Global sensitivity and uncertainty analysis of dynamical systems using variance-based methods – GSUA Matlab toolbox

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Description of toolbox

This GSUA Matlab toolbox allows the global sensitivity analysis and uncertainty analysis of dynamical systems, implemented as Simulink models, using variance-based methods (brute-force, Sobol, Jansen, and Saltelli) with combinations of parameters generated by Monte Carlo method, and with the next sampling methods: uniform distribution, Latin hypercube. Several figures are generated: (1) temporal responses with all sets of parameters, highlighting the nominal or experimental response; (2) scatter plots of parameters (to see the distribution of parameters) and output (to see the dependence of output on every parameter in specific times; (3) time-dependent fractional sensitivity indices; (4) normalized time-dependent total sensitivity indices; (5) pie and bar plot of sensitivity indices for every parameter in specific times; (6) fractional end total sensitivity indices of minimum square error (MSE) function (difference between every time response and nominal time response).

All functions of toolbox are optimized as three main functions: sensitivity methods, uncertainty analysis and plotting. The percentage of progress and estimated processing time are displayed. Parallel computing may be used.

Steps for running the toolbox

The script sens_main can be run to implement each step of

```
HELP:
     The Simulink model has to be set correctly (see Configurtion parameters of Simulink examples):
     1. The name of parameters are p(1), p(2),...
     2. Connect an "Out block" to the output
     3. In "Configuration parameters | Data Import/Export" check these options:
        Time (tout), Output (yout), Save simulation output as single object
     Format of cell of parameters {Parameter1, Uncertainty_model1, uncertainty_value1; Parameter2, ...}:
     This tool needs the Statistics Toolbox

Select the model name (examples: 'SIR', 'Pendulum', 'PID'): 'sens_example_sir_sim'
Give the parameters: {'beta','percent',[0.15 30];'alpha','percent',[0.45 30];'I0','percent',[0.1 30]}
Give the name of experimental time-response vector ('' or ENTER for using nominal time response):
Select the sensitivity method ('brute-force', 'Sobol', 'Jansen', 'Saltelli'): 'Jansen'
Select the sample method ('Uniform', 'LatinHypercube'): 'Uniform'
Sample size: 20
Do use parallel computing? (Yes:1, No: 0): 0
```

1. **Diagramming of Simulink model (model)**. The Simulink model has to be set correctly:
    a. The name of parameters are p(1), p(2),...
    b. Connect an "Out block" to the output
    c. In "Configuration parameters | Data Import/Export" check these options: Time (tout), Output (yout), Save simulation output as single object; use fixed-step solver.
    d. If there are problems with rate transition, then fix the problem in Configuration | Diagnostics | Sample time.

2. **Set of parameter values and intervals (Par)**. Set the model parameters as a cell (Npx1) with information about Np parameters: {'Parameter_name', 'Uncertainty_mode', uncertainty_value}. There are three possibilities:

    a. Uncertainty_mode = 'range', uncertainty_value = [min, max]
    b. Uncertainty_mode = 'std', uncertainty_value = [nominal, standard_deviation]
    c. Uncertainty_mode = 'percent', uncertainty_value = [nominal, percent_(0,100)]

    Example:

    ```
    Par = {'beta','percent', [0.15 30];
           'alpha','std', [0.45 0.05];
           'I0','range', [0.05 0.15]};
    ```

3. **Selection of sensitivity method (SensMethod)**. Several variance-based methods are implemented: Saltelli, Sobol, Jansen, and brute-force. See this reference: Saltelli, A., Annoni, P., Azzini, I., Campolongo, F., Ratto, M., & Tarantola, S. (2010). Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index. Computer Physics Communications, 181(2), 259–270.

4. **Selection of sample method (SampleMethod)**. These sample methods are implemented: Uniform, Latin Hypercube

5. **Selection of sample size (N)**. Select the sample size N, i.e. the number of sets of parameters to test. If N is large the results are more accurate, but the processing time is also large. A balance between the accuracy of the calculations and the computational time must be find.

6. **Specification (optional) of experimental time response (y_nom)**. Specify the experimental time response for computation of sensitivity indices of minimum square error (MSE) function (difference between every time response and nominal time response). If it is not specified, the nominal time response is used.

7. **Calculation of time-dependent and scalar sensitivity indices from Monte Carlo simulation**. Function:

    [S_vec, ST_vec, SJ, STJ, Y, t, M, y_nom] = sens_methods (model, Par, N, SensMethod, SampleMethod, y_nom)

    The function uses the information of steps 1-6 to give the following information:

    S_vec: matrix (Nd×Np) with time-dependent fractional sensitivity indices (one column by factor and one row by time instant). Nd is the number of simulation time instants.
    ST_vec: matrix (Nd×Np) with time-dependent total sensitivity indices (one column by factor and one row by time instant)
    SJ: vector (Np×1) of fractional sensitivity indices for mean squared error (MSE) function
    STJ: vector (Np×1) of total sensitivity indices for mean squared error (MSE) function
    Y: matrix (NxNd) with time responses in rows to every set of parameters
    t: vector (1×Nd) with the time instants
    M: matrix (N×Np) of input parameters (one parameter by column and one sample by row)

    The progress percentage, remaining time and other information is shown:

    ```
    Progress: 2%
    Estimated processing time (h:m:s): 0:13:21
    Remaining time (h:m:s): 0:13:6
    Elapsed time (h:m:s): 0:0:15
    Estimated stop time (h:m:s): 12:33:20
    ```

8. **Plotting**. Different type of figures can be plotted using only one function sens_plot, but with different input arguments.

   Filled area plot of time-dependent fractional sensitivity indices

   sens_plot('FractionalSensitivityArea', Par, SensMethod, S, t)

   Plot of every time-dependent fractional sensitivity index

   sens_plot('FractionalSensitivityPlots', Par, SensMethod, S, t)

   Filled area plot of time-dependent normalized (divided by the sum) fractional sensitivity indices

   sens_plot('TotalSensitivityArea', Par, SensMethod, S, t)

   Plot of every time-dependent total sensitivity index

   sens_plot('TotalSensitivityPlots', Par, SensMethod, S, t)

   Monte Carlo simulation for uncertainty analysis

   sens_plot('UncertaintyAnalysis', Par, Y t, y_nom)

   Scatter plot of one parameter in function of other

   sens_plot('ScatterParameter', Par, SensMethod, M) - Scatter plot of pair of parameters

   Scatter plot of output in function of each parameter in a specific time tref

   sens_plot('ScatterOutput', Par, t, SensMethod, Y, M, tref)

   Bar chart of fractional sensitivity indices in specific time instants given in vector tref

   sens_plot('Bar', Par, SensMethod, ST, t, tref) (time-dependent case)

   sens_plot('Bar', Par, SensMethod, SJ) (scalar case)

   Pie chart of fractional sensitivity indices in specific time instants given in vector tref

   sens_plot('Pie', Par, SensMethod ,ST, t, tref) (time-dependent case)

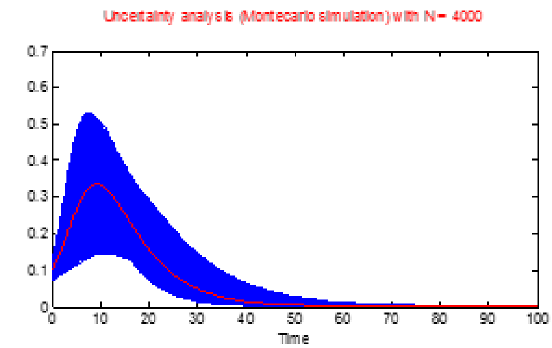
   sens_plot('Pie', Par, SensMethod, SJ) (scalar case)

