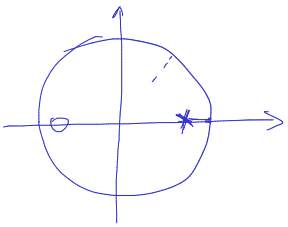
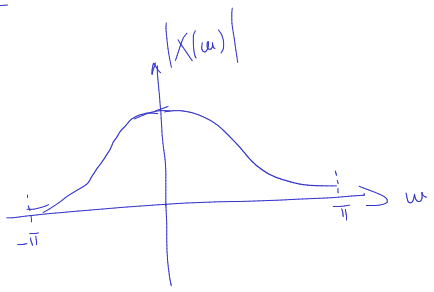
Exercises Week 12

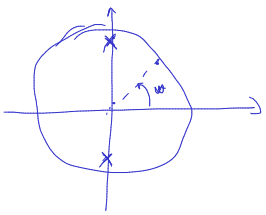
(1)

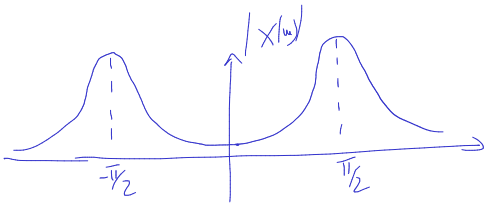
a)



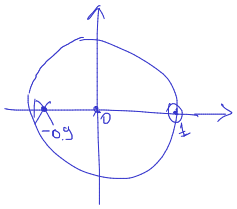


b).

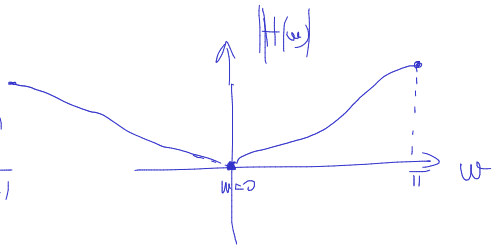




(2)



$$H(z) = C \cdot \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}$$



$$H(z) = C \cdot \frac{z - 1}{z + 0.9} = C \cdot \frac{1 - z^{-1}}{1 + 0.9 z^{-1}}$$

b). $|H(\omega)|$, $\angle H(\omega)$

$$H(\omega) = H(z)\Big|_{z = e^{j\omega}} = C \cdot \frac{e^{j\omega} - 1}{e^{j\omega} + 0.9} = C \cdot \frac{\cos\omega + j\sin\omega - 1}{\cos\omega + j\sin\omega + 0.9}$$

$$|H(\omega)| = |C| \cdot \frac{\sqrt{(\cos\omega - 1)^2 + (\sin\omega)^2}}{\sqrt{(\cos\omega + 0.9)^2 + (\sin\omega)^2}}$$

$$\angle H(\omega) = \angle C + \operatorname{atan}\frac{\sin\omega}{\cos\omega - 1} - \operatorname{atan}\frac{\sin\omega}{\cos\omega + 0.9}$$

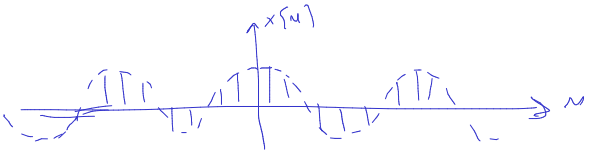
c). Normalize = find C

$$|H(\pi)| = 1 \iff |H(\omega)|_{\omega = \pi} = 1 \iff |C| \cdot \frac{\sqrt{4 + 0}}{\sqrt{0.1^2 + 0}} = 1$$

$$\iff |C| \cdot \frac{2}{0.1} = 1 \Rightarrow |C| = \frac{1}{20} \Rightarrow C = \pm\frac{1}{20}$$

d). $x[n] = 2 \cdot \cos\left(\underbrace{\frac{\pi}{6}}_{\omega} n + \pi/4\right)$, $n \in \mathbb{Z}$

$y[n] = ?$



$$y[n] = 2 \cdot |H(\pi/6)| \cdot \cos\left(\frac{\pi}{6} n + \frac{\pi}{4} + \angle H(\pi/6)\right)$$

$$|H(\pi/6)| = |H(\omega)|_{\omega=\pi/6} = \frac{1}{20} \cdot \frac{\sqrt{\left(\frac{\sqrt{3}}{2} - 1\right)^2 + 1/4}}{\sqrt{\left(\frac{\sqrt{3}}{2} + 0.9\right)^2 + \frac{1}{4}}} = \dots$$

$$\angle H(\pi/6) = \begin{cases} 0 & , C > 0 \\ \text{or} & \\ \pi & , C < 0 \end{cases} + \operatorname{atan}\left(\frac{1/2}{\frac{\sqrt{3}}{2} - 1}\right) - \operatorname{atan}\left(\frac{1/2}{\frac{\sqrt{3}}{2} + 0.9}\right) = \dots$$

$\angle 7 = 0$

$\angle -7 = \pi$

$(-7 = 7 \cdot e^{j \cdot \pi})$