DSP Exam 2024-01-30 No.1

Exercises

1. (4.5p) Characterize the following system:

$$y[n] = (x[n])^2 + x[n-1] - x[n-2]$$

with respect to:

- Memory
- Linearity
- Time invariance

Justify the answers.

2. (4.5p) A causal LTI system has the impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]$$

a. (1p) Compute the system function $H(z)$;
b. (1p) Draw the pole-zero diagram, specify the ROC, and justify if the system is stable or not;
c. (1p) Write the system equation;
d. (1.5p) Compute the output signal $y[n]$ if the input signal is the unit step $x[n] = u[n]$.

3. (3p) Consider a periodic signal with period $[2, -2, 0, 0]$.

Find the DFT coefficients of the signal.

4. (3p) Consider the **causal** system with system function

$$H(z) = \frac{1 - 2z^{-1}}{1 + 0.5z^{-1}}$$

a. Find the expression of the amplitude response and the phase response of the filter;
b. Find the output signal $y[n]$, in permanent regime, if the input is:

$$x[n] = 3\cos(2\pi \cdot 1/16 \cdot n), \forall n \in \mathbb{Z}$$

c. Draw the Direct-Form I implementation of the system.

Theory

1. (2p) An ADC circuit has a sampling frequency $F_s = 48kHz$. What anti-alias filter should we use? (specify filter type and cutoff frequency). Justify.

2. (2p) What are causal, anti-causal, bilateral signals?

3. (1p) Can the ROC of a Z transform contain poles? Justify your answer.

4. (3p) Prove that, if a signal $x[n]$ has the Z transform $X(z)$, then $x[n-k]$ has the Z transform $z^{-k}X(z)$.

5. (4p) Explain the similarities and differences between the Discrete Fourier Transform (DFT) and the Discrete-Time Fourier Transform (DTFT):

   - Their definitions;
   - To what type of signals do we apply each one;
   - How is the shape of the resulting spectrum (continuous, discrete);
   - What is the relation between their values;

6. (3p) What are linear-phase filters? Explain:

   - what they are;
   - what conditions they must satisfy;
   - give an example of $H(z)$ for a linear-phase filter;

Notes: Obtain 30p for grade 10. 3p are awarded from start. Time available: 2h

Known:

$$\begin{aligned}
\delta[n] &\overset{\text{Z}}{\longleftrightarrow} 1 \\
a^n \cdot u[n] &\overset{\text{Z}}{\longleftrightarrow} \frac{1}{1 - a \cdot z^{-1}} = \frac{z}{z-a}, ROC : |z| > |a| \\
-a^n \cdot u[-n-1] &\overset{\text{Z}}{\longleftrightarrow} \frac{1}{1 - a \cdot z^{-1}} = \frac{z}{z-a}, ROC : |z| < |a|
\end{aligned}$$