Embedded System Design and Modeling

II. Modeling of discrete systems

Actor model of systems

A system can be decomposed as inter-connected building blocks, called "actors"

- Each actor has:
  - 0, 1 or more input ports
  - 0, 1 or more output ports
  - an internal computation / function / what it does
- Connections = Signals

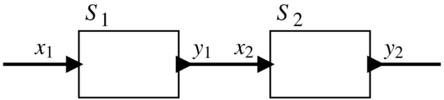


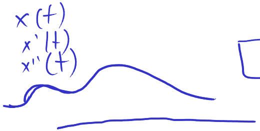
Figure 1: Actor model of systems[1]

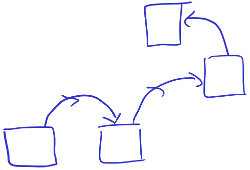
[1](Image from Lee & Seshia 2017)

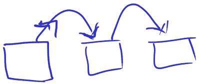
Actor dynamics

How to describe what a component does?

- Continuous dynamics (previous lecture)
- Discrete dynamics (from now on)







Ancient philosophy debate: Heraclitus (continuous) vs Parmenides (discrete)

Discrete dynamics

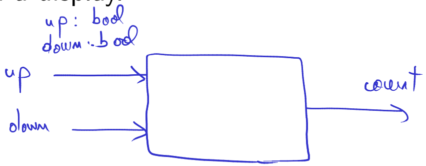
$u(t) = L \cdot \frac{di(t)}{dt}$

- **Dynamic system** = system whose state evolves in time
- **Discrete dynamics** = the system operates in a sequence of discrete steps
  - there are no continuous changes (no continuous signals)
  - like digital circuits (values change only on clock front)
- It's more a mathematical model (real-life is continuous), but still extremely useful

Sample discrete system

Example of discrete system model:

- Sense the cars which enter and leave a parking area (e.g. at barriers), and display the current number of cars inside the parking on a display.



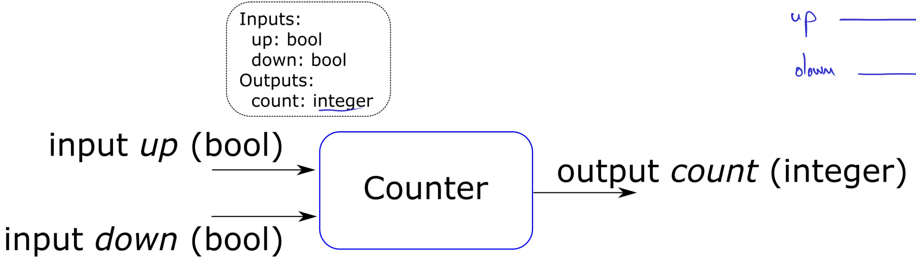


Figure 2: Parking system

State, reaction, transitions

- **State** of the system = condition of system at a particular point in time
  - The state encompasses everything in the past that has any influence at the current moment
- When any input is True, the system **reacts**
- **Reaction** means that the system changes its internal state, and enters a new state
- Moving from one state to the next state means a **transition**.

Finite State Machine representation

- Finite State Machine = a system whose operation is described as a set of states and transitions

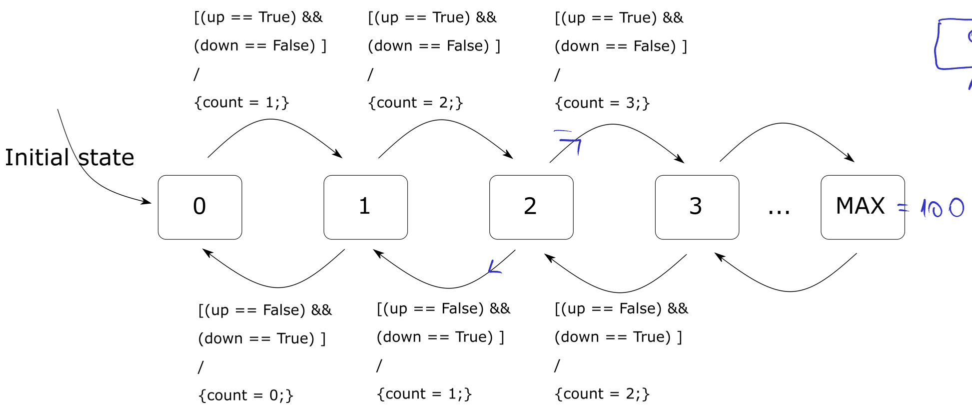
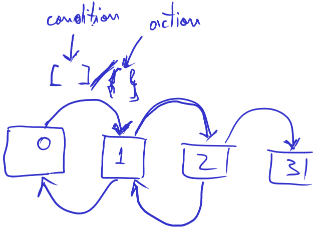


