Exercises Week 5

(2)

$$X(z) = \frac{7}{(1-2z^{-1})(1+0.5z^{-1})} \quad \begin{matrix} \cdot z^2 \\ \cdot z^2 \\ \cdot z \cdot z \end{matrix}$$

$$= \frac{7z^2}{(z-2)(z+0.5)}$$

$$\Rightarrow Az + A\cdot 0.5 + Bz - 2B = 7z \Rightarrow \begin{cases} A+B=7 \\ 0.5A - 2B = 0 \end{cases}$$

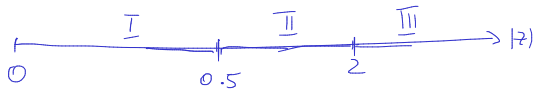
$$\frac{X(z)}{z} = \frac{7\cdot z}{(z-2)(z+0.5)} = \frac{\overset{z+0.5}{A}}{z-2} + \frac{\overset{z-0.2}{B}}{z+0.5} \quad \Big| \cdot z$$

$$A = \frac{7z}{(z+0.5)}\Big|_{z=2} = \frac{7\cdot 2}{2+0.5} = \frac{14}{2.5} = 5.6$$

$$B = \frac{7\cdot z}{z-2}\Big|_{z=-0.5} = \frac{-0.5\cdot 7}{-2.5} = 3.5/2.5 = 1.4$$

$$X(z) = A\cdot \underbrace{\frac{z}{z-2}}_{a=2} + B\cdot \underbrace{\frac{z}{z+0.5}}_{a=-0.5} = \overset{A}{5.6}\,\frac{z}{z-2} + \overset{B}{1.4}\cdot\frac{z}{z+0.5}$$

$|z|<2$ $\quad$ $|z|>2$ $\quad$ $|z|<0.5$ $\quad$ $|z|>0.5$

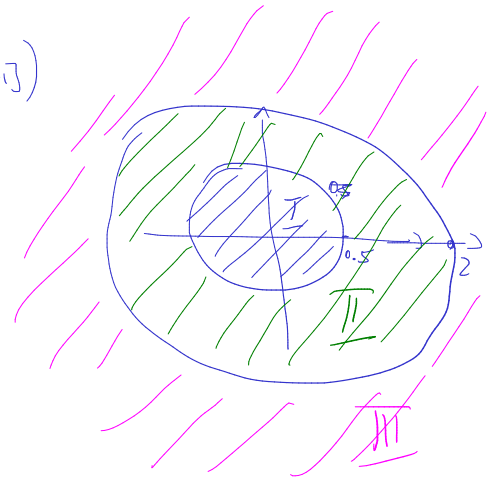


ROC: I $\quad |z| < 0.5 < 2$

anti-causal $\quad x[n] = \underbrace{\overset{A}{5.6}\cdot(-2^n u[-n-1])}_{\text{anti-causal}} + \underbrace{\overset{B}{1.4}\cdot(-(-0.5)^n u[-n-1])}_{\text{anti-causal}}$

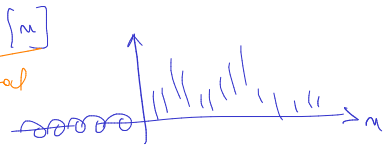
II $\quad 0.5 < |z| < 2$

bilateral $\quad x[n] = \underbrace{\overset{A}{5.6}\cdot(-2^n u[-n-1])}_{\text{anti-causal}} + \underbrace{\overset{B}{1.4}\cdot(-0.5)^n u[n]}_{\text{causal}}$



III $\quad 0.5 < 2 < |z|$

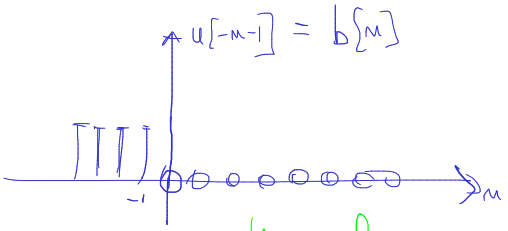
causal! $\quad x[n] = \underbrace{\overset{A}{5.6}\cdot 2^n\cdot u[n]}_{\text{causal}} + \underbrace{\overset{B}{1.4}\cdot(-0.5)^n u[n]}_{\text{causal}}$



$2^n\cdot u[n]$:

causal

$u[-n-1] = b[n]$



anti-causal