$\varphi :=$ change of orientation $/2$

if $\varphi > \frac{\kappa^2}{2\sigma}$: Euler $\to$ circ $\to$ Euler

if $\varphi \leq \frac{\kappa^2}{2\sigma}$: Euler $\to$ Euler

If $\varphi_{max} = \frac{\kappa^2_{max}}{2\sigma}$

$$x_d = \sqrt{\frac{\pi}{\sigma}}\, C_F\left(\sqrt{\frac{2}{\pi}}\, \varphi_{max}\right)$$

$$y_d = \sqrt{\frac{\pi}{\sigma}}\, S_F\left(\sqrt{\frac{2}{\pi}}\, \varphi_{max}\right)$$

$$X_R = R_{\cancel{max}\ min} \sin(\varphi - \varphi_{max})$$

$$Y_R = 2R_{max} \sin^2\left(\frac{\varphi - \varphi_{max}}{2}\right)$$

$$X_1 = \cos(\varphi - \varphi_{max})\left(x_d + y_d \tan(\varphi - \varphi_{max})\right)$$

$$Y_1 = X_1 \tan(\varphi)$$

$A = (-X_1, Y_1)$

$\cancel{B = (-X_R, Y_R)}$

$B = (-X_2, Y_2)$

$X_2 = X_R$

$Y_2 = \sin(\varphi)(x_d + y_d \tan(\varphi)) - \frac{y_d}{\cos(\varphi)}$

$C = (0, Y_2 - Y_R)$

$D = (X_2, Y_2)$

$E = (X_1, Y_1)$

Once the calculation of the point position is finished, it is possible to fit a Hermite-polinom on them.

If $\varphi \leq \frac{\kappa^2_{max}}{2\sigma}$

$$x_d = \sqrt{\frac{\pi}{\sigma}}\, C_F\left(\sqrt{\frac{2}{\pi}}\, \varphi\right)$$

$$y_d = \sqrt{\frac{\pi}{\sigma}}\, S_F\left(\sqrt{\frac{2}{\pi}}\, \varphi\right)$$

$B = \left(0, \frac{y_d}{\cos\varphi}\right) = C = D$

$A = (-X_1, Y_1)$

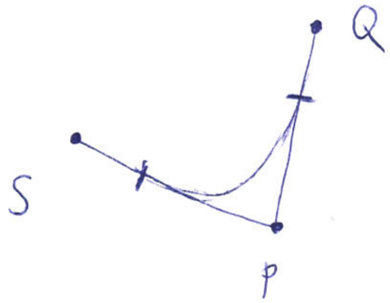
$X_1 = y_d \cdot \sin(\varphi) + x_d \cdot \cos(\varphi)$

$Y_1 = y_d \cdot \cos\varphi + x_d \sin(\varphi)$

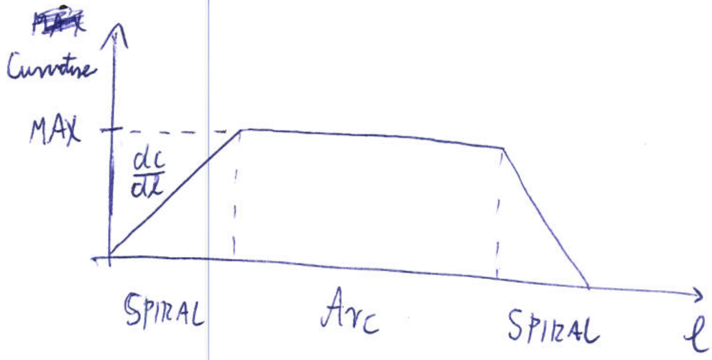
$E = (X_1, Y_1)$

LOCAL PLANNER

① Generate Shape



Clotoide - Arc - Clotoide



$\frac{dc}{dl}$:= MAX Possible Curvature change

(e.g. Turning speed at steering wheel)

2D kinematic model:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{\mathcal{K}} \end{bmatrix} = \begin{bmatrix} \cos\psi \\ \sin\psi \\ \mathcal{K} \\ 0 \end{bmatrix} V + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \beta$$

