

Embedded System Design and Modeling

II. Modeling of discrete systems

Discrete dynamics

- ▶ **Dynamic system** = system whose state evolves in time
- ▶ **Discrete dynamics** = the system operates in a sequence of discrete steps
 - ▶ there are no continuous changes (no continuous signals)
 - ▶ like digital circuits (values change only on clock front)
- ▶ It's more a mathematical model (real-life is continuous), but still extremely useful

Components of discrete signal models

Finite State Machine

- ▶ Systems = Actors
- ▶ States, Transitions, Conditions, Actions
- ▶ Properties

Sample discrete system

$$\text{boolean} = \{T, F\}$$

Example of discrete system model:

- Sense the cars which enter and leave a parking area (e.g. at barriers), and display the current number of cars inside the parking on a display.

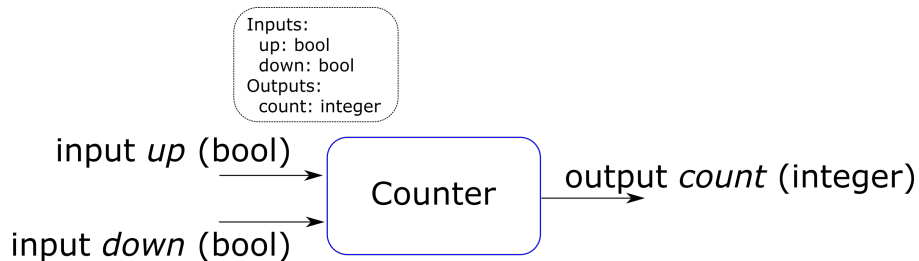


Figure 1: Parking system

State, reaction, transitions

- ▶ State of the system = condition of system at a particular point in time
 - ▶ The state encompasses everything in the past that has any influence at the current moment
- ▶ When any input is True, the system **reacts**
- ▶ Reaction means that the system changes its internal state, and enters a new state
- ▶ Moving from one state to the next state means a transition.

Finite State Machine representation

- Finite State Machine = a system whose operation is described as a set of states and transitions

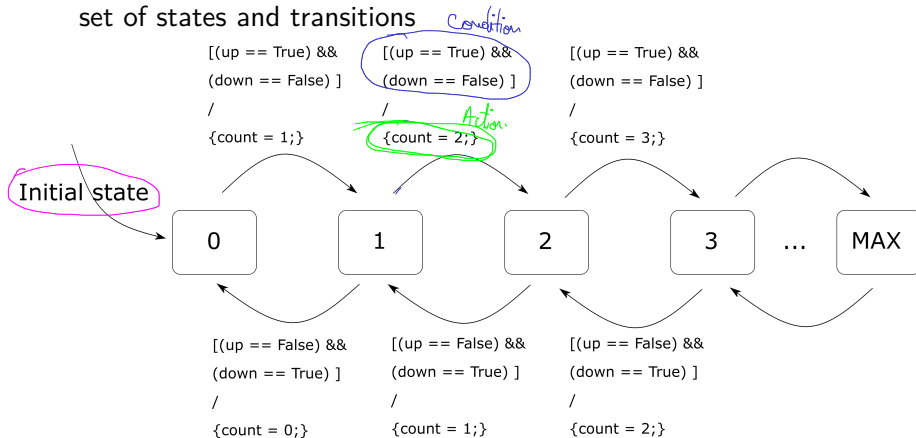
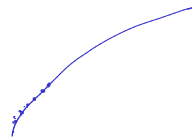
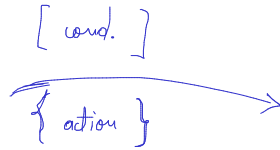


Figure 2: Parking system FSM

FSM



Components of a FSM representation



- ▶ States = the “bubbles”
- ▶ Transitions = the arrows
- ▶ Conditions (guards) = the conditions on the transitions are taken (inside “[]”)
- ▶ Actions = the instructions executed when a transition is taken (after “/”, inside “{ }”)

FSM notations

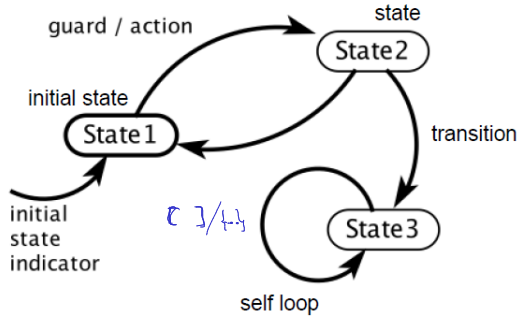


Figure 3: FSM Notations (image from Seshia slides)

Conditions and actions

- ▶ A transition is taken when its condition becomes True
- ▶ When a transition is taken, the actions are executed
- ▶ It is possible that no transition is taken, so the system preserves its state (“default transition”)
- ▶ The **initial transition** indicates which is the starting states

FSM mathematical model

A FSM is a tuple (States, Inputs, Outputs, update, initialState) consisting of the following:

- ▶ States = a set $0, 1, \dots, M$
- ▶ Inputs = a set of variables with their data types
- ▶ Outputs = a set of variables with their data types
- ▶ update = a function $f : \text{States} \times \text{Inputs} \rightarrow \text{States} \times \text{Outputs}$
 - ▶ the function takes as inputs = old state + current input values
 - ▶ the function outputs = new state + current output values
- ▶ initialState = the initial state

If all of the above is known, everything is known about the model.

