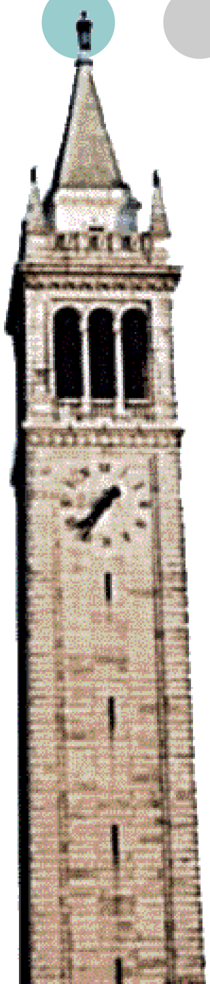

Introduction to Embedded Systems

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EECS 149/249A

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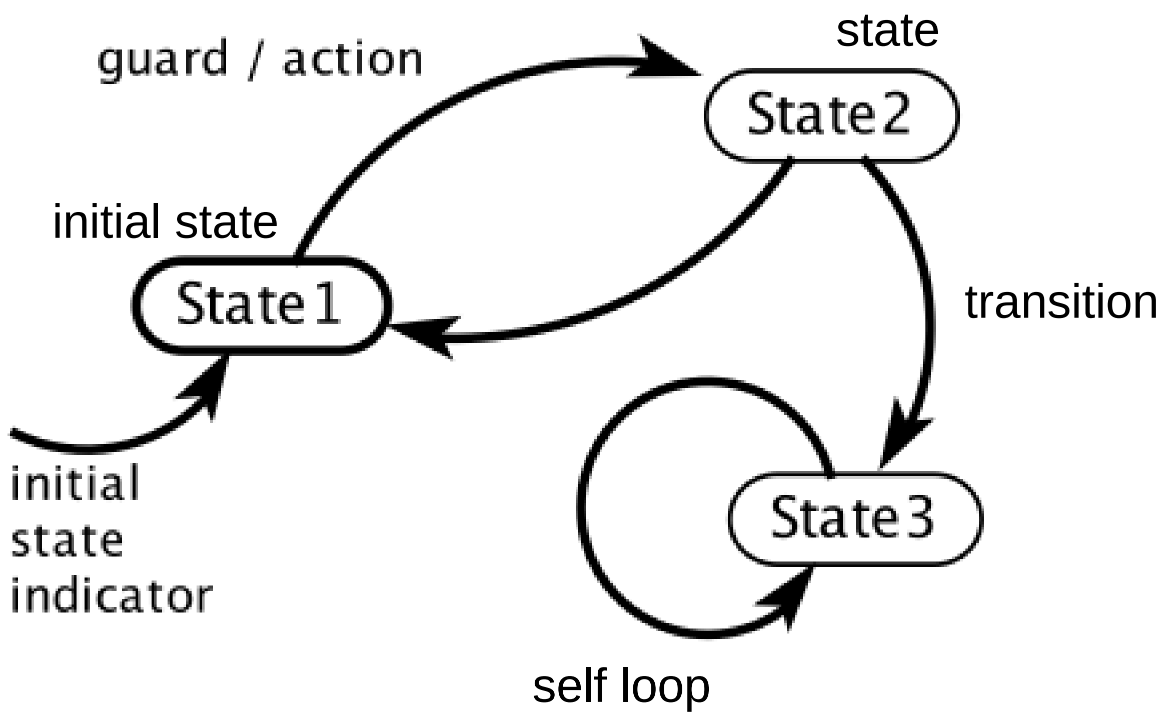
Chapter 3, 4: Extended Finite State Machines, Timed Automata, Hybrid Automata

