

Embedded System Design and Modeling

III. Extended FSMs and Timed Automata

FSM example

► Recall the previous FSM example

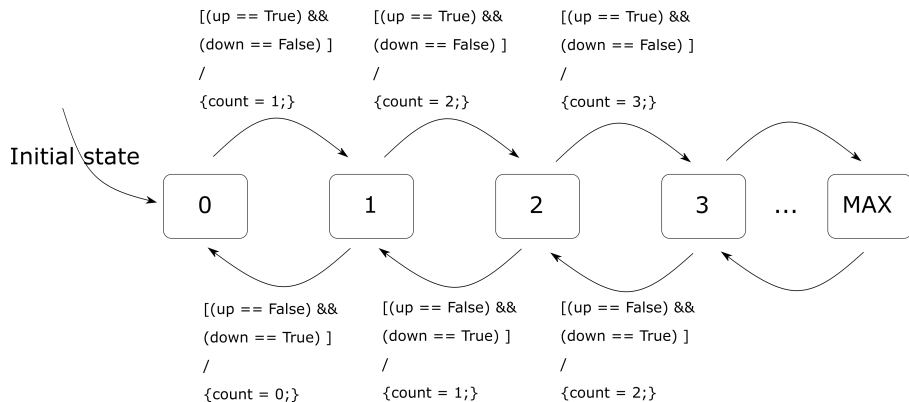


Figure 1: Parking system FSM

► Can we make it is simpler to draw?

Extended FSMs

► Extended FSM = FSM with **internal variables**

Inputs:

up: bool

down: bool

Outputs:

count: integer (0, MAX)

Variables:

count: integer (0, MAX)

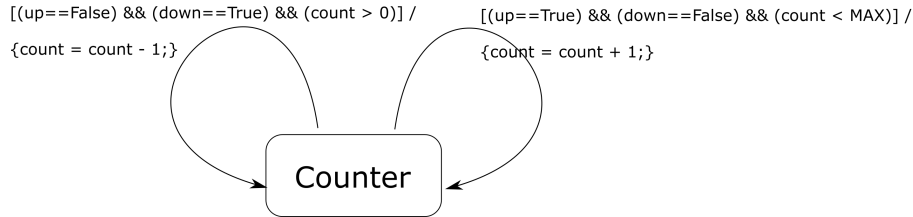


Figure 2: Extended FSM with variable “count”

- ▶ The state of the model = the current “bubble” and the values of **all the internal variables**
- ▶ Example: OS hibernation in Windows:
 - ▶ state of computer = all the RAM memory values
 - ▶ if all memory is written down on HDD, and reloaded tomorrow, the system effectively resumes operation from where it left off
- ▶ State is not anymore “the number of bubbles”
 - ▶ there is only one “bubble” in our FSM
 - ▶ but there are $\text{MAX}+1$ states (all possible values of the count variable)

Declarations

- ▶ Always make explicit declaration of:
 - ▶ model inputs
 - ▶ model outputs
 - ▶ model internal variables
 - ▶ and their data types

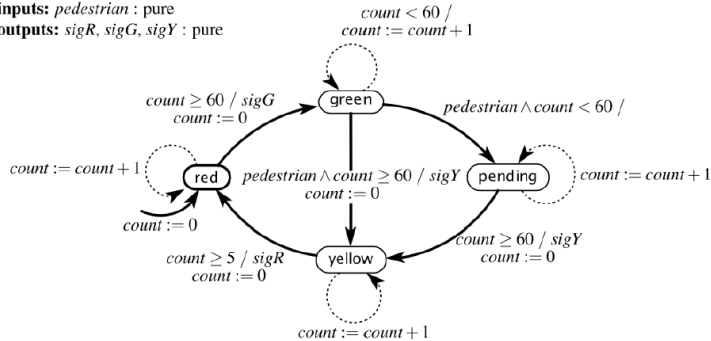
Measure time

- ▶ Extended FSM are useful for modeling **time-based** conditions:
 - ▶ measure passage of time: increment a variable every *tick*
 - ▶ only works if the FSM is time-triggered

