Embedded System Design and Modeling

VI. Composition of State Machines

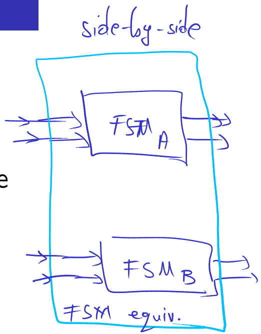
Composition of state machines

- How to combine multiple smaller FSMs into a bigger one?
- What problems arise?
- Two types of compositions:
  1. **Spatial** composition: how are the components connected?
  2. **Temporal** composition: how do the components react in time?

Spatial composition

Spatial composition = how are two components connected, how does the information flow between the components

- **Side-by-side** composition = no common inputs/outputs, no shared data
- **Cascade** composition = Outputs of one FSM are inputs to another one
- **Feedback** composition = (Some) outputs of a FSM are inputs to the same FSM, or to some other component which is in front



Side-by-side composition

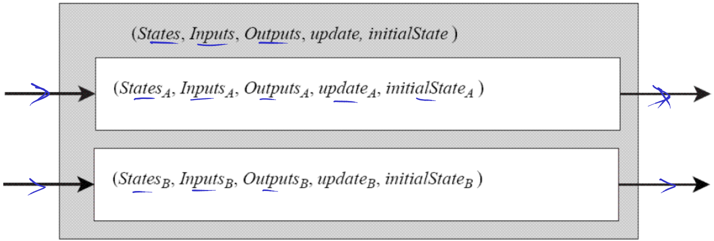


Figure 1: Side-by-side composition

Cascade composition

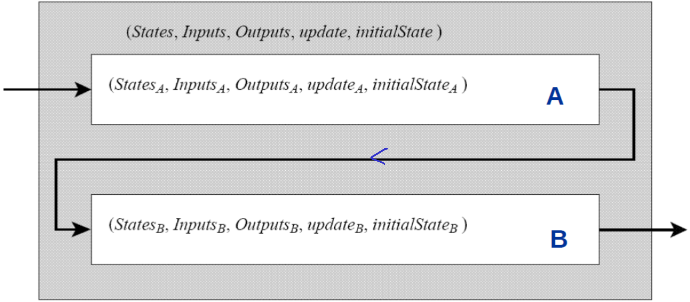
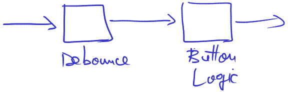


Figure 2: Cascade composition



- Outputs of FSM A are inputs to FSM B

Feedback composition

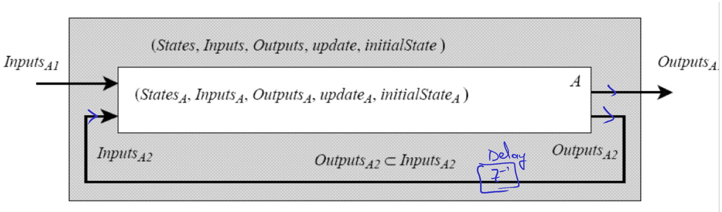
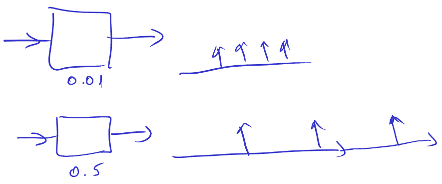


Figure 3: Feedback composition

- Some outputs of the FSM are coming back as inputs

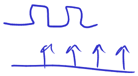
Temporal composition

time step 0.01



Temporal composition = when do two components react?