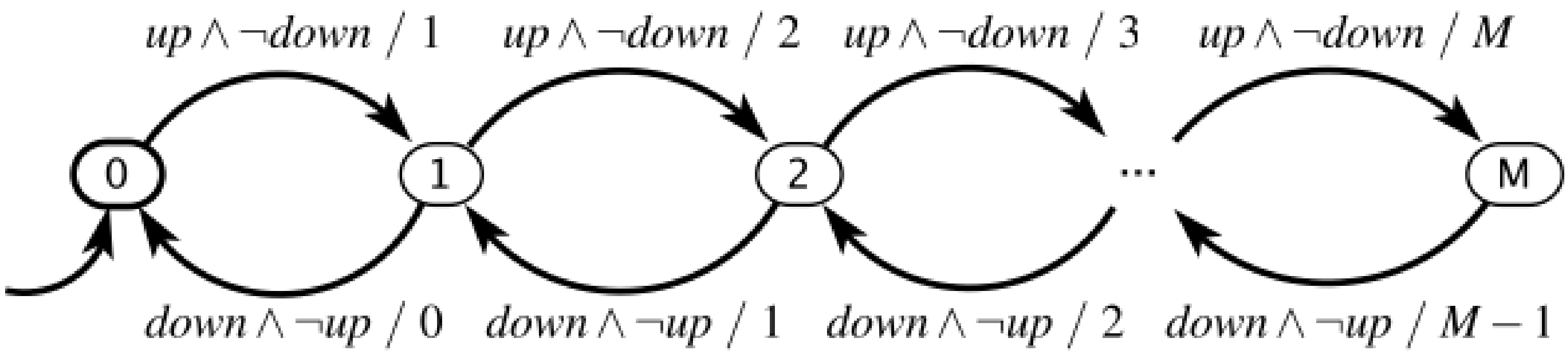
Recall FSM Notation



Garage Counter Example

Recall this example, which counts cars in a parking garage:

inputs: $up, down \in \{present, absent\}$
output $\in \{0, \cdots, M\}$

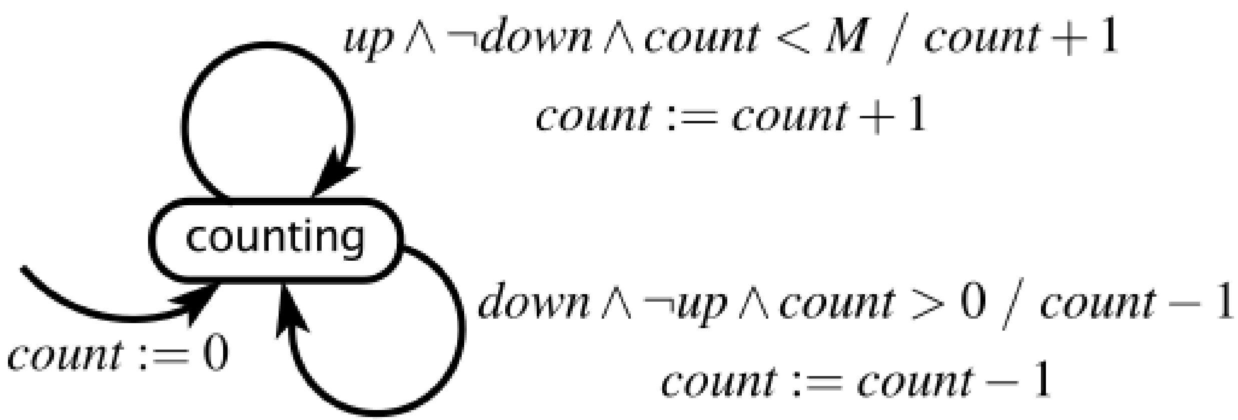


Every state is represented separately!

Extended State Machines

Extended state machines augment the FSM model with *variables* that may be read or written. E.g.:

variable: $count \in \{0, \cdots, M\}$
inputs: $up, down \in \{present, absent\}$
output $\in \{0, \cdots, M\}$



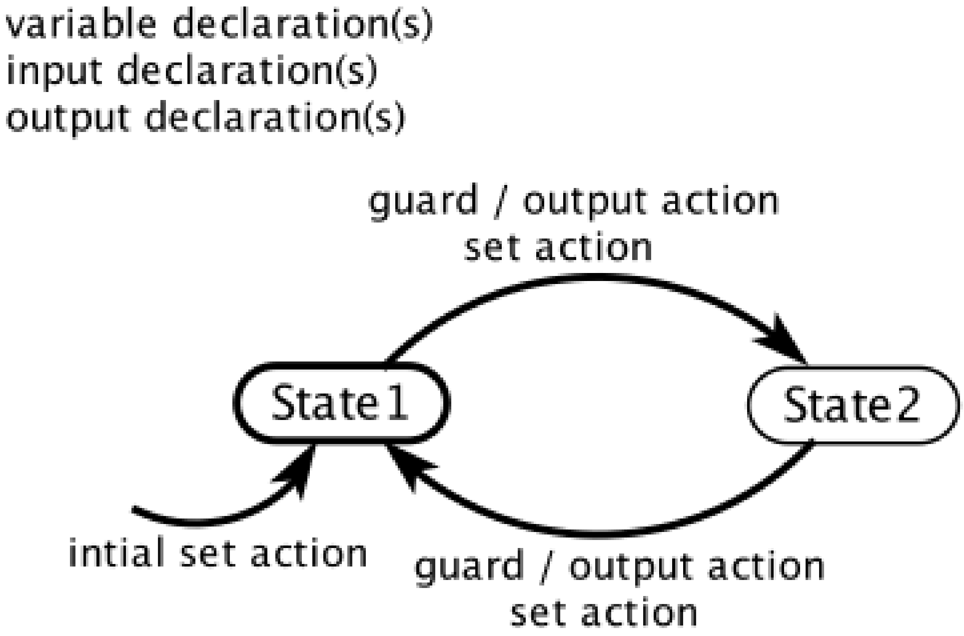
Question: What is the size of the state space?

