Exercises Week 4

①

$$I = \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 2 & 2 \\ \hline 3 & 3 & 3 & 3 & 3 \\ \hline \end{array} \quad * \quad H = \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 2 & 20 \\ \hline \end{array}$$

$$Y = \begin{array}{|c|c|c|c|c|} \hline -1 & 0 & 0 & 0 & -1 \\ \hline -2 & 0 & 0 & 0 & -2 \\ \hline -7 & -4 & -4 & -4 & -7 \\ \hline \end{array}$$

$1\cdot(-4) + 1\cdot 1 + 2\cdot 1 + 2\cdot 0$

②

$$x_1[n] = \{\dots 0, \overset{z^{1}}{1}, \underset{\uparrow}{\overset{z^{0}}{2}}, \overset{z^{-1}}{3}, \overset{z^{-2}}{4}, 0 \dots\} \overset{Z}{\longleftrightarrow} X_1(z) = z + 2 + 3z^{-1} + 4z^{-2}$$

$$x_2[n] = \{\dots 0, \overset{z^{2}}{2}, \overset{z^{1}}{2}, \underset{\uparrow}{\overset{z^{0}}{3}}, \overset{z^{-1}}{3}, 0 \dots\} \overset{Z}{\longleftrightarrow} X_2(z) = 2z^2 + 2z + 3 + 3z^{-1}$$

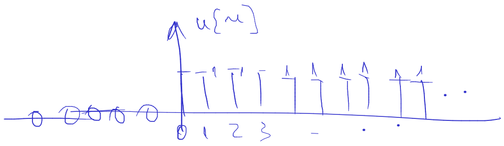
$$X_1(z)\cdot X_2(z) = 2\cdot z^3 + z^2\underbrace{(4+2)}_{6} + z\underbrace{(3+4+6)}_{13} + z^0(3+\underbrace{6+6+8}_{23}) +$$

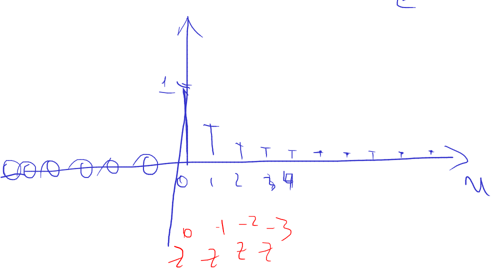
$$+ z^{-1}\underbrace{(6+9+8)}_{23} + z^{-2}\underbrace{(9+12)}_{21} + z^{-3}\cdot\underbrace{12}_{12}$$

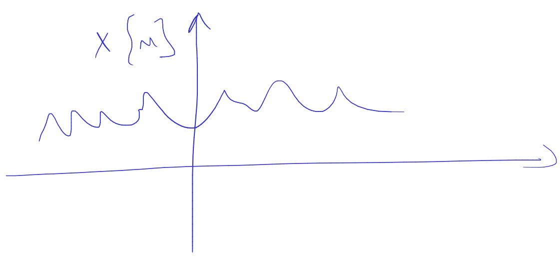
$$\{\dots 0, 2, 6, 13, \underset{\uparrow}{23}, 23, 21, 12, 0, \dots\}$$

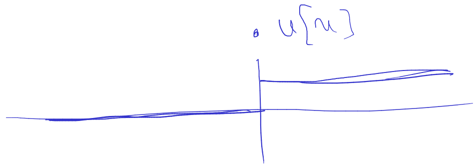
③

a) $x[n] = \left(\frac{1}{3}\right)^n \cdot u[n]$



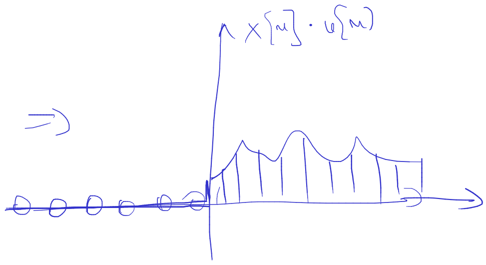






$\Rightarrow$

causal



$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n \cdot z^{-n} = \sum \left(\frac{1}{3}\right)^n \cdot \left(\frac{1}{z}\right)^n$$

$$= \sum_{n=0}^{\infty} \underbrace{\left(\frac{1}{3z}\right)}_{q}^{n} = 1 + q + q^2 + q^3 + \dots =$$

sum of geometric series

$$\boxed{= \frac{1}{1-q}, \quad |q| < 1}$$

$$= \frac{1}{1 - \frac{1}{3z}} = \frac{3z}{3z-1} = \boxed{\frac{z}{z-\frac{1}{3}}} = \boxed{\frac{1}{1-\frac{1}{3}z^{-1}}}$$

only when $|q| < 1$

$$\left|\frac{1}{3z}\right| < 1 \Leftrightarrow 1 < |3z| \Leftrightarrow \boxed{|z| > \frac{1}{3}}$$

$$\boxed{a^n \cdot u[n] \xleftrightarrow{z} \frac{z}{z-a}, \quad \underline{\text{ROC}}: |z| > |a|}$$