Coordinate : x, y

Orientation : ψ

Curvature : $\mathcal{K}$

$\frac{d \text{ Curvature}}{dt}$: β

Max speed: V_{max}

Min radius: $R_{min} = \frac{1}{\mathcal{K}_{max}}$

$$\frac{d\mathcal{K}}{dt} = \beta$$

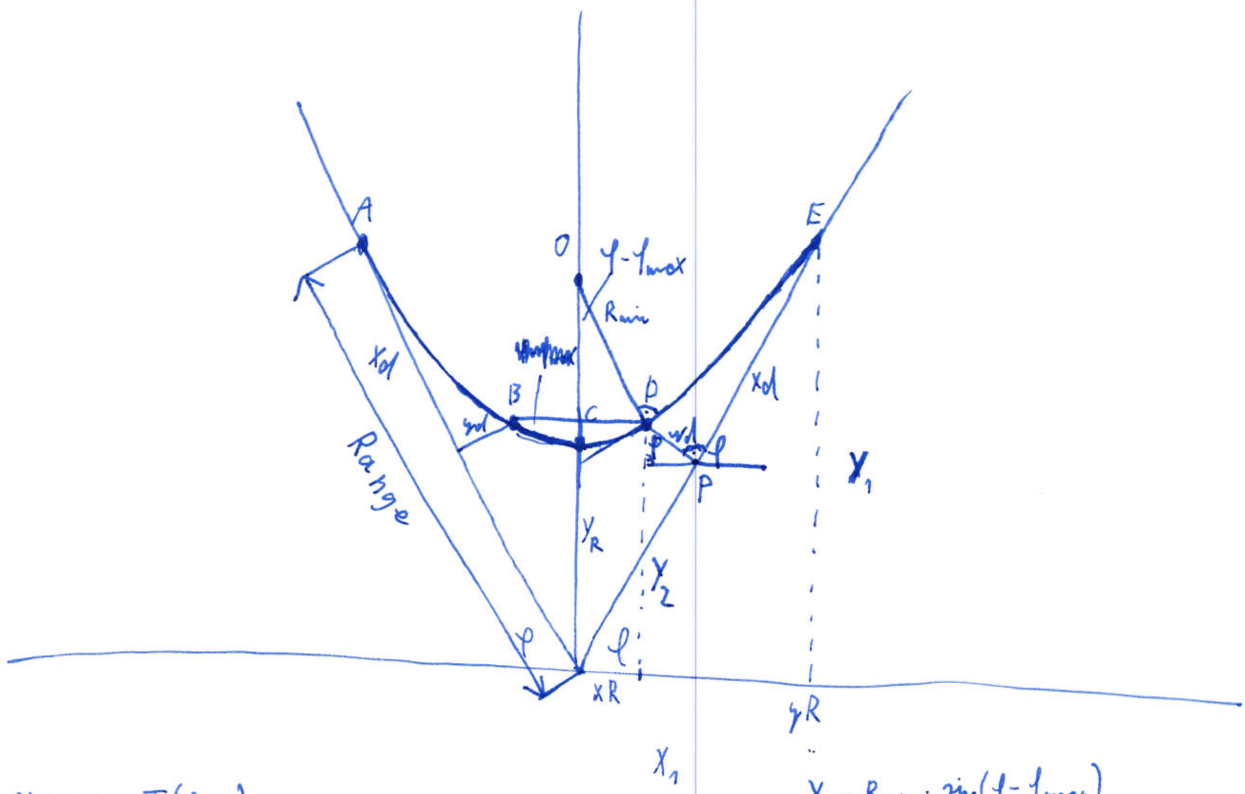
During a turn, the following conditions must be met:

$$\mathcal{K} \leq \mathcal{K}_{max}$$

$$\beta \leq \beta_{max}$$

$R_x(t)$ is the autocorrelation function of a WSS random process $X(t)$ if, and only if $R_x(\tau)$ is a positive semi-definite function

LOCAL PLANNER



$\underline{x_d, y_d = F(\varphi_{max})}$

$$x_d = \sqrt{\frac{\pi}{\sigma}}\, C_F(\varphi_{max})$$

$$y_d = \sqrt{\frac{\pi}{\sigma}}\, S_F(\varphi_{max})$$

$$\cancel{X_R = R_{min} \cdot \sin(\varphi - \varphi_{max})}$$

$$\cancel{Y_R = 2 R_{min} \cdot \sin^2\left(\frac{\varphi - \varphi_{max}}{2}\right)}$$

$$\cancel{X_1 = \cos(\varphi - \varphi_{max})(x_d + y_d \tan(\varphi - \varphi_{max}))}$$

$$\cancel{Y_1 = X_1 \tan(\varphi)}$$

$$\cancel{X_2 = X_R}$$

$$\cancel{Y_2 = \sin(\varphi) \cdot (x_d + y_d \tan(\varphi)) - \frac{y_d}{\cos\varphi}}$$

$$X_R = R_{min} \cdot \sin(\varphi - \varphi_{max})$$

$$P_X = X_R + y_d \cdot \sin(\varphi)$$

$$X_1 = P_X + x_d \cdot \cos(\varphi)$$

$$Y_1 = X_1 \cdot \tan(\varphi)$$

$$P_y = Y_1 - x_d \sin(\varphi)$$

$$Y_2 = P_y + y_d \cdot \cos(\varphi)$$

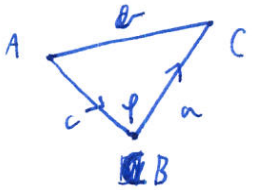
$$O_y = R_{min} \cdot \cos(\varphi - \varphi_{max}) + Y_2$$

$$R_y = O_y - R_{min}$$

$$X_2 = X_R$$

RANDOM REFERENCES

#1. Calculate angles:



