Embedded System Design and Modeling

III. Extended FSMs and Timed Automata

FSM example

- Recall the previous FSM example

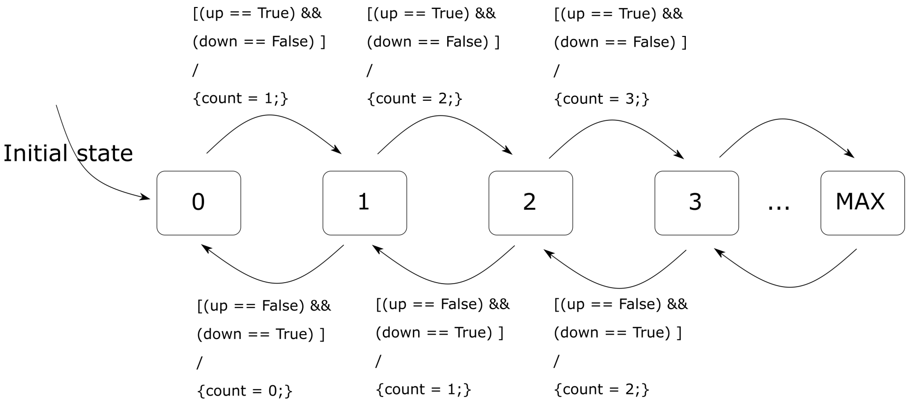


Figure 1: Parking system FSM

- Can we make it is simpler to draw?

Extended FSMs

- **Extended FSM** = FSM with **internal variables**

Inputs:
- up: bool
- down: bool

Outputs:
- count: integer (0, MAX)

internal Variables:
- count: integer (0, MAX)

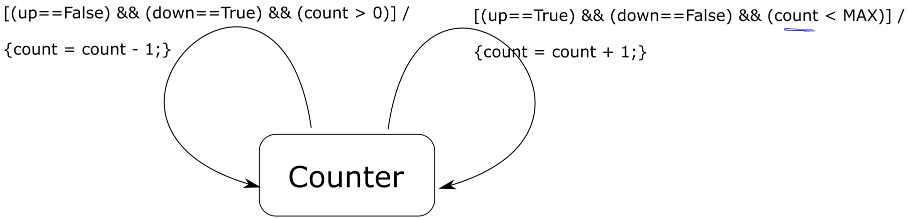
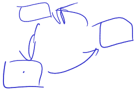


Figure 2: Extended FSM with variable "count"

How many states does this model have?

Extended FSM



- The state of the model = the current "bubble" and the values of **all the internal variables**
- Example: OS hibernation in Windows:
  - state of computer = all the RAM memory values
  - if all memory is written down on HDD, and reloaded tomorrow, the system effectively resumes operation from where it left off
- State is not anymore "the number of bubbles"
  - there is only one "bubble" in our FSM
  - but there are MAX+1 states (all possible values of the count variable)

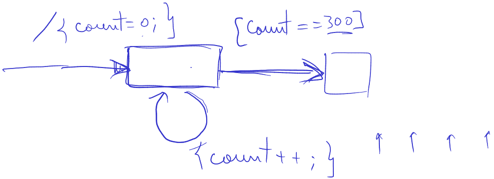
Declarations

- Always make explicit declaration of:
  - model inputs
  - model outputs
  - model internal variables
  - and their data types

Measure time

```
for ( count = 0 ; count < 300 ; count++ )
    wait 10 ms ;
```

$\Leftrightarrow$



- Extended FSM are useful for modeling **time-based** conditions:
  - measure passage of time: increment a variable every *tick*
  - only works if the FSM is time-triggered

$T_s = 10\ ms$

$300 \cdot T_s = 3$ seconds

Example: pedestrian crossing light

- How is time measured in the model below?
- How many states does the model below have?

