Embedded System Design and Modeling

VI. Composition of State Machines

Composition of state machines

- How to combine multiple smaller FSMs into a bigger one?

- What problems arise?

- Two types of compositions:
    1. **Spatial** composition: how are the components connected?
    2. **Temporal** composition: how do the components react in time?

Spatial composition

Spatial composition = how are two components connected, how does the information flow between the components

- ▶ **Side-by-side** composition = no common inputs/outputs, no shared data
- ▶ **Cascade** composition = Outputs of one FSM are inputs to another one
- ▶ **Feedback** composition = (Some) outputs of a FSM are inputs to the same FSM, or to some other component which is in front
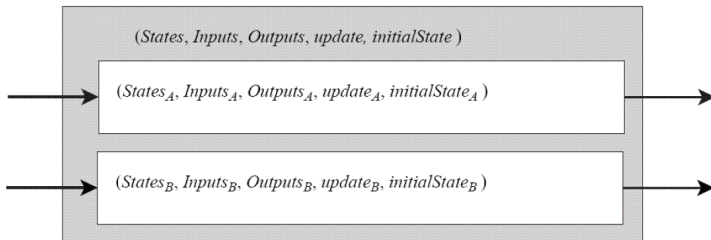
Side-by-side composition

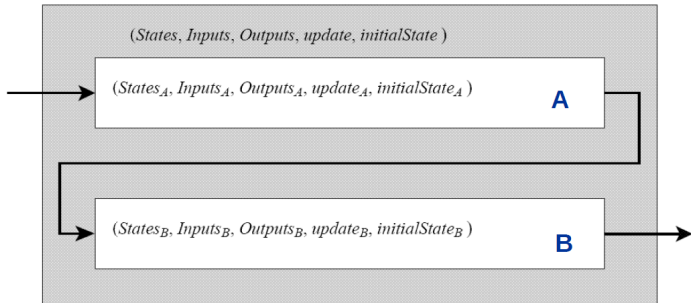Figure 1: Side-by-side composition

Cascade composition

Figure 2: Cascade composition

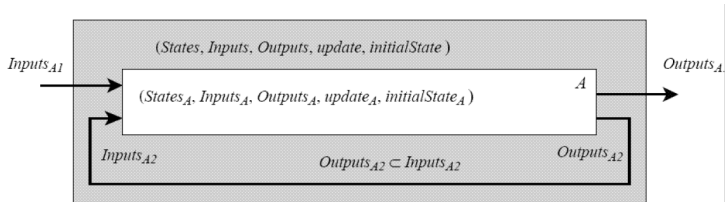- Outputs of FSM A are inputs to FSM B

Figure 3: Feedback composition

▶ Some outputs of the FSM are coming back as inputs

Temporal composition

Temporal composition = when do two components react?

- **Sequential vs Parallel** composition:
    - **Sequential** = the two FSM do not work at the same time
    - **Parallel** = the two FSM work at the same time

- **Asynchonous vs Synchronous** composition = only for parallel composition

    - **Synchronous** = transitions are taken at the same time in both FSMs

    - **Asynchronous** = transitions are taken at independent times in the FSMs
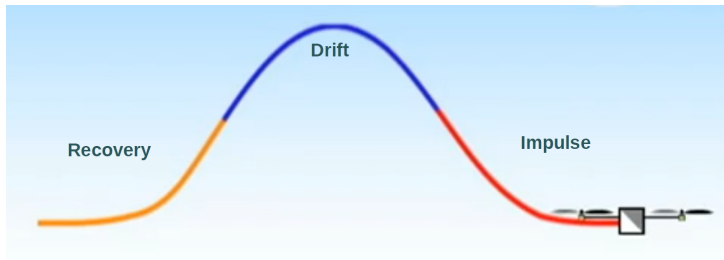
Sequential composition

Figure 4: Example of Sequential composition

- 13579/:https://wwwoutubeom/watch?vD3QgGpzzIM
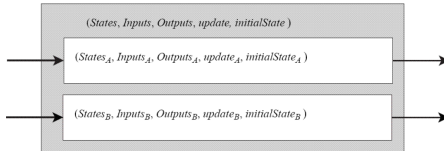- The drone has three modes of operation, working **in sequence**

Parallel composition

Figure 5: Side-by-side composition

▶ The two FSMs form an **equivalent model**

▶ When do the transitions in these FSM take place?