Figure 3: Parking system FSM



Components of a FSM representation

- States = the "bubbles"
- Transitions = the arrows
- Conditions (guards) = the conditions on the transitions are taken (inside "[]")
- Actions = the instructions executed when a transition is taken(after "/", inside "{}")

FSM notations

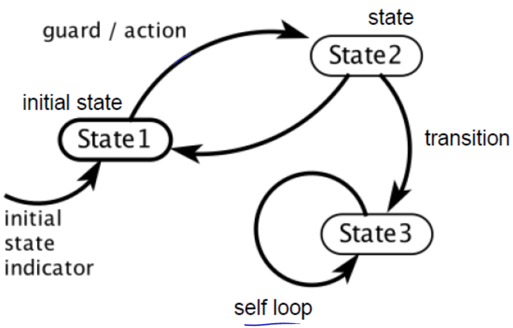


Figure 4: FSM Notations [2]

[2] image from Seshia's slides

Conditions and actions

- A transition is taken when its condition becomes True
- When a transition is taken, the actions are executed
- It is possible that no transition is taken, so the system preserves its state ("default transition")
- The **initial transition** indicates which is the starting states

FSM mathematical model

A FSM is a tuple (States, Inputs, Outputs, update, initialState) consisting of the following:

- States = a set $0, 1, \ldots M$
- Inputs = a set of variables with their data types
- Outputs = a set of variables with their data types
- update = a function $f : States \times Inputs \rightarrow States \times Outputs$
  - the function takes as inputs = old state + current input values
  - the function outputs = new state + current output values
- initialState = the initial state

$\boxed{f\,(\text{old State},\ \text{Inputs}) \longrightarrow \text{New State},\ \text{New outputs}}$

If all of the above is known, everything is known about the model.

Conditions and transitions

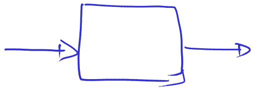
- Conditions and transitions can be written in many ways
- Here we use a simple C / Matlab instructions:
  - use == to check equality
  - ! means negation
  - True, False = boolean values
- Examples:
  - [a == True]
  - [!a == True]
  - [x >= 3]
  - [x < b]
  - etc ...

[a == True && b > 5]

[!(b <= 5]

Thermostat

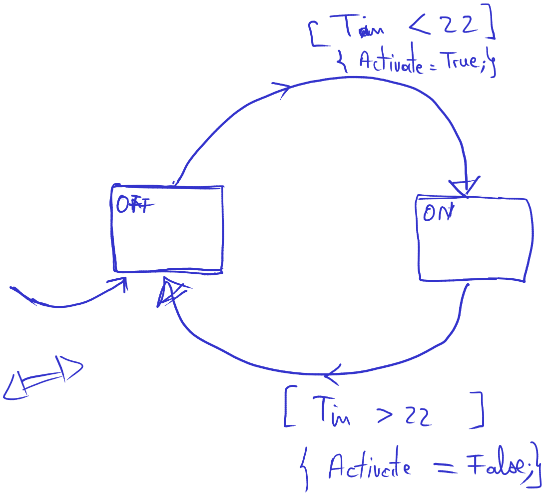
Inputs : Tin : real
Output : Activate : bool



Model example: thermostat

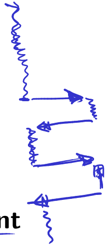
[To draw]

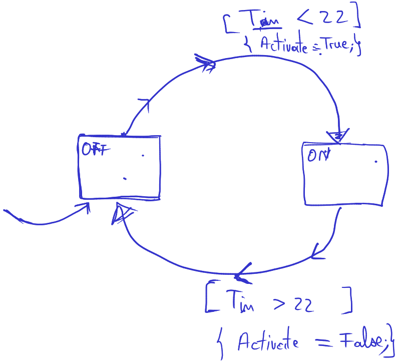
```
while (True)
    if (Tin < 22)
        Activate := True ;
    if (Tin > 22)
        Activate := False ;
    sleep (1 sec.)
```

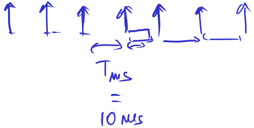


When does a reaction occur?