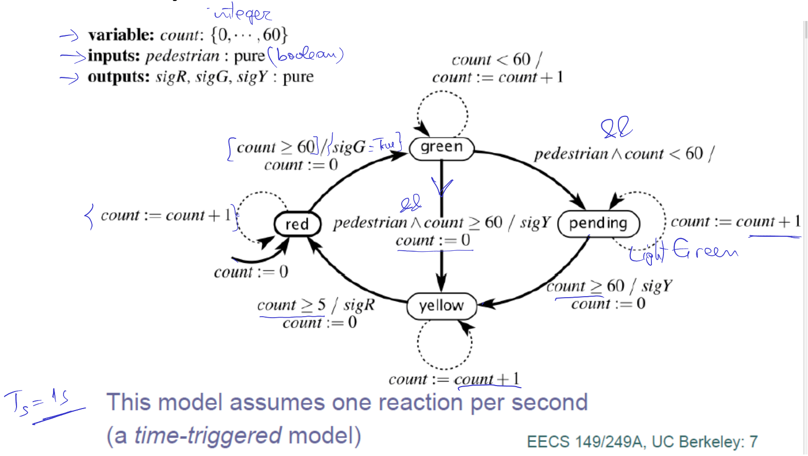


Figure 3: Extended FSM with time measuring [1]

[1] image from Seshia' slides

Hybrid systems

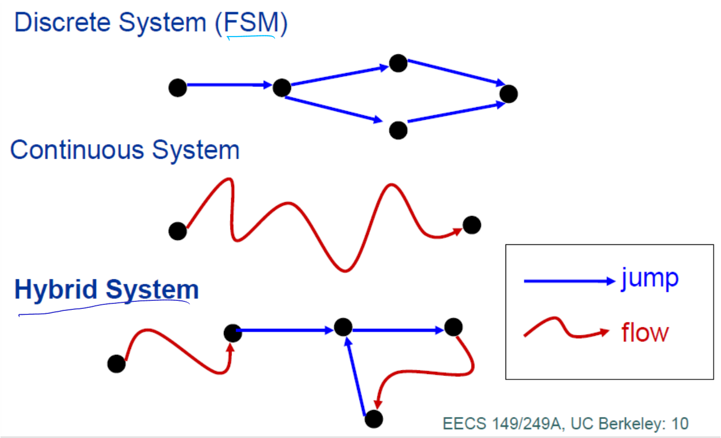
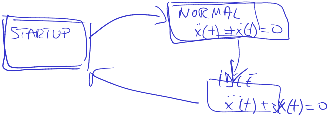


Figure 4: Hybrid systems [2]

[2] image from Seshia' slides

Hybrid systems

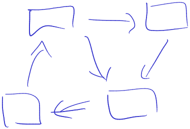


- **Hybrid systems** = system with mixes discrete and continuous behavior
- Example: a PID controller with different modes:
  - a set of distinct functioning model (e.g. Startup / Normal / Idle)
  - each state is a sub-system implemented with continuous dynamics
- State **refinement** = a lower-level implementation of a state

Types of hybrid systems

automaton

- **Timed automata** = hybrid system where every state refinement just measures passage of time (differential equation of degree 1)
- **Higher-order systems** = hybrid system where every state refinement uses higher-order differential equation (2 or more)
- **Two-level control systems** = complex controllers with two levels of operation
  - high-level discrete modes of operation (e.g. ECU Power Modes: Normal / Startup / Sleep Mode 1 / Sleep Mode 2)
  - low-level refinements with continuous dynamics



Timed automata

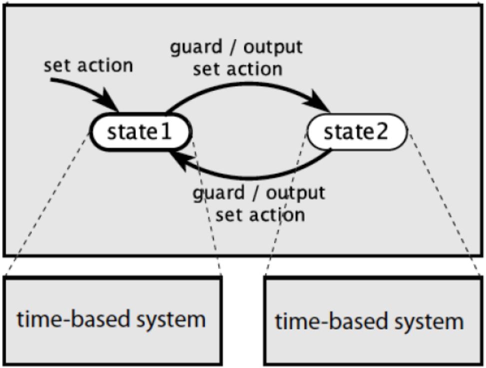
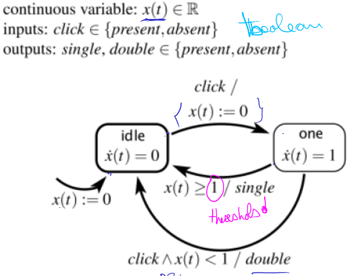