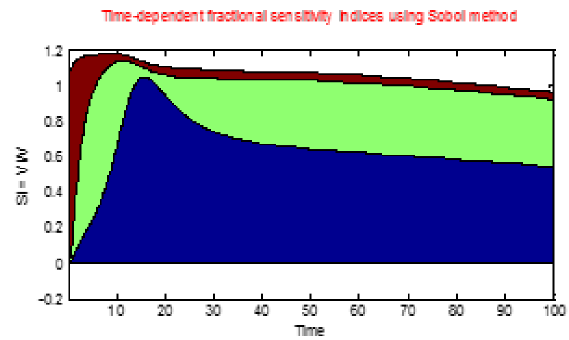
Examples

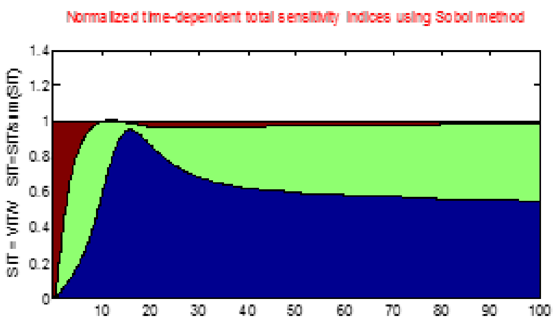
Example 1 (Sobol, SIR, N = 4000, non-parallel computing)

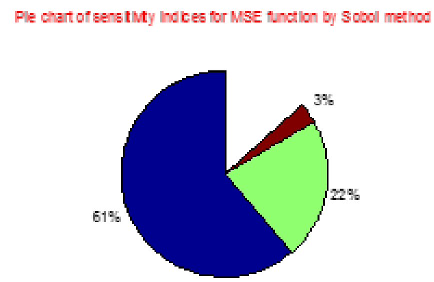
```
Progress: 2%
Estimated processing time (h:m:s): 0:13:21
Remaining time (h:m:s): 0:13:6
Elapsed time (h:m:s): 0:0:15
Estimated stop time (h:m:s): 12:33:20
```

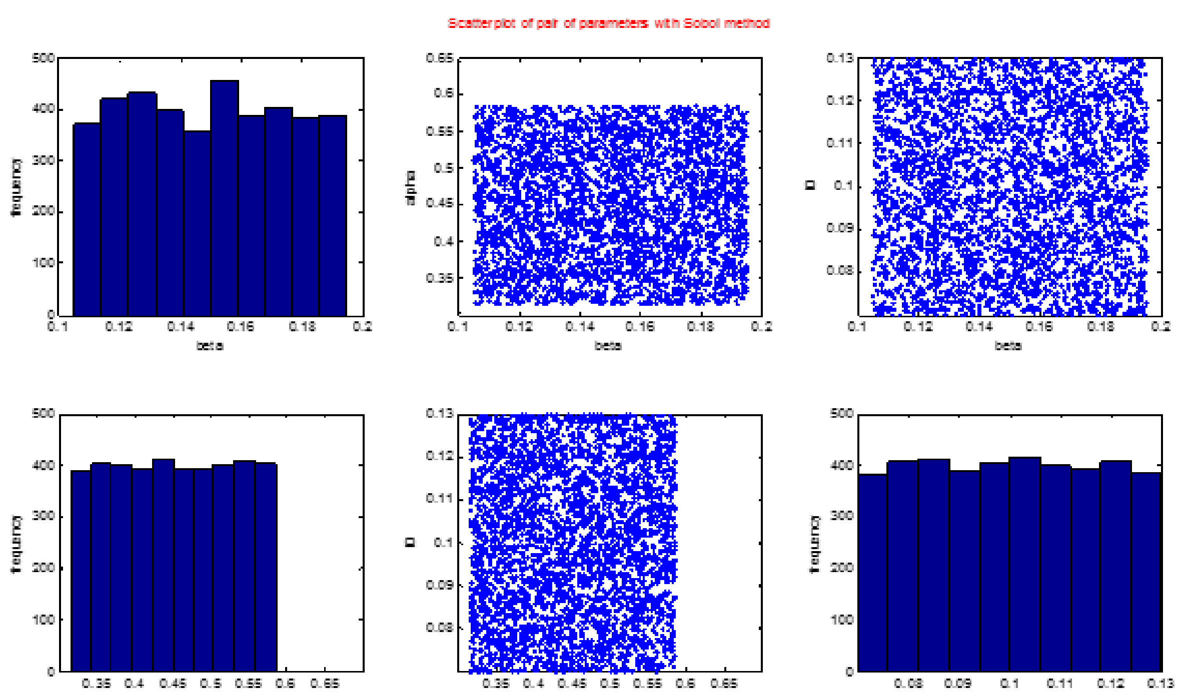
```
Progress: 100%
Estimated processing time (h:m:s): 0:13:15
Remaining time (h:m:s): 0:0:0
Elapsed time (h:m:s): 0:13:15
Estimated stop time (h:m:s): 12:33:14
```

