

## Embedded System Design and Modeling

## VI. Composition of State Machines

# Composition of state machines

- ▶ How to combine multiple smaller FSMs into a bigger one?
- ▶ What problems arise?
- ▶ Two types of compositions:
  1. **Spatial** composition: how are the components connected?
  2. **Temporal** composition: how do the components react in time?

# Spatial composition

**Spatial** composition = how are two components connected, how does the information flow between the components

- ▶ **Side-by-side** composition = no common inputs/outputs, no shared data
- ▶ **Cascade** composition = Outputs of one FSM are inputs to another one
- ▶ **Feedback** composition = (Some) outputs of a FSM are inputs to the same FSM, or to some other component which is in front

# Side-by-side composition

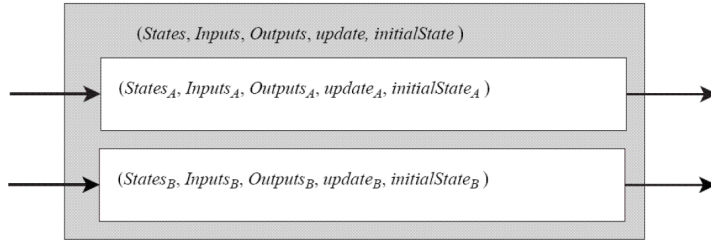


Figure 1: Side-by-side composition

# Cascade composition

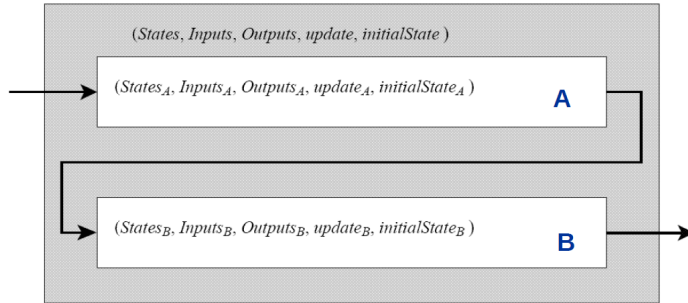


Figure 2: Cascade composition

- Outputs of FSM A are inputs to FSM B

# Feedback composition

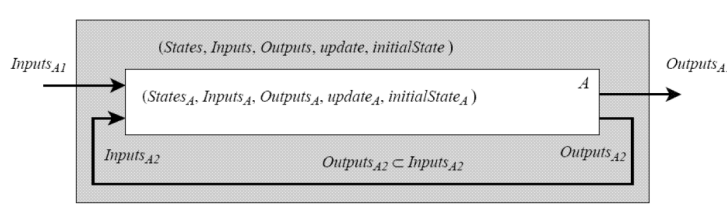


Figure 3: Feedback composition

- Some outputs of the FSM are coming back as inputs

# Temporal composition

**Temporal** composition = when do two components react?

▶ **Sequential vs Parallel** composition:

- ▶ **Sequential** = the two FSM do not work at the same time
- ▶ **Parallel** = the two FSM work at the same time

▶ **Asynchronous vs Synchronous** composition = only for parallel composition

- ▶ **Synchronous** = transitions are taken at the same time in both FSMs
- ▶ **Asynchronous** = transitions are taken at independent times in the FSMs



# Sequential composition

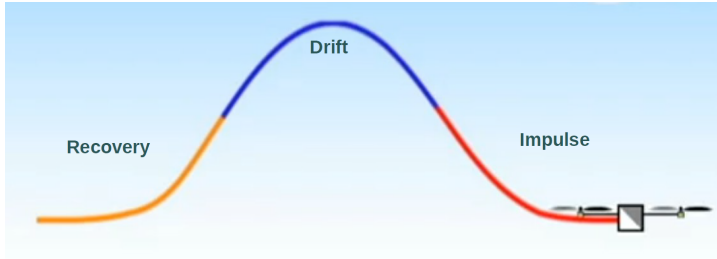


Figure 4: Example of Sequential composition

- ▶ <https://www.youtube.com/watch?v=iD3QgGpzzlM>
- ▶ The drone has three modes of operation, working **in sequence**

# Parallel composition

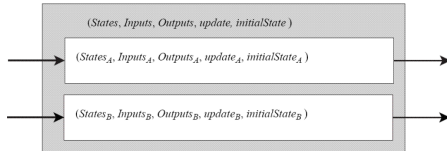


Figure 5: Side-by-side composition

- ▶ The two FSMs form an **equivalent model**
- ▶ When do the transitions in these FSM take place?
  - ▶ Synchronous: simultaneously
  - ▶ Asynchronous: independently

# Synchronous composition

- ▶ Consider the two FSM on the left (A and B)
- ▶ The equivalent model is on the right

