $N_R > N_I$ and $\sigma^2 = 0$, the receive antennas can cancel every interfering signal.

When σ^2 is neglected, N_{ij} can be approximated as in (14) and can be substituted in (11) to obtain the final MGF of SINR. The SER can then be obtained using (6). If the number of non-zero eigen values $L = 0$ as is the case for Rayleigh fading, then

$$M_\eta(s) = c|\mathbf{N}_{\sigma^2=0,L=0}|, \tag{15}$$

where

$$c = \frac{((n_2-n_1)!)^{-n_1}}{\prod_{i=1}^{n_1}(n_1-i)!}(-1)^{N_R}(\sigma^2/E_I)^{(N_R-n_1)}, \tag{16}$$

and the value of $|\mathbf{N}_{\sigma^2=0,L=0}|$ is evaluated using (14) at $L = 0$. Note that in c, we do not neglect σ^2. The expression can be further simplified as shown in Appendix C to obtain,

$$M_\eta(s) = \frac{(-1)^{N_R+1}(\sigma^2/E_I)^{(N_R-n_1)}n_2!}{(n_2-n_1)!\prod_{i=1}^{n_1}(n_1-i)!}\sum_{i=1}^{n_1}(-1)^{i+1}A(i)\frac{\prod_{j=1}^{n_1}(j-1)!}{(n_1-i)!(i-1)!},$$

where $A(i)$ is given in Appendix C. This can be further substituted in (6), to obtain the expression for SER as,

$$SER \approx \sum_{l=1}^{5} a_l \frac{(-1)^{N_R+1}(\sigma^2/E_I)^{(N_R-n_1)}n_2!}{(n_2-n_1)!\prod_{i=1}^{n_1}(n_1-i)!}\sum_{i=1}^{n_1}(-1)^{i+1}A(i)\frac{\prod_{j=1}^{n_1}(j-1)!}{(n_1-i)!(i-1)!}|_{s=-b_l}. \tag{17}$$

$$\mathbf{N}_{i,j}=\begin{cases}\sum_{k=0}^{T_2}\frac{w_i^k}{q_k k!}\Bigg[\sum_{l=0}^{T_1}\frac{u^l}{(N_R)_l}\Bigg[-\Gamma(k+n_2-N_R+2)U(l+1,l-k-n_2+N_R,-v)\\ -p\Gamma(k+n_2-N_R+1)U(l+1,-k-n_2+N_R+l+1,-v)\Bigg]\Bigg]\\ -\ \sum_{t=1}^{N_R-n_1}\frac{{}_1F_1(t;N_R;u/v)}{v^t}\ \times\Bigg[p\Gamma(t+n_2-N_R)\,{}_1F_1(t+n_2-N_R;q;w_i)\\ \quad+\Gamma(t+n_2-N_R+1){}_1F_1(t+n_2-N_R+1;q;w_i)\Bigg],\\ \qquad\qquad\qquad\qquad\qquad j=1,\ i=1,...,L,\\ \frac{(n_2-n_1)!}{(n_2-i)!}\Bigg[\sum_{l=0}^{T_1}\frac{u^l}{(N_R)_l}\Bigg[-\Gamma(n_2-N_R+n_1-i+2)\\ U(l+1,l-n_2+N_R-n_1+i,-v)\\ -p\Gamma(n_2-N_R+n_1-i+1)U(l+1,-n_2+N_R-n_1+i+l+1,-v)\Bigg]\\ -\ \sum_{t=1}^{N_R-n_1}\frac{{}_1F_1(t;N_R;u/v)}{v^t}\Bigg[p\Gamma(t+n_2+n_1-N_R-i)\\ \quad+\Gamma(t+n_2+n_1-N_R-i+1)\Bigg]\Bigg],\\ \qquad\qquad\qquad\qquad\qquad j=1,\ i=L+1,...,n_1,\\ p\,{}_1F_1(n_2-j+1;q;w_i)\Gamma(n_2-j+1)+\Gamma(n_2-j+2){}_1F_1(n_2-j+2;q;w_i),\\ \qquad\qquad\qquad\qquad\qquad j=2,...,n_1\,i=1,...,L.\\ \frac{(n_2-n_1)!}{(n_2-i)!}[p\Gamma(n_2+n_1-i-j+1)+\Gamma(n_2+n_1-i-j+2)],\\ \qquad\qquad\qquad\qquad\qquad j=2,...,n_1\,i=L+1,...,n_1.\end{cases}\tag{12}$$

$$\mathbf{N}_{i,j}|_{L=0}=\begin{cases}\Bigg[\sum_{l=0}^{T_1}\frac{u^l}{(N_R)_l}\Bigg[-\Gamma(n_2-N_R+n_1-i+2)U(l+1,l-n_2+N_R-n_1+i,-v)\\ -p\Gamma(n_2-N_R+n_1-i+1)U(l+1,-n_2+N_R-n_1+i+l+1,-v)\Bigg]\\ -\ \sum_{t=1}^{N_R-n_1}\frac{{}_1F_1(t;N_R;u/v)}{v^t}\Bigg[p\Gamma(t+n_2+n_1-N_R-i)\\ \qquad+\Gamma(t+n_2+n_1-N_R-i+1)\Bigg]\Bigg],\qquad j=1,\ i=1,...,n_1,\\ p\Gamma(n_2+n_1-i-j+1)+\Gamma(n_2+n_1-i-j+2),\ j=2,...,n_1,\ i=1,...,n_1.\end{cases}\tag{13}$$

$$
\mathbf{N}_{i,j}|_{\sigma^2=0} = \begin{cases}
\sum_{k=0}^{T_2} \frac{w_i^k}{(n_2-n_1+1)_k k!}\Bigg[\sum_{l=0}^{T_1} \frac{(aN_R sE_D/E_I)^l}{(N_R)_l}\Bigg[-\Gamma(k+n_2-N_R+2) \\
U(l+1, l-k-n_2+N_R, -bsE_D/E_I)\Bigg]\Bigg] \\
- \sum_{t=1}^{N_R-n_1} \frac{{}_1F_1(t;N_R;aN_R sE_D/(-bsE_D)}{(bsE_D/E_I)^t}\Gamma(t+n_2-N_R+1) \\
\times \quad {}_1F_1(t+n_2-N_R+1; n_2-n_1+1; w_i), \qquad j=1,\, i=1,...,L, \\
\frac{(n_2-n_1)!}{(n_2-i)!}\Bigg[\sum_{l=0}^{T_1} \frac{(aN_R sE_D/E_I)^l}{(N_R)_l}\Bigg[-\Gamma(n_2-N_R+n_1-i+2) \\
U(l+1, l-n_2+N_R-n_1+i, -bsE_D/E_I)\Bigg] \\
- \sum_{t=1}^{N_R-n_1} \frac{{}_1F_1(t;N_R;aN_R sE_D/(-bsE_D)}{(bsE_D/E_I)^t} \\
\quad \times\Bigg[\Gamma(t+n_2+n_1-N_R-i+1)\Bigg]\Bigg], \qquad j=1,\, i=L+1,...,n_1, \\
\Gamma(n_2-j+2)_1F_1(n_2-j+2; n_2-n_1+1; w_i), \qquad j=2,...,n_1\, i=1,...,L. \\
\frac{(n_2-n_1)!}{(n_2-i)!}[\Gamma(n_2+n_1-i-j+2)], \qquad j=2,...,n_1,\, i=L+1,...,n_1.
\end{cases} \tag{14}
$$

For an interference limited scenario ($N_I > N_R$ and $\sigma^2 = 0$), by substituting $n_1 = N_R$ and $n_2 = N_I$, the SER becomes,

$$
SER \approx \sum_{l=1}^{5} a_l \frac{(-1)^{N_R+1} N_I!}{(N_I-N_R)! \prod_{i=1}^{N_R}(N_R-i)!} \sum_{i=1}^{N_R} (-1)^{i+1} A(i) \frac{\prod_{j=1}^{N_R}(j-1)!}{(N_R-i)!(i-1)!}|_{s=-b_l}. \tag{18}
$$

Ours is the first work to obtain SER expression in an interference limited scenario, for Rayleigh faded interferers in a closed form. All existing work, so far, require an explicit evaluation of the determinant. Further, the expression derived also gives an approximation of SER, for $N_I > N_R$ for very low noise values $\sigma^2 \approx 0$. Note that the dependence of c term on σ^2 is not present for $N_I > N_R$. On the other hand, the σ^2 term exists in c term for $N_R > N_I$. We derive in the succeeding subsection, an SER approximation for $N_R > N_I$, by substituting $\sigma^2 = 0$ only in the determinant.

SER approximation for $N_R > N_I$: If we substitute $n_1 = N_I$ and $n_2 = N_R$ in (11) and ignore the σ^2 term inside the determinant, we also obtain an approximation for the SER as,

$$
SER \approx \sum_{l=1}^{5} a_l \frac{(-1)^{N_R+1} (\sigma^2/E_I)^{(N_R-N_I)} N_R!}{(N_R-N_I)! \prod_{i=1}^{N_I}(N_I-i)!} \sum_{i=1}^{N_I} (-1)^{i+1} A(i) \frac{\prod_{j=1}^{N_I}(j-1)!}{(N_I-i)!(i-1)!}|_{s=-b_l}. \tag{19}
$$

Note that SER expressions obtained for Rayleigh interferers in [2] involve not only an explicit evaluation of determinants but also numerical integration, while results here require neither. This approximation works very well when σ^2 is actually small or when interferer powers are large compared to σ^2.

B. Moments of Rician-Rician SINR

Similar to [2], we can obtain the moments of the SINR η. The l^{th} moment of SINR for Rician faded user and Rician faded interferers is given by,

$$\mu_l^{Ric-Ric} = \frac{d^l}{ds^l} M_\eta(s)|_{s=0} = \alpha_l^{Ric} c \sum_{k=1}^{n_1} (-1)^k |Y_k| d_l = \alpha_l^{Ric} \mu_l^{Ray-Ric}$$

where $c = \frac{e^{-tr(\mathbf{\Omega})}((n_2-n_1)!)^{-n_1}}{\prod_{i<j}^{n_1}(w_i-w_j)} (-1)^{N_R} (\sigma^2/E_I)^{(N_R-n_1)}$, d_l is given by (51), Y_k is the matrix formed by omitting the k^{th} row and 1^{st} column of the matrix $Y_{i,j} = \frac{\sigma^2}{E_I} {}_1F_1(n_2-j+1; n_2-n_1+1; w_k)\Gamma(n_2-j+1) + \Gamma(n_2-j+2) {}_1F_1(n_2-j+2; n_2-n_1+1; w_i)$ and $\alpha_l^{Ric} = b^l \sum_{k=0}^{l} \binom{l}{k} \frac{(aN_R/b)^k}{(N_R)_k}$. The derivation is given in Appendix D. From the above relation it is clear that the remarks 1-5 in [2] hold true, irrespective of whether the interferers undergo Rician or Rayleigh fading.

