Exercises Week 9
DSP

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4. Consider a periodic signal $x[n]$ with period $N = 5$ and the DFT coefficients:

X_k = [15.0000 + 0.0000i , -2.5000 + 3.4410i , -2.5000 + 0.8123i , -2.5000 - 0.8123i , -2.5000 - 3.4410i]

Write $x[n]$ as a sum of sinusoids.

$$X_k = \{ \underset{X_0}{15}, \underset{X_1}{-2.5 + 3.441j}, \underset{X_2}{-2.5 + 0.8123j}, \underset{X_3 = X_{-2}}{-2.5 - 0.8123j}, \underset{X_4 = X_{-1}}{-2.5 - 3.441j} \}$$

$N = 5$

$$x[n] = \frac{1}{N} X_0 + \frac{1}{N} \sum_{k=1}^{(N-1)/2} 2|X_k| \cos(2\pi (k/N) n + \angle X_k)$$

$X_0 = 15$

$X_1 = -2.5 + 3.441j$

$|X_1| = \sqrt{2.5^2 + 3.441^2} = 4.25$

$\angle X_1 = \operatorname{atan} \frac{3.441}{-2.5} = -0.94$

$X_2 = -2.5 + 0.8123j$

$|X_2| = 2.62$

$\angle X_2 = \operatorname{atan} \frac{0.8123}{-2.5} = -0.31$

$X_3 = X_2^* \Rightarrow |X_3| = |X_2|$, $\angle X_3 = -\angle X_2$

$X_4 = X_1^* \Rightarrow |X_4| = |X_1|$, $\angle X_4 = -\angle X_1$

Formula:

$$x[n] = 3 + \frac{1}{5} \cdot 2 \cdot 4.25 \cos\left(2\pi \frac{1}{5} n - 0.94\right) + \frac{1}{5} \cdot 2 \cdot 2.62 \cos\left(2\pi \frac{2}{5} n + -0.31\right)$$

✓

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5. Find the DFT coefficients of the periodic signal with period $\{1, 1, 0, 0\}$, and write the signal as a sum of sinusoidal components.

$$x[n] = [\overset{x[0]}{1}, \overset{x[1]}{1}, \overset{x[2]}{0}, \overset{x[3]}{0}] = 1 \text{ period} \quad N=4$$

$$X_k = ?$$

$$\text{DFT}: \quad X_k = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \frac{k}{N} \cdot n} \quad N = 4$$

$$X_0 = \sum_n x[n] \cdot \underbrace{e^{-j \cdot 0}}_{1} = \sum x[n] = 2 = x[0]\cdot 1 + x[1]\cdot 1 + x[2]\cdot 1 + x[3]\cdot 1$$

$$X_1 = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \frac{1}{4} \cdot n} = \underbrace{x[0]}_{1} \cdot \underbrace{e^{-j \cdot 2\pi \frac{1}{4} \cdot 0}}_{1} + \underbrace{x[1]}_{1} \cdot e^{-j 2\pi \frac{1}{4} \cdot 1}$$

$$= 1 + e^{-j\pi/2} = 1 + \underbrace{\cos(-\pi/2)}_{0} + j \underbrace{\sin(-\pi/2)}_{-1}$$

$$= \cancel{1 + 0 +} = 1 - j$$

$$X_2 = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \cdot \frac{2}{4} \cdot n} = \underbrace{x[0]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{2}{4} \cdot 0}}_{1} + \underbrace{x[1]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{2}{4} \cdot 1}}_{e^{-j\pi}} + \underbrace{x[2]}_{0} \cdots + \underbrace{x[3]}_{0}$$

$$e^{-j\pi} = \underbrace{\cos(-\pi)}_{-1} + j \underbrace{\sin(-\pi)}_{0}$$

$$= 1 + (-1) = 0$$

$$(e^{-j\pi} = -1)$$

$$X_3 = \sum_{n=0}^{3} x[n] \cdot e^{-j 2\pi \frac{3}{4} \cdot n} = \underbrace{x[0]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{3}{4} \cdot 0}}_{1} + \underbrace{x[1]}_{1} \cdot \underbrace{e^{-j 2\pi \frac{3}{4} \cdot 1}}_{e^{-j\frac{3}{2}\pi}}$$

$$e^{-j\frac{3}{2}\pi} = \underbrace{\cos(-\tfrac{3}{2}\pi)}_{0} + j \underbrace{\sin(-\tfrac{3}{2}\pi)}_{1} = j$$

$$= X_{-1} = X_1^* = 1 + j$$

(3 − 4)
(3 − N)

Write as sinusoids:

$$|X_1| = \sqrt{1+1} = \sqrt{2}$$
$$\angle X_1 = \text{atan}(-1) = -45° = -0.78$$

$$\boxed{x[n] = \frac{1}{2} + \frac{1}{4} \cdot 2 \cdot \sqrt{2} \cdot \cos\left(2\pi \frac{1}{4} \cancel{+} 0.78\right) + 0}$$

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$$\underset{X_k}{\begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix}} = \underset{W}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-j2\pi\frac{1}{4}\cdot 1} & e^{-j2\pi\frac{1}{4}\cdot 2} & e^{-j2\pi\frac{1}{4}\cdot 3} \\ 1 & e^{-j2\pi\frac{2}{4}\cdot 1} & e^{-j2\pi\frac{2}{4}\cdot 2} & e^{-j2\pi\frac{2}{4}\cdot 3} \\ 1 & e^{-j2\pi\frac{3}{4}\cdot 1} & e^{-j2\pi\frac{3}{4}\cdot 2} & e^{-j2\pi\frac{3}{4}\cdot 3} \end{bmatrix}} \cdot \underset{x}{\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}$$

4×4 $\quad$ 4×1

$$\boxed{X}_k = \boxed{W} \cdot \boxed{x}$$