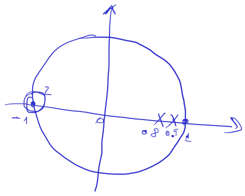
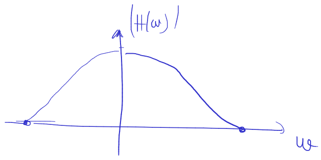
Exercises Week 13

① **Low-pass filter**:

2 poles $\rightarrow$ 0.9 , 0.8

2 zeros $\rightarrow$ -1 , -1





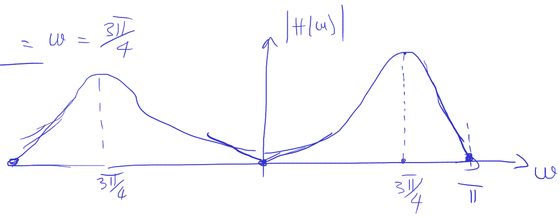
$$H(z) = \underbrace{C}_{=1} \cdot \frac{(z+1)(z+1)}{(z-0.9)(z-0.8)} = \frac{(z+1)\cdot(z+1)}{(z-0.9)(z-0.8)} \qquad y[n] = \ldots$$

$$H(z) = \frac{\overset{b_0}{1}\cdot z^2 + \overset{b_1}{2}z + \overset{b_2}{1}}{\underset{\substack{a_0 \\ =1 \\ \text{Must be } 1}}{1}\cdot z^2 \underset{a_1}{-1.7}z + \underset{a_2}{0.72}}$$

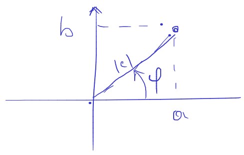
$\Rightarrow$ $y[n] = -a_1 y[n-1] - a_2 y[n-2] - \ldots + b_0 x[n] + b_1 x[n-1] + \ldots$

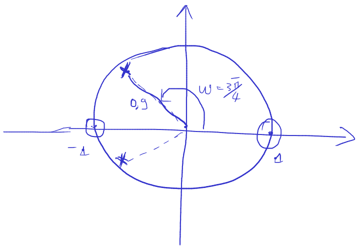
$$\boxed{y[n] = 1.7\, y[n-1] - 0.72\, y[n-2] + x[n] + 2x[n-1] + x[n-2]}$$

Band-pass filter, central $= \omega = \frac{3\pi}{4}$



$c = a + jb$
$= |c| \cdot e^{j\varphi}$





Zeros : 1 , -1

poles : $0.9 \cdot e^{j\,3\pi/4} = \ldots$

$0.9 \cdot e^{-j\,3\pi/4} = \ldots$

$$H(z) = \underbrace{C}_{=1} \cdot \frac{(z-1)(z+1)}{(z-0.9\cdot e^{j\,3\pi/4})(z-0.9\, e^{-j\,3\pi/4})} = \frac{z^2 - 1}{z^2 - z\cdot 0.9\cdot e^{j\,3\pi/4} - z\cdot 0.9\cdot e^{-j\,3\pi/4} + 0.9\cdot 0.9\cdot \underbrace{e^0}_{1}}$$

$$= \frac{z^2 - 1}{z^2 - z\cdot 0.9\cdot \underbrace{\left(e^{j\,3\pi/4} + e^{-j\,3\pi/4}\right)}_{\left(\frac{e^{jx}+e^{-jx}}{2} = \cos x\right)} + 0.9^2}$$

$2\cdot \cos\frac{3\pi}{4} = \cancel{2}\cdot \frac{-\sqrt{2}}{\cancel{2}}$

$$= \frac{\overset{b_0}{1}\cdot z^2 \overset{b_1}{+0}\cdot z + \overset{b_2}{-1}}{\underset{a_0}{1}\, z^2 + \underset{a_1}{0.9\cdot\sqrt{2}}\cdot z + \underset{a_2}{0.81}}$$

$$\Rightarrow \boxed{y[n] = -0.9\sqrt{2}\, y[n-1] - 0.81\, y[n-2] + x[n] - x[n-2]}$$

2

a. $H(z) = 7 + 3z^{-1} + z^{-2} + 7z^{-3} + 3z^{-4} + z^{-5}$
b. $H(z) = \frac{1+2z^{-1}+z^{-2}}{1-2z^{-1}+z^{-2}}$
c. $H(z) = 1 + 2z^{-1} + z^{-2}$
d. $H(z) = 1 - 2z^{-1} + z^{-2}$
e. $H(z) = 1 - 2z^{-1} - 2z^{-2} + z^{-3}$
f. $H(z) = 1 + 2z^{-1} + 7z^{-2} - 2z^{-2} - z^{-3}$
g. $H(z) = 1 - z^{-1}$
h. $H(z) = 1 - z^{-2}$

FIR filter

$h[0]$

$h[n] = [7, 3, 1, 7, 3, 1]$
$z^{0} \; z^{-1} \; z^{-2} \; z^{-3} \; z^{-4} \; z^{-5}$

Linear-phase ?

a). $h[n] = [7 \; 3 \; 1 \; 7 \; 3 \; 1]$ ⟹ NO, because symmetry not good

Not good

b). $H(z) = \dfrac{1+2z^{-1}+z^{-2}}{1-2z^{-1}+z^{-2}}$ ⟹ NO, because not an FIR

c). $h[n] = [1 \; 2 \; 1]$ ⟹ YES, positive symmetry

d). $h[n] = [1 \; -2 \; 1]$ ⟹ YES ——— // ———

e) $h[n] = [1 \; -2 \; -2 \; 1]$ ⟹ YES, ——— // ———

f) $h[n] = [1 \; 2 \; 7 \; -2 \; -1]$ ⟹ NO, because 7 doesn't follow the same rule as the others

Not good (should be 0)

g). $H(z) = 1 - z^{-1}$ ⟹ $h[n] = [1 \; -1]$ ⟹ YES, negative symmetry

h). $H(z) = 1 - z^{-2}$ ⟹ $h[n] = [1 \; 0 \; -1]$ ⟹ YES, ——— // ———
$z^{0} \quad z^{-1} \quad z^{-2}$