IV. SER EXPRESSIONS FOR CORRELATED INTERFERERS AND UNEQUAL POWER INTERFERERS

In the previous section, SER expressions are derived for the case of equal power uncorrelated interferers. But, in practice, the interferers can have different power and/or can be correlated. In a practical cellular systems, there can be one or more of the following: a) Receiver side correlation, b) Interferer correlation, c) Unequal power interferers.

The general non-central Wishart matrix $\mathbf{W}$ is written as $\mathbf{W} = \mathbf{C}'\mathbf{C}'^H$, where $\mathbf{C}' \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma} \otimes \mathbf{\Psi})$. Here, the $N_R \times N_R$ matrix $\mathbf{\Sigma}$ denotes the receive correlation and the $N_I \times N_I$ matrix $\mathbf{\Psi}$ denotes the transmit correlation or interferer correlation in our case.

Suppose, we consider only receive side correlation and assume that the interferer correlation is not present, i.e., $\mathbf{\Psi}$ is an identity matrix. This reduces to the non-central Wishart matrix denoted by $\mathbf{W} \sim \mathcal{CW}(N_I, \mathbf{\Sigma}, \mathbf{\Sigma}^{-1}\mathbf{M}\mathbf{M}^H)$. This case, where $\mathbf{\Psi}$ is assumed to be an identity matrix, is widely discussed in literature. The eigen-value distribution of this case, i.e., a non-central Wishart matrix with a covariance matrix $\mathbf{\Sigma}$ which is not an identity matrix, is analyzed in [38] in terms of zonal polynomials. However, using this eigen-value distribution to obtain the MGF expression of

η and hence derive the SER expressions becomes mathematically intractable. Hence, considering receive correlation is beyond the scope of this work.

On the other hand, the cases of $\mathbf{\Psi}$ being a diagonal matrix, i.e., unequal power interferers or $\mathbf{\Psi}$ being a full matrix, i.e., correlated interferers, have barely received attention in statistic literature. There do not even exist matrix variate and eigen-value distribution results for this case. However, we do provide results for this case by considering the problem as two sub-problems a) for $N_R \geq N_I$ exact results are provided, b) for $N_R < N_I$ approximate results are provided. In short, in this section we derive SER expressions for $\mathbf{C}' \sim \mathcal{CN}(\mathbf{M}, \mathbf{I}_{N_R} \otimes \mathbf{\Psi})$.

Note $\mathbf{W} = \mathbf{C}'\mathbf{C}'^H$ can be decomposed into $\mathbf{W} = \mathbf{C}\mathbf{\Psi}\mathbf{C}^H$, such that $\mathbf{C} \sim \mathcal{CN}(\mathbf{M}\mathbf{\Psi}^{-\frac{1}{2}}, \mathbf{I}_{N_R} \otimes \mathbf{I}_{N_I})$ [39]. Let us first consider the case of correlated interferers. The covariance matrix of the interference term plus the noise term is given by

$$\mathbf{R} = \mathbf{C}\mathbf{\Psi}\mathbf{C}^H + \sigma^2\mathbf{I}. \tag{20}$$

The received SINR for the OC is given by [8],

$$\eta = E_D\mathbf{c}^H\mathbf{R}^{-1}\mathbf{c}, \tag{21}$$

where E_D is the mean energy of the user signal. We will consider this problem as two cases: a) $N_R \geq N_I$, b) $N_R < N_I$.

A. $N_R \geq N_I$

Like in the case of equal power uncorrelated interferers, we consider the following general expression for the MGF $M_\eta(s)$ of SINR η, from [2],

$$M_\eta(s) = (-1)^{N_R}(\sigma^2)^{(N_R-n_1)}\mathbb{E}_{\mathbf{\Lambda}_R}\left[\left(\prod_{i=1}^{n_1}\frac{\sigma^2+\lambda_i}{\lambda_i^{(N_R-n_1)}}\right)\frac{det(\mathbf{J})}{V_{n_1}(\mathbf{\Lambda}_R)}\right], \tag{22}$$

where $V_{n_1}(\mathbf{\Lambda}_R)$ is the determinant of the Vandermonde matrix formed by eigen values of non-central Wishart matrix $\mathbf{C}\mathbf{\Psi}\mathbf{C}^H$. $\mathbf{J}$ is an $n_1 \times n_1$ matrix with elements,

$$\mathbf{J}_{i,j} = \begin{cases} h_1(s,\lambda_i) - \sum_{t=1}^{N_R-n_1} h_t(s,0)\lambda_i^{t-1}, & j = 1, \\ \lambda_i^{N_R-j}, & j = 2, ..., n_1, \end{cases}$$

and

$$h_t(s,x) = \frac{{}_1F_1(t; N_R; \frac{aN_Rs}{x/E_D+\sigma^2/E_D-bs})}{(bsE_D-\sigma^2-x)^t}. \tag{23}$$

Recall that, in the case of equal power uncorrelated interferers, we used the above MGF expression of η and simplified the expression using the eigen value distribution of the non-central Wishart matrix. But for the case of correlated interferers, there exists no matrix variate distribution forumla in the open literature and deriving one requires an integration over the Steifel manifold [38]. Also, there exists no eigen value distribution for this case. Hence, initially, we consider the case of Rayleigh-faded interferers, i.e., $\mathbf{M} = \mathbf{0}$.

1) Rayleigh faded correlated interferers: We exploit the property that $\mathbf{W} = \mathbf{C}\mathbf{\Psi}\mathbf{C}^H$ has the same non-zero eigen values as that of $\mathbf{\Psi}^{\frac{1}{2}}\mathbf{C}^H\mathbf{C}\mathbf{\Psi}^{\frac{1}{2}}$, where $\mathbf{C}^H\mathbf{C}$ is also a Wishart matrix. The eigen value distribution of $\mathbf{\Psi}^{\frac{1}{2}}\mathbf{C}^H\mathbf{C}\mathbf{\Psi}^{\frac{1}{2}}$, is given by [40],

$$f(\lambda_1, ..., \lambda_s) = c_1|\mathbf{\Upsilon}| \prod_{i<j}^{N_I}(\lambda_i - \lambda_j) \prod_{k=1}^{N_I} \lambda_k^{N_R - N_I}, \tag{24}$$

where $\mathbf{\Upsilon}$ is a $N_I \times N_I$ matrix whose $(i,j)^{th}$ entry $\forall i, j = 1, ..., N_I$ is given by,

$$\mathbf{\Upsilon}_{i,j} = e^{-\frac{\lambda_i}{r_j}}$$

and

$$c_1 = (-1)^{\frac{1}{2}N_I(N_I-1)} \frac{|\mathbf{\Psi}|^{-N_R}}{\prod_{i<j}^{N_I}(\frac{1}{r_i} - \frac{1}{r_j})\prod_{k=1}^{N_I}(N_R - k)!}.$$

Note that r_is are the ordered N_I distinct non-zero eigen-values of $\mathbf{\Psi}$. Further simplification of the MGF using (24) is given in Appendix E. The MGF after simplification becomes,

$$M_\eta(s) = c|\mathbf{N}|, \tag{25}$$

$$c = (-1)^{N_R}(\sigma^2)^{(N_R - N_I)}(-1)^{\frac{1}{2}N_I(N_I-1)} \frac{|\mathbf{\Psi}|^{-N_R}}{\prod_{i<j}^{N_I}(\frac{1}{r_i} - \frac{1}{r_j})\prod_{k=1}^{N_I}(N_R - k)!},$$

and N is given by

$$\mathbf{N}_{i,j} = \begin{cases} \sum_{l=0}^{\infty} \frac{(aN_R s E_D)^l}{(N_R)_l}(\sigma^2|bsE_D - \sigma^2|^{-l} U(1, 1-l, r_i|bsE_D - \sigma^2|) \\ \quad +|bsE_D - \sigma^2|^{-l+1}U(2, 2-l, r_i|bsE_D - \sigma^2|)) \\ \quad - \sum_{t=1}^{N_R - N_I} \frac{{}_1F_1(t; N_R; \frac{aN_R s}{\sigma^2/E_D - bs})}{(bsE_D - \sigma^2)^t}(\sigma^2 r_i^t \Gamma(t) + r_i^{t+1}\Gamma(t+1)], \quad j = 1,\, i = 1, ..., N_I, \\ \sigma^2 r_i^{N_R - j + 1}\Gamma(N_R - j + 1) + r_i^{N_R - j + 2}\Gamma(N_R - j + 2), \quad j = 2, ..., N_I\, i = 1, ..., N_I. \end{cases} \tag{26}$$

where r_i, $1 \le i \le N_I$ are the eigen values of $\mathbf{\Psi}$. We can truncate the converging infinite series for $j = 1$ at a finite value, with an arbitrarily small truncation error. The convergence proof is similar to the one given in Appendix A. Recall that the the SER can then be obtained substituting (25) in (6).