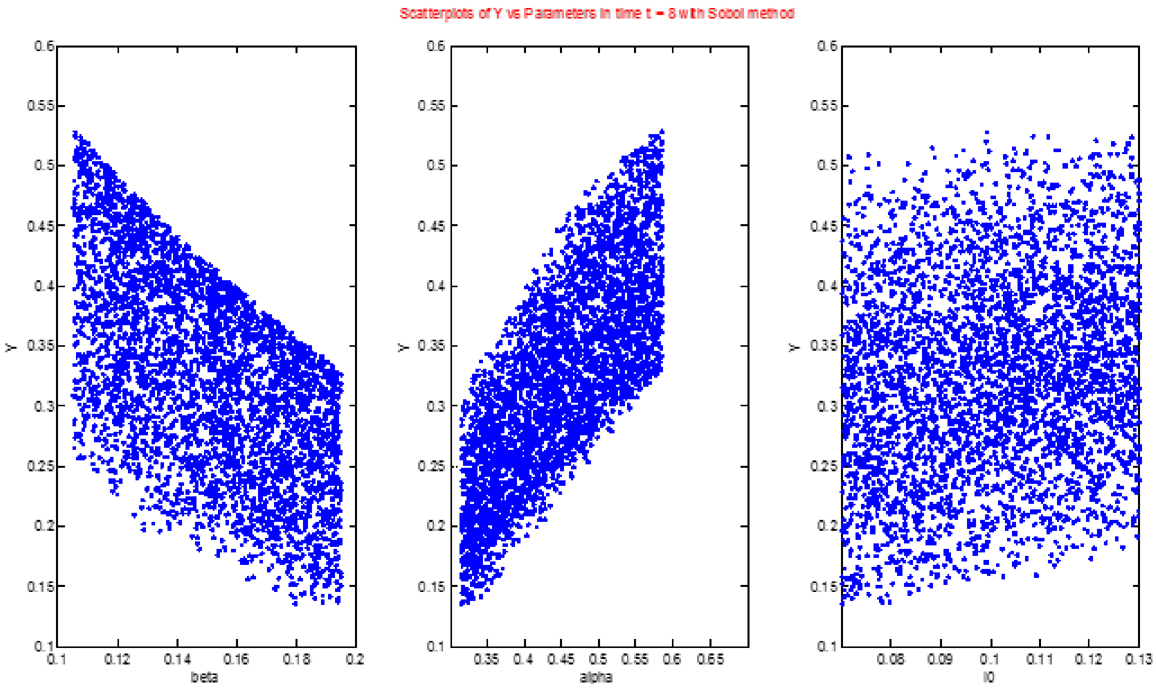


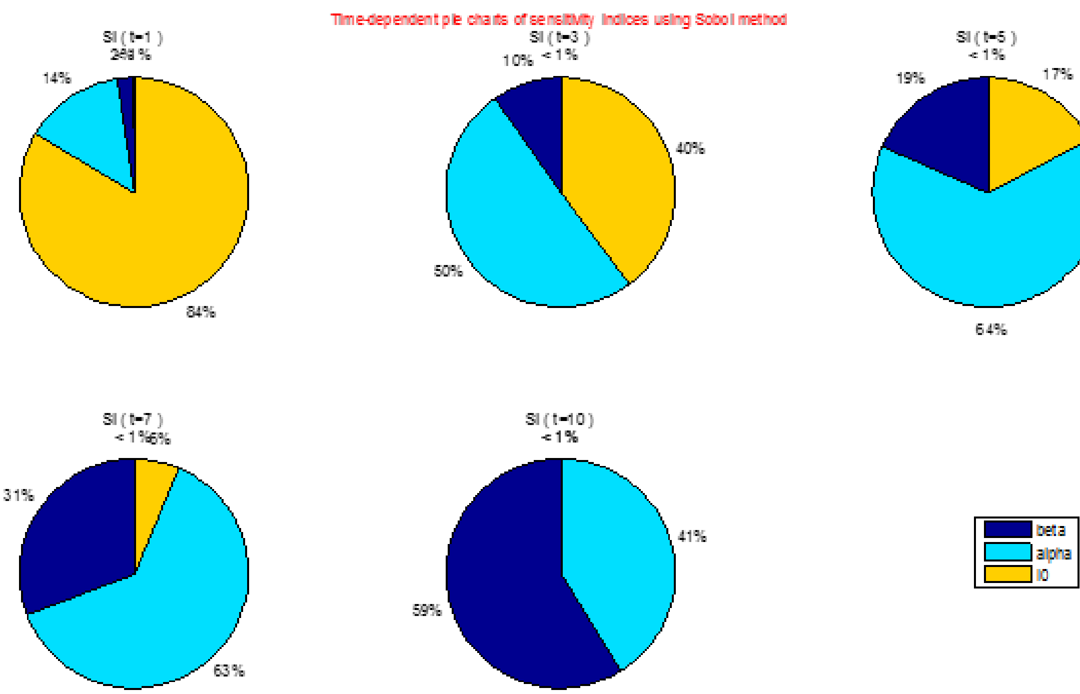








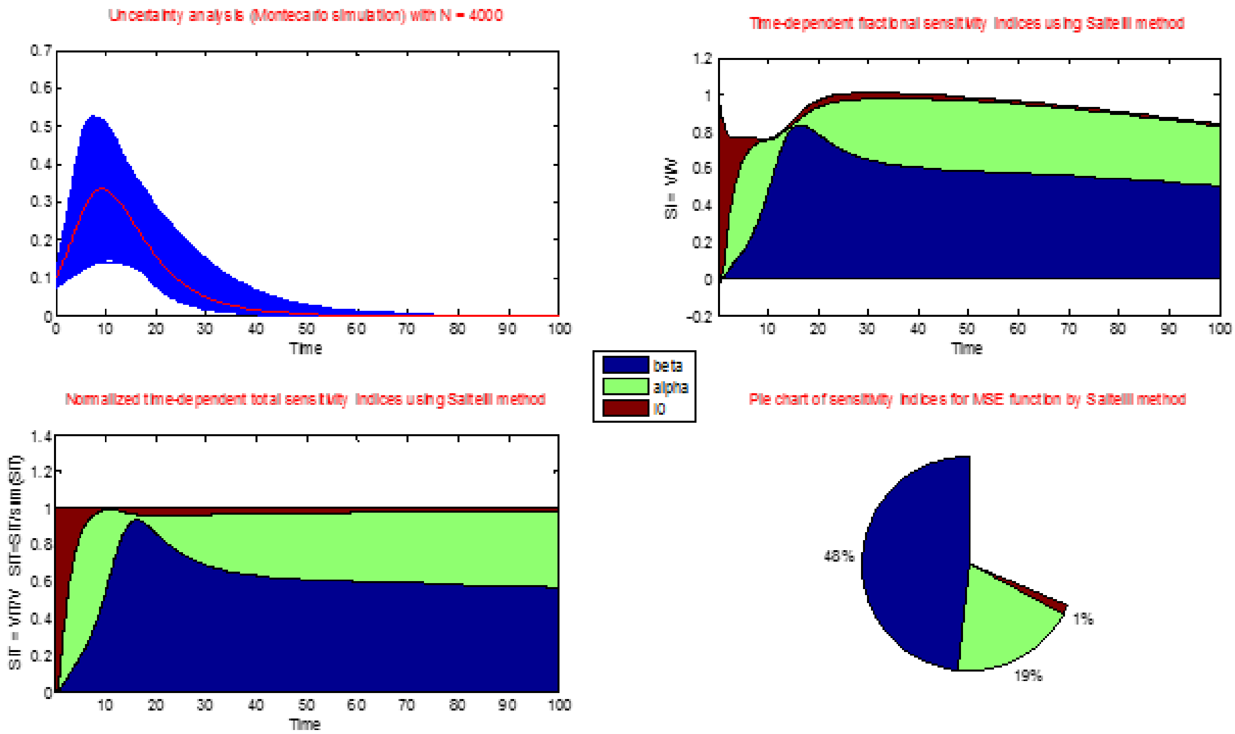

Scatterplots of Y vs Parameters in time t = 8 with Sobol method
Y
beta
alpha
I0

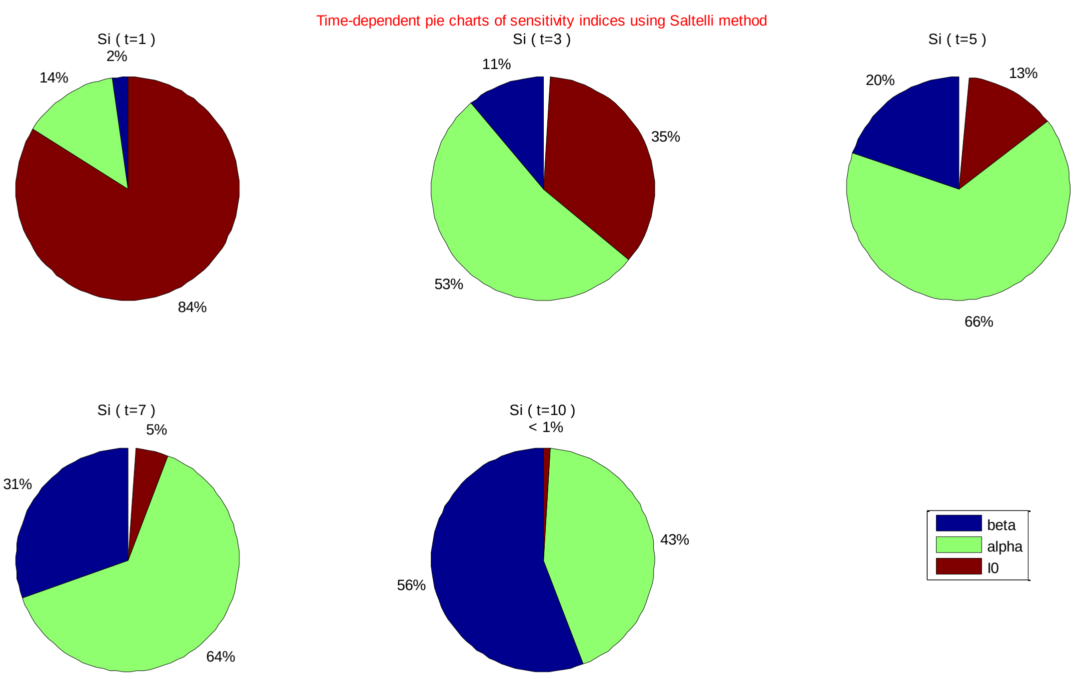

Time-dependent pie charts of sensitivity indices using Sobol method
SI (t=1)
14%
84%
SI (t=3)
10%
< 1%
40%
50%
SI (t=5)
< 1%
19%
17%
64%
SI (t=7)
31%
63%
SI (t=10)
< 1%
41%
59%
beta
alpha
I0

