$\sum x[n] z^{-n}$ **Exercises Week 7** $a^n u[n]$

Causal $\hookrightarrow$ $h[n]$ = causal

(1) $x[n] = \ldots \longleftrightarrow X(z)$

$y[n] = \ldots \longleftrightarrow Y(z)$

$\Rightarrow H(z) = \frac{Y(z)}{X(z)}$

$a[n] = \left(\frac{1}{3}\right)^n u[n] \longleftrightarrow \frac{1}{1-\frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$

$a[n-1] = \left(\frac{1}{3}\right)^{n-1} u[n-1] \longleftrightarrow z^{-1} \cdot \frac{1}{1-\frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$

$x[n] = \left(\frac{1}{3}\right)^n \cdot u[n] - \frac{1}{4} \cdot \underbrace{\left(\frac{1}{3}\right)^n \cdot u[n-1]}_{\frac{1}{3}\cdot\left(\frac{1}{3}\right)^{n-1}\cdot u[n-1]}$

$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{4}\cdot\frac{1}{3}\cdot z^{-1}\cdot\frac{1}{1-\frac{1}{3}z^{-1}} = \frac{z}{z-\frac{1}{3}} - \frac{1}{12}\cdot\frac{1}{z}\cdot\frac{z}{z-\frac{1}{3}} = \frac{\left(z-\frac{1}{12}\right)}{\left(z-\frac{1}{3}\right)}$

R.O.C. $|z| > \frac{1}{3}$ $\qquad |z| > \frac{1}{3}$

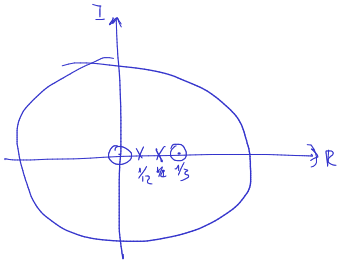
$y[n] = \left(\frac{1}{4}\right)^n \cdot u[n] \Rightarrow Y(z) = \frac{1}{1-\frac{1}{4}z^{-1}} = \frac{z}{z-\frac{1}{4}}$

R.O.C. $|z| > \frac{1}{4}$ $\qquad |z| > \frac{1}{4}$

$H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z-\frac{1}{4}} \cdot \frac{z-\frac{1}{3}}{z-\frac{1}{12}} = \frac{z\cdot\left(z-\frac{1}{3}\right)}{\left(z-\frac{1}{4}\right)\cdot\left(z-\frac{1}{12}\right)}$

$|z| > \frac{1}{4}$ $\qquad |z| > \frac{1}{3}$ $\Rightarrow$ $|z| > \frac{1}{3}$

$\Rightarrow$



z: $z_1 = 0$, $z_2 = \frac{1}{3}$

p: $p_1 = \frac{1}{4}$, $p_2 = \frac{1}{12}$

b). $h[n] = \mathcal{Z}^{-1}\{H(z)\}$

$H(z) = \frac{z\cdot\left(z-\frac{1}{3}\right)}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{12}\right)}$, R.O.C.: $|z| > \frac{1}{3}$

$A = \frac{\frac{1}{4}-\frac{1}{3}}{\frac{1}{4}-\frac{1}{12}} = \frac{\frac{-1}{12}}{\frac{2}{12}} = \frac{-1}{2}$

$B = \frac{\frac{1}{12}-\frac{1}{3}}{\frac{1}{12}-\frac{1}{4}} = \frac{\frac{-3}{12}}{\frac{-2}{12}} = \frac{3}{2}$

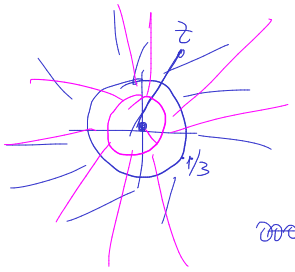
$\frac{H(z)}{z} = \frac{z-\frac{1}{3}}{\left(z-\frac{1}{4}\right)\cdot\left(z-\frac{1}{12}\right)} = \frac{A}{z-\frac{1}{4}} + \frac{B}{z-\frac{1}{12}} \quad |\cdot z$

$H(z) = A\cdot\frac{z}{z-\frac{1}{4}} + B\cdot\frac{z}{z-\frac{1}{2}}$

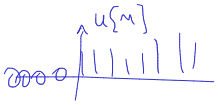
$\mathcal{Z}^{-1}\downarrow$

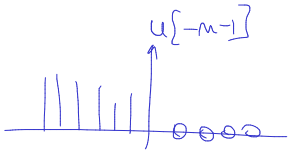
$|z| > \frac{1}{2}$ $\qquad |z| < \frac{1}{2}$

$|z| > \frac{1}{4}$ $\qquad |z| < \frac{1}{4}$



$\frac{z}{z-\frac{1}{4}} \leftrightarrow \frac{1}{1-\frac{1}{4}z^{-1}}$





$h[n] = \frac{-1}{2}\cdot\left(\frac{1}{4}\right)^n \cdot u[n] + \frac{3}{2}\cdot\left(\frac{1}{2}\right)^n \cdot u[n]$

$-\left(\frac{1}{2}\right)^n u[-n-1]$ $\quad \frac{1}{3} < |z| < \frac{1}{2}$

$\left(\frac{1}{2}\right)^n u[n]$ $\quad \frac{1}{3} < \frac{1}{2} < |z|$ $\quad |z| > \frac{1}{3}$

H = causal $\Rightarrow$ $h[n]$ = causal $\Rightarrow$