- **Sequential vs Parallel** composition:
  - **Sequential** = the two FSM do not work at the same time
  - **Parallel** = the two FSM work at the same time
- **Asynchonous vs Synchronous** composition = only for parallel composition



  - **Synchronous** = transitions are taken at the same time in both FSMs

    All FSM are driven by the same clock signal

  - **Asynchronous** = transitions are taken at independent times in the FSMs

Sequential composition

$\to \boxed{A} \to$

$\to \boxed{B} \to$

$\to \boxed{C} \to$

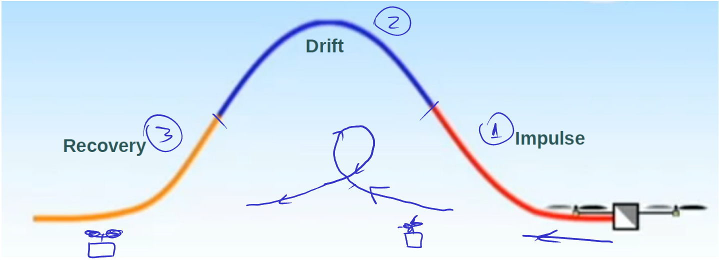


Figure 4: Example of Sequential composition

- 13579/:https://wwwoutubeom/watch?vD3QgGpzzIM
- The drone has three modes of operation, working **in sequence**

Parallel composition

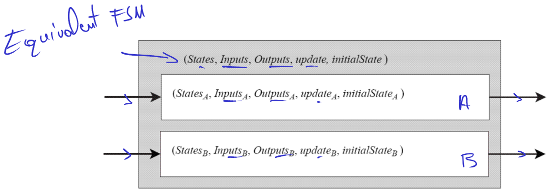


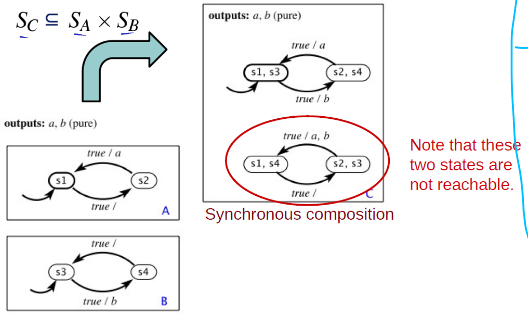
Figure 5: Side-by-side composition

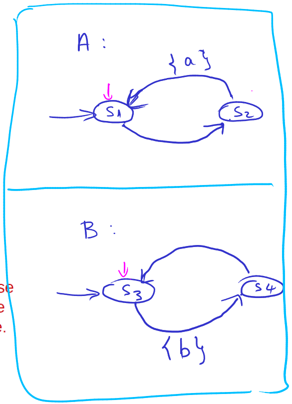
- The two FSMs form an **equivalent model**
- When do the transitions in these FSM take place?
    - Synchronous: simultaneously
    - Asynchronous: independently

Synchronous composition

- Consider the two FSM on the left (A and B)
- The equivalent model is on the right

$S_C \subseteq S_A \times S_B$





Equivalent finite state machine (FSM):

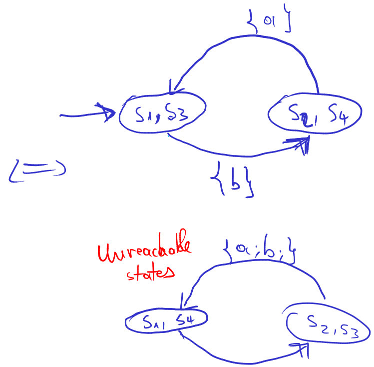


Figure 6: Synchronous composition

Synchronous composition

See Above

Redraw here

Synchronous composition

- In the equivalent model:
  - States = combination of states of the two FSMs
  - Transition = transition in FSM A and FSM B, happening simultaneously.
  - There might exist unreachable states in the equivalent model (states that will never be reached)

Asynchronous composition

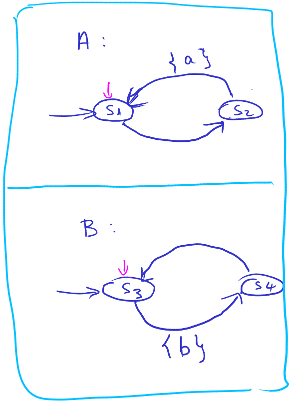
Transitions in A and B not simultaneous

Variant a). you never have trans. in A and B simult.

