KrySBAS: Krylov Subspace-Based Adaptive Solvers

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Introduction

In many applications arising in scientific computing, we must solve a linear system of the type $A\mathbf{x} = \mathbf{b}$, where A is a sparse, often non-symmetric and potentially ill-conditioned matrix, $\mathbf{x}$ is the unknown solution and $\mathbf{b}$ is the vector of known data. See Figure 1 for a typical system.

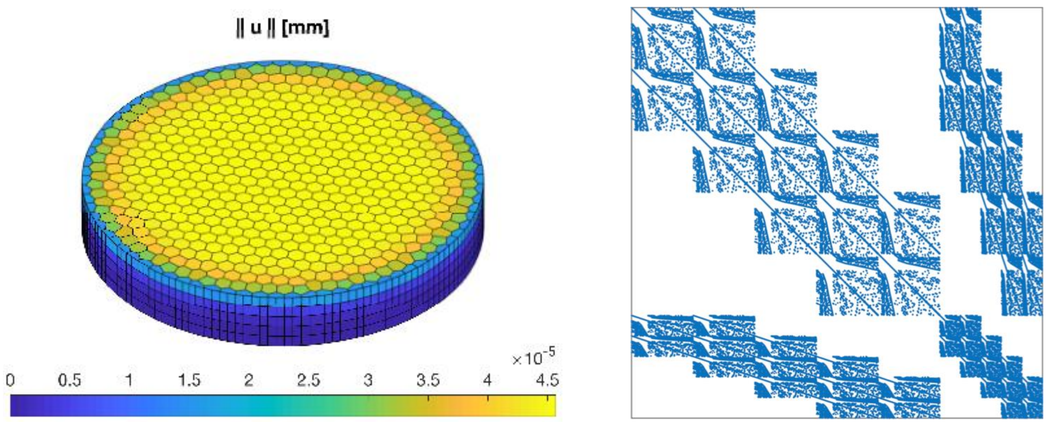

Figure 1: Left: Magnitude of the displacement field after solving the equations of unsaturated flow in deformable porous media in the case of evaporation [1]. Right: Sparsity of the linearized system of equations resulting from a multipoint flux / multipoint stress finite volume discretization [1].

Solving large systems with direct solvers is not feasible due to computational limitations. One therefore employs indirect solvers such as GMRES or its restarted version GMRES(m) [1]. However, when the linear systems are ill-conditioned, GMRES(m) can exhibit low convergence at best or stagnation at worst. One potential remedy to this problem is to employ *adaptive* solvers.

The need for KrySBAS

Several adaptive solvers has been proposed in the last three decades. However, many of such solvers are currently not available to the scientific community. We were therefore motivated to develop KrySBAS, an open-source toolbox written for MATLAB/OCTAVE containing a collection of adaptive solvers based on Krylov subspaces. KrySBAS is developed under GPL v.3.0 license and is available at:

`www.github.com/nidtec-una/krysbas-dev`

Installation

To install KrySBAS, simply clone the GitHub repository, and add the folder to your MATLAB or OCTAVE path.

Available solvers

KrySBAS currently provides three adaptive solvers:

- PD-GMRES [2]
- LGMRES [3]
- GMRES-E [4]

New solvers will be included shortly.

Testing, coverage and documentation

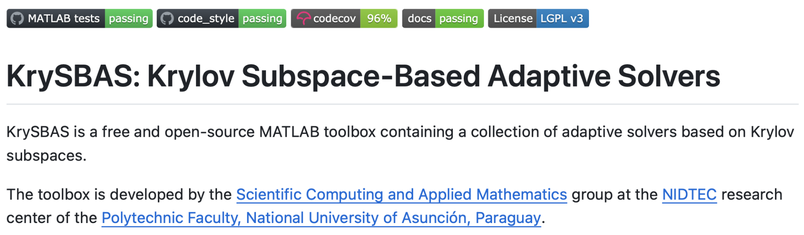

Figure 2: KrySBAS is thoroughly tested with automated workflows and coverage reports. In addition, all the funcitionality is carefully documented and available through read-the-docs.