Figure 5: Timed automaton example [3]

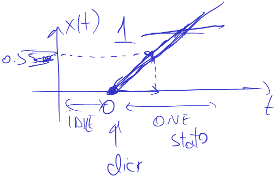
[3] image from Seshia' slides

Example



$\dot{x}(t) = 0$

$x'(t) = 0$



$\dot{x}(t) = 1$

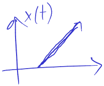


Figure 6: Mouse Double-click detector model [4]

- Here $\dot{x}(t) = 1$ means "x(t) increases linearly with time", so it measures time
- How many states does this model have? ∞ because $x(t) \in \mathbb{R}$

[4]image from Seshia' slides

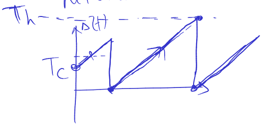
Example: Another Thermostat

Temperature threshold is 20 with minimum times T_c and T_h in each mode

Input $\tau(t)$ Temperature

Output $h(t)$

Internal $s(t)$



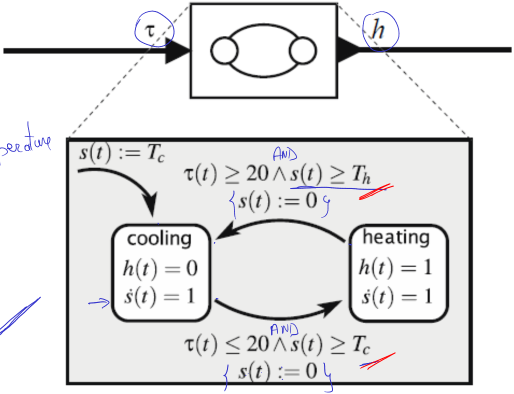


Figure 7: - Another thermostat model as a Timed Automaton [5]

[5] image from Seshia' slides

Example: Another Thermostat

Timed automaton

Temperature threshold is 20 with minimum times T_c and T_h in each mode

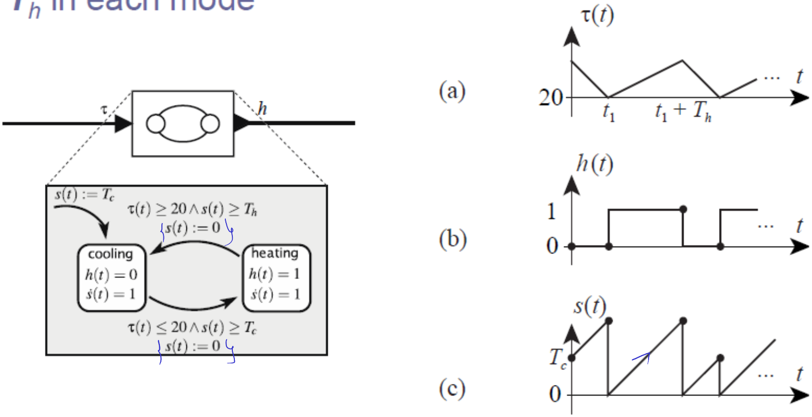
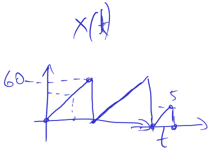


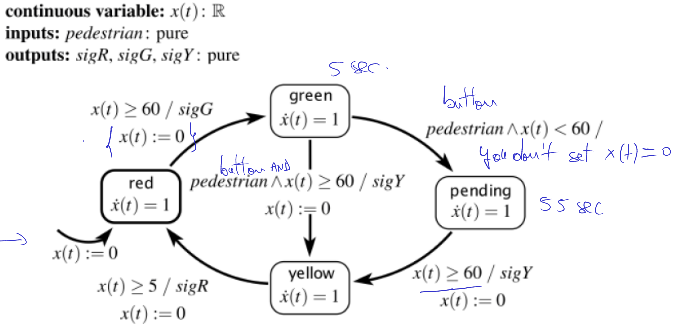
Figure 8: Input and output signals [6]