Example 2 (Saltelli, SIR, N = 4000, parallel computing)

```
Progress: 29%
Estimated processing time (h:m:s): 0:5:17
Remaining time (h:m:s): 0:3:45
Elapsed time (h:m:s): 0:1:32
Estimated stop time (h:m:s): 16:38:26
```

```
Progress: 100%
Estimated processing time (h:m:s): 0:4:27
Remaining time (h:m:s): 0:0:0
Elapsed time (h:m:s): 0:4:27
Estimated stop time (h:m:s): 16:37:36
```

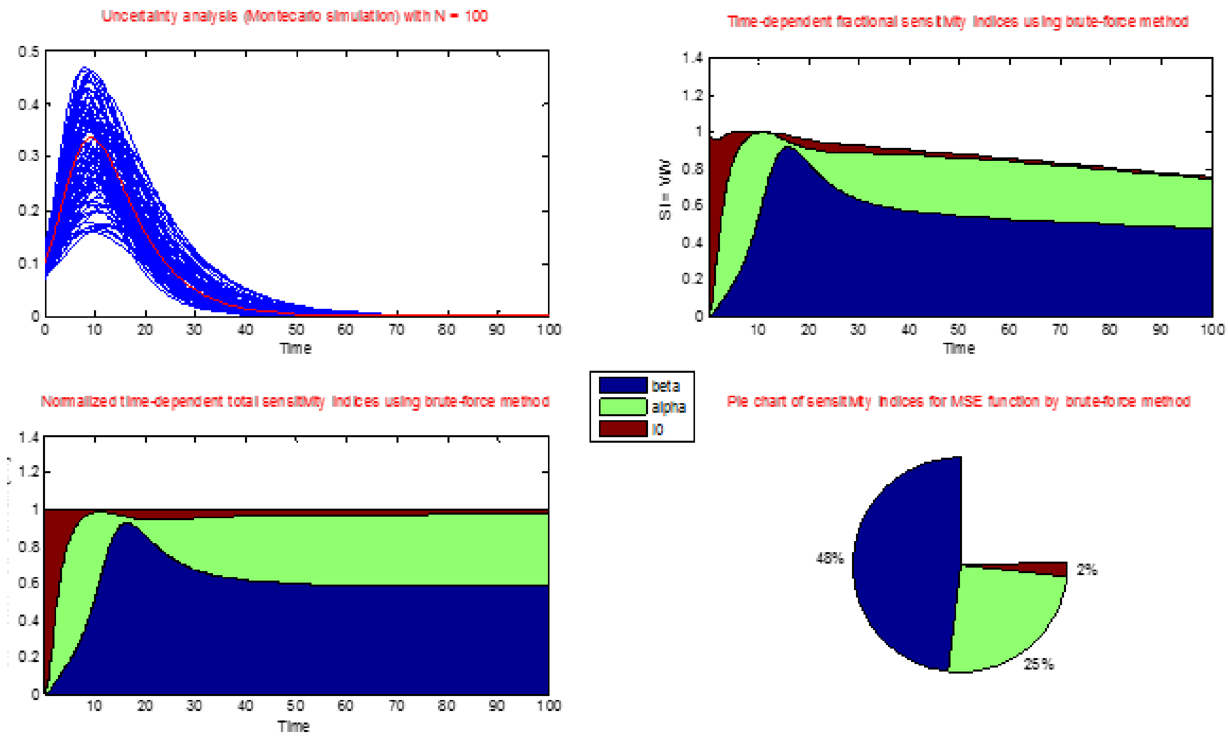


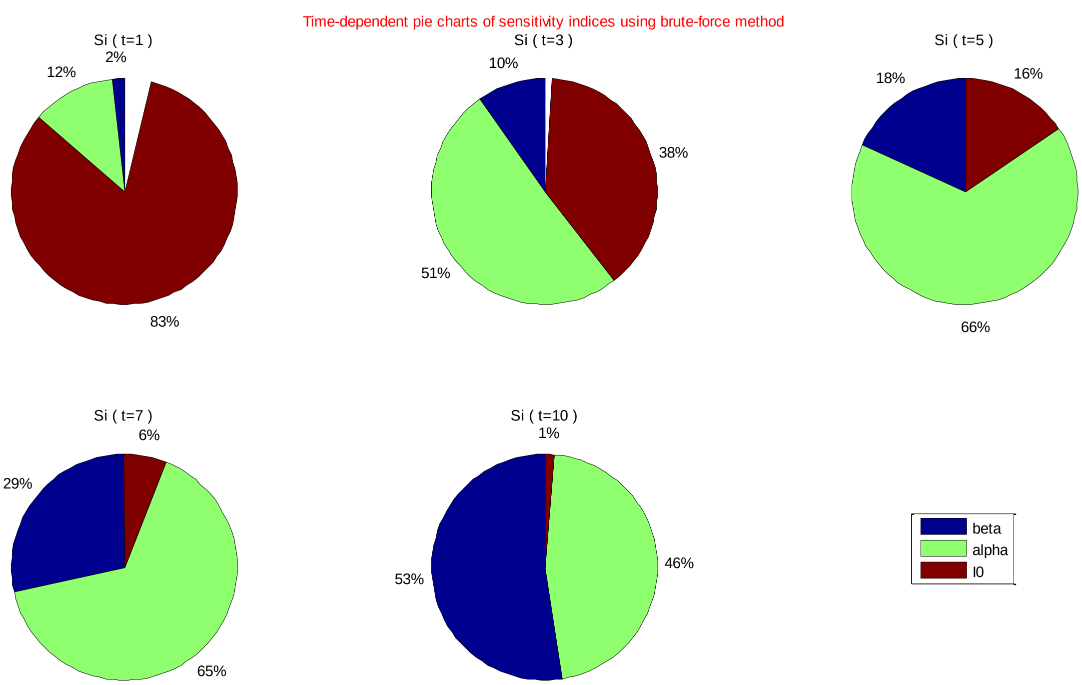


Example 3 (Brute-force, SIR, N = 100, parallel computing)

```
Progress: 5%
Estimated processing time (h:m:s): 0:16:47
Remaining time (h:m:s): 0:16:0
Elapsed time (h:m:s): 0:0:46
Estimated stop time (h:m:s): 8:20:55
```

