Exercises Week 10

(4) $x[n]$, $N = 5$

X_0 X_1 X_2 X_3

$X_k = [15.0000 + 0.0000i\ ,\ -2.5000 + 3.4410i\ ,\ -2.5000 + 0.8123i\ ,\ -2.5000 - 0.8123i\ ,\ -2.5000 - 3.4410i]$

X_4

$$x[n] = \frac{1}{N} \cdot X_0 + \frac{1}{N} \cdot \sum_{k=1}^{2} 2 \cdot |X_k| \cdot \cos\left(2\pi \frac{k}{N} n + \angle X_k\right)$$

$$= \frac{1}{N} \cdot X_0 + \frac{1}{N} \cdot 2 \cdot |X_1| \cdot \cos\left(2\pi \frac{1}{N} n + \angle X_1\right) + \frac{1}{N} \cdot 2 \cdot |X_2| \cdot \cos\left(2\pi \frac{2}{N} \cdot n + \angle X_2\right)$$

$N = 5$

$X_1 = -2.5 + 3.441 \cdot j \Rightarrow |X_1| = \sqrt{(-2.5)^2 + (3.441)^2} = 4.25$

$\Rightarrow \angle X_1 = \operatorname{atan} \frac{3.441}{-2.5} = -0.94$

$X_2 = -2.5 + 0.8123\,j \Rightarrow |X_2| = \sqrt{(-2.5)^2 + (0.8123)^2} = 2.62$

$\Rightarrow \angle X_2 = \operatorname{atan} \frac{0.8123}{-2.5} = -0.31$

$$x[n] = \underbrace{\frac{1}{5} \cdot 15}_{\text{D.C.}} + \underbrace{\frac{1}{5} \cdot 2 \cdot 4.25}_{A_1} \cdot \cos\left(2\pi \underbrace{\frac{1}{5}}_{f_1} n \underbrace{- 0.94}_{\varphi_1}\right) + \underbrace{\frac{1}{5} \cdot 2 \cdot 2.62}_{A_2} \cdot \cos\left(2\pi \underbrace{\frac{2}{5}}_{f_2} n \underbrace{- 0.31}_{\varphi_2}\right)$$

($f_0 = 0$)

(5) $x[n] = [\underset{x[0]}{1}\ \underset{x[1]}{1}\ \underset{x[2]}{0}\ \underset{x[3]}{0}]\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0 \ldots$

$N = 4$

DFT:

$$X_k = \sum_{n=0}^{N-1} x[n]\, e^{-j 2\pi \frac{k}{N} \cdot n}$$

$$e^{jx} = \cos(x) + j \sin(x)$$

$k = 0$:

$$X_0 = \sum_{n=0}^{3} x[n] \cdot \underbrace{e^{-j 2\pi \frac{0}{N} \cdot n}}_{1} = 2$$

$k = 1$:

$$X_1 = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \frac{1}{4} n} = \underbrace{x[0]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{1}{4} 0}}_{1} + \underbrace{x[1]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{1}{4} 1}}_{e^{-j\pi/2}} + \underbrace{x[2]}_{0} \cdot e^{-j 2\pi \frac{1}{4} 2} + \underbrace{x[3]}_{0} \cdot e^{-j 2\pi \frac{1}{4} 3}$$

$$= 1 + e^{-j\pi/2}$$

$$= 1 + \underbrace{\cos\left(-\pi/2\right)}_{0} + j\underbrace{\sin\left(-\pi/2\right)}_{-1} = \boxed{1 - j}$$

$k = 2 \qquad X_2 = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \frac{2}{4} n} = \underbrace{x[0]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{2}{4} 0}}_{1} + \underbrace{x[1]}_{1} \underbrace{e^{-j 2\pi \frac{2}{4} 1}}_{e^{-j\pi}} + \underbrace{x[2]}_{0} e^{-j 2\pi \frac{2}{4} 2} + \underbrace{x[3]}_{0} \cdot e^{-j 2\pi \frac{2}{4} 3}$

$$= 1 + e^{-j\pi} = 1 + \underbrace{\cos(-\pi)}_{-1} + j \cdot \underbrace{\sin(-\pi)}_{0} = 0$$

$X_3 = X_{-1} = X_1^*$

$$X_3 = X_{3-4} = X_{-1} = X_1^* = \boxed{1 + j}$$

OR with the def.:

$k = 3 \qquad X_3 = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \frac{3}{4} n} = \underbrace{x[0]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{3}{4} 0}}_{1} + \underbrace{x[1]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{3}{4} 1}}_{e^{-j\frac{6\pi}{4}}} + \underbrace{x[2]}_{0} \cdot e^{-j 2\pi \frac{3}{4} 2} + \underbrace{x[3]}_{0} \cdot e^{-j 2\pi \frac{3}{4} 3}$

$$= 1 + e^{-j\frac{3\pi}{2}} = 1 + \underbrace{\cos\left(-\frac{3\pi}{2}\right)}_{\pi/2} + j \cdot \underbrace{\sin\left(-\frac{3\pi}{2}\right)}_{\pi/2} = 1 + j$$

$$\underbrace{\cos\left(-\frac{3\pi}{2}\right)}_{0} \qquad \underbrace{\sin\left(-\frac{3\pi}{2}\right)}_{1}$$

	X_0	X_1	X_2	X_3
X_k :	2 ,	$1-j$,	0 ,	$1+j$

$N = 4 = \text{even}$ $\qquad (N-2)/2 = 1$

$$x[n] = \frac{1}{N} X_0 + \frac{1}{N} \sum_{k=1}^{} 2 |X_k| \cdot \cos\left(2\pi \frac{k}{N} n + \angle X_k\right) + \frac{1}{N} \cdot X_{N/2} \cdot \cos(n\pi)$$

$2\pi f n, \; f = \frac{2}{4}$

$$= \frac{1}{4} \cdot \underbrace{X_0}_{2} + \frac{1}{4} \cdot 2 \cdot |X_1| \cdot \cos\left(2\pi \frac{1}{4} n + \angle X_1\right) + \frac{1}{4} \cdot \underbrace{X_2}_{0} \cdot \cos(n\pi)$$

$$|X_1| = \sqrt{1+1} = \sqrt{2}$$

$$\angle X_1 = \operatorname{atan} \frac{-1}{1} = -\pi/4 = -0.78$$

$$\Rightarrow \boxed{x[n] = 0.5 + \frac{\sqrt{2}}{2} \cdot \cos\left(2\pi \frac{1}{4} n - \pi/4\right)}$$

$$\underline{Ex}: \; x[1] = 0.5 + \frac{\sqrt{2}}{2} \cdot \underbrace{\cos\left(\pi/2 - \pi/4\right)}_{\sqrt{2}/2} = 1$$

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