[6] image from Seshia' slides

Example: Another Traffic Light

Timed automaton model of a traffic light controller





This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

Figure 9: Traffic Light controller Timed Automaton [7]

[7] image from Seshia' slides

Example: Tick generator

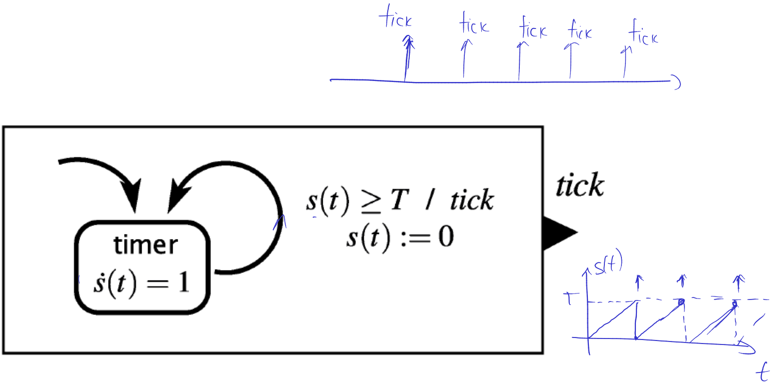


Figure 10: Timed Automaton to generate a *tick* event every T seconds [8]

[8] image from Seshia' slides

Example: Bouncing Ball

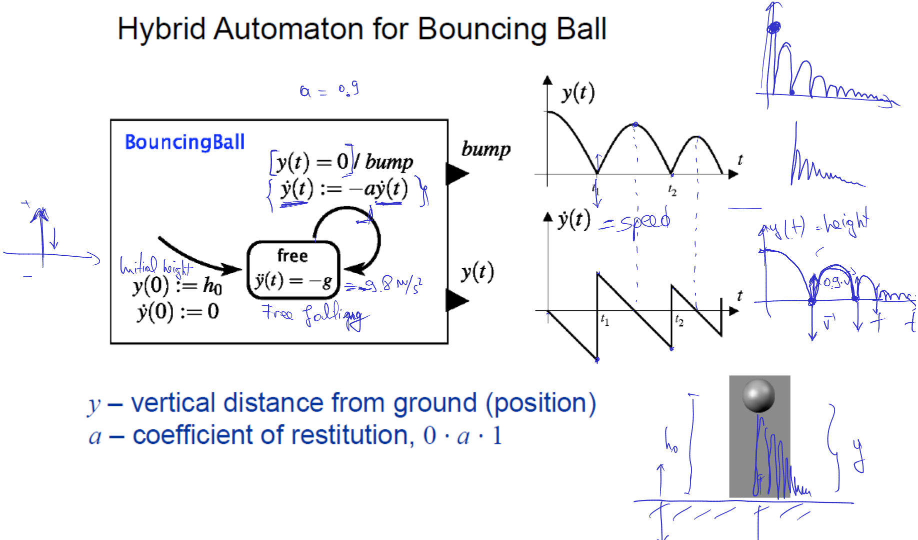


Figure 11: Timed Automaton to simulate a bouncing ball movements [9]

[9] image from Seshia' slides

FSM simulation software

- FSM simulation software
- Used in this class: Stateflow (Simulink / Matlab)
- Features:
  - State Actions
  - Temporal Logic
  - Other events
  - ... other ...