Example: pedestrian crossing light

- ▶ How is time measured in the model below?
- ▶ How many states does the model below have?

variable: $count: \{0, \dots, 60\}$
inputs: $pedestrian: \text{pure}$
outputs: $sigR, sigG, sigY: \text{pure}$



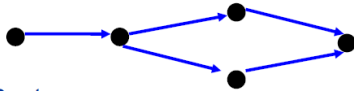
This model assumes one reaction per second
(a *time-triggered* model)

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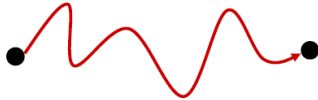
Figure 3: Extended FSM with time measuring (image from Seshia' slides)

Hybrid systems

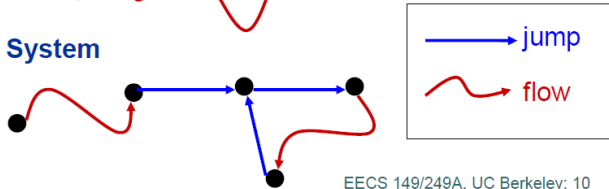
Discrete System (FSM)



Continuous System



Hybrid System



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Figure 4: Hybrid systems (image from Seshia' slides)

Hybrid systems

- ▶ **Hybrid systems** = system with mixes discrete and continuous behavior
- ▶ Example: a PID controller with different modes:
 - ▶ a set of distinct functioning model (e.g. Startup / Normal / Idle)
 - ▶ each state is a sub-system implemented with continuous dynamics
- ▶ State **refinement** = a lower-level implementation of a state

Types of hybrid systems

- ▶ **Timed automata** = hybrid system where every state refinement just measures passage of time (differential equation of degree 1)
- ▶ **Higher-order systems** = hybrid system where every state refinement uses higher-order differential equation (2 or more)
- ▶ **Two-level control systems** = complex controllers with two levels of operation
 - ▶ high-level discrete modes of operation (e.g. ECU Power Modes: Normal / Startup / Sleep Mode 1 / Sleep Mode 2)
 - ▶ low-level refinements with continuous dynamics

Timed automata

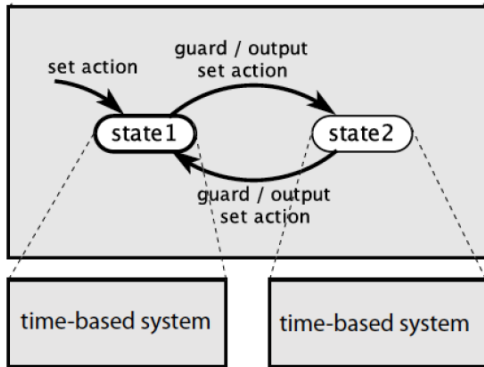


Figure 5: Timed automaton example (image from Seshia's slides)

Example

► Mouse Double-click detector model

continuous variable: $x(t) \in \mathbb{R}$

inputs: $click \in \{present, absent\}$

outputs: $single, double \in \{present, absent\}$

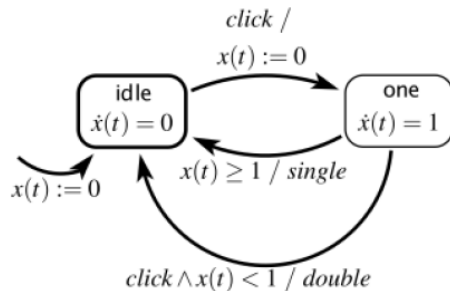


Figure 6: (image from Seshia's slides)

- Here $\dot{x}(t) = 1$ means “ $x(t)$ increases linearly with time”, so it measures time

