$$
\begin{aligned}
\frac{(m+M)\,\ddot{X}}{Ml} =& -\left(\mathrm{s}_\phi \mathrm{c}_\chi \mathrm{s}_\theta - \mathrm{c}_\phi \mathrm{s}_\chi\right)\left(\dot{\phi}^2 + \dot{\chi}^2\right) - \mathrm{s}_\phi \mathrm{c}_\chi \mathrm{s}_\theta \dot{\theta}^2 \\
&- 2\left(\mathrm{c}_\phi \mathrm{s}_\chi \mathrm{s}_\theta - \mathrm{s}_\phi \mathrm{c}_\chi\right)\dot{\phi}\dot{\chi} \\
&+ 2\mathrm{c}_\theta \dot{\theta}\left(\mathrm{c}_\phi \mathrm{c}_\chi \dot{\phi} - \mathrm{s}_\phi \mathrm{s}_\chi \dot{\chi}\right) \\
&+ \left(\mathrm{c}_\phi \mathrm{c}_\chi \mathrm{s}_\theta + \mathrm{s}_\phi \mathrm{s}_\chi\right)\ddot{\phi} \\
&- \left(\mathrm{s}_\phi \mathrm{s}_\chi \mathrm{s}_\theta + \mathrm{c}_\phi \mathrm{c}_\chi\right)\ddot{\chi} \\
&+ \mathrm{s}_\phi \mathrm{c}_\chi \mathrm{c}_\theta \ddot{\theta} + \lambda_1 / Ml
\end{aligned}
$$

$$
\begin{aligned}
\frac{(m+M)\ddot{Y}}{Ml} =& (\mathrm{c}_\phi \mathrm{c}_\chi \mathrm{s}_\theta + \mathrm{s}_\phi \mathrm{s}_\chi)(\dot{\phi}^2 + \dot{\chi}^2) + \mathrm{c}_\phi \mathrm{c}_\chi \mathrm{s}_\theta \dot{\theta}^2 \\
&- 2(\mathrm{s}_\phi \mathrm{s}_\chi \mathrm{s}_\theta + \mathrm{c}_\phi \mathrm{c}_\chi)\dot{\phi}\dot{\chi} \\
&+ 2\mathrm{c}_\theta \dot{\theta}(\mathrm{s}_\phi \mathrm{c}_\chi \dot{\phi} + \mathrm{c}_\phi \mathrm{s}_\chi \dot{\chi}) \\
&+ (\mathrm{s}_\phi \mathrm{c}_\chi \mathrm{s}_\theta - \mathrm{c}_\phi \mathrm{s}_\chi)\ddot{\phi} \\
&+ (\mathrm{c}_\phi \mathrm{s}_\chi \mathrm{s}_\theta - \mathrm{s}_\phi \mathrm{c}_\chi)\ddot{\chi} \\
&- \mathrm{c}_\theta \mathrm{c}_\phi \mathrm{c}_\chi \ddot{\theta} + \lambda_2 / Ml
\end{aligned}
$$

$$
\begin{aligned}
\left(mr^2 \mathrm{c}_\theta^2 + Mr\mathrm{s}_\theta^2\left(r + 2l\mathrm{c}_\chi\right) + Ml^2 \mathrm{c}_\chi^2 + I_{\mathrm{tr}}\right)\ddot{\theta} =& \mathrm{c}_\theta \mathrm{s}_\theta \Big\{\left(Ml^2 \mathrm{c}_\chi^2 + I_{\mathrm{ax}} - I_{\mathrm{tr}}\right)\dot{\phi}^2 \\
&\quad - Mlr\mathrm{c}_\chi \dot{\chi}^2 \\
&\quad + \left(mr^2 - Mr^2 - 2rl\mathrm{c}_\chi\right)\dot{\theta}^2\Big\} \\
&- 2Ml^2 \mathrm{c}_\chi^2 \mathrm{c}_\theta \dot{\phi}\dot{\chi} + 2Ml\left(l\mathrm{c}_\chi + r\mathrm{s}_\theta^2\right)\mathrm{s}_\chi \dot{\theta}\dot{\chi} - \mathrm{c}_\theta I_{\mathrm{ax}} \dot{\phi}\dot{\psi} \\
&+ Ml\mathrm{c}_\theta\left(\mathrm{s}_\phi \mathrm{c}_\chi \ddot{X} - \mathrm{c}_\phi \mathrm{c}_\chi \ddot{Y} - l\mathrm{s}_\chi \mathrm{c}_\chi \ddot{\phi} - r\mathrm{s}_\chi \mathrm{s}_\theta \ddot{\chi}\right) \\
&+ g\left(mr + M\left(r + l\mathrm{c}_\chi\right)\right)\mathrm{s}_\theta \\
&+ r\mathrm{c}_\theta(\mathrm{s}_\phi \lambda_1 - \mathrm{c}_\phi \lambda_2)
\end{aligned}
$$