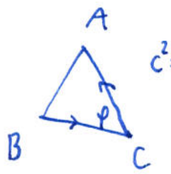
Law of cosines:

$$b^2 = a^2 + c^2 - 2ab\cos(\varphi)$$

$$\varphi = \operatorname{acos} \frac{a^2 + c^2 - b^2}{2ac}$$

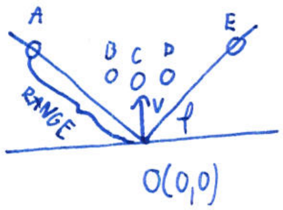
$$\varphi = \operatorname{acos}\left[\frac{(B_x - C_x)^2 + (B_y - C_y)^2 + (A_x - B_x)^2 + (A_y - B_y)^2 - (A_x - C_x)^2 - (A_y - C_y)^2}{2\sqrt{(B_x - C_x)^2 + (B_y - C_y)^2} \cdot \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}}\right]$$

Coded with the following notation:

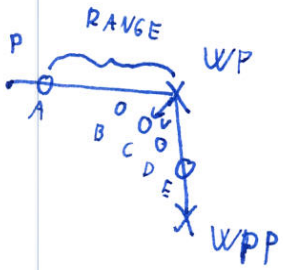


$$c^2 = a^2 + b^2 - 2ab\cos(\varphi)$$

#2: Linear transform of Poly-Point coordinates



$\xrightarrow{T}$



$$\underline{R_a} = \overrightarrow{OA} = (A_x, A_y)$$

$$\underline{R_e} = \overrightarrow{OE} = (E_x, E_y)$$

$$\underline{v} = \underline{R_a} + \underline{R_e} = (A_x + E_x, A_y + E_y)$$

$$\alpha = \operatorname{atan}\left(\frac{A_y + E_y}{A_x + E_x}\right)$$

$$\left(\alpha = \frac{\pi}{2}\right)$$

$$\underline{R_P} = \overrightarrow{WP\,P} = (P_x - WP_x, P_y - WP_y)$$

$$\underline{R_{WPP}} = \overrightarrow{WP\,WPP} = (WPP_x - WP_x, WPP_y - WP_y)$$

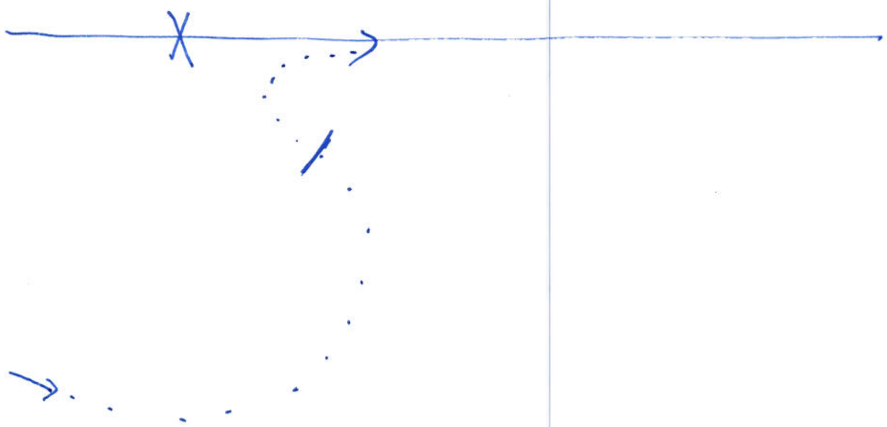
$$\underline{v} = \underline{R_P} + \underline{R_{WPP}} = (P_x + WPP_x - 2WP_x, P_y + WPP_y - 2WP_y)$$

$$\delta = \operatorname{atan}\left(\frac{P_y + WPP_y - 2WP_y}{P_x + WPP_x - 2WP_x}\right)$$

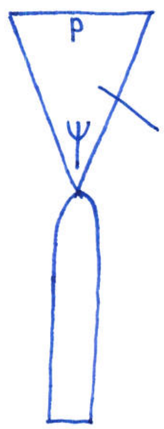
$$T = \left[\operatorname{rot}(\delta - \alpha) + \operatorname{offset}(WP - O)\right]$$

Deviation from Path:

No points in the scope ⟶ Need to generate some



NAVIGATION



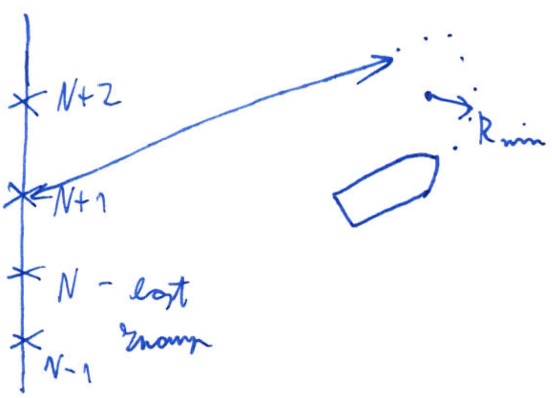
OUTDATED

- The ship always follows the Sub-W/P with the highest Order in the scope.

- If there is no Sub-W/P in the scope, the ship has left the path and must be guided back, therefore we generate new Sub-W/Ps into the scope.

Deviation:

- The last seen highest Sub-W/P order is known.



- The ship is set to a circular path until it faces N+1 **OR** it **SPOTS** a Sub-W/P $\geq N+1$

- Then a dummy W/P is generated towards N+1, and a local path is generated with N+1 as a turn point