Note that this is an exact MGF expression and is novel for the case of Rayleigh faded correlated interferers with Rician faded users. Earlier works like [2], considers only equal power uncorrelated interferers, while recent works like [10] consider only Rayleigh faded user. An approximation which works for $\sigma^2 \approx 0$ is also derived in Appendix E. The expression is as follows:

$$M_\eta(s) \approx c \sum_{i=1}^{N_I} (-1)^{i+1} A(i) r_i^{-N_R+N_I-2} |V^i(\mathbf{r})|, \tag{27}$$

where,

$$c = (-1)^{N_R} (\sigma^2)^{(N_R-N_I)} (-1)^{\frac{1}{2}N_I(N_I-1)} \frac{|\mathbf{\Psi}|^{-N_R}}{\prod_{i<j}^{N_I} (\frac{1}{r_i} - \frac{1}{r_j}) \prod_{k=1}^{N_I} (N_R-k)!},$$

$$\begin{aligned} A(i) = &\sum_{l=0}^{\infty} \frac{(aN_R s E_D)^l}{(N_R)_l} (|bsE_D|^{-l+1} U(2, 2-l, r_i|bsE_D|)) \\ &- \sum_{t=1}^{N_R-N_I} \frac{{}_1F_1(t; N_R; \frac{aN_R s}{-bs})}{(bsE_D)^t} r_i^{t+1} \Gamma(t+1), \end{aligned}$$

and $V^i(\mathbf{r})$ denotes the Vandermonde matrix formed from all elements of $\mathbf{r} = (r_1, r_2, .., r_{N_I})$ except the i^{th} element. This can be further substituted in (6), to obtain the approximate expression for SER as,

$$\begin{aligned} SER \approx &\sum_{l=1}^{5} a_l (-1)^{N_R} (\sigma^2)^{(N_R-N_I)} (-1)^{\frac{1}{2}N_I(N_I-1)} \frac{|\mathbf{\Psi}|^{-N_R}}{\prod_{i<j}^{N_I} (\frac{1}{r_i} - \frac{1}{r_j}) \prod_{k=1}^{N_I} (N_R-k)!} \\ &\times \sum_{i=1}^{N_I} (-1)^{i+1} A(i)_{s=-b_l} r_i^{-N_R+N_I-2} |V^i(\mathbf{r})|. \end{aligned} \tag{28}$$

2) Rician faded correlated interferers: For the case of Rician faded interferers, the above approach is not possible, even for $N_R \geq N_I$. This is because, we have to use the zonal-polynomial based eigen value distribution from [38] to simplify the MGF, which is not mathematically tractable. Nevertheless, we arrive at a mathematically tractable solution, wherein we propose to approximate the non-central Wishart matrix by a central Wishart matrix through moment matching and then use the derived expression given in (25).

Expected value of any matrix of the form $\mathbf{A}^H\mathbf{A}$, if $\mathbf{A} \sim \mathcal{CN}(\mathbf{M}, \mathbf{I}_{N_I} \otimes \mathbf{I}_{N_R})$, with N_R degrees of freedom, is given by $E[\mathbf{A}^H\mathbf{A}] = N_R + \mathbf{M}^H\mathbf{M}$. This implies that for $\mathbf{W} = \mathbf{\Psi}^{\frac{1}{2}}\mathbf{C}^H\mathbf{C}\mathbf{\Psi}^{\frac{1}{2}}$, the first moment is given by, $E[\mathbf{W}] = N_R\mathbf{\Psi} + \mathbf{\Psi}^{1/2}\mathbf{M}^H\mathbf{M}\mathbf{\Psi}^{1/2}$. The first moment of any central Wishart matrix $\mathbf{W}_1 \sim \mathcal{CW}(N_R, \mathbf{\Phi})$ with the same degree of freedom N_R, is given by $E[\mathbf{W}_1] = N_R\mathbf{\Phi}$. When the first moments of the $\mathbf{W}$ and $\mathbf{W}_1$ are equated, we obtain

$$N_R\mathbf{\Phi} = N_R\mathbf{\Psi} + \mathbf{\Psi}^{1/2}\mathbf{M}^H\mathbf{M}\mathbf{\Psi}^{1/2}$$
$$\mathbf{\Phi} = \mathbf{\Psi} + \frac{1}{N_R}\mathbf{\Psi}^{1/2}\mathbf{M}^H\mathbf{M}\mathbf{\Psi}^{1/2}.$$

We have now obtained a central Wishart approximation of the non-central Wishart matrix. Hence, $\mathbf{W}$ can be approximated as a central Wishart $\mathcal{CW}(N_R, \mathbf{\Psi} + \frac{1}{N_R}\mathbf{\Psi}^{1/2}\mathbf{M}^H\mathbf{M}\mathbf{\Psi}^{1/2})$. A similar approximation is performed in [41]. Now that we have a central Wishart matrix, the expressions derived for the case of Rayleigh faded interferers holds, but with the matrix $\mathbf{\Psi}$ in (25) replaced by $\mathbf{\Psi} + \frac{1}{N_R}\mathbf{\Psi}^{1/2}\mathbf{M}^H\mathbf{M}\mathbf{\Psi}^{1/2}$. The simplified MGF expression is now given by

$$M_\eta(s) \approx c|\mathbf{N}|, \tag{29}$$
$$c = (-1)^{N_R}(\sigma^2)^{(N_R-N_I)}(-1)^{\frac{1}{2}N_I(N_I-1)}\frac{|\mathbf{\Psi} + \frac{1}{N_R}\mathbf{\Psi}^{1/2}\mathbf{M}^H\mathbf{M}\mathbf{\Psi}^{1/2}|^{-N_R}}{\prod_{i<j}^{N_I}(\frac{1}{r_i} - \frac{1}{r_j})\prod_{k=1}^{N_I}(N_R-k)!}$$

and N is given by

$$\mathbf{N}_{i,j} = \begin{cases} \sum_{l=0}^{\infty}\frac{(aN_RsE_D)^l}{(N_R)_l}(\sigma^2|bsE_D-\sigma^2|^{-l}U(1,1-l,r_i|bsE_D-\sigma^2|) \\ \quad +|bsE_D-\sigma^2|^{-l+1}U(2,2-l,r_i|bsE_D-\sigma^2|)) \\ \quad -\sum_{t=1}^{N_R-N_I}\frac{{}_1F_1(t;N_R;\frac{aN_Rs}{\sigma^2/E_D-bs})}{(bsE_D-\sigma^2)^t}(\sigma^2 r_i^t\Gamma(t)+r_i^{t+1}\Gamma(t+1)], \quad j=1,\, i=1,...,N_I, \\ \sigma^2 r_i^{N_R-j+1}\Gamma(N_R-j+1)+r_i^{N_R-j+2}\Gamma(N_R-j+2), \quad j=2,...,N_I\, i=1,...,N_I, \end{cases} \tag{30}$$

where r_i $1 \le i \le N_I$ are the eigen values of $\mathbf{\Psi} + \frac{1}{N_R}\mathbf{\Psi}^{1/2}\mathbf{M}^H\mathbf{M}\mathbf{\Psi}^{1/2}$. The SER approximation given by (28), can also be used, since we are anyway approximating non-central Wishart matrix by central Wishart matrix. But the tightness of the SER approximation given by (28), now also depends on how good is the non-central Wishart to central Wishart matrix for the specific case.

3) Unequal power interferers: All the above analysis holds for a general $\mathbf{\Psi}$. For the case of unequal power interferers, $\mathbf{\Psi}$ is just a diagonal matrix, with the interferer powers occupying the diagonal. Hence, the MGF expressions (25) and (29) can be used for unequal power Rayleigh-faded and Rician-faded interferers respectively.

B. $N_I > N_R$

1) Rayleigh faded correlated interferers: For the case of Rayleigh-faded correlated interferers, for $N_I > N_R$, the covariance matrix of the interference term plus the noise term is given by

$$\mathbf{R} = \mathbf{C}\mathbf{\Psi}\mathbf{C}^H + \sigma^2\mathbf{I}. \tag{31}$$

Here, $\mathbf{C} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_R} \otimes \mathbf{I}_{N_I})$. From [42], the distribution of $\mathbf{W} = \mathbf{C}\mathbf{\Psi}\mathbf{C}^H$ is same as that of $\sum_{i=1}^{N_I} \lambda_i \mathbf{W}_i$, where λ_i are the eigen values of $\mathbf{\Psi}$ and $\mathbf{W}_i \sim \mathcal{CW}(1, \mathbf{I}_{N_R})$. Though this method works for $N_R \geq N_I$, we use the accurate analysis given in the previous subsection for calculating SER.

From [43], sum of central Wishart matrices can be approximated by another central Wishart matrix. In our case, from [43], $\mathbf{W} \approx \mathbf{S}\frac{\sum_{i=1}^{N_I} \lambda_i}{p_s}$, where $\mathbf{S} \sim \mathcal{CW}(p_s, \mathbf{I}_{N_R})$ and $p_s = \left\lceil \frac{(\sum_{i=1}^{N_I} \lambda_i)^2}{\sum_{i=1}^{N_I} \lambda_i^2} \right\rfloor$ rounded to the nearest integer. Note that, this has reduced to a case of a Wishart matrix with an identity covariance matrix. Hence, the MGF expression derived for the case of equal power Rayleigh interferers, i.e., expressions corresponding to $L = 0$ given in Section III, can now be used. Also, the determinant simplification that has been derived in the case of equal power Rayleigh faded interferers holds for this case.

2) Rician faded interferers for $N_I > N_R$: In case, of correlated or unequal power Rician faded interferers for $N_I > N_R$, it is mathematically intractable to give an MGF expression and hence derive an SER expression. Nevertheless, the existing SER expressions in Section III derived for the case of equal power uncorrelated interferers can be used as an upper bound. If we consider all the interferers to have the same power as that of the maximum-power interferer, our expression gives an upper bound for the actual SER, i.e., our expressions give the worst case SER. Similarly, the expressions for uncorrelated case gives the worst case SER, i.e., a good upper bound on for the actual SER of correlated interferers. This is because, correlated interferers cause partial interference alignment [10] and hence the receive antennas can cancel the interferers better, leading to lower SER when compared to the uncorrelated case.

V. Numerical Results

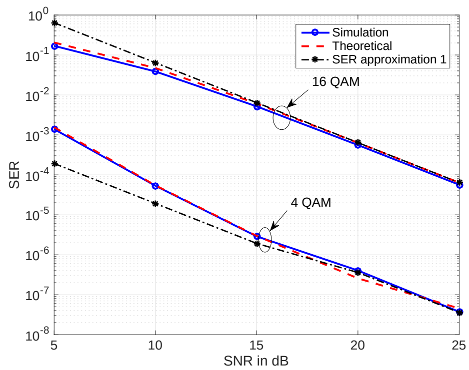
The derived SER expressions are first verified using Monte-Carlo simulations, for both $N_R > N_I$ and $N_R < N_I$, for the case of equal power uncorrelated interferers. The total interference power is denoted as E_I'', from which average interference power per interferer is obtained as $E_I' = E_I''/N_I$. The mean energy of the received signal, E_D, is taken to be unity without loss of generality. Signal to interference noise ratio (SIR) is given by E_D/E_I''. The determinant of $\mathbf{N}$ matrix whose entries are given by (12) is determined with the infinite summation truncated to $T_1 = T_2 = 100$. For $T_1 = T_2 = 100$, the truncation error has negligible magnitude, as evinced by our simulations. By substituting the determinant of $\mathbf{N}$ matrix in (11), the MGF is evaluated for

$s = b_l$, $\forall l = 1 \; to \; 5$. These MGF values are substituted in (6) and theoretical SER is calculated for values of signal to noise ratio (SNR) in the range 5 dB to 25 dB.