- When are transitions checked? (when do the reactions happen)?
- Two variants:
  - **Event-triggered** model
  - **Time-triggered** model
- Event-triggered model:
  - The reaction can take time **any time**.
  - The environment triggers the transition, via an **event**
  - Works like an **interrupt** in microcontrollers
- Time-triggered model:
  - The reaction occurs periodically, on the global *tick* of an **external clock**
  - e.g. everything runs at $T_s = 10\text{ms}$, 20ms etc.







Time-triggered models

- Simplest case = time-triggered models
- How it works:
  - the clock ticks, the FSM "wakes up" in a certain state
  - the inputs are read
  - the outgoing transitions from the current state are verified
  - if a transition is true, it is executed, the system enters a new state
  - the system "goes to sleep" until the next tick

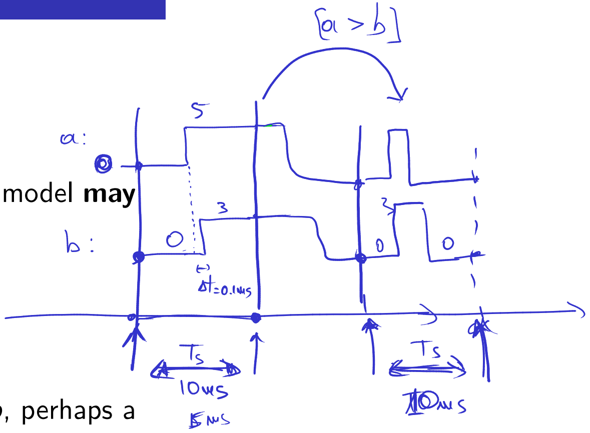
Event vs time-triggered models

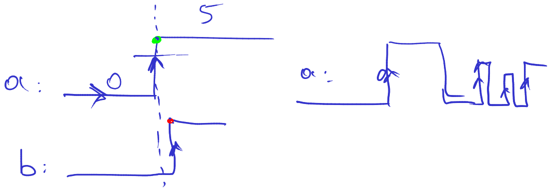
Advantages/disadvantages of time-triggered models:

- Bad: if a input changes very fast, within a T_s interval, the model **may not see it**
- Good: all inputs are read simultaneously
- Good: simple to understand

Advantages/disadvantages of event-triggered models:

- Bad: the inputs are not synchronized (in a condition $a > b$, perhaps a changes 1ms faster than b, and this leads to a wrong result
- Good: no risk that values are lost
- Bad: difficult to analyze, difficult to understand

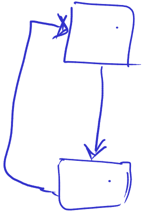


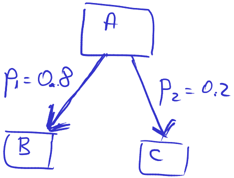


Properties of discrete models

Properties of discrete models

- **Determinism**: In every state, for all possible input values, at most one transition is enabled
  - if you know the initial state and all the inputs' evolution, you know the complete behavior of the system
- **Non-determinism**: Models unknown behavior (unknown inputs), or random transitions





Determinism computation tree

For a fixed input sequence and initial state:

- A deterministic system exhibits a single behavior
- A non-deterministic system exhibits a set of behaviors, visualized as a **computation tree**

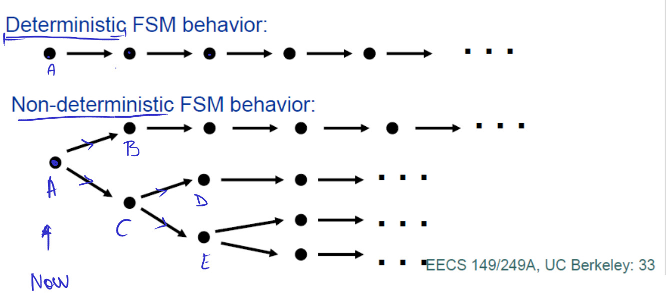


Figure 5: Computation tree [3]

[3] image from Seshia's slides