Week 8

1

$x_1[n] = [1, 3, 1, 3]$

$x_2[n] = [2, 2, 5, 5]$

$x_1 \circledast_4 x_2 =$

2	2	5	5
15	6	6	15
5	5	2	2
6	15	15	6
28	28	28	28

2

$x_1[n] = [1, 3, 1, 3]\ 0\ 0,$ periodic

$x_2[n] = [2, 2, 5, 5]$

in $N = 6$ points

2 2 5 5 0 0

0 6 6 15 15 0

0 0 2 2 5 5

15 0 0 0 6 6 15

$x_1 \circledast_6 x_2 =$ 17 8 13 28 26 20

in $N =$ ~~7~~ points

2 2 5 5 _ _ _

6 6 15 15

2 2 5 5

6 6 15 15

2 8 13 28 26 20 15

3

$x[n]$, $N = 6$

$X_k = [\underbrace{21.0000 + 0.0000i}_{X_0}, \underbrace{-3.0000 + 5.1962i}_{X_1}, \underbrace{-3.0000 + 1.7321i}_{X_2}, \underbrace{-3.0000 + 0.0000i}_{X_3}, \underbrace{-3.0000 - 1.7321i}_{X_4 = X_{-2}}, \underbrace{-3.0000 - 5.1962i}_{X_5 = X_{-1}}]$

$$x[n] = \frac{1}{N}\sum_{k=0}^{5} X_k \cdot e^{j 2\pi \frac{k}{N} n}$$ TFD inverse

$\boxed{f = f - 1}$

$\boxed{\frac{4}{6} = \frac{-2}{6}}$

$\boxed{5/6 = \frac{-1}{6}}$

$$= \frac{1}{6}\left(\underbrace{X_0}_{21} \cdot \underbrace{e^{j0}}_{1} + \underbrace{X_1}_{} \cdot e^{j2\pi \frac{1}{6} n} + X_2 \cdot e^{j2\pi \frac{2}{6} n} + X_3 \cdot e^{j2\pi \frac{3}{6} n} + \underbrace{X_4}_{X_{-2}} \cdot e^{j2\pi \overset{-2/6}{\frac{4}{6}} n} + \underbrace{X_5}_{X_{-1}} \cdot e^{j2\pi \overset{-1/6}{\frac{5}{6}} n}\right)$$

$$= \frac{1}{6} \cdot 21 + \frac{1}{6}\left(X_1 \cdot e^{j 2\pi \frac{1}{6} n} + X_2 \cdot e^{j 2\pi \frac{2}{6} n} + \underbrace{X_3}_{-3} \cdot e^{j 2\pi \frac{3}{6} n} + X_{-2} \cdot e^{j 2\pi \left(\frac{-2}{6}\right) n} + X_{-1} \cdot e^{j 2\pi \left(\frac{-1}{6}\right) n} \right)$$

$$\angle = \operatorname{arctg} \frac{b}{a}$$

$$X_1 = -3 + 5.1962 j \Rightarrow |X_1| = 6$$

$$\angle X_1 = \operatorname{arctg} \frac{5.1962}{-3} = -1.04$$

$$= 6 \cdot e^{-j \cdot 1.04}$$

$$e^{jx} = \cos x + j \sin x$$

$$X_{-1} = -3 - 5.1962 j \Rightarrow |X_{-1}| = 6$$

$$\angle X_{-1} = \operatorname{arctg} \frac{-5.1962}{-3} = 1.04$$

$$X_2 = -3 + 1.7321 j \Rightarrow |X_2| = 3.46$$

$$\angle X_2 = \cancel{\operatorname{arctg}}\ \operatorname{atan} = \frac{1.7321}{-3} = -0.52$$

$$X_{-2} = -3 - 1.7321 j \Rightarrow |X_{-2}| = 3.46$$

$$\angle X_{-2} = +0.52$$

$$x[n] = \frac{1}{6} \cdot 21 + \frac{1}{6}\left(\underbrace{6 \cdot e^{-j \cdot 1.04}}_{X_1} \cdot e^{j 2\pi \frac{1}{6} n} + \underbrace{3.46 \cdot e^{-j 0.52}}_{X_2} \cdot e^{j 2\pi \frac{2}{6} n} + \underbrace{(-3)}_{X_3} \cdot \underbrace{e^{j 2\pi \frac{3}{6} n}} + \right.$$

$$e^{j n\pi} = \cos(n\pi) + \underbrace{j \sin(n\pi)}_{0} = \cos(n\pi) = \cos\left(2\pi \frac{1}{2} n\right)$$

$$\left. + \underbrace{3.46 \cdot e^{j 0.52}}_{X_4 = X_{-2}} \cdot e^{-j 2\pi \frac{2}{6} n} + \underbrace{6 \cdot e^{j \cdot 1.04}}_{X_5 = X_{-1}} \cdot e^{-j 2\pi \frac{1}{6} n} \right)$$

$$= \frac{1}{6} \cdot 21 + \frac{1}{6}\left(6 \left(\underbrace{e^{j\left(2\pi \frac{1}{6} n - 1.04\right)} + e^{-j\left(2\pi \frac{1}{6} n - 1.04\right)}}_{2 \cdot \cos\left(2\pi \frac{1}{6} n - 1.04\right)} \right) + 3.46 \left(\underbrace{e^{j\left(2\pi \frac{2}{6} n - 0.52\right)} + e^{-j\left(2\pi \frac{2}{6} n - 0.52\right)}}_{2 \cos\left(2\pi \frac{2}{6} n - 0.52\right)} \right) + (-3) \cdot \cos\left(2\pi \frac{1}{2} n\right) \right)$$

$$x[n] = \frac{1}{6} \cdot 21 \underbrace{}_{\cdot \cos(f=0)} + \frac{1}{6} \cdot \underbrace{6}_{|X_1|} \cdot 2 \cdot \cos\left(2\pi \frac{1}{6} n \underbrace{-1.04}_{\angle X_1}\right) + \frac{1}{6} \cdot \underbrace{3.46}_{|X_2|} \cdot 2 \cdot \cos\left(2\pi \frac{2}{6} n \underbrace{-0.52}_{\angle X_2}\right) + \frac{1}{6} \cdot \underbrace{(-3)}_{X_3} \cdot \cos\left(2\pi \underbrace{\frac{1}{2}}_{3/6} n \underbrace{+0}_{\angle X_3}\right)$$

$$x[n] = \frac{1}{N} \cdot X_0 + \frac{1}{N}\left(\sum_{k=1}^{\lfloor \frac{N-1}{2} \rfloor} |X_k| \cdot 2 \cdot \cos\left(2\pi \frac{k}{N} \cdot n + \angle X_k\right) + \underbrace{X_{\frac{N}{2}} \cdot \cos\left(2\pi \frac{1}{2} n\right)}_{\text{only if } N = \text{even}}\right)$$

$\frac{k}{N}$ not over $\frac{1}{2}$