For the Monte-Carlo simulation, the deterministic matrix $\mathbf{M}'$ is first obtained with unit magnitude and uniform phase, satisfying the condition $tr(\mathbf{M}'\mathbf{M}'^H) = N_R N_I$ and the $\mathbf{M}$ matrix is then obtained from $\mathbf{M}'$ using the relation $\mathbf{M} = \sqrt{\kappa_i}\mathbf{M}'$. This matrix is fixed during a set of simulations. For each Monte-Carlo simulation, the random matrix $\mathbf{C} \sim \mathcal{CN}(\mathbf{M}, \mathbf{I}_{N_R})$ is generated and the random covariance matrix $\mathbf{R}$ of the interference terms plus the noise term is calculated using (2). The i.i.d Rician user channel $\mathbf{c}$ is then generated and using (3), SINR η can be calculated. Finally, the simulated SER is obtained by substituting the value of SINR η in (4) and evaluating it. The number of Monte-Carlo simulations for each SNR value is of the order of 10^7. This is repeated for various combinations of parameters N_R, N_I, E_I'', κ_s, and κ_i and numerical results obtained are compared with the simulation for each case.

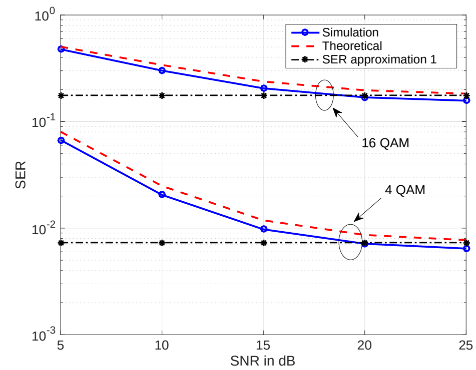
For the case of equal power and uncorrelated interferers, we see that there is a close match between the theoretical and simulated SER as seen from Fig. 1 and Fig. 1b. The exponential approximation $Q(x) \approx \frac{1}{12}e^{-\frac{1}{2}x^2} + \frac{1}{4}e^{-\frac{2}{3}x^2}$, from [29] provides a very tight upper bound for values of $x > 0.5$ and the bound becomes tighter as x increases. Since, for $N_I > N_R$, the average SINR is much lower when compared to the case $N_R > N_I$, we can observe a small mismatch between the theoretical and the simulated SER in Fig. 1 (b). Also, the SER approximation 1 plot in Fig. 1, calculated by using the N_{ij} matrix whose entries are given by (14), is tight beyond 15 dB. For Rayleigh interferers, the SER approximation computed using (19), match with the simulation results at high SNR, for $N_R > N_I$ as seen from Fig. 2(a). We can also see from Fig. 2(a) that, when the interference power dominates the noise power, as is the case when $E_I'' = -2$ dB, the SER approximation is tight even at 10 dB SNR. For $N_I > N_R$, the high SNR approximation computed using (18) is compared with Monte-Carlo simulations in Fig. 2 (b) for an SNR of 20 dB and a series of SIR values. An excellent match is observed in an interference limited scenario.

Similar Monte-Carlo simulations are performed for the case of correlated and/or unequal power interferers. The difference is that, random covariance matrix $\mathbf{R}$ of the interference terms plus the noise term is now calculated using (20). Also note that the determinant of $\mathbf{N}$ matrix whose entries are given by (26) and (30), for Rayleigh and Rician faded interferers respectively, is determined with the infinite summation truncated to $T_1 = 400$. From Fig. 3 (a), we can observe that for unequal power Rayleigh faded interferers, SER computed by means of (25) and (26) matches



(a)

$N_R = 8, N_I = 6, E_I'' = -1dB, \kappa_s = 3, \kappa_I = 6$



(b)

$N_R = 2, N_I = 3, E_I'' = -8dB, \kappa_s = 2, \kappa_I = 3$

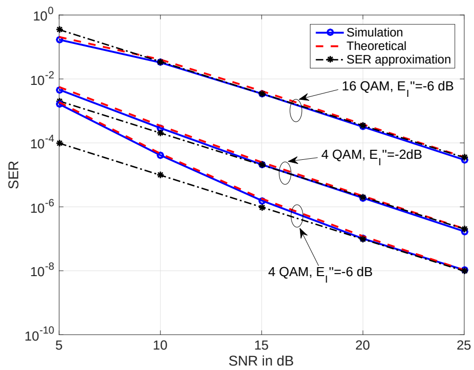
Fig. 1: SER for Rician interferers

the simulated values. We can also see that the approximation for $\sigma^2 \approx 0$ computed using (28) gives good match with the simulated results for higher values of SNR. For the case of correlated interferers, we consider exponential correlation between interferers [10], i.e., $\mathbf{\Psi}(i,j) = \rho^{|i-j|}$ and $0 \leq \rho \leq 1$, where $\mathbf{\Psi}$ is the interferer covariance matrix. A good match between the theoretical and simulation results, is observed for the case of correlated Rayleigh faded interferers, in Fig. 3 (b).

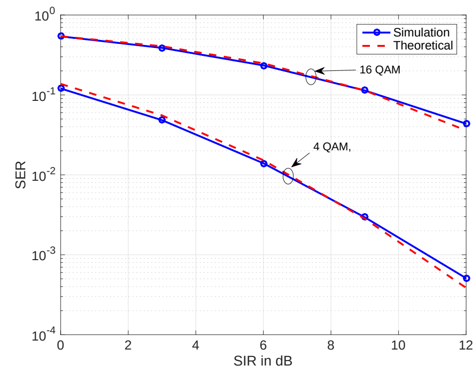
In Fig. 4 (a), we studied the case of mix of Rayleigh and Rician faded unequal power interferers. The theoretical SER computed by means of (29) and (30) match the simulation results. Also, the SER approximation given by (28) gives a good match to the simulated SER for high SNR values. For the case of only Rician faded interferers, the simulated value matches the theoretical value for some cases as seen in Fig.4(b) and doesn't perform very well for some cases as seen from Fig.5 (a). This is so because, the expressions (29) and (30) used for Rician faded interferers are obtained using moment matching approximation. For Rayleigh faded interferers with unequal power and $N_I > N_R$, the approximation in Section IV.B, gives a fairly good match to the theoretical values as seen from Fig.5 (b), as long as the interferer powers do not vary widely.

VI. Conclusions

In this paper, we considered receive diversity systems in the presence of multiple equal power correlated interferers. Approximate SER expressions have been derived for optimum combining

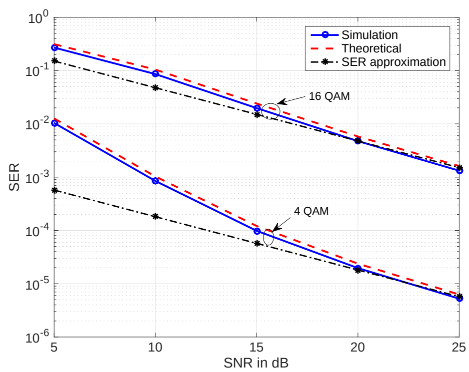


(a) $N_R = 6, N_I = 4, \kappa_s = 3$

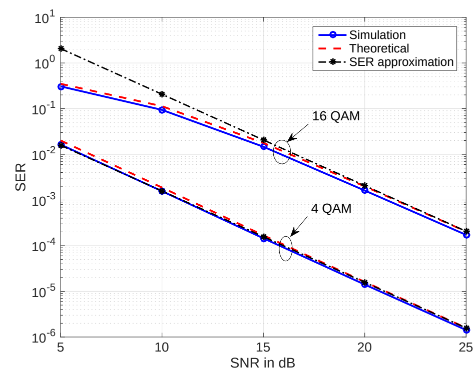


(b) Interference dominated scenario $N_R = 2, N_I = 4, \kappa_s = 4$

Fig. 2: SER for Rayleigh interferers



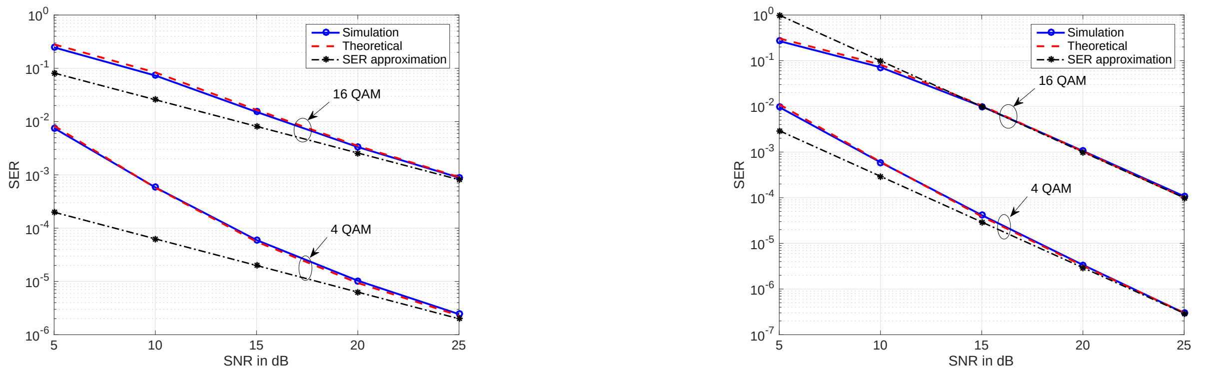
(a) unequal power Rayleigh interferers $\kappa_s = 2, N_R = 4, N_I = 3, E_I'' = -5, -8, -10dB$



(b) correlated Rayleigh interferers $\kappa_s = 3, N_R = 4, N_I = 2, E_I'' = -1dB, \rho = 0.6$

Fig. 3: SER for correlated/unequal power Rayleigh interferers

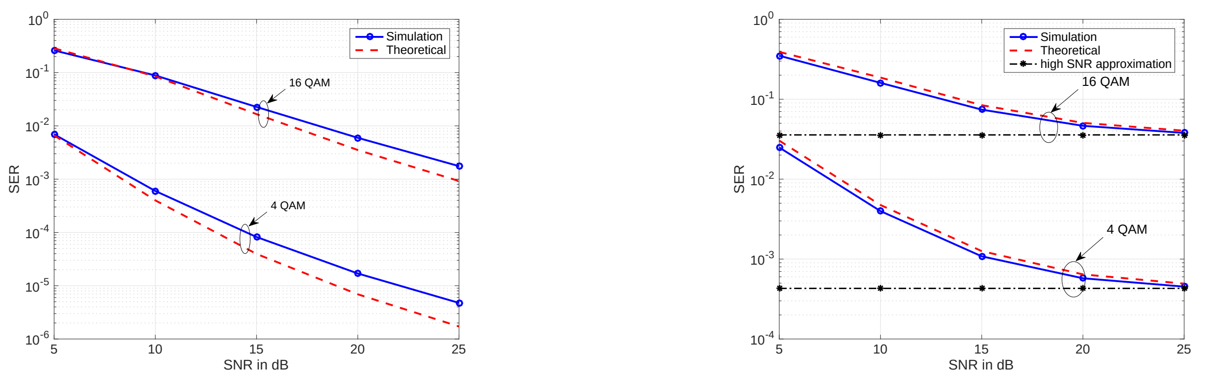
system for the Rician-Rician case, i.e., when the desired and interfering signals is subjected to Rician fading and interferers can be a mix of Rayleigh and Rician faded signals. SER is also derived for an interference limited scenario and the expressions obtained are significantly simpler than existing expressions. Both, interferer correlation and unequal power interferers are also considered. The Monte-Carlo simulation closely match the derived results. We believe extending this analysis to take into account receiver side correlation may be an interesting future work.



