1 Tableaux for classical propositional logic
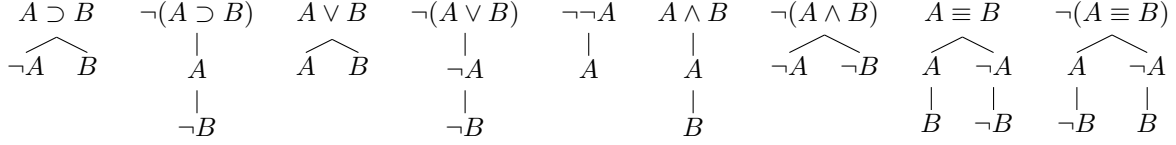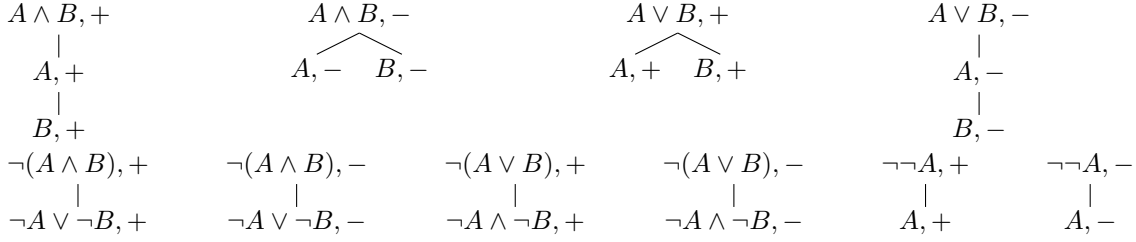
Initial list for $A_1, ... A_n \models B$ is with $\neg B$. Closure condition: A and $\neg A$ for some formula A occur on a branch.

$$
\begin{array}{ccccccccc}
A \supset B & \neg(A \supset B) & A \vee B & \neg(A \vee B) & \neg\neg A & A \wedge B & \neg(A \wedge B) & A \equiv B & \neg(A \equiv B) \\
\end{array}
$$

$A \supset B$ branches to $\neg A \quad B$

$\neg(A \supset B)$: A, then $\neg B$

$A \vee B$ branches to $A \quad B$

$\neg(A \vee B)$: $\neg A$, then $\neg B$

$\neg\neg A$: A

$A \wedge B$: A, then B

$\neg(A \wedge B)$ branches to $\neg A \quad \neg B$

$A \equiv B$ branches to (A, B) $\quad$ ($\neg A$, $\neg B$)

$\neg(A \equiv B)$ branches to (A, $\neg B$) $\quad$ ($\neg A$, B)

2 Semantic tableaux for FDE

Initial list for $A_1, ... A_n \models B$ is with all $A_n, +$ and $B, -$. Closure condition: $A, +$ and $A, -$ for some formula A occur on a branch.

$A \wedge B, +$: $A, +$ then $B, +$

$A \wedge B, -$ branches to $A, - \quad B, -$

$A \vee B, +$ branches to $A, + \quad B, +$

$A \vee B, -$: $A, -$ then $B, -$

$\neg(A \wedge B), +$: $\neg A \vee \neg B, +$

$\neg(A \wedge B), -$: $\neg A \vee \neg B, -$

$\neg(A \vee B), +$: $\neg A \wedge \neg B, +$

$\neg(A \vee B), -$: $\neg A \wedge \neg B, -$

$\neg\neg A, +$: $A, +$

$\neg\neg A, -$: $A, -$

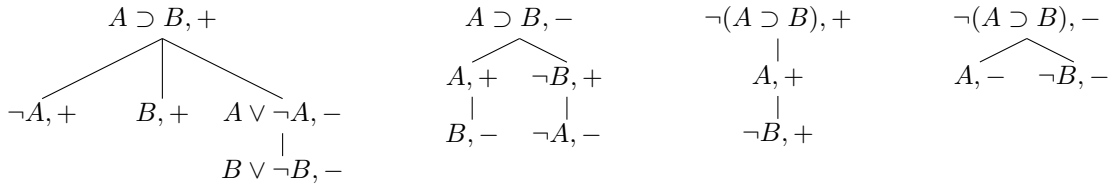# 3 Semantic tableaux for K_3, LP, L_3, RM_3

3.1 K_3

Same as for FDE with extra closure condition: A branch also closes if it contains $A, +$ and $\neg A, +$ for some formula A.

3.2 LP

Same as for FDE with extra closure condition: A branch also closes if it contains $A, -$ and $\neg A, -$ for some formula A.
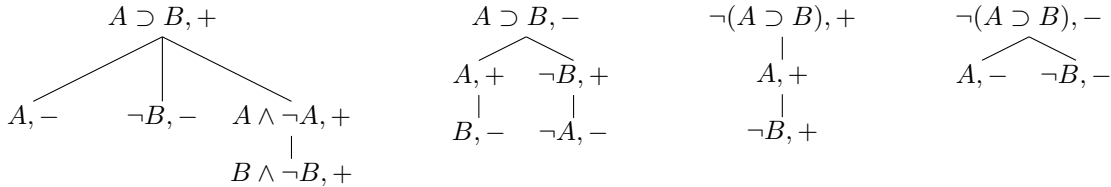
3.3 L_3
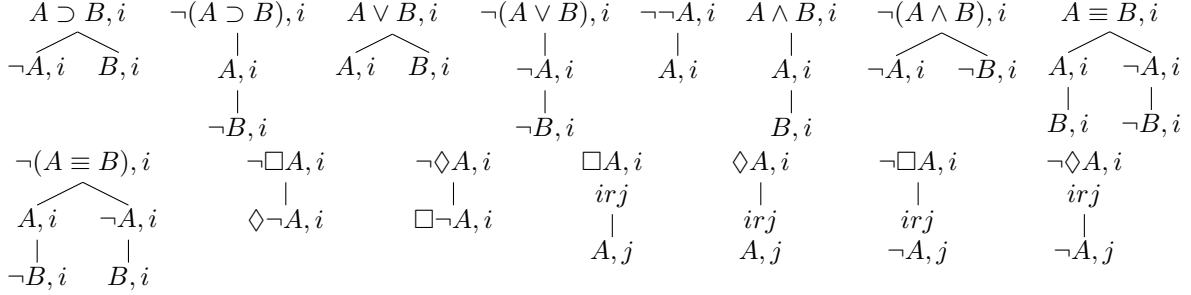
Like K_3 but with the following rules for $\supset$:

