Exercises Week 11

$$e^{jx} = \cos x + j \sin x$$

(1) $x[n] = \ldots 0, \underset{\uparrow}{1}, 1, 0, 0, 0 \ldots$ ($x[0]$, $x[1]$)

DTFT: $X(\omega) = ?$

DTFT: $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n} = 1 \cdot \underbrace{e^{-j\omega \cdot 0}}_{1} + 1 \cdot e^{-j\omega \cdot 1} = 1 + e^{-j\omega}$

$= 1 + \underbrace{\cos(-\omega)}_{\cos(\omega)} + j \underbrace{\sin(-\omega)}_{-\sin(\omega)}$

$= 1 + \cos(\omega) - j \sin(\omega)$

Last Week:

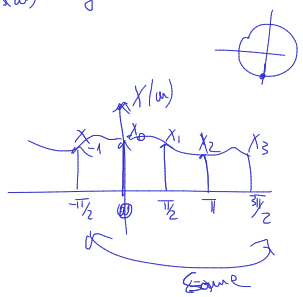
DFT

$X_0 = 2 = X(\omega)\big|_{\omega=0,\ f=0,\ \omega=2\pi f}$

$X_1 = 1-j = X(\omega)\big|_{\omega=\pi/2,\ f=\frac{1}{4},\ \omega=2\pi f}$

$X_2 = 0 = X(\omega)\big|_{\omega=\pi,\ f=\frac{2}{4},\ \omega=2\pi f}$

$X_3 = 1+j = X(\omega)\big|_{\omega=\frac{3\pi}{2},\ f=\frac{3}{4},\ \omega=2\pi f}$



$$X_k = X(\omega)\Big|_{\omega = 2\pi \frac{k}{N}}$$

(2) $x[n]$

$X(z)$ $\quad p_1 = -0.5$, $z_1 = 0.9$

$|X(\omega)|_{\omega=\pi} = 1$

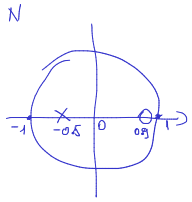
a) $X(z) = ?$

b) $|X(\omega)|$, $\angle X(\omega) = ?$

c).

d).

$$X(z) = C \cdot \frac{(z - z_1)}{(z - p_1)} = C \cdot \frac{z-0.9}{z+0.5}$$



$z = e^{j\omega}$

$$X(\omega) = C \cdot \frac{e^{j\omega} - 0.9}{e^{j\omega} + 0.5} = C \cdot \frac{\cos(\omega) - 0.9 + j \cdot \sin(\omega)}{\cos(\omega) + 0.5 + j \cdot \sin(\omega)}$$

$\omega = \pi \Rightarrow X(\pi) = C \cdot \frac{-1-0.9}{-1+0.5} = C \cdot \frac{-1.9}{-0.5} = 3.8 \cdot C$

$|X(\pi)| = 1 \Rightarrow |3.8 \cdot C| = 1 \Rightarrow C = \pm \frac{1}{3.8}$

a) $X(z) = \pm \frac{1}{3.8} \cdot \frac{z+0.9}{z+0.5}$

b) $|X(\omega)| = \left|\pm \frac{1}{3.8}\right| \cdot \frac{|\cos(\omega) - 0.9 + j \sin(\omega)|}{|\cos\omega + 0.5 + j \sin(\omega)|}$

$= \frac{1}{3.8} \cdot \frac{\sqrt{(\cos(\omega) - 0.9)^2 + \sin^2(\omega)}}{\sqrt{(\cos(\omega) + 0.5)^2 + \sin^2(\omega)}}$

$\angle X(\omega) = \angle C + \angle(\cos(\omega) - 0.9 + j \sin\omega) - \angle(\cos\omega + 0.5 + j \sin(\omega))$

$= \frac{0}{\pi} + \operatorname{atan} \frac{\sin(\omega)}{\cos(\omega) - 0.9} - \operatorname{atan} \frac{\sin(\omega)}{\cos(\omega) + 0.5}$

$|a \cdot b| = |a| \cdot |b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a + jb| = \sqrt{a^2 + b^2}$

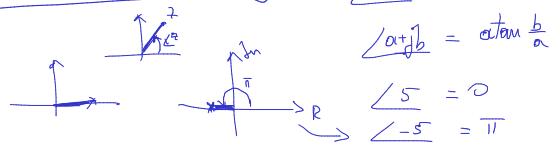
$\angle a \cdot b = \angle a + \angle b$

$\angle \frac{a}{b} = \angle a - \angle b$

$\angle a + jb = \operatorname{atan} \frac{b}{a}$

$\angle 5 = 0$

$\angle -5 = \pi$



c) $\left|X\left(\frac{\pi}{2}\right)\right| = \frac{1}{3.8} \cdot \frac{\sqrt{(-0.9)^2 + 1^2}}{\sqrt{0.5^2 + 1^2}} = \dots$

$\left|X\left(-\frac{\pi}{2}\right)\right| = \left|X\left(\frac{\pi}{2}\right)\right| = \text{same}$

$\left|X(0)\right| = \frac{1}{3.8} \cdot \frac{\sqrt{0.1^2 + 0^2}}{\sqrt{1.5^2 + 0}} = \dots$

d)