(a) mixture of two Rician and two Rayleigh interferers $\kappa_s = 2, \kappa_i = 5, N_R = 5, N_I = 4, E_I'' = -1, -1, 5, -2, -2.5dB$

(b) unequal power Rician interferers $\kappa_s = 3, \kappa_i = 2, N_R = 4, N_I = 2, E_I'' = -3, -10dB$

Fig. 4: SER for correlated/unequal power Rayleigh interferers



(a) unequal power Rician interferers $\kappa_s = 5, \kappa_i = 4, N_R = 5, N_I = 4, E_I'' = -2, -5, -1, -9dB$

(b) unequal power Rayleigh interferers $\kappa_s = 2, N_R = 3, N_I = 5, E_I'' = -6, -9, -10, -10, -10dB$

Fig. 5: SER for unequal power interferers

Appendix A
Simplification of $N_{i,j}$

We will simplify N_{ij} substantially and obtain a series expansion by exploiting various properties of special functions. We first substitute the value of $h(t, x)$ from (8) for N_{ij} $j = 1$ and then use the following identities [44], for $p < q$ and $Re(s) > 0$,

$$\int_0^\infty e^{-x} x^{s-1} {}_pF_q(a_1, .., a_p; b_1, .., b_q; ax)dx = \Gamma(s) {}_{p+1}F_q(s, a_1, .., a_p; b_1, .., b_q; a), \quad (32)$$

and

$$\int_0^\infty e^{-x} x^{s-1} dx = \Gamma(s), \quad (33)$$

to solve the integrals in $N_{i,j}$ entries for $j = 2, .., n_1$ in (11). For $j = 1$, $i = 1, ...L$ and $i = L+1, ..., n_1$, the integrals to be solved are of the form,

$$I = \int_0^\infty \frac{p+x}{v-x} e^{-x} x^z ({}_0F_1(q, w_i x))({}_1F_1(1; N_R; \frac{u}{x-v}))dx, \quad (34)$$

where $p = \sigma^2/E_I$, $q = n_2 - n_1 + 1$, $u = aN_R sE_D/E_I$, $v = bsE_D/E_I - \sigma^2/E_I$ and z is a positive integer greater than zero. To obtain a solution for I, we substitute the series expansion for ${}_0F_1$ and ${}_1F_1$, and interchange summations and integration. The integral to be solved becomes,

$$I = \sum_{k=0}^\infty \frac{w_i^k}{(q)_k k!} \bigg[\sum_{l=0}^\infty \frac{u^l (1)_l}{(N_R)_l l!} \int_0^\infty \frac{p+x}{v-x} e^{-x} \frac{x^{k+z}}{(x-v)^l} dx \bigg]. \quad (35)$$

The justification for the interchange of summations and integration is provided in Appendix B. Let $A1 = \int_0^\infty \frac{p}{v-x} e^{-x} \frac{x^{k+z}}{(x-v)^l} dx$ and $A2 = \int_0^\infty \frac{x}{v-x} e^{-x} \frac{x^{k+z}}{(x-v)^l} dx$. The Tricomi function or confluent Hyper-geometric function of the second kind is given by [44],

$$U(\alpha, \gamma, z) = \frac{1}{\Gamma(\alpha)} \int_0^\infty e^{-zt} t^{\alpha-1} (1+t)^{\gamma-\alpha-1} dt \quad [Re(\alpha) > 0, Re(z) > 0]. \quad (36)$$

In our case $-v > 0$ and $k + z > 0$. Hence using the above identity, A_1 and A_2 can be simplified as,

$$A1 = -p\Gamma(k+z+1)(-v)^{l-k-z} U(k+z+1, k+z+1-l, -v), \quad (37)$$

$$A2 = -\Gamma(k+z+2)(-v)^{l-k-z-1} U(k+z+2, k+z+2-l, -v). \quad (38)$$

Further, using the functional identity $U(a, b, z) = z^{1-b} U(a-b+1, 2-b, z)$ from [45], (37) and (38) are simplified and substituted back in (35) to obtain,

$$I = \sum_{k=0}^\infty \frac{w_i^k}{(q)_k k!} \Bigg[\sum_{l=0}^\infty \frac{u^l (1)_l}{(N_R)_l l!} \bigg[-\Gamma(k+z+2) U(l+1, l-k-z, -v) - p\Gamma(k+z+1) U(l+1, -k-z+l+1, -v) \bigg] \Bigg] \quad (39)$$

To the best of our knowledge, there are no known identities available in open literature that give a closed form expression for the above double infinite summation.

Convergence of the infinite summations

To prove the convergence of the above infinite summation, first consider the summation $I_1 = \sum_{k=0}^{\infty} \frac{w_i^k}{(q)_k k!}\left[\sum_{l=0}^{\infty} \frac{|u|^l (1)_l}{(N_R)_l l!}\Gamma(k+z+2)U(l+1, l-k-z, -v)\right]$. From Theorem 3 in [46] we get the identity $U(a,b,x) < x^{-a}$ for $x > 0,\ a > 0$ and $a - b + 1 > 0$. In our case, we can see that $a = l + 1 > 0$ and $a - b + 1 = k + z + 2 > 0$ and $x = -v > 0$. Therefore,

$$\begin{aligned} I_1 &< \sum_{k=0}^{\infty} \frac{w_i^k}{(q)_k k!} \sum_{l=0}^{\infty} \frac{|u|^l (1)_l}{(N_R)_l l!}\Gamma(k+z+2)(-v)^{l+1} \\ &= (-v)\Gamma(z+2) \sum_{k=0}^{\infty} \frac{w_i^k (z+2)_k}{(q)_k k!} \sum_{l=0}^{\infty} \frac{|uv|^l (1)_l}{(N_R)_l l!} \\ &= (-v)\Gamma(z+2)\, {}_1F_1(z+2, q, w_i)\, {}_1F_1(1, N_R, |uv|) \end{aligned}$$

The last equality is obtained from the series expansion definition of ${}_1F_1$ Hypergeometric function [45]. A similar argument can be used to prove the absolute convergence of the other infinite summation. Hence, I is convergent, which implies that we can truncate the double summation to T_1 and T_2 values such that $I - \sum_{l=0}^{T_1} \sum_{k=0}^{T_2} \frac{u^l}{(N_R)_l} \frac{w_i^k}{(q)_k}\Bigg[-(k+1)U(l+1, l-k, -v) - pU(l+ 1, -k+l+1, -v)\Bigg] \leq \epsilon$ for any $\epsilon > 0$. Hence we approximate I by

$$\begin{aligned} I_{T_1,T_2} = \sum_{k=0}^{T_2} \frac{w_i^k}{(q)_k k!}\Bigg[\sum_{l=0}^{T_1} \frac{u^l}{(N_R)_l}\Bigg[&-\Gamma(k+z+2)U(l+1, l-k-z, -v) \\ &- p\Gamma(k+z+1)U(l+1, -k-z+l+1, -v)\Bigg]\Bigg] \end{aligned} \tag{40}$$

with arbitrarily low approximation error. Hence, the simplified $N_{i,j}$ entry is given by (12). The $N_{i,j}$ entries for $L = 0$ is given in (13).

Appendix B
Interchange of Integration and Summation

From (34) we have,

$$I = \int_0^{\infty} \frac{p+x}{v-x} e^{-x} x^z ({}_0F_1(q, w_i x))({}_1F_1(1; N_R; \frac{u}{x-v}))dx.$$

Substituting the series expansion of ${}_0F_1$ term and using the property ${}_1F_1(a;b;z) = e^z\, {}_1F_1(b - a; b; -z)$ [44], we obtain,

$$I = \int_0^{\infty} \sum_{k=0}^{\infty} \frac{w_i^k}{(q)_k k!}\left[\frac{p+x}{v-x} e^{-x} x^{k+z} k e^{\frac{u}{x-v}}\, {}_1F_1(N_R - 1; N_R; \frac{-u}{x-v})\right]dx.$$

We see that $v < 0$ and $u < 0$ and hence $v = -|v|$ and the argument in confluent hypergeometric term $\frac{-u}{x-v}$ is positive. By application of a special case of Tonelli's theorem [47], according to which, if $f_n(x) \geq 0, \forall n, x,$ then $\sum \int f_n(x)dx = \int \sum f_n(x)dx$, we can interchange the summation and integration in I since all the terms inside the integration are positive. After interchanging, again using the property ${}_1F_1(a;b;z) = e^{-z}\,{}_1F_1(b-a;b;-z)$ [44], we get,

$$I = -\sum_{k=0}^{\infty} \frac{w_i^k}{(q)_k k!} \int_0^{\infty} \frac{p+x}{|v|+x} e^{-x} x^{k+z}\,{}_1F_1(1;N_R;\frac{u}{x-v})dx.$$

Writing ${}_1F_1$ in terms of its series expansion, we get,

$$I = \sum_{k=0}^{\infty} \frac{w_i^k}{(q)_k k!} \left[\int_0^{\infty} \sum_{l=0}^{\infty} \frac{u^l (1)_l}{(N_R)_l l!} \frac{p+x}{v-x} e^{-x} \frac{x^{k+z}}{(x-v)^l} dx \right].$$

Since $u < 0$, u^l alternates between positive and negative values. Therefore, Tonelli's theorem cannot be applied as in the previous interchange. So we use Lebesgue dominated convergence theorem to interchange the inner summation and integration. By Lebesgue dominated convergence theorem [48], if a sequence of measurable functions f_l are such that $|f_l(x)| \leq d_l(x)\ \forall l, x$, $\sum d_l(x)$ converges $\forall x$ and $\sum \int g_l(x)dx = \sum J_l$ converges $\forall x$, then $\int \sum f_l(x)dx = \sum \int f_l(x)dx$.

