

# Exercises Week 1

$N = \text{integer}$

① a)  $x[n] = \cos\left(\pi \frac{3}{10} n\right) = x[n+N] = \cos\left(\pi \frac{3}{10} (n+N)\right)$

Periodic?

Yes, with  $N=20$

$$= \cos\left(\pi \cdot \frac{3}{10} n + \pi \cdot \frac{3}{10} N\right)$$

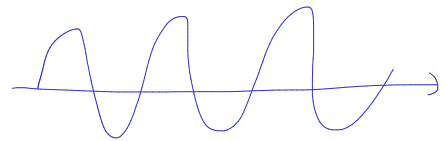
$20, 40, 60, \dots$   
 $6\pi$

b)  $x[n] = \cos(7.2 \cdot \pi \cdot n)$

$x[n+N] = \cos(7.2 \cdot \pi \cdot n + 7.2 \cdot \pi \cdot N)$   
 $N=10$   
 $N=5$

c)  $x[n] = \sin(3 \cdot n)$

$x[n+N] = \sin(3n + 3N)$   
 $N \in \mathbb{Q} = \frac{a}{b}$   
 $N \in \mathbb{Q}$  is periodic  
 $2\pi \notin \mathbb{Q}$  quasi-periodic



d)  $x[n] = \sin\left(\pi \frac{n}{2}\right) + \cos\left(\frac{3\pi n}{4}\right)$

$x[n+N] = \sin\left(\frac{\pi n}{2} + \frac{\pi N}{2}\right) + \cos\left(\frac{3\pi n}{4} + \frac{3\pi N}{4}\right)$   
 $N_1=4$   
 $N_2=8$   
 $N=8$

$x[n] = \sin\left(\frac{\pi n}{2}\right) + \cos\left(\frac{3\pi n}{4}\right)$   
 $N_1=4$   
 $N_2=6$   
 $N=12$

②  $x_a(t) = \left(1 + 0.5 \cdot \cos(400 \pi t)\right) \cdot \cos(8000 \pi t)$   
 $2\pi F_1 t$   
 $F_1 = 200 \text{ Hz}$   
 $2\pi F_2 t$   
 $F_2 = 4000 \text{ Hz}$

a)  $F_s \text{ minimal} = ?$   
 $8000 \text{ Hz}$   
 $= 2 \cdot F_{\max} = 8400 \text{ Hz}$

$\cos a \cdot \cos b = \cos(a+b) + \cos(a-b)$

$= \cos(8000 \pi t) + 0.5 \cos(400 \pi t) \cdot \cos(8000 \pi t)$

$= \cos(8000 \pi t) + 0.5 \cdot \frac{1}{2} \cos(8400 \pi t) + 0.5 \cdot \frac{1}{2} \cos(7600 \pi t)$   
 $F_1 = 4000 \text{ Hz}$   
 $F_2 = 4200 \text{ Hz}$   
 $F_3 = 3800 \text{ Hz}$

b)  $F_s = 8000$  ~~Hz~~  $\text{Hz}$

$x[n] = ?$

$$t \rightarrow n \cdot T_s = \frac{n}{F_s}$$

$$x[n] = \cos\left(\frac{8000\pi n}{8000}\right) + 0.25 \cos\left(\frac{8400\pi n}{8000}\right) + 0.25 \cos\left(\frac{7600\pi n}{8000}\right)$$

$$= \cos(\pi n) + 0.25 \cos\left(\frac{8.4}{8} \pi n\right) + 0.25 \cos\left(\frac{7.6}{8} \pi n\right)$$

$f = \frac{1}{2}$   
✓

$f = \frac{4.2}{8} = 0.525$   
 $> \frac{1}{2}$

$f = \frac{3.8}{8} = 0.475$   
✓

Always do modification

$$f = 0.525 = 0.525 - 1 = -0.475$$

$$= \cos(\pi n) + \underbrace{0.25 \cdot \cos(2\pi(-0.475)n)}_{0.25 \cdot \cos\left(\frac{7.6}{8} \pi n\right)} + 0.25 \cos\left(\frac{7.6}{8} \pi n\right)$$

c). Reconstruct:  $n \rightarrow t \cdot F_s$   $\text{8000}$

$$x_n(t) = \cos(\pi \cdot t \cdot 8000) + 0.25 \cos(2\pi(-0.475) \cdot 8000 \cdot t) + 0.25 \cos\left(\frac{7.6}{8} \pi \cdot t \cdot 8000\right)$$