Exercises Week 6

(1) $y[n] = \underbrace{-0.1}_{-a_1} y[n-1] + \underbrace{0.2}_{-a_2} y[n-2] + \underbrace{1}_{b_0} x[n] + \underbrace{1}_{b_1} x[n-1]$

a) $H(z) = \dfrac{\overbrace{1}^{b_0} + \overbrace{1 \cdot z^{-1}}^{b_1}}{1 + 0.1 \cdot z^{-1} - 0.2 z^{-2}} = \dfrac{1 + z^{-1}}{1 + 0.1 z^{-1} - 0.2 z^{-2}} \cdot \dfrac{(\cdot z^2)}{(\cdot z^2)} = \dfrac{z^2 + z}{z^2 + 0.1 z - 0.2}$

$z^2 + z = 0 \Leftrightarrow z(z+1) = 0$

Z: $z_1 = 0$, $z_2 = -1$

P: $z^2 + 0.1 z - 0.2 = 0$

$= \dfrac{-0.1 \pm \sqrt{0.1^2 + 0.8}}{2} = \dfrac{-0.1 \pm 0.9}{2}$

$= \begin{cases} 0.4 = p_1 \\ -0.5 = p_2 \end{cases}$

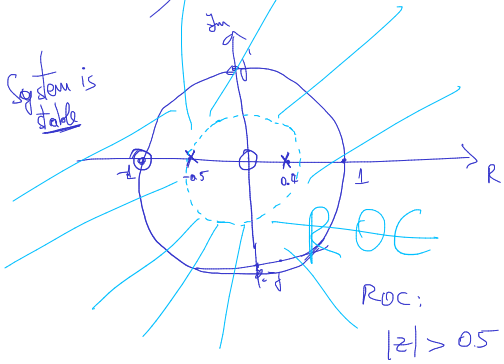
$y[n] = -0.1\, y[n-1] + 0.2\, y[n-2] + x[n] + x[n-1]$

$Y(z) = -0.1 \cdot z^{-1} \cdot Y(z) + 0.2 \cdot z^{-2} \cdot Y(z) + X(z) + z^{-1} X(z)$

$Y(z)\left(1 + 0.1 z^{-1} - 0.2 z^{-2}\right) = X(z)\left(1 + z^{-1}\right) \Rightarrow H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1 + z^{-1}}{1 + 0.1 z^{-1} - 0.2 z^{-2}}$

$(z - 0.4)(z + 0.5)$

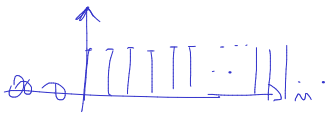
System is causal
Poles inside unit circle
$\Rightarrow$ System is stable



ROC: $|z| > 0.5$

b). like c)

c). $x[n] = u[n]$



$y[n] = ?$

$Y(z) = X(z) \cdot H(z) = \dfrac{1}{1 - z^{-1}} \cdot \dfrac{z^2 + z}{z^2 + 0.1 z - 0.2} = \dfrac{z}{z - 1} \cdot \dfrac{z(z+1)}{(z - 0.4)(z + 0.5)}$

$x[n] = u[n]$, $X(z) = \dfrac{1}{1 - z^{-1}}$, ROC: $|z| > 1$

$|z| > 0.5$

$= \dfrac{z^2 (z+1)}{(z-1)(z-0.4)(z+0.5)}$, ROC: $|z| > 1$

$\dfrac{Y(z)}{z} = \dfrac{z(z+1)}{(z-1)(z-0.4)(z+0.5)} = \dfrac{A}{z-1} + \dfrac{B}{z-0.4} + \dfrac{C}{z+0.5} \quad | \cdot z$

$Y(z) = A \cdot \underbrace{\dfrac{z}{z-1}}_{\frac{1}{1 - z^{-1}}} + B \cdot \underbrace{\dfrac{z}{z-0.4}}_{\frac{1}{1 - 0.4 z^{-1}}} + C \cdot \underbrace{\dfrac{z}{z+0.5}}_{\frac{1}{1 + 0.5 z^{-1}}}$

$y[n] = A \cdot u[n] + B \cdot (0.4)^n u[n] + C \cdot (-0.5)^n u[n]$

A, B, C: below:

$a^n u[n] \longleftrightarrow \dfrac{z}{z-a} = \dfrac{1}{1 - a z^{-1}}$, $|z| > |a|$

$-a^n u[-n-1] \longleftrightarrow$ ---, $|z| < |a|$

$$A = \frac{1 \cdot 2}{0.6 \cdot 1.5} = \frac{2}{0.6 \cdot 1.5} = \dots$$

$$B = \frac{0.4 \cdot 1.4}{-0.6 \cdot 0.9} = \dots$$

$$C = \frac{-0.5 \cdot 0.5}{(-1.5) \cdot (-0.9)} = \dots$$

d). $x[n] = \left(\frac{1}{3}\right)^n u[n] \longrightarrow X(z) = \frac{1}{1 - \frac{1}{3} z^{-1}}$, ROC $|z| > \left|\frac{1}{3}\right|$

$$= \frac{z}{z - \frac{1}{3}}$$

$$Y(z) = X(z) \cdot H(z) = \frac{z}{z - \frac{1}{3}} \cdot \frac{z(z+1)}{(z-0.4)(z+0.5)}$$

$\dots$ similar

$y[n] = \dots$