Assume $f_l = \frac{u^l(1)_l}{(N_R)_l l!} \frac{p+x}{v-x} e^{-x} \frac{x^{k+z}}{(x-v)^l}$. $|\frac{p+x}{(v-x)}| < 1$ because $v = -p-y$, where $y > 0$. Therefore, $|f_l| < d_l = \frac{|u|^l(1)_l}{(N_R)_l l!} e^{-x} \frac{x^{k+z}}{(x-v)^l}$. Applying ratio test, according to which, the series $\sum a_n$ converges if $\limsup_{l\to\infty} |\frac{a_{l+1}}{a_l}| < 1$, we get, $\lim_{l\to\infty} |\frac{d_{l+1}}{d_l}| = \lim_{l\to\infty} \frac{|u|}{N_R+l} = 0$. Therefore $\sum d_l$ converges. Let,

$$J_l = \int_0^{\infty} \frac{|u|^l (1)_l}{(N_R)_l l!} e^{-x} \frac{x^{k+z}}{(x-v)^l} dx \tag{41}$$

Using (36) , equation (41) can be simplified as,

$$J_l = \frac{|u|^l (1)_l}{(N_R)_l l!} \Gamma(k+z+1) U(l, l-k-z, -v).$$

Since $U(a, b-k, x)$ is monotonically decreasing with k [20],

$$|J_l| < H_l = \frac{|u|^l (1)_l}{(N_R)_l l!} \Gamma(k+z+1) U(l, l+1, -v).$$

Using the identity $U(a, a+1, z) = z^{-a}$,

$$|J_l| < H_l = \frac{|u|^l (1)_l}{(N_R)_l l!} \Gamma(k++z1)(-v)^{-l}.$$

It can be seen that, $\lim_{l\to\infty} |\frac{H_{l+1}}{H_l}| = \lim_{l\to\infty} \frac{|u|}{(N_R+l)(-v)} = 0$. Hence by ratio test, $\sum H_l$ converges. Since $|J_l|$ is upper bounded by H_l and $\sum H_l$ converges, $\sum J_l$ converges. In our case, since $|f_l| \leq d_l\ \forall l$ and $\sum d_l$ converges and $\sum \int d_l(x)dx = \sum J_l$ converges, the integration and the inner summation can be interchanged.

Appendix C
SER approximation for Rayleigh interferers

Consider the expression to be simplified, $M_\eta(s) = c|\mathbf{N}_{\sigma^2=0,L=0}|$, where

$$c = \frac{((n_2-n_1)!)^{-n_1}}{\prod_{i=1}^{n_1}(n_1-i)!}(-1)^{N_R}(\sigma^2/E_I)^{(N_R-n_1)}$$

and $\mathbf{N}_{\sigma^2=0,L=0}$ is from (14) for $L=0$. First the common terms inside each column or row of the determinant is taken out of the determinant and canceled with the existing terms in the constant c. All columns from $j=2,...,n_1$ are flipped and all rows from $i=1,..,n_1$ are flipped. $\Gamma(n_2-n_1+i+j-1)$ is then removed from each row, to obtain $\tilde{\mathbf{N}}$. Now, $M_\eta(s)=c|\tilde{\mathbf{N}}|$, where

$$c = \frac{\prod_{i=1}^{n_1}(n_2-n_1+i)!}{\prod_{i=1}^{n_1}(n_1-i)!\prod_{i=1}^{n_1}(n_2-i)!}(-1)^{N_R+1}(\sigma^2/E_I)^{(N_R-n_1)}, \tag{42}$$

$$\tilde{\mathbf{N}}_{i,j} = \begin{cases} A(i) & j=1,\, i=1,...,n_1, \\ 1, & j=2\, i=1,...,n_1. \\ \prod_{k=1}^{j-2}(n_2-n_1+i+k), & j=3,...,n_1, 1\le i\le n_1 \end{cases} \tag{43}$$

where

$$A(i) = \frac{1}{(n_2-n_1+i)!}\Bigg[\sum_{l=0}^{T_1}\frac{(aN_RsE_D/E_I)^l}{(N_R)_l}\Bigg[-\Gamma(n_2-N_R+i+1)$$
$$U(l+1,l-n_2+N_R-i+1,-bsE_D/E_I)\Bigg] - \sum_{t=1}^{N_R-n_1}\frac{{}_1F_1(t;N_R;aN_RsE_D/(-bsE_D)}{(bsE_D/E_I)^t}$$
$$\times\Bigg[\Gamma(t+n_2-N_R+i)\Bigg]\Bigg]$$

From [23], shifted factorials are defined by,

$$(z)_{s;n} = \begin{cases} 1 & ;n=0, \\ z(z+s)....(z+(n-1)s) & ;n=1,2,... \end{cases} \tag{44}$$

A special case of this is the Pochhammer's symbols, when $s=1$.

$$(z)_n = (z)_{1;n} = \begin{cases} 1 & ;n=0, \\ z(z+1)....(z+(n-1)) & ;n=1,2,... \end{cases} \tag{45}$$

In our case, in the $\tilde{\mathbf{N}}$ matrix, we have such Pochhammer's symbols in all columns except in the first. From [23, Lemma.1], we have the relation that determinant of a matrix with ij^{th} element for $0\le i,j\le n-1$, being a shifted factorial $(z_j)_{s;i}$ is given by $|(z_j)_{s;i}| = \Delta_n(\mathbf{z})$, where

$\Delta_n(\mathbf{z}) = \prod_{0\leq i<j\leq n-1}(z_j - z_i)$. Evaluating $|\tilde{\mathbf{N}}|$ by Laplace expansion along the first column and using the above relation from [23], we get

$$|\tilde{\mathbf{N}}| = \sum_{i=1}^{n_1}(-1)^{i+1}A(i)\Delta^i_{n_1-1}(\mathbf{z}), \tag{46}$$

where $\mathbf{z} = [n_2-n_1+1+1, n_2-n_1+1+2, ..., n_2-n_1+1+n_1]$ and $\Delta^i_{n_1-1}(z)$ is the Vandermonde determinant formed by all elements of the vector $\mathbf{z}$ except the i^{th} element. Any Vandermonde determinant remains unchanged if from each element of the matrix, one subtracts the same constant, i.e., $\Delta_n(\mathbf{z}+c) = \prod_{0\leq k<j\leq n-1}((z_j+c)-(z_k+c)) = \prod_{0\leq k<j\leq n-1}(z_j - z_k)$. Hence, the constant $n_2 - n_1 + 1$ can be subtracted from each element of the vector $\mathbf{z}$. Hence,

$$|\tilde{\mathbf{N}}| = \sum_{i=1}^{n_1}(-1)^{i+1}A(i)\Delta^i_{n_1-1}(\mathbf{z}), \tag{47}$$

where $\mathbf{z} = [1, ..., n_1]$. The Vandermonde determinant $\Delta_n(\mathbf{z})$, whose nodes are given by first n_1 integers, i.e., $\mathbf{z} = [1, ..., n_1]$, is given by $\Delta_n(\mathbf{z}) = \prod_{1\leq k<j\leq n_1}(j-k)$. For simplifying this expression, we expand the double product as follows:

$$\Delta_n(\mathbf{z}) = (n_1-1)!(n_1-2)!...(1)! = \prod_{j=1}^{n_1}(j-1)!. \tag{48}$$

However, we actually want to evaluate $\Delta^i_{n_1-1}(\mathbf{z})$ and not $\Delta_n(\mathbf{z})$. Note that the Vandermonde determinant $\Delta^i_{n_1-1}(\mathbf{z})$ in which the i^{th} element is missing, is given by,

$$\Delta^i_{n_1-1}(\mathbf{z}) = \prod_{1\leq k<j\leq n_1; k,j\neq i}(j-k)$$

Note that the above expression is difficult to evaluate. Hence to obtain a simplified expression we multiply and divide the expression for $\Delta^i_{n_1-1}(\mathbf{z})$ by the terms that are present in $\Delta_{n_1}(\mathbf{z})$, but are missing in $\Delta^i_{n_1-1}(\mathbf{z})$. We thus obtain,

$$\Delta^i_{n_1-1}(\mathbf{z}) = \frac{\prod_{1\leq k<j\leq n_1;}(j-k)}{(i-1)!(n_1-i)!.}$$

Substituting (48) in the above expression, we obtain, $\Delta^i_{n_1-1}(\mathbf{z})$ in terms of $\Delta_{n_1}(\mathbf{z})$ as,

$$\Delta^i_{n_1-1}(\mathbf{z}) = \frac{\Delta_{n_1}(\mathbf{z})}{(i-1)!(n_1-i)!} = \frac{\prod_{j=1}^{n_1}(j-1)!}{(i-1)!(n_1-i)!}.$$

Hence the final expression becomes $M_\eta(s) = c|\tilde{\mathbf{N}}|$ where, $c = \frac{(-1)^{N_R+1}(\sigma^2/E_I)^{(N_R-n_1)}n_2!}{(n_2-n_1)!\prod_{i=1}^{n_1}(n_1-i)!}$ and

$$|\tilde{\mathbf{N}}| = \sum_{i=1}^{n_1}(-1)^{i+1}A(i)\frac{\prod_{j=1}^{n_1}(j-1)!}{(n_1-i)!(i-1)!}. \tag{49}$$