State actions

- Actions can exist not only on transitions, but also **inside states**
- Three main types of **State Actions**:
  - **entry (en)**: executed only when a **state is entered**
  - **exit (ex)**: executed only when a **state is exited**
  - **during (du)**: executed when we are in state which is neither entered, not exited

State actions

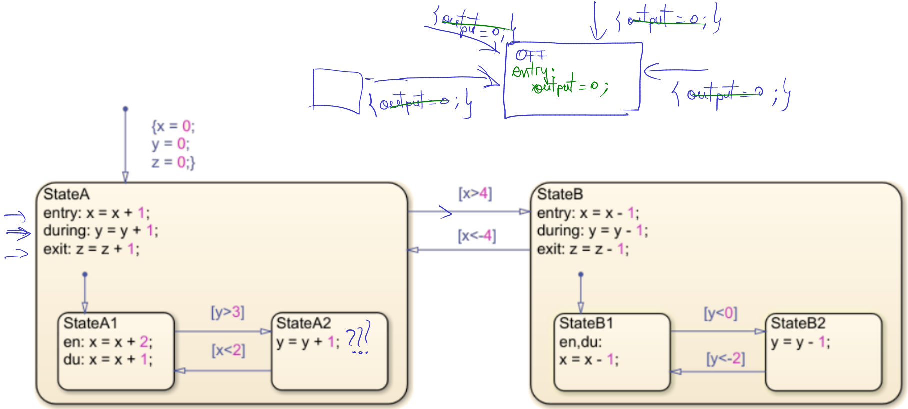


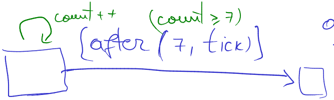
Figure 12: State Actions example [10]

[10] image from Matlab docs

State actions

- State actions can be avoided (use only transitions actions), but sometimes one or the other are more convenient

Temporal logic



0.07 seconds
3.5 seconds

$T_s = 0.01$

$T_s = 0.5$

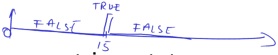
- For time-based conditions, states certain predefined variables, which can be used to **measure time spent in a state**
  - ***tick***: measures time steps
    - is incremented at **every time step**
    - is reset to 0 every time a state is exited or entered
    - actual duration **depends** on model step size
  - ***sec / msec***: measures seconds or miliseconds
    - is incremented every second / milisecond
    - is reset to 0 every time a state is exited or entered
    - actual duration is **independent** on model step size

~~count~~

Temporal logic

- Temporal operators **after()**, **on()**, **every()** can generate events which can be used in conditions
- Examples:

on(15, sec)



  - ***after(10, tick)***:
    - event is fired after 10 time steps spent in a state
    - evaluates to FALSE for the first 9 steps, is TRUE every time after that
  - ***on(x, tick)***:
    - event is fired only **once**, exactly after x time steps spent in a state
    - evaluates to FALSE for the first $x-1$ time moments, is TRUE only once at the x-th moment, is FALSE after that
  - ***every(x, tick)***:
    - event is fired pariodically after x time steps
    - evaluates to FALSE for the first $x-1$ time moments, is TRUE once at the x-th moment, then FALSE for the next $x-1$ time moments, then TRUE again, and so on

Temporal logic

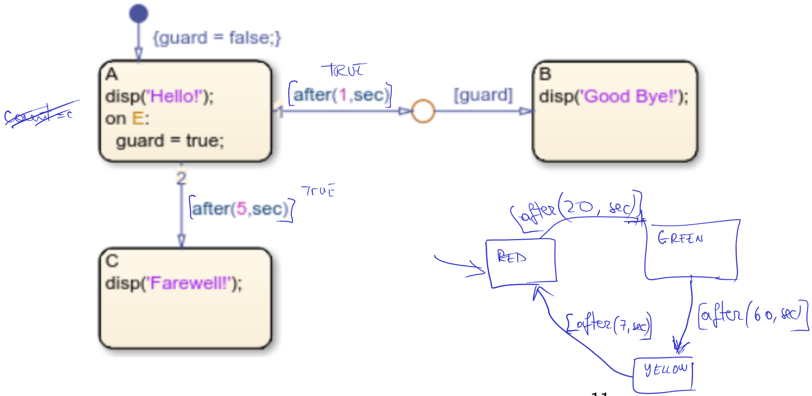


Figure 13: Temporal Logic example [11]

[11]image from Matlab docs