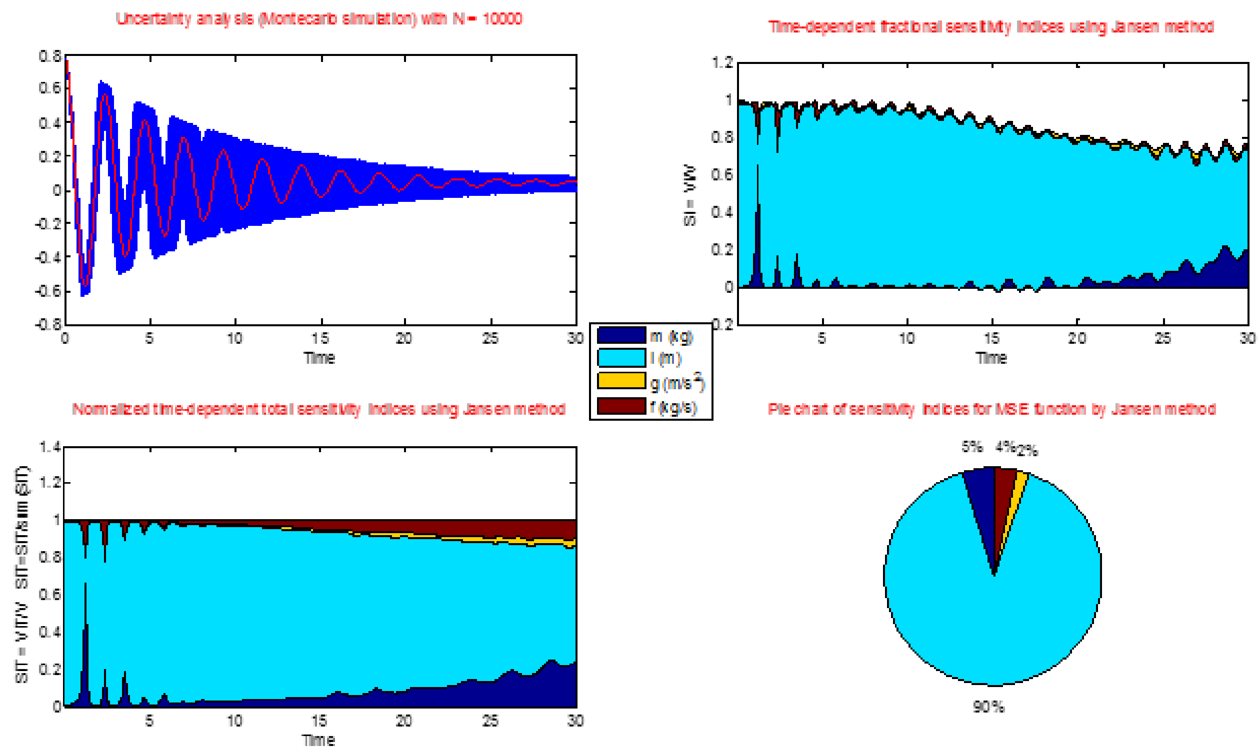
```
Progress: 100%
Estimated processing time (h:m:s): 0:15:41
Remaining time (h:m:s): 0:0:0
Elapsed time (h:m:s): 0:15:41
Estimated stop time (h:m:s): 8:19:49
```