Appendix D
Moments of SINR

For the case of $L = n_1$ we will derive the l^{th} moment. The mgf equation for this case can be written as

$$M_\eta(s) = c\sum_{k=1}^{n_1}(-1)^{k+1}\rho_k|Y_k|, \tag{50}$$

where $\rho_k = \int_0^\infty(\frac{\sigma^2}{E_I}+x)e^{-x}x^{n_2-N_R}\,{}_0F_1(n_2-n_1+1;w_kx)\frac{{}_1F_1(1;N_R;\frac{aN_Rs}{xE_I/E_D+\sigma^2/E_D-bs})}{(bsE_D/E_I-\sigma^2/E_I-x)}dx$ $-\ \sum_{t=1}^{N_R-n_1}\frac{{}_1F_1(t;N_R;aN_RsE_D/(\sigma^2-bsE_D)}{(bsE_D/E_I-\sigma^2/E_I)^t}\ \times\left[\frac{\sigma^2}{E_I}\Gamma(t+n_2-N_R)\,{}_1F_1(t+n_2-N_R;n_2-n_1+1;w_k)\right.$ $\left.+\ \Gamma(t+n_2-N_R+1){}_1F_1(t+n_2-N_R+1;n_2-n_1+1;w_i)\right]$, $Y_{i,j} = \frac{\sigma^2}{E_I}\,{}_1F_1(n_2-j+1;n_2-n_1+1;w_k)\Gamma(n_2-j+1)+\Gamma(n_2-j+2){}_1F_1(n_2-j+2;n_2-n_1+1;w_i)$ and Y_k is the matrix Y with k^{th} row and first column removed. The l^{th} moment is given by $\mu_l = \frac{d^l}{ds^l}M_\eta(s)|_{s=0}$. We need to evaluate $\frac{d^l}{ds^l}\rho_k$. We use the relations in [2] to evaluate the differential and obtain,

$$\begin{aligned}\frac{d^l}{ds^l}\rho_k|_{s=0} &= -\alpha_l^{Ric}l!(\frac{E_D}{E_I})^l\int_0^\infty(\frac{\sigma^2}{E_I}+x)^{-l}e^{-x}x^{n_2-N_R}\,{}_0F_1(n_2-n_1+1;w_kx)dx\\ &\quad-\sum_{t=1}^{N_R-n_1}\alpha_l^{Ric}\left[\frac{\sigma^2}{E_I}\Gamma(t+n_2-N_R)\,{}_1F_1(t+n_2-N_R;n_2-n_1+1;w_k)\right.\\ &\quad\left.+\Gamma(t+n_2-N_R+1){}_1F_1(t+n_2-N_R+1;n_2-n_1+1;w_i)\right](t)_l(-\frac{E_I}{\sigma^2})^t(\frac{E_D}{\sigma^2})^l\end{aligned}$$

where $\alpha_l^{Ric} = b^l\sum_{k=0}^{l}\binom{l}{k}\frac{(aN_R/b)^k}{(N_R)_k}$. The integral $\int_0^\infty(\frac{\sigma^2}{E_I}+x)^{-l}e^{-x}x^{n_2-N_R}\,{}_0F_1(n_2-n_1+1;w_kx)dx$ can only be solved by expanding the hypergeometric function and exchanging the integration and summation to obtain, $\frac{d^l}{ds^l}\rho_k|_{s=0} = -\alpha_l^{Ric}d_l$ where

$$\begin{aligned}d_l &= l!(\frac{E_D}{E_I})^l\sum_{n=0}^{\infty}\frac{(w_k)^n(n_2-n_1+1)_n}{n!}\Gamma(n_2-N_R+n+1)U(l,l-n_2+N_R-n,\frac{\sigma^2}{E_I})\\ &+\sum_{t=1}^{N_R-n_1}\left[\frac{\sigma^2}{E_I}\Gamma(t+n_2-N_R)\,{}_1F_1(t+n_2-N_R;n_2-n_1+1;w_k)\right.\\ &\left.+\Gamma(t+n_2-N_R+1){}_1F_1(t+n_2-N_R+1;n_2-n_1+1;w_i)\right](t)_l(-\frac{E_I}{\sigma^2})^t(\frac{E_D}{\sigma^2})^l\end{aligned} \tag{51}$$

APPENDIX E

CORRELATED AND/OR UNEQUAL POWER INTERFERERS

Using (24) to simplify (22), we obtain the MGF of η as

$$M_\eta(s) = c_1(-1)^{N_R}(\sigma^2)^{(N_R-N_I)} \int_0^\infty \left[\left(\prod_{i=1}^{n_1} \frac{\sigma^2+\lambda_i}{\lambda_i^{(N_R-N_I)}}\right)\frac{det(\mathbf{J})}{V_{N_I}(\mathbf{\Lambda}_R)}\right]|\mathbf{\Upsilon}|$$
$$\times \prod_{i<j}^{N_I}(\lambda_i-\lambda_j)\prod_{k=1}^{N_I}\lambda_k^{N_R-N_I}d\lambda_1...d\lambda_{N_I}$$
$$= (-1)^{N_R}(\sigma^2)^{(N_R-N_I)}\int_0^\infty c_1\prod_{i=1}^{n_1}(\sigma^2+\lambda_i)|\mathbf{J}||\mathbf{\Upsilon}|d\lambda_1...d\lambda_{N_I}. \tag{52}$$

From Theorem 2 in Appendix of [32], it can be observed that, for two arbitrary $m\times m$ matrices $\boldsymbol{\phi(y)}$ and $\boldsymbol{\psi(y)}$ with ij^{th} element $\phi_i(y_j)$ and $\psi_i(y_j)$ and arbitrary function g(.), where $\mathbf{y} = [y_1, y_2, ...y_m]^T$, the following identity holds:

$$\int\cdots\int_{b\geq y_i\geq a}|\boldsymbol{\phi(y)}||\boldsymbol{\psi(y)}|\prod_{k=1}^{m}g(y_k)dy_1dy_2...dy_m = det\left(\int_a^b\{\phi_i(y)\psi_j(y)g(y)dy\}_{i,j=1,..,m}\right).$$

Using the above property to simplify MGF in (52), we get,

$$M_\eta(s) = c|\mathbf{N}|, \tag{53}$$

where

$$c = (-1)^{N_R}(\sigma^2)^{(N_R-N_I)}(-1)^{\frac{1}{2}N_I(N_I-1)}\frac{|\mathbf{\Psi}|^{-N_R}}{\prod_{i<j}^{N_I}(\frac{1}{r_i}-\frac{1}{r_j})\prod_{k=1}^{N_I}(N_R-k)!},$$

and

$$\mathbf{N}_{i,j} = \begin{cases}\int_0^\infty(\sigma^2+x)e^{-\frac{x}{r_i}}[h_1(s,x)-\sum_{t=1}^{N_R-n_1}h_t(s,0)x^{t-1}]dx, & j=1,\ i=1,...,N_I,\\ \int_0^\infty(\sigma^2+x)e^{-\frac{x}{r_i}}x^{N_R-j}dx, & j=2,...,N_I\ i=1,...,N_I.\end{cases}$$

Solving the integrals using the identity $\int_0^\infty e^{-px}x^{s-1}dx = p^{-s}\Gamma(s)$ from [44], we obtain

$$\mathbf{N}_{i,j} = \begin{cases}\int_0^\infty(\sigma^2+x)e^{-\frac{x}{r_i}}\frac{{}_1F_1(1;N_R;\frac{aN_Rs}{x/E_D+\sigma^2/E_D-bs})}{(bsE_D-\sigma^2-x)}dx \\ \qquad -\sum_{t=1}^{N_R-N_I}\frac{{}_1F_1(t;N_R;\frac{aN_Rs}{\sigma^2/E_D-bs})}{(bsE_D-\sigma^2)^t}(\sigma^2r_i^t\Gamma(t)+r_i^{t+1}\Gamma(t+1)], & j=1,\ i=1,...,N_I,\\ \sigma^2r_i^{N_R-j+1}\Gamma(N_R-j+1)+r_i^{N_R-j+2}\Gamma(N_R-j+2), & j=2,...,N_I,\ i=1,...,N_I.\end{cases}$$

We can solve the integral for $j=1$, by expanding the ${}_1F_1$ hypergeometric series and interchanging the integration and summation. The approach followed in Appendix A can be followed here. After making simplifications similar to the ones made for (35), we will obtain the final $\mathbf{N}$ matrix as (26).

Approximation for $\sigma^2 \approx 0$

The determinant evaluation of $|\mathbf{N}|$ can be significantly simplified for $\sigma^2 \approx 0$. We first substitute $\sigma^2 = 0$ in $N_{i,j}$, to obtain

$$\mathbf{N}_{i,j} = \begin{cases} A(i), \quad j = 1,\, i = 1, ..., N_I, \\ r_i^{N_R - j + 2}\Gamma(N_R - j + 2), \qquad j = 2, ..., N_I,\, i = 1, ..., N_I. \end{cases}$$

where $A(i) = \sum_{l=0}^{\infty} \frac{(aN_r s)^l}{(N_R)_l}(|bsE_D|^{-l+1}U(2, 2-l, r_i|bsE_D|)) - \sum_{t=1}^{N_R - N_I} \frac{{}_1F_1(t; N_R; \frac{aN_R s}{-bs})}{(bsE_D)^t}(r_i^{t+1}\Gamma(t+1))$. By taking $r_i^{N_r - N_I + 2}$ and common gamma terms outside the determinant term we obtain,

$$M_\eta(s) \approx c|\mathbf{N}|, \tag{54}$$

$$c = (-1)^{N_R}(\sigma^2)^{(N_R - N_I)}(-1)^{\frac{1}{2}N_I(N_I-1)} \frac{|\mathbf{\Psi}|^{-N_R}}{\prod_{i<j}^{N_I}(\frac{1}{r_i} - \frac{1}{r_j}) \prod_{k=1}^{N_I}(N_R - k)!}$$
$$\times \prod_{i=1}^{N_I} r_i^{N_R - N_I + 2} \prod_{j=2}^{N_I} \Gamma(N_R - j + 2),$$

$$\mathbf{N}_{i,j} = \begin{cases} A(i) r_i^{-N_R + N_I - 2}, \quad j = 1,\, i = 1, ..., N_I, \\ r_i^{N_I - j}, \qquad j = 2, ..., N_I,\, i = 1, ..., N_I. \end{cases}$$

Expanding along the first column, we obtain an approximation for the MGF for $\sigma^2 = 0$ as,

$$M_\eta(s) \approx c \sum_{i=1}^{N_I} (-1)^{i+1} A(i) r_i^{-N_R + N_I - 2} |V^i(\mathbf{r})|, \tag{55}$$

where $V^i(\mathbf{r})$ denotes the Vandermonde matrix formed from all elements of $\mathbf{r} = (r_1, r_2, .., r_{N_I})$ except the i^{th} element. Note that, we do not substitute $\sigma^2 \approx 0$ in the c term but only in the $|\mathbf{N}|$ term, to obtain the approximation.

References

[1] J. Winters, "Optimum Combining in Digital Mobile Radio with Cochannel Interference," *IEEE Journal on Selected Areas in Communications*, vol. 2, no. 4, pp. 528–539, July 1984.

[2] M. McKay, A. Zanella, I. Collings, and M. Chiani, "Error Probability and SINR Analysis of Optimum Combining in Rician Fading," *IEEE Transactions on Communications*, vol. 57, no. 3, pp. 676–687, March 2009.

[3] O. B. S. Ali, C. Cardinal, and F. Gagnon, "Performance of Optimum Combining in a Poisson Field of Interferers and Rayleigh Fading Channels," *IEEE Transactions on Wireless Communications*, vol. 9, no. 8, pp. 2461–2467, August 2010.