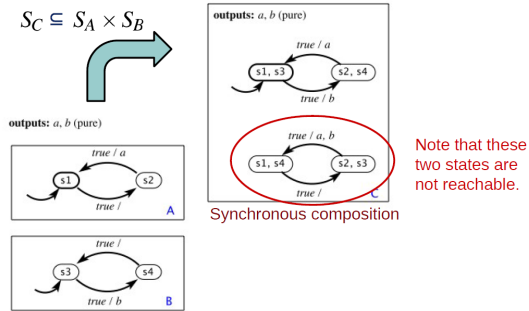


Figure 6: Synchronous composition

# Synchronous composition

Redraw here

# Synchronous composition

- ▶ In the equivalent model:
  - ▶ States = combination of states of the two FSMs
  - ▶ Transition = transition in FSM A and FSM B, happening simultaneously.
  - ▶ There might exist unreachable states in the equivalent model (states that will never be reached)

# Asynchronous composition

Draw here

# Asynchronous composition

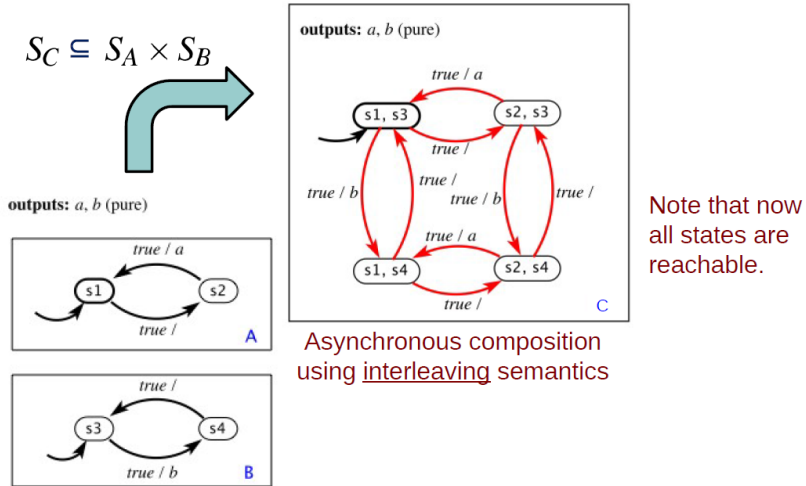


Figure 7: Asynchronous composition

# Asynchronous composition

- ▶ In the equivalent model:
  - ▶ States = combination of states of the two FSMs
  - ▶ Transitions in the two FSMs can take place at irregular and independent (not synchronized) times
  - ▶ All states are reachable
    - ▶ because one model can be much faster than the other



# Asynchronous composition

## Flavors of asynchronous composition

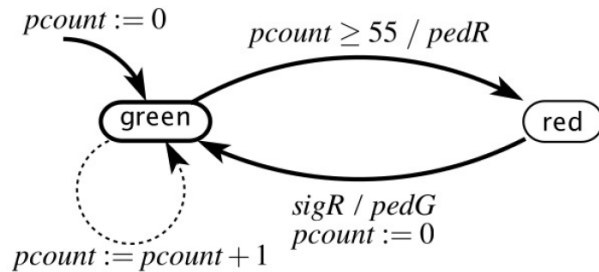
- ▶ How are simultaneous transitions handled?
- ▶ **Interleaving** semantics:
  - ▶ simultaneous transition in models A and B is not allowed (we may have either a transition in model A, or a transition in B)
  - ▶ i.e. transition from A takes place first, then transition from B takes place after a non-zero time delay (or vice-versa)
- ▶ **Simultaneous** semantics:
  - ▶ simultaneous transition in models A and B is allowed
  - ▶ for example, we may have either
    - ▶ transition only in model A
    - ▶ transition only in model B
    - ▶ Simultaneous transition in models A and B

## Example

**variable:**  $pcount: \{0, \dots, 55\}$

**input:**  $sigR$ : pure

**outputs:**  $pedG, pedR$ : pure



This light stays green for 55 seconds, then goes red.  
Upon receiving a  $sigR$  input, it repeats the cycle.

Figure 8: Composition - Pedestrian Light

# Example

**variable:**  $count: \{0, \dots, 60\}$

**inputs:**  $pedestrian : \text{pure}$

**outputs:**  $sigR, sigG, sigY : \text{pure}$

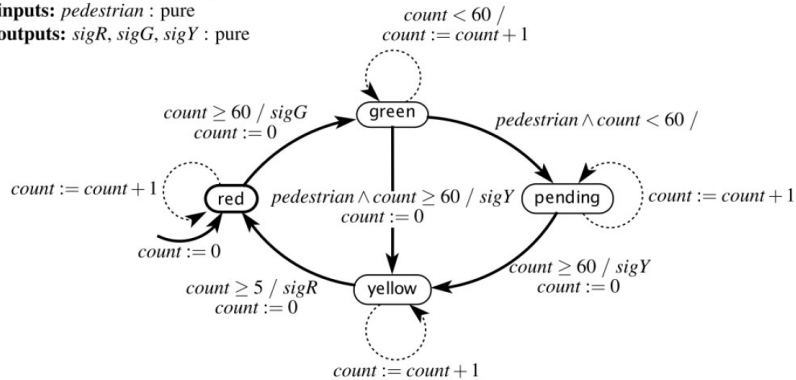
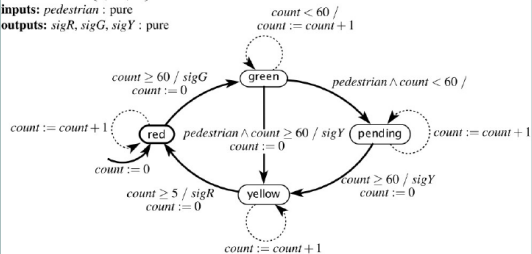