$A \supset B, +$ branches to $\neg A, + \quad B, + \quad (A \vee \neg A, - \text{ then } B \vee \neg B, -)$

$A \supset B, -$ branches to ($A, +$ then $B, -$) $\quad$ ($\neg B, +$ then $\neg A, -$)

$\neg(A \supset B), +$: $A, +$ then $\neg B, +$

$\neg(A \supset B), -$ branches to $A, - \quad \neg B, -$

3.4 RM_3

Like LP but with the following rules for $\supset$:

$A \supset B, +$ branches to $A, - \quad \neg B, - \quad (A \wedge \neg A, + \text{ then } B \wedge \neg B, +)$

$A \supset B, -$ branches to ($A, +$ then $B, -$) $\quad$ ($\neg B, +$ then $\neg A, -$)

$\neg(A \supset B), +$: $A, +$ then $\neg B, +$

$\neg(A \supset B), -$ branches to $A, - \quad \neg B, -$

4 Tableaux for modal logic

Initial list for $A_1, ...A_n \models B$ is with all $A_n, 0$ and $\neg B, 0$. Closure condition: A, i and $\neg A, i$ for some formula A occur on a branch with the same number i.

$$
\begin{array}{llllllll}
A \supset B, i & \neg(A \supset B), i & A \vee B, i & \neg(A \vee B), i & \neg\neg A, i & A \wedge B, i & \neg(A \wedge B), i & A \equiv B, i \\
\diagup\!\!\diagdown & | & \diagup\!\!\diagdown & | & | & | & \diagup\!\!\diagdown & \diagup\!\!\diagdown \\
\neg A, i \quad B, i & A, i & A, i \quad B, i & \neg A, i & A, i & A, i & \neg A, i \quad \neg B, i & A, i \quad \neg A, i \\
& \neg B, i & & \neg B, i & & B, i & & B, i \quad \neg B, i
\end{array}
$$

$$
\begin{array}{llllllll}
\neg(A \equiv B), i & \neg\square A, i & \neg\lozenge A, i & \square A, i & \lozenge A, i & \neg\square A, i & \neg\lozenge A, i \\
\diagup\!\!\diagdown & | & | & irj & | & | & irj \\
A, i \quad \neg A, i & \lozenge\neg A, i & \square\neg A, i & | & irj & irj & | \\
\neg B, i \quad B, i & & & A, j & A, j & \neg A, j & \neg A, j
\end{array}
$$

4.1 Restrictions

Reflexive (ρ):

$$
\frac{}{iri}
$$

Symmetric (σ):

$$
\frac{irj}{jri}
$$

Extendable (η):

$$
\frac{}{irj}
$$

Transitive (τ):

$$
\frac{irj \quad jrk}{irk}
$$

Universal (υ):

$$
\frac{\lozenge A, i}{A, j} \qquad \frac{\square A, i}{A, j}
$$

Dense (δ):

$$
\frac{irj}{irk \quad krj}
$$

Extra rules for $=$:

$$
\frac{\alpha(i) \quad i = j}{\alpha(j)} \qquad \frac{\alpha(i) \quad j = i}{\alpha(j)}
$$

Forward convergent (φ):

$$
\frac{irj \quad irk}{jrk \quad j = k \quad krj}
$$

Backward convergent (β):

$$
\frac{jri \quad kri}{jrk \quad j = k \quad krj}
$$

4.2 Tense logic

$$
\begin{array}{llllllll}
[F]A, i & \langle F\rangle A, i & \neg[F]A, i & \neg\langle F\rangle A, i & [P]A, i & \langle P\rangle A, i & \neg[P]A, i & \neg\langle P\rangle A, i \\
irj & | & | & | & jri & | & | & | \\
| & irj & \langle F\rangle\neg A, i & [F]\neg A, i & | & jri & \langle P\rangle\neg A, i & [P]\neg A, i \\
A, j & A, j & & & A, j & A, j
\end{array}
$$

5 First-order modal logic

5.1 CK

$$
\begin{array}{llll}
\neg\exists x A, i & \neg\forall x A, i & \forall x A, i & \exists x A, i \\
| & | & | & | \\
\forall x \neg A, i & \exists x \neg A, i & A_x(a), i & A_x(c), i \\
& & \text{where } a \text{ is old (if possible);} & \text{where } c \text{ is new} \\
& & \text{otherwise introduce an } a
\end{array}
$$

5.2 VK

$$
\begin{array}{ll}
\forall x A, i & \exists x A, i \\
\diagup\!\!\diagdown & | \\
\neg\mathfrak{E}A, i \quad A_x(a), i & \mathfrak{E}c, i \\
& | \\
& A_x(c), i
\end{array}
$$

a is old, if possible; otherwise introduce a new a, c is a new variable