= "interleaving semantics"

Draw here

b). "simultaneous semantics"



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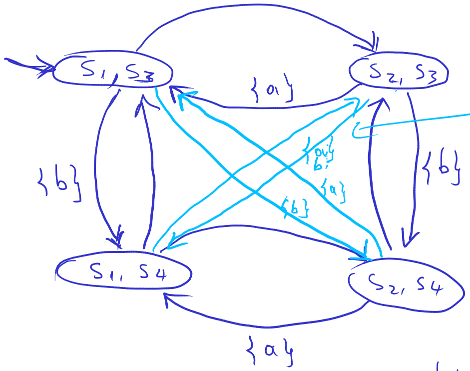
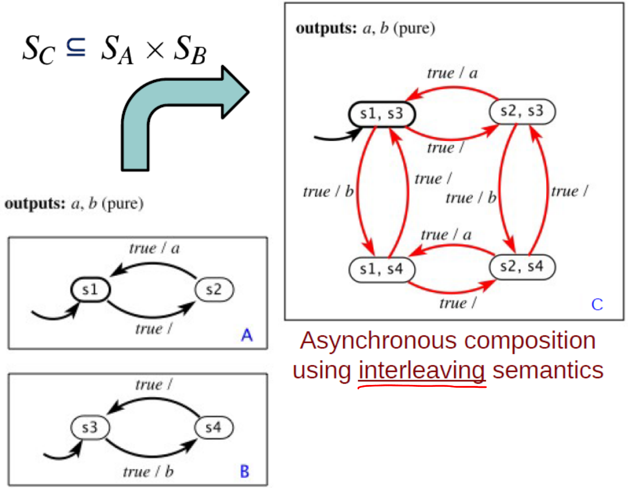
Equivalent FSM :



with simultaneous semantics

All states are reachable

Asynchronous composition



Note that now all states are reachable.

Figure 7: Asynchronous composition

Asynchronous composition

- In the equivalent model:
  - States = combination of states of the two FSMs
  - Transitions in the two FSMs can take place at irregular and independent (not synchronized) times
  - All states are reachable
    - because one model can be much faster than the other

Asynchronous composition

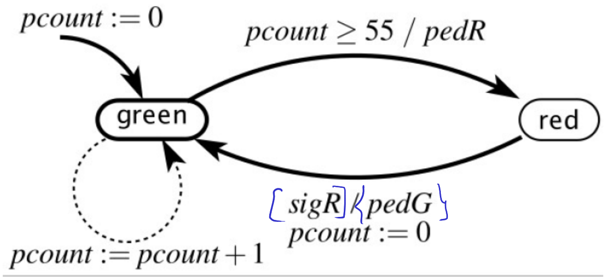
Flavors of asynchronous composition

- How are simultaneous transitions handled?
- **Interleaving** semantics:
  - simultaneous transition in models A and B is not allowed (we may have either a transition in model A, or a transition in B)
  - i.e. transition from A takes place first, then transition from B takes place after a non-zero time delay (or vice-versa)
- **Simultaneous** semantics:
  - simultaneous transition in models A and B is allowed
  - for example, we may have either
    - transition only in model A
    - transition only in model B
    - Simultaneous transition in models A and B

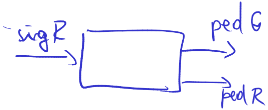
you may have:
- a transition in A, not in B
- a transition in B, not in A
- a trans. in A and one in B simultaneously

Example

variable: *pcount*: $\{0, \cdots, 55\}$
→ **input:** *sigR*: pure
→ **outputs:** *pedG*, *pedR*: pure



Pedestrian traffic light



This light stays green for 55 seconds, then goes red. Upon receiving a sigR input, it repeats the cycle.

Figure 8: Composition - Pedestrian Light

Example

Car
traffic
light

variable: *count*: $\{0, \cdots, 60\}$
inputs: *pedestrian* : pure
outputs: *sigR*, *sigG*, *sigY* : pure