Extended State Machines

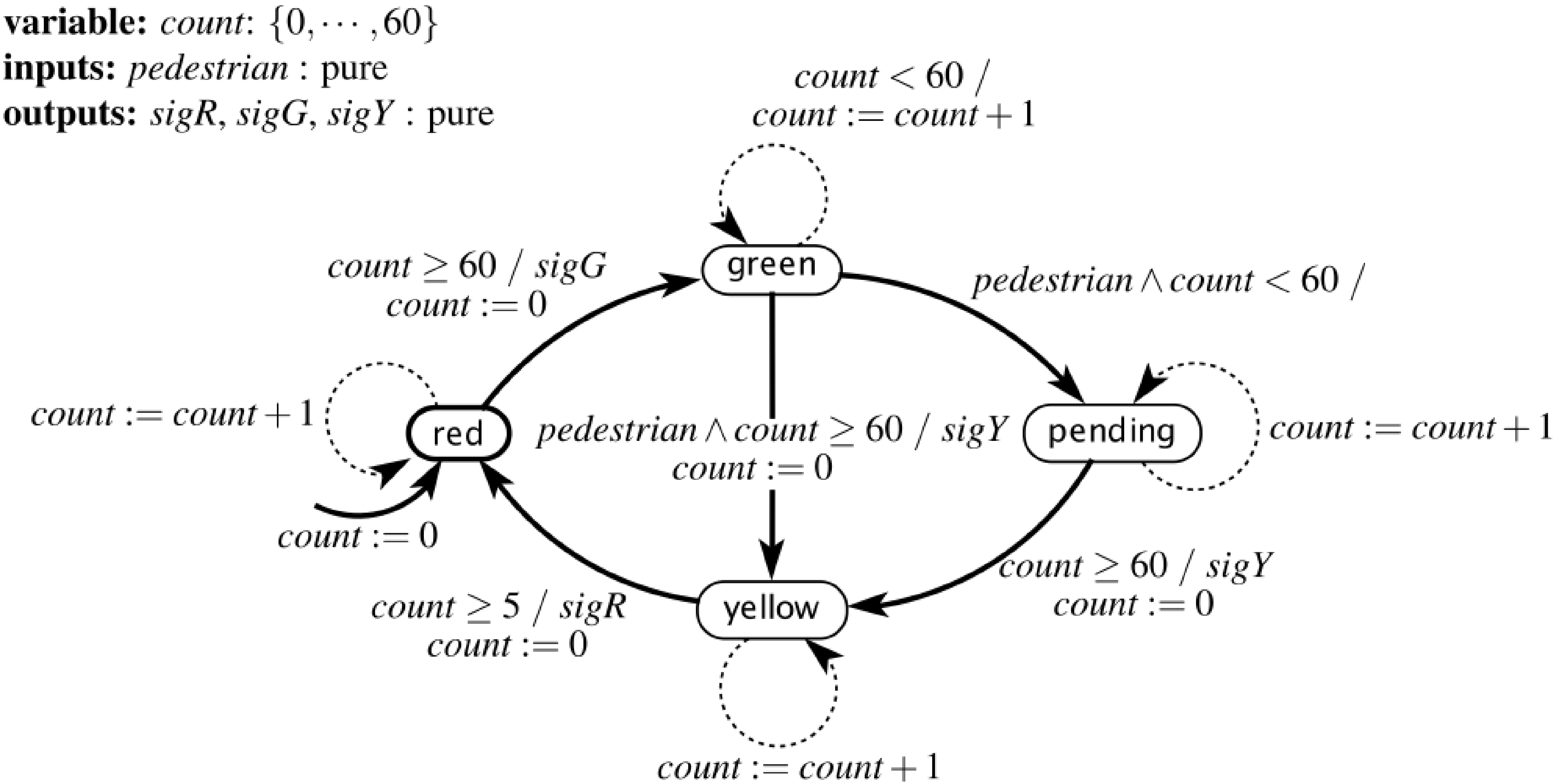
- State of an Extended State Machine = the current "bubble" **and** the value of all variables (internal data)
  - Same "bubble" but different variable value means different states
- Extended State Machines are useful for modeling time-based conditions:
  - Increment a variable every *tick* <=> measure passage of time
  - If the State Machine is *time-triggered* (e.g. driven by an external clok)

General Notation for Extended State Machines

We make explicit declarations of variables, inputs, and outputs to help distinguish the three.