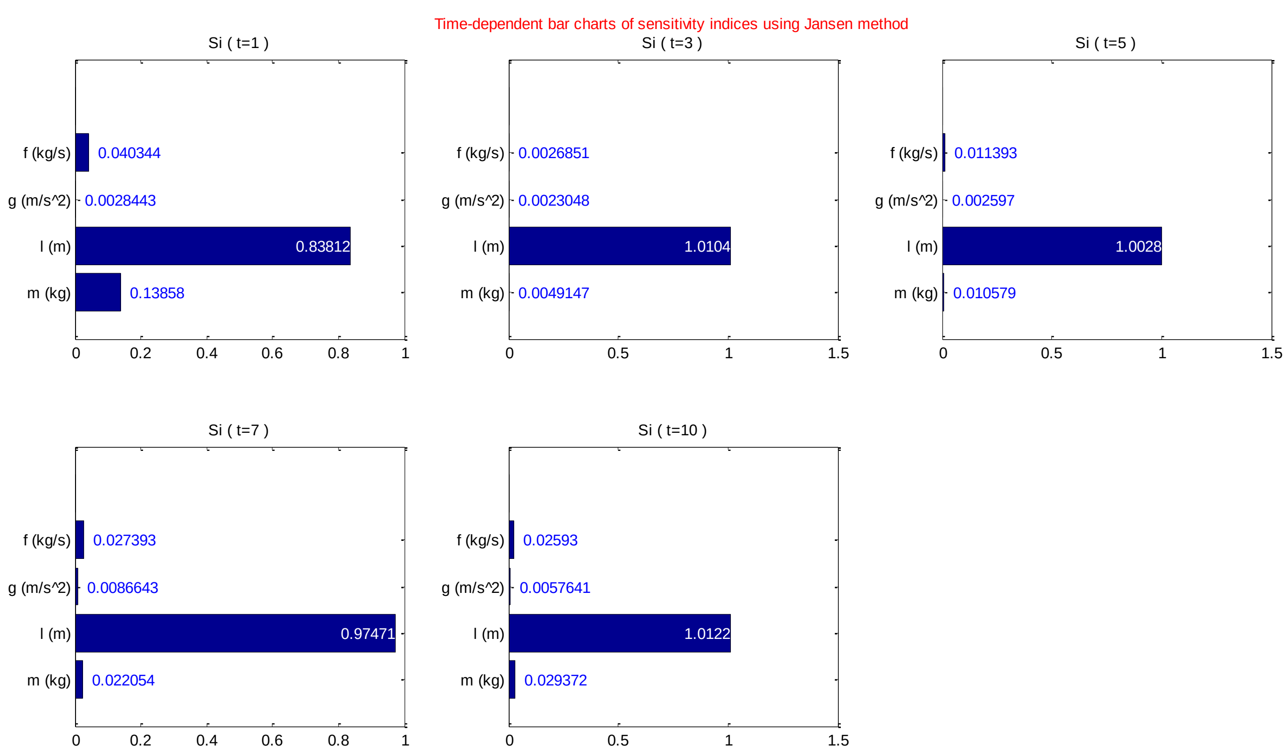


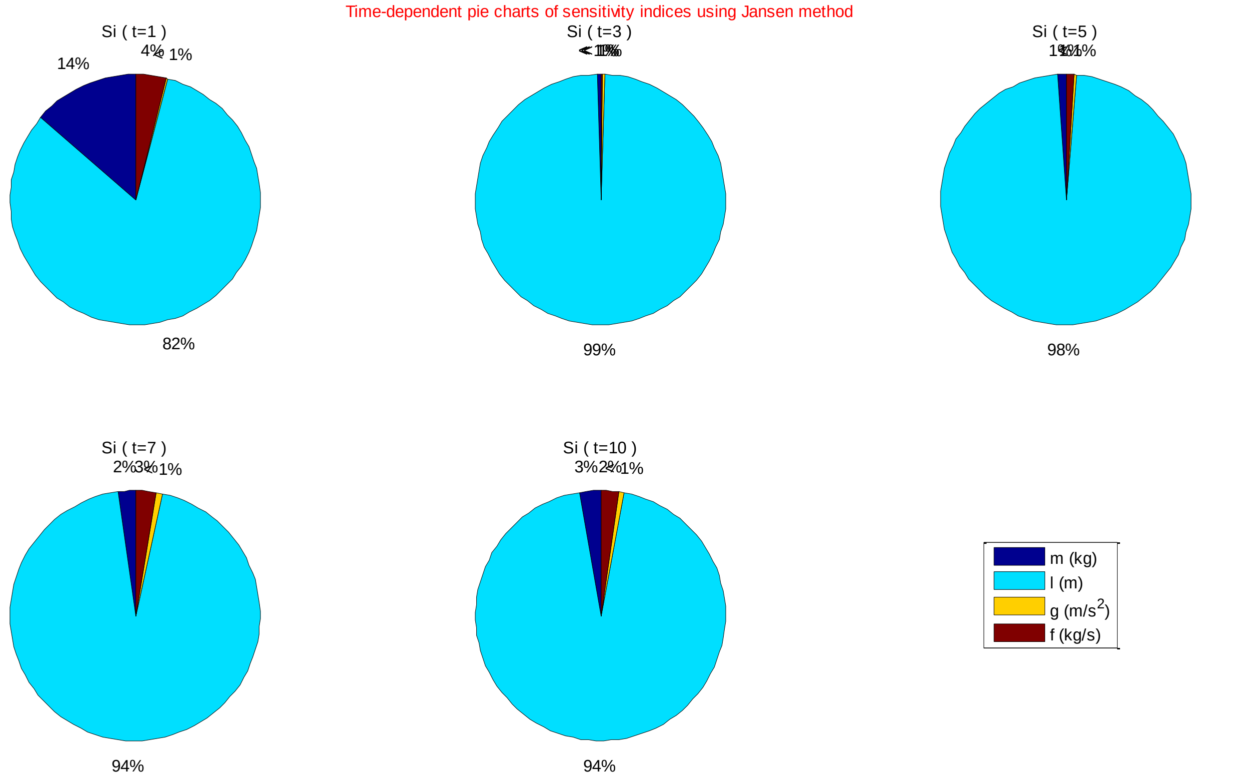


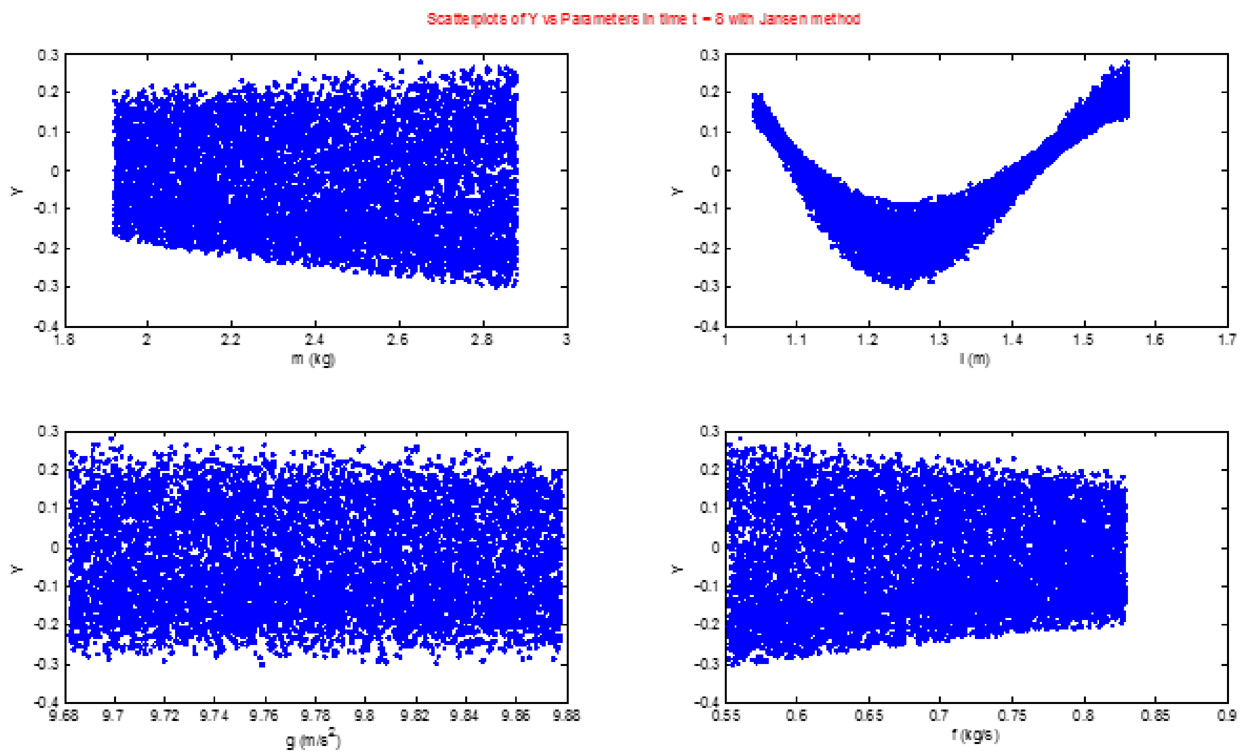
Example 4 (Jansen, pendulum, N = 10000, non-parallel computing)

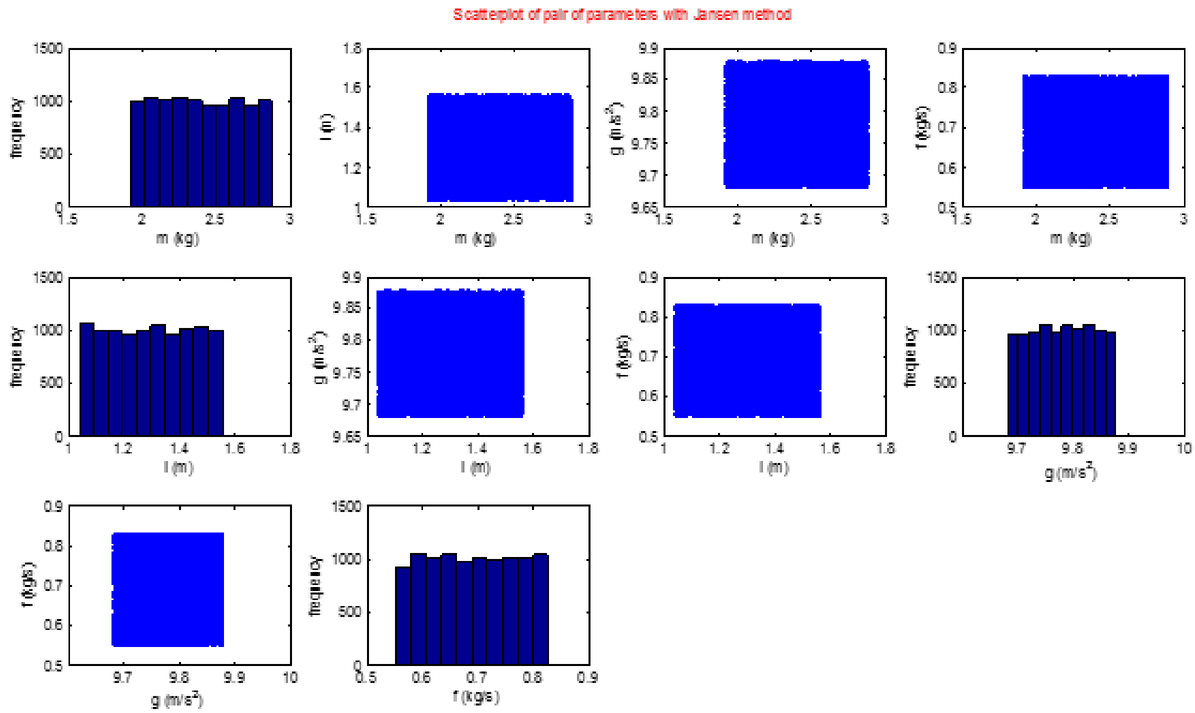
```
Progress: 100%
Estimated processing time (h:m:s): 0:18:30
Remaining time (h:m:s): 0:0:0
Elapsed time (h:m:s): 0:18:30
Estimated stop time (h:m:s): 17:20:31
```