[4] P. D. Rahimzadeh and N. C. Beaulieu, "Limits to Performance of Optimum Combining with Dense Multiple Correlated Antennas," *IEEE Transactions on Communications*, vol. 58, no. 7, pp. 2014–2022, July 2010.

[5] K. Chung, “An Analytical Expression for Performance of Optimum Combining with Multiple Correlated CCIs and Two Antennas,” *IEEE Communications Letters*, vol. 16, no. 4, pp. 458–461, April 2012.

[6] A. Shah and A. Haimovich, “Performance Analysis of Optimum Combining in Wireless Communications with Rayleigh Fading and Cochannel Interference,” *IEEE Transactions on Communications*, vol. 46, no. 4, pp. 473–479, Apr 1998.

[7] J. S. Kwak and J. H. Lee, “Performance Analysis of Optimum Combining for Dual-Antenna Diversity with Multiple Interferers in a Rayleigh Fading Channel,” *IEEE Communications Letters*, vol. 6, no. 12, pp. 541–543, Dec 2002.

[8] M. Chiani, M. Win, A. Zanella, R. Mallik, and J. Winters, “Bounds and Approximations for Optimum Combining of Signals in the Presence of Multiple Cochannel Interferers and Thermal Noise,” *IEEE Transactions on Communications*, vol. 51, no. 2, pp. 296–307, Feb 2003.

[9] R. Mallik, M. Win, M. Chiani, and A. Zanella, “Bit-Error Probability for Optimum Combining of Binary Signals in the Presence of Interference and Noise,” *IEEE Transactions on Wireless Communications*, vol. 3, no. 2, pp. 395–407, March 2004.

[10] S. Menon and S. Kalyani, “SER for Optimal Combining in the Presence of Multiple Correlated Co-channel Interferers,” *IEEE Communications Letters*, vol. PP, no. 99, pp. 1–1, 2015.

[11] D. Yue, X. Wang, and F. Xu, “Performance Analysis for Optimum Combining of Rayleigh Fading Signals with Correlated Rayleigh Interferers and Noise,” *IEEE Signal Processing Letters*, vol. 13, no. 5, pp. 269–272, May 2006.

[12] Q. Zhang and X. Cui, “Outage Probability for Optimum Combining of Arbitrarily Faded Signals in the Presence of Correlated Rayleigh Interferers,” *IEEE Transactions on Vehicular Technology*, vol. 53, no. 4, pp. 1043–1051, July 2004.

[13] N. Suraweera and N. C. Beaulieu, “Optimum Combining for Cooperative Relaying in a Poisson Field of Interferers,” *IEEE Transactions on Communications*, vol. 63, no. 9, pp. 3132–3142, Sept 2015.

[14] ——, “Optimum Combining in Dual-Hop AF Relaying for Maximum Spectral Efficiency in the Presence of Co-Channel Interference,” *IEEE Transactions on Communications*, vol. 63, no. 6, pp. 2071–2080, June 2015.

[15] ——, “Optimum Combining With Joint Relay and Antenna Selection for Multiple-Antenna Relays in the Presence of Co-Channel Interference,” *IEEE Communications Letters*, vol. 18, no. 8, pp. 1459–1462, Aug 2014.

[16] A. Afana, S. Ikki, T. M. N. Ngatched, and O. A. Dobre, “Performance analysis of cooperative networks with optimum combining and Co-channel interference,” in *2015 IEEE International Conference on Communication Workshop (ICCW)*, June 2015, pp. 949–954.

[17] Y. Zhong, W. Zhang, and M. Haenggi, “Managing Interference Correlation Through Random Medium Access,” *IEEE Transactions on Wireless Communications*, vol. 13, no. 2, pp. 928–941, February 2014.

[18] S. Kumar and S. Kalyani, “Impact of Correlation between Interferers on Coverage Probability and rate in Cellular Systems,” *arXiv preprint arXiv:1707.08802*, 2017.

[19] ——, “Impact of Correlated Interferers on Coverage and Rate of FFR and SFR Schemes,” *IEEE Transactions on Vehicular Technology*, vol. 65, no. 1, pp. 434–440, Jan 2016.

[20] J. Zhang, Z. Tan, H. Wang, Q. Huang, and L. Hanzo, “The Effective Throughput of MISO Systems Over κ-μ Fading Channels,” *IEEE Transactions on Vehicular Technology*, vol. 63, no. 2, pp. 943–947, Feb 2014.

[21] M. You, H. Sun, J. Jiang, and J. Zhang, “Effective Rate Analysis in Weibull Fading Channels,” *IEEE Wireless Communications Letters*, vol. 5, no. 4, pp. 340–343, Aug 2016.

[22] J. Zhang, L. Dai, Z. Wang, D. W. K. Ng, and W. H. Gerstacker, “Effective Rate Analysis of MISO Systems over α-μ Fading Channels,” in *2015 IEEE Global Communications Conference (GLOBECOM)*, Dec 2015, pp. 1–6.

[23] J.-M. Normand, “Calculation of some determinants using the s -shifted factorial,” *Journal of Physics A: Mathematical and General*, vol. 37, no. 22, p. 5737, 2004. [Online]. Available: http://stacks.iop.org/0305-4470/37/i=22/a=003

[24] H. V. Poor and S. Verdu, "Probability of error in MMSE multiuser detection," *IEEE Transactions on Information Theory*, vol. 43, no. 3, pp. 858–871, May 1997.

[25] A. Giorgetti and M. Chiani, "Influence of Fading on the Gaussian Approximation for BPSK and QPSK with Asynchronous Cochannel Interference," *Wireless Communications, IEEE Transactions on*, vol. 4, no. 2, pp. 384–389, March 2005.

[26] J. Proakis and M. Salehi, *Fundamentals of Communication Systems*. Pearson Education, 2007.

[27] M. Chiani, M. Win, A. Zanella, and J. Winters, "Exact Symbol Error Probability for Optimum Combining in the Presence of Multiple Co-channel Interferers and Thermal Noise," in *Global Telecommunications Conference, 2001. GLOBECOM '01. IEEE*, vol. 2, 2001, pp. 1182–1186 vol.2.

[28] ——, "A Simple and Asymptotically Tight Upper Bound on the Symbol Error Probability of Adaptive Antennas with Optimum Combining," in *Communications, 2002. ICC 2002. IEEE International Conference on*, vol. 3, 2002, pp. 1878–1881 vol.3.

[29] M. Chiani, D. Dardari, and M. K. Simon, "New Exponential Bounds and Approximations for the Computation of Error Probability in Fading Channels," *IEEE Transactions on Wireless Communications*, vol. 2, no. 4, pp. 840–845, July 2003.

[30] H. Exton, *Multiple hypergeometric Functions and Applications*, ser. Ellis Horwood series in mathematics and its applications. E. Horwood, 1976.

[31] S. Jin, M. McKay, X. Gao, and I. Collings, "MIMO Multichannel Beamforming: SER and Outage Using New Eigenvalue Distributions of Complex Noncentral Wishart Matrices," *IEEE Transactions on Communications*, vol. 56, no. 3, pp. 424–434, March 2008.

[32] M. Chiani, M. Win, and A. Zanella, "On the Capacity of Spatially Correlated MIMO Rayleigh-Fading Channels," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2363–2371, Oct 2003.

[33] J. F. Paris, "Outage Probability in η-μ/ η-μ and κ-μ/ η-μ Interference-Limited Scenarios," *IEEE Transactions on Communications*, vol. 61, no. 1, pp. 335–343, January 2013.

[34] S. Kumar and S. Kalyani, "Coverage Probability and Rate for κ -μ/η-μ Fading Channels in Interference-Limited Scenarios," *IEEE Transactions on Wireless Communications*, vol. 14, no. 11, pp. 6082–6096, Nov 2015.

[35] S. Kumar, G. Chandrasekaran, and S. Kalyani, "Analysis of Outage Probability and Capacity for κ- μ/η- μ Faded Channel," *IEEE Communications Letters*, vol. 19, no. 2, pp. 211–214, Feb 2015.

[36] G. Zhou, W. Xu, and G. Bauch, "A practical scheme to achieve sum capacity for strong interference-limited scenarios," in *2015 IEEE International Conference on Communications (ICC)*, June 2015, pp. 2797–2802.

[37] C. Chayawan and V. A. Aalo, "On the outage probability of optimum combining and maximal ratio combining schemes in an interference-limited Rice fading channel," *IEEE Transactions on Communications*, vol. 50, no. 4, pp. 532–535, Apr 2002.

[38] T. Ratnarajah, R. Vaillancourt, and M. Alvo, "Complex random matrices and Rician channel capacity," *Problems of Information Transmission*, vol. 41, no. 1, pp. 1–22, 2005. [Online]. Available: http://dx.doi.org/10.1007/s11122-005-0006-6

[39] M. R. McKay and I. B. Collings, "General Capacity Bounds for Spatially Correlated Rician MIMO Channels," *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3121–3145, Sept 2005.

[40] R. Couillet and M. Debbah, *Random Matrix Methods for Wireless Communications*. Cambridge University Press, 2011. [Online]. Available: https://books.google.co.in/books?id=_j7pT9HjKAUC

[41] R. Xu, Z. Zhong, and J. M. Chen, "Approximation to the Capacity of Rician Fading MIMO Channels," in *VTC Spring 2009 - IEEE 69th Vehicular Technology Conference*, April 2009, pp. 1–5.

[42] M. Ohlson and T. Koski, "On the Distribution of Matrix Quadratic Forms," *Communications in Statistics-Theory and Methods*, vol. 41, no. 18, pp. 3403–315.

[43] S. Kumar, G. F. Pivaro, G. Fraidenraich, and C. F. Dias, “On the exact and approximate eigenvalue distribution for sum of Wishart matrices,” *arXiv preprint arXiv:1504.00222*, 2015.

[44] A. Jeffrey and D. Zwillinger, *Table of Integrals, Series, and Products*, ser. Table of Integrals, Series, and Products Series. Elsevier Science, 2007.

[45] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, ser. Applied mathematics series. U.S. Government Printing Office, 1972.

[46] J. A. C.M. Joshi, “Some inequalities for the gauss and Kummer Hypergeometric Functions ,” *Indian Journal of Pure and Applied Mathematics*, vol. 22, no. 8, pp. 637–644.

[47] T. Tao, *An Introduction to Measure Theory*, ser. Graduate studies in mathematics. American Mathematical Society, 2011.

[48] T. Apostol, *Mathematical Analysis*, ser. Addison-Wesley series in mathematics. Addison-Wesley Publishing Company, 1974.