Extended state machine model of a traffic light controller at a pedestrian crossing:

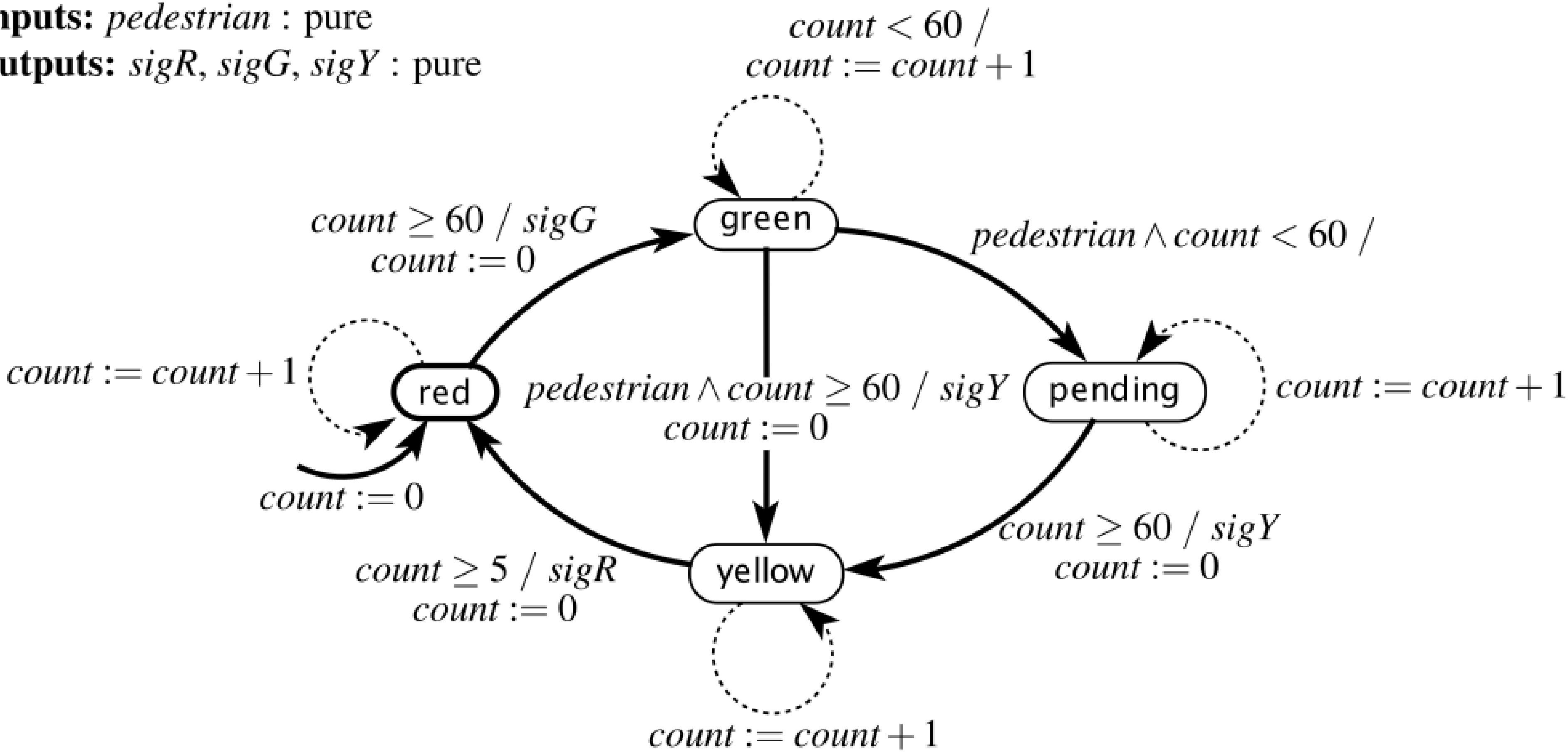


This model assumes one reaction per second

(a *time-triggered* model)

Quiz: What is the Size of the State Space for the Traffic Light Controller?