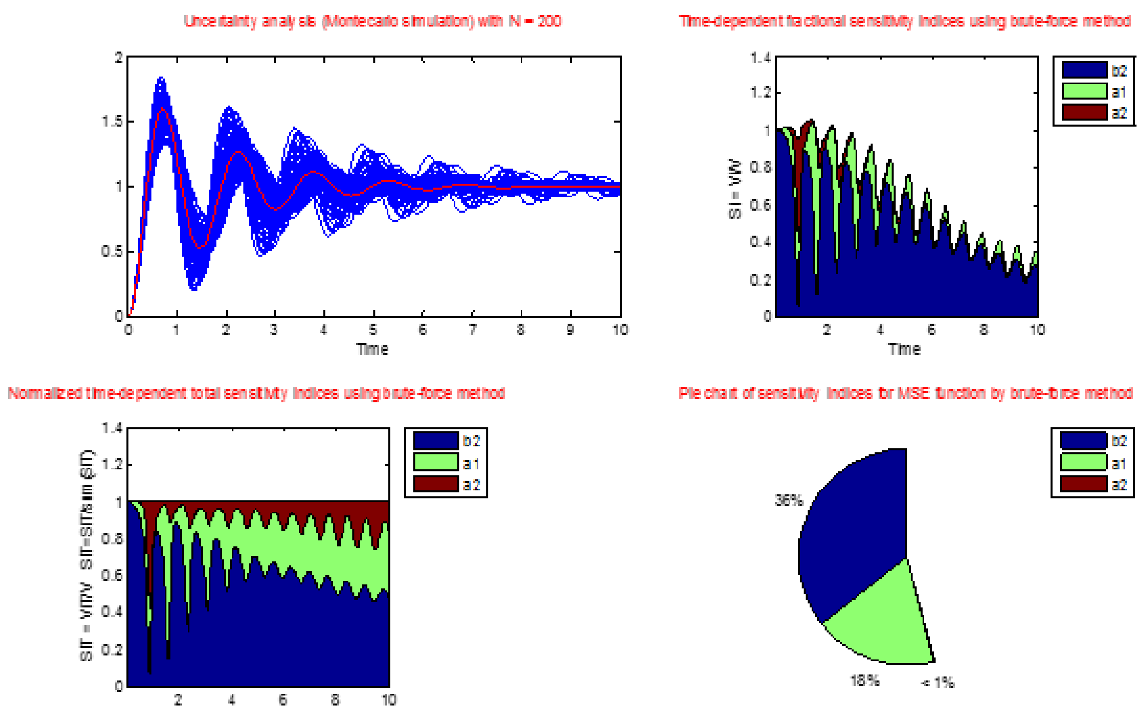

Time-dependent pie charts of sensitivity indices using Jansen method
Si (t=1)
14%
4%
< 1%
82%
Si (t=3)
99%
Si (t=5)
98%
Si (t=7)
2%3%< 1%
94%
Si (t=10)
3%2%< 1%
94%
m (kg)
l (m)
g (m/s2)
f (kg/s)

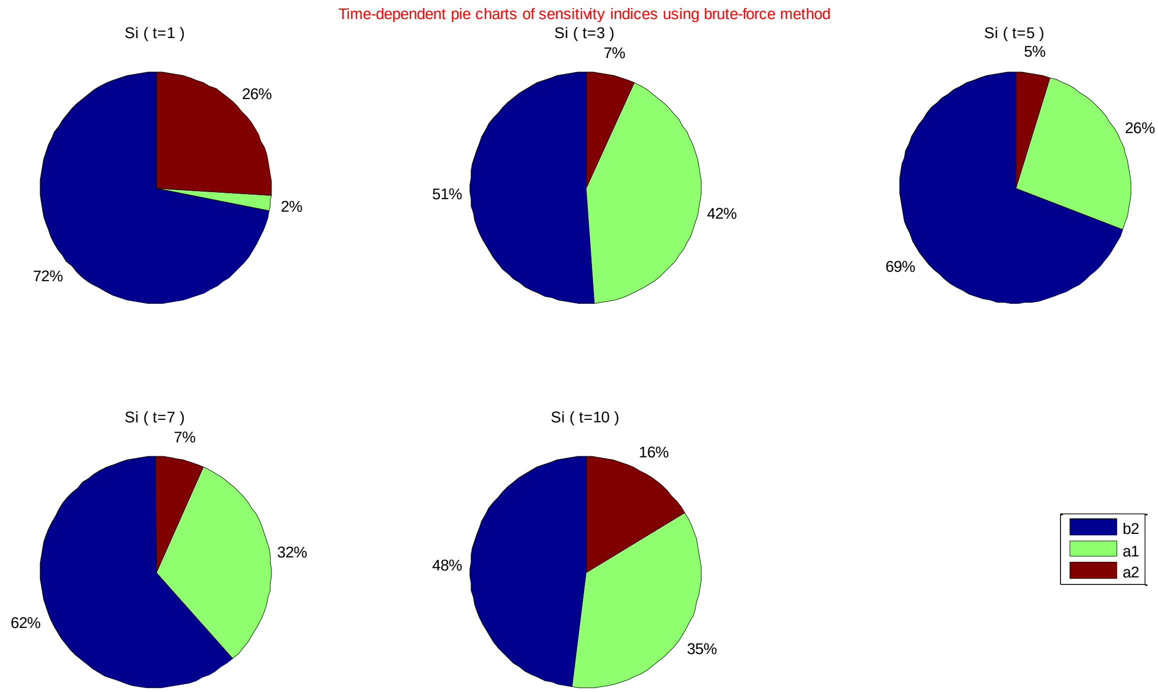

Scatterplots of Y vs Parameters in time t = 8 with Jansen method
Y
m (kg)
l (m)
g (m/s2)
f (kg/s)



Example 5 (brute-force, PID, N = 200, non-parallel computing)

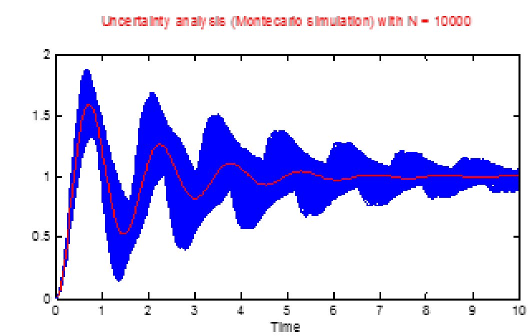
```
Progress: 100%
Estimated processing time (h:m:s): 1:5:7
Remaining time (h:m:s): 0:0:0
Elapsed time (h:m:s): 1:5:7
Estimated stop time (h:m:s): 10:1:13
```