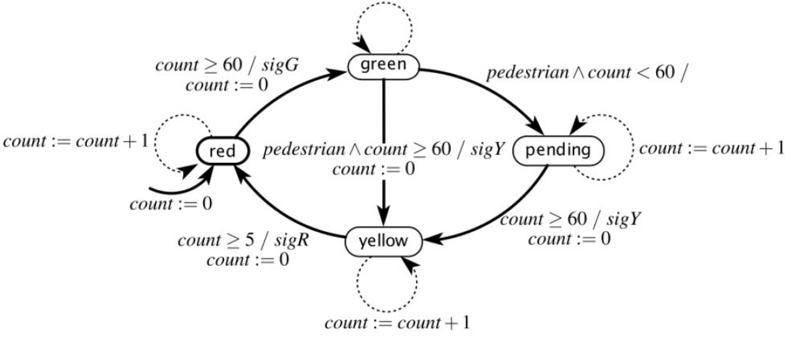
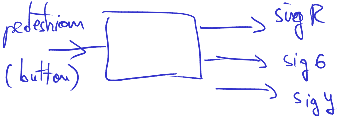
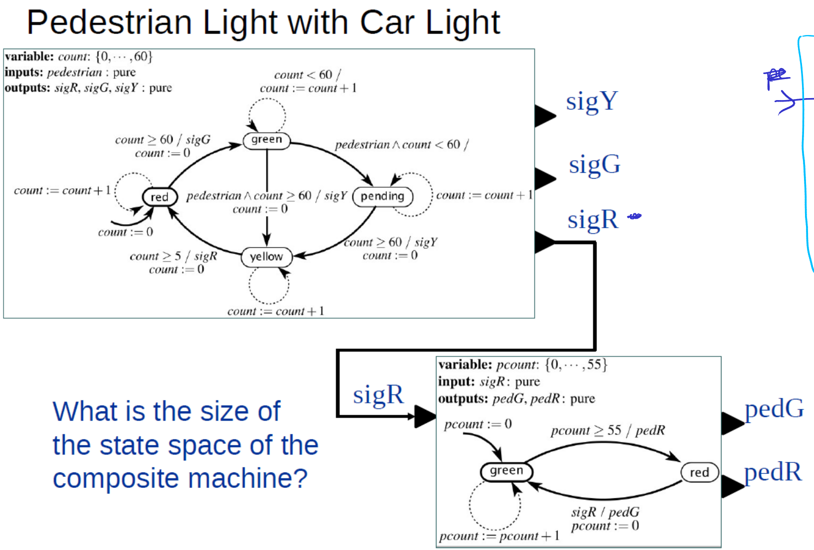


Figure 9: Composition - Car Light



Example



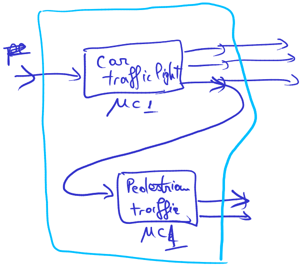
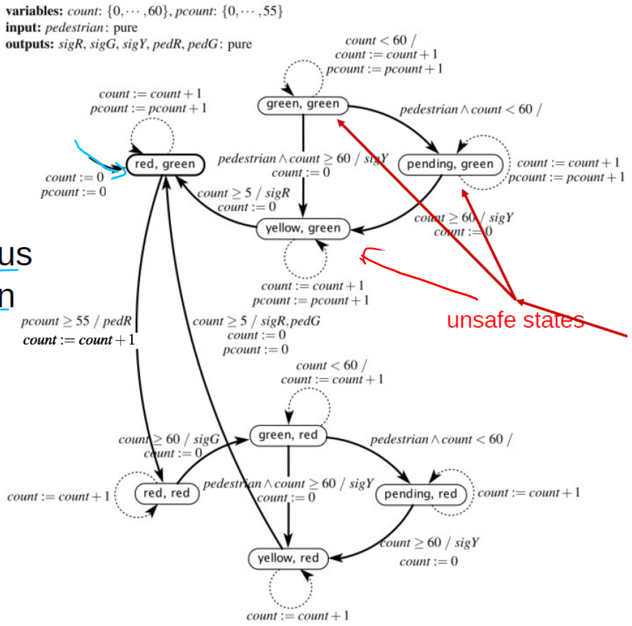
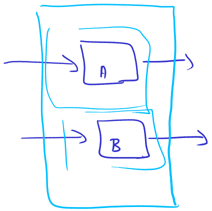


Figure 10: Cascade Composition - Both Lights

Example

Synchronous composition





Example

Synchronous composition