 ▶ Synchronous: simultaneously
 ▶ Asynchronous: independently

Synchronous composition

▶ Consider the two FSM on the left (A and B)
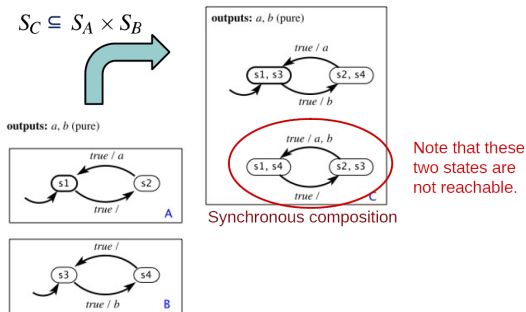
▶ The equivalent model is on the right

$$S_C \subseteq S_A \times S_B$$

outputs: a, b (pure)

Synchronous composition

Note that these two states are not reachable.

outputs: a, b (pure)

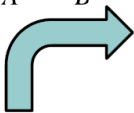Figure 6: Synchronous composition

Synchronous composition

Redraw here

- ▶ In the equivalent model:
    - ▶ States = combination of states of the two FSMs
    - ▶ Transition = transition in FSM A and FSM B, happening simultaneously.
    - ▶ There might exist unreachable states in the equivalent model (states that will never be reached)
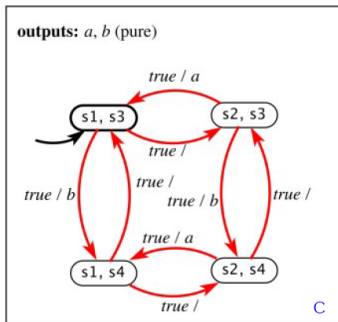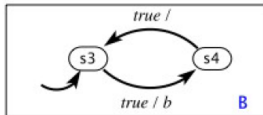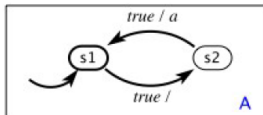
Asynchronous composition

Draw here

Asynchronous composition

Figure 7: Asynchronous composition

- In the equivalent model:
    - States = combination of states of the two FSMs
    - Transitions in the two FSMs can take place at irregular and independent (not synchronized) times
    - All states are reachable
        - because one model can be much faster than the other

Asynchronous composition

Flavors of asynchronous composition

- ▶ How are simultaneous transitions handled?

- ▶ **Interleaving** semantics:

    - ▶ simultaneous transition in models A and B is not allowed (we may have either a transition in model A, or a transition in B)
    - ▶ i.e. transition from A takes place first, then transition from B takes place after a non-zero time delay (or vice-versa)

- ▶ **Simultaneous** semantics:

    - ▶ simultaneous transition in models A and B is allowed

    - ▶ for example, we may have either

        - ▶ transition only in model A
        - ▶ transition only in model B
        - ▶ Simultaneous transition in models A and B

variable: *pcount*: $\{0, \cdots, 55\}$
input: *sigR*: pure
outputs: *pedG*, *pedR*: pure
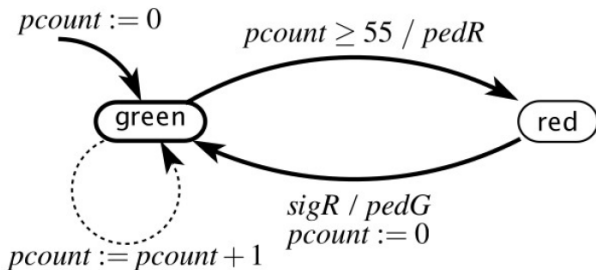
This light stays green for 55 seconds, then goes red.
Upon receiving a sigR input, it repeats the cycle.

Figure 8: Composition - Pedestrian Light

Figure 9: Composition - Car Light

Example

Pedestrian Light with Car Light
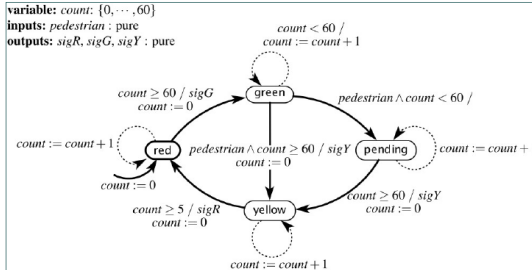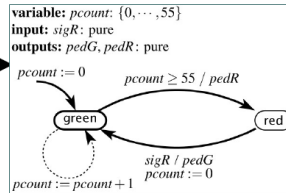
Figure 10: Cascade Composition - Both Lights

Example

Synchronous composition

unsafe states

Example

Synchronous composition

variables: *count*: $\{0, \cdots, 60\}$, *pcount*: $\{0, \cdots, 55\}$
input: *pedestrian*: pure
outputs: *sigR, sigG, sigY, pedR, pedG*: pure

unsafe states