Figure 9: Composition - Car Light

# Example

## Pedestrian Light with Car Light

variable:  $count: \{0, \dots, 60\}$   
inputs:  $pedestrian: pure$   
outputs:  $sigR, sigG, sigY: pure$



sigY

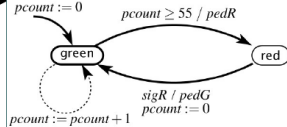
sigG

sigR

What is the size of the state space of the composite machine?

sigR

variable:  $pcount: \{0, \dots, 55\}$   
input:  $sigR: pure$   
outputs:  $pedG, pedR: pure$



pedG

pedR

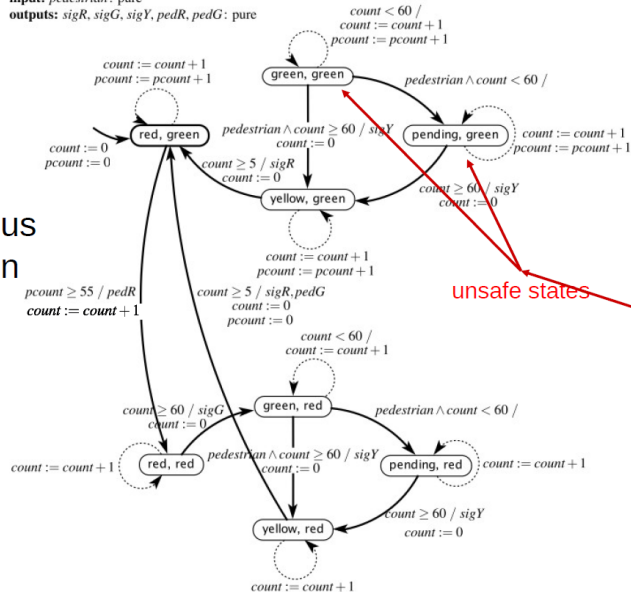
Figure 10: Cascade Composition - Both Lights

# Example

**variables:**  $count: \{0, \dots, 60\}, pcount: \{0, \dots, 55\}$

**input:**  $pedestrian: \text{pure}$

**outputs:**  $sigR, sigG, sigY, pedR, pedG: \text{pure}$



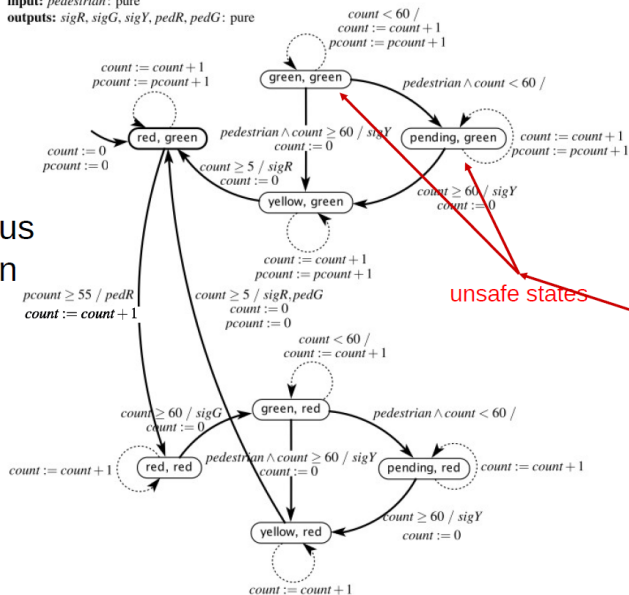
Synchronous  
composition

# Example

**variables:**  $count: \{0, \dots, 60\}, pcount: \{0, \dots, 55\}$

**input:**  $pedestrian: \text{pure}$

**outputs:**  $sigR, sigG, sigY, pedR, pedG: \text{pure}$



Synchronous  
composition

Other possibilities for model composition:

- ▶ **Shared** variables = variables which can be written / read by both models
  - ▶ Analysis much harder
  - ▶ Potential problems: What happens if both models try to access (read or write) the variable at the same time?
    - ▶ Answer: something bad. Might end up with an incorrect value
    - ▶ Solution: access to shared variable must be via **atomic operations** and guarded with a **mutex**

- ▶ **Atomic** operation = an operation that is indivisible (once it starts, it can't be interrupted until it ends)
- ▶ **Mutex** = a mechanism for ensuring only one process accesses a given resource (e.g. variable) at one time
  - ▶ A process first **acquires** the mutex, if it is available
  - ▶ Only afterwards it accesses the variable
  - ▶ While the mutex is acquired, no other process can access it
  - ▶ The process **releases** the mutex when it's done with the variable