Numerical examples

Convergence of available solvers for two benchmark matrices arising in fluid dynamics problems: sherman4 and sherman5.

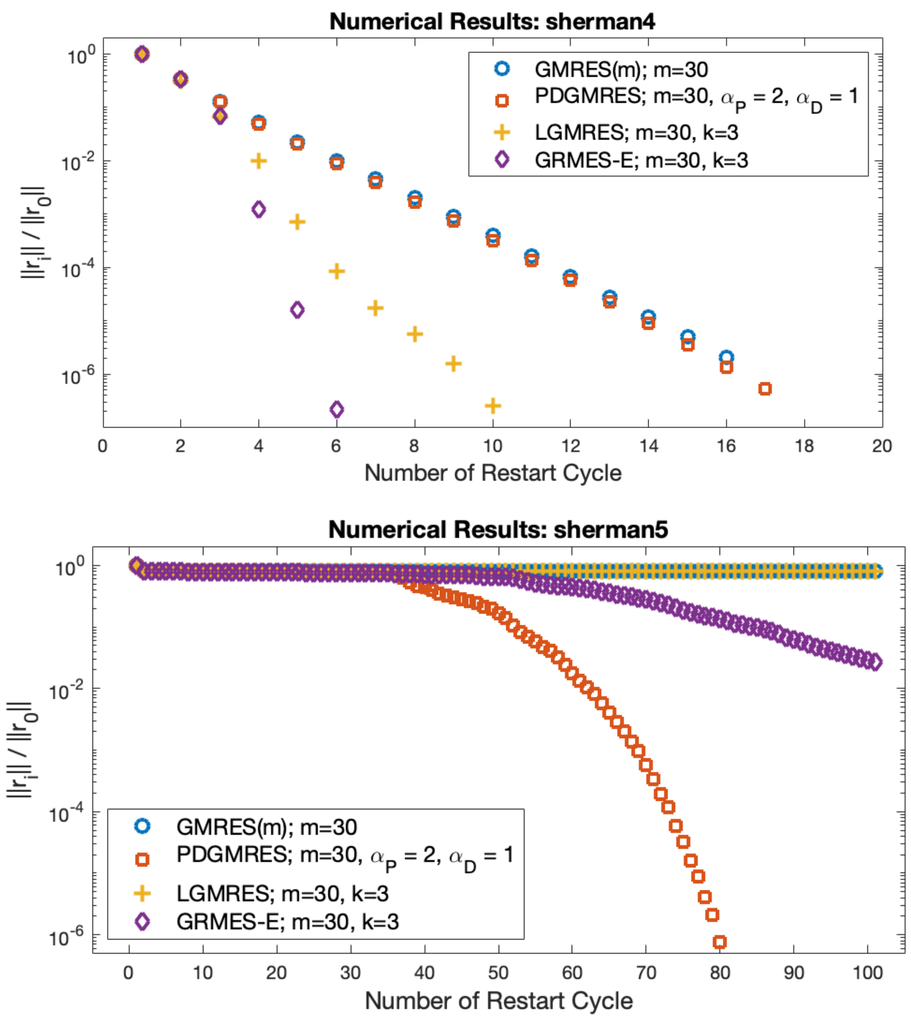

Figure 3: Relative residual norms as a function of the outter iterations for GMRES(m), PDGMRES, LGMRES, and GMRES-E. Top: Analysis for the matrix sherman4. Bottom: Analysis for sherman5.

References

[1] Varela, J., Gasda, S. E., Keilegavlen, E., & Nordbotten, J. M. (2021). A finite-volume-based module for unsaturated poroelasticity. *Advanced Modeling with the MATLAB Reservoir Simulation Toolbox*.
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Acknowledgements

JV is funded by CONACYT trough the project PRIA01-8 and the FPUNA . GE is funded by CONACYT through the Incentive for the Training of Researchers in National Postgraduate Programs.