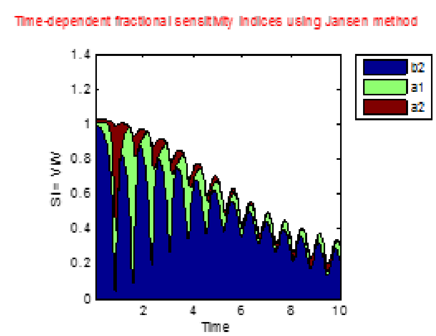


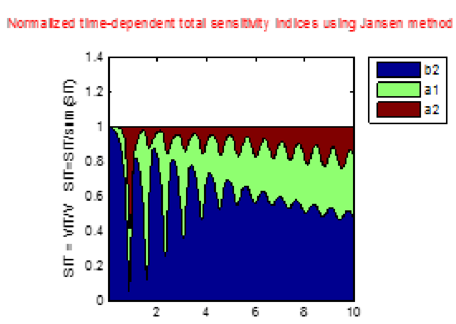


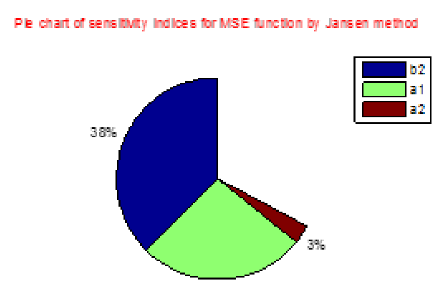
Example 6 (Jansen, PID, N = 10000, non-parallel computing)

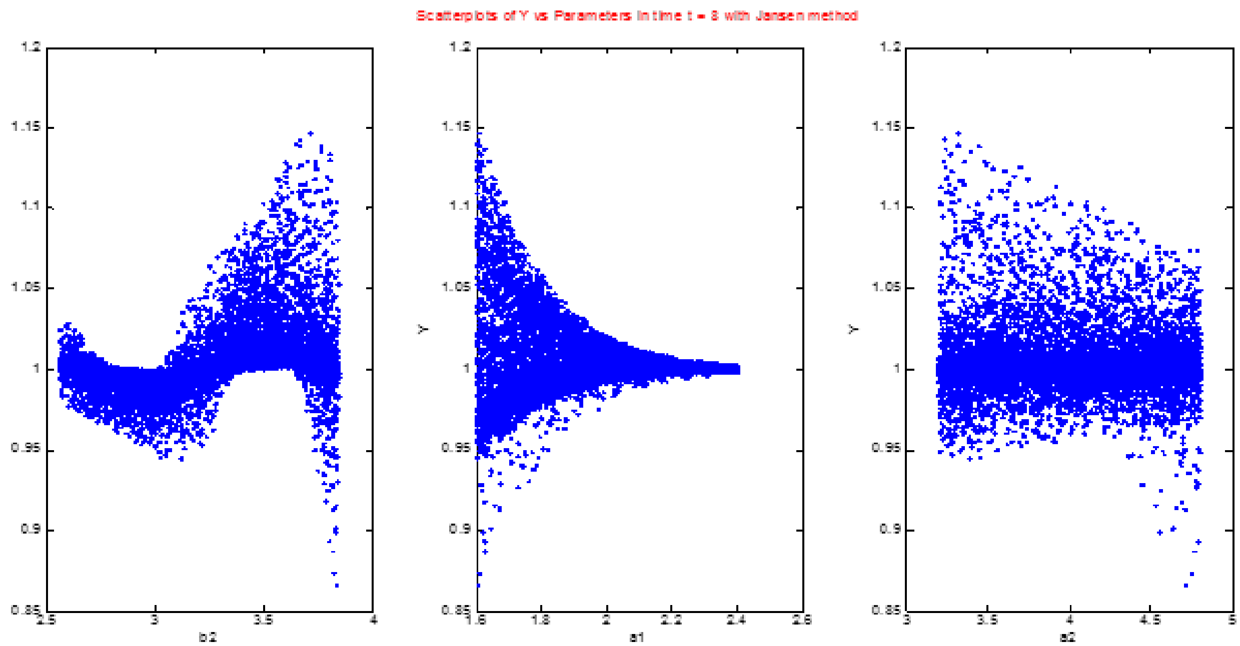
```
Progress: 100%
Estimated processing time (h:m:s): 0:12:12
Remaining time (h:m:s): 0:0:0
Elapsed time (h:m:s): 0:12:12
Estimated stop time (h:m:s): 11:17:7
```

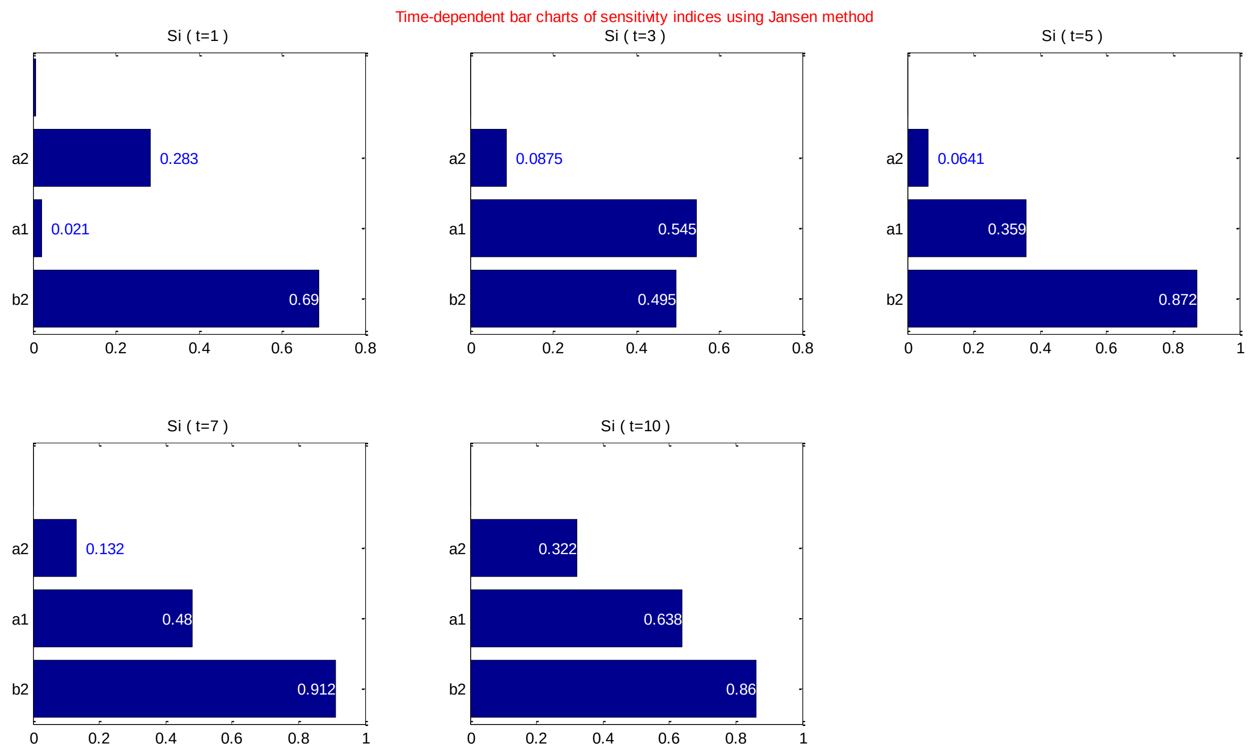









Scatterplots of Y vs Parameters in time t = 8 with Jansen method
Y
b2
a1
a2


Time-dependent bar charts of sensitivity indices using Jansen method
Si (t=1)
a2
a1
b2
0.283
0.021
0.69
Si (t=3)
0.0875
0.545
0.495
Si (t=5)
0.0641
0.359
0.872
Si (t=7)
0.132
0.48
0.912
Si (t=10)
0.322
0.638
0.86