Conditions and transitions

- ▶ Conditions and transitions can be written in many ways
- ▶ Here we use a simple C / Matlab instructions:
 - ▶ use `==` to check equality
 - ▶ `!` means negation
 - ▶ `True`, `False` = boolean values
- ▶ Examples:
 - ▶ `[a == True]`
 - ▶ `[!a == True]`
 - ▶ `[x >= 3]`
 - ▶ `[x < b]`
 - ▶ etc ...

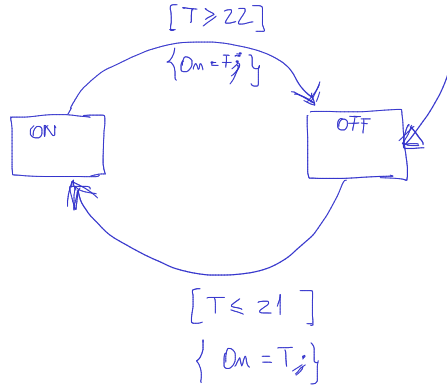
Thermostat

Input : T : read
Output : ~~Off~~ : bool ($T_{\text{set}} \neq$)
 On

Model example: thermostat

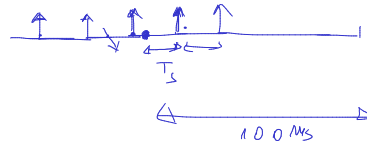
[To draw]

$21^{\circ} \rightarrow 22^{\circ}$



When does a reaction occur?

- ▶ When are transitions checked? (when do the reactions happen)?
- ▶ Two variants:
 - **Event-triggered** model
 - **Time-triggered** model
- ▶ Event-triggered model:
 - ▶ The reaction can take time any time.
 - ▶ The environment triggers the transition, not the model
 - ▶ Works **like an interrupt**
- ▶ Time-triggered model:
 - ▶ The reaction occurs on the tick of an **external clock**
 - ▶ e.g. everything runs at $T_s = 10\text{ms}$, 20ms etc.



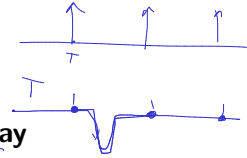
Time-triggered models

- ▶ Simplest case = time-triggered models
- ▶ How it works:
 - ▶ the clock ticks, the FSM “wakes up”
 - ▶ the inputs are read
 - ▶ the transitions from the current state are verified
 - ▶ if a transition is true, it is executed, the system enters a new state
 - ▶ the system “goes to sleep” until the next tick

Event vs time-triggered models

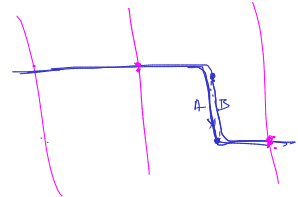
Advantages/disadvantages of time-triggered models:

- ▶ Bad: if a input changes very fast, within a T_s interval, the model may not see it
- ▶ Good: all inputs are read simultaneously
- ▶ Good: simple to understand



Advantages/disadvantages of event-triggered models:

- ▶ Bad: the inputs are not synchronized (in a condition $a > b$, perhaps a changes 1ms faster than b , and this leads to a wrong result)
- ▶ Good: no risk that values are lost
- ▶ Bad: difficult to analyze, difficult to understand



Traffic light controller

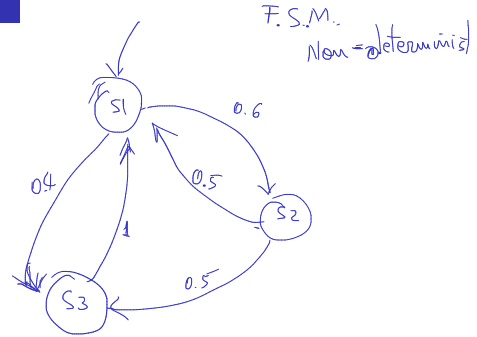
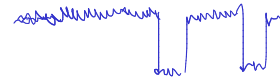
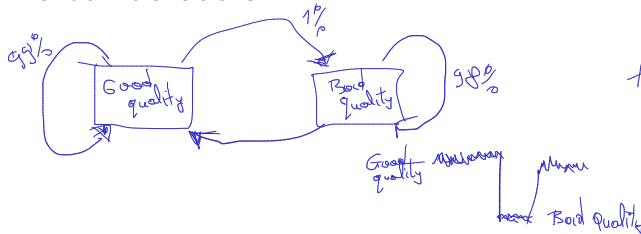
Model example: car traffic light controller

[To draw]

Properties of discrete models

Properties of discrete models

- ▶ **Determinism**: In every state, for all possible input values, at most one transition is enabled
 - ▶ if you know the initial state and all the inputs' evolution, you know the complete behavior of the system
- ▶ **Non-determinism**: Models unknown behavior (unknown inputs), or random transitions

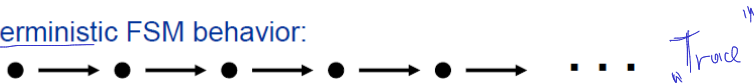


Determinism computation tree

For a fixed input sequence and initial state:

- ▶ A deterministic system exhibits a single behavior
- ▶ A non-deterministic system exhibits a set of behaviors, visualized as a computation tree

Deterministic FSM behavior:



Non-deterministic FSM behavior:

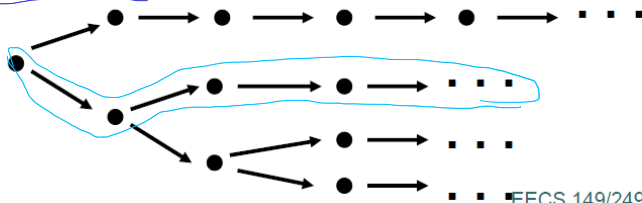


Figure 4: Computation tree (image from Seshia's slides)