$$
\begin{aligned}
\left(I_{\mathrm{tr}}\mathrm{c}_\theta^2 + I_{\mathrm{ax}}\mathrm{s}_\theta^2 + Ml^2\left(\mathrm{c}_\chi^2 \mathrm{s}_\theta^2 + \mathrm{s}_\chi^2\right)\right)\ddot{\phi} =& I_{\mathrm{tr}}\mathrm{c}_{2\theta}\dot{\theta}\dot{\phi} + I_{\mathrm{ax}}\left(\mathrm{c}_\theta \dot{\theta}\dot{\psi} + \mathrm{s}_\theta \ddot{\psi} - \mathrm{c}_{2\theta}\dot{\theta}\dot{\phi}\right) \\
&+ Ml\left(\mathrm{c}_\phi \mathrm{c}_\chi \mathrm{s}_\theta + \mathrm{s}_\phi \mathrm{s}_\chi\right)\ddot{X} \\
&+ Ml\left(\mathrm{s}_\phi \mathrm{c}_\chi \mathrm{s}_\theta - \mathrm{c}_\phi \mathrm{s}_\chi\right)\ddot{Y} \\
&+ Ml^2\left(\mathrm{c}_\chi \mathrm{s}_\chi \mathrm{s}_\theta \dot{\theta}^2 - \mathrm{s}_{2\chi}\mathrm{c}_\theta^2 \dot{\phi}\dot{\chi} - \mathrm{c}_\chi^2 \mathrm{s}_{2\theta}\dot{\phi}\dot{\theta} + 2\mathrm{s}_\chi^2 \mathrm{c}_\theta \dot{\chi}\dot{\theta} - \mathrm{c}_\chi \mathrm{s}_\chi \mathrm{c}_\theta \ddot{\theta} + \mathrm{s}_\theta \ddot{\chi}\right) \\
&+ r\mathrm{s}_\theta\left(\mathrm{c}_\phi \lambda_1 - \mathrm{s}_\phi \lambda_2\right)
\end{aligned}
$$

$$
I_{\mathrm{ax}}\ddot{\psi} = I_{\mathrm{ax}}\left(\mathrm{c}_\theta \dot{\theta}\dot{\phi} + \mathrm{s}_\theta \ddot{\phi}\right) - r\left(\mathrm{c}_\phi \lambda_1 + \mathrm{s}_\phi \lambda_2\right)
$$

$$
\begin{aligned}
l\ddot{\chi} =& l\mathrm{s}_\theta \ddot{\phi} - r\mathrm{s}_\theta \mathrm{c}_\theta \mathrm{s}_\chi \ddot{\theta} \\
&- (\mathrm{s}_\phi \mathrm{s}_\chi \mathrm{s}_\theta + \mathrm{c}_\phi \mathrm{c}_\chi)\ddot{X} + (\mathrm{c}_\phi \mathrm{s}_\chi \mathrm{s}_\theta - \mathrm{s}_\phi \mathrm{c}_\chi)\ddot{Y} \\
&- (r\mathrm{c}_\theta^2 \mathrm{s}_\chi + l\mathrm{c}_\chi \mathrm{s}_\chi)\dot{\theta}^2 - l\mathrm{s}_\chi \mathrm{c}_\chi \mathrm{c}_\theta^2 \dot{\phi}^2 \\
&- 2l\mathrm{c}_\chi^2 \mathrm{c}_\theta \dot{\theta}\dot{\phi} + g\mathrm{c}_\theta \mathrm{s}_\chi
\end{aligned}
$$