Example: Another Thermostat

- ▶ Another thermostat model as a Timed Automaton

Temperature threshold is 20 with minimum times T_c and T_h in each mode

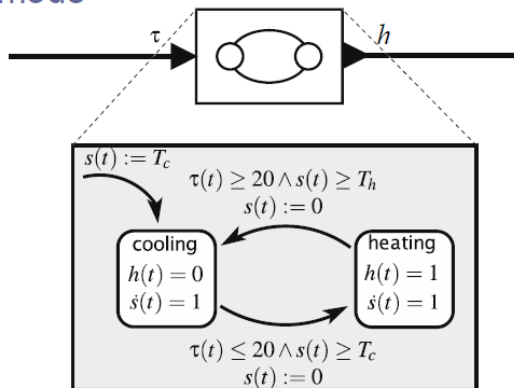


Figure 7: (image from Seshia's slides)

Example: Another Thermostat

- ▶ Another thermostat model as a Timed Automaton

Temperature threshold is 20 with minimum times T_c and T_h in each mode

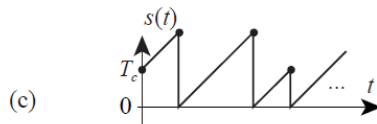
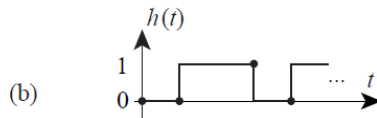
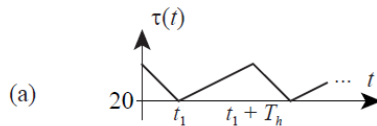
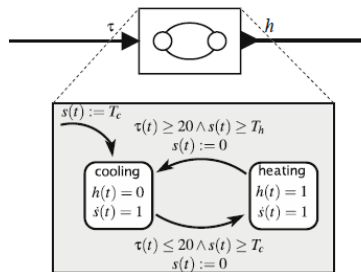


Figure 8: (image from Seshia's slides)

Example: Another Traffic Light

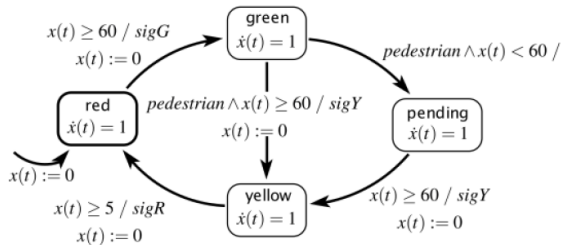
► Traffic Light controller Timed Automaton

Timed automaton model of a traffic light controller

continuous variable: $x(t) : \mathbb{R}$

inputs: *pedestrian*: pure

outputs: *sigR*, *sigG*, *sigY*: pure



This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

Figure 9: (image from Seshia's slides)

Example: Tick generator

- ▶ Timed Automaton to generate a *tick* every T seconds

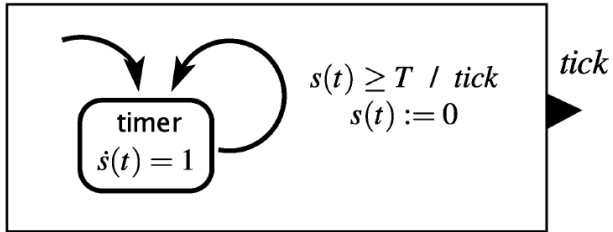
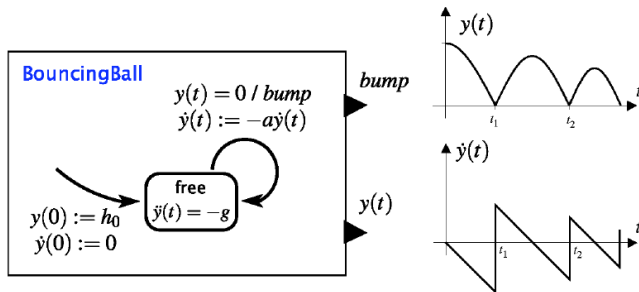


Figure 10: (image from Seshia's slides)

Example: Bouncing Ball

- Timed Automaton to simulate a bouncing ball movements

Hybrid Automaton for Bouncing Ball



y – vertical distance from ground (position)

a – coefficient of restitution, $0 \cdot a \cdot 1$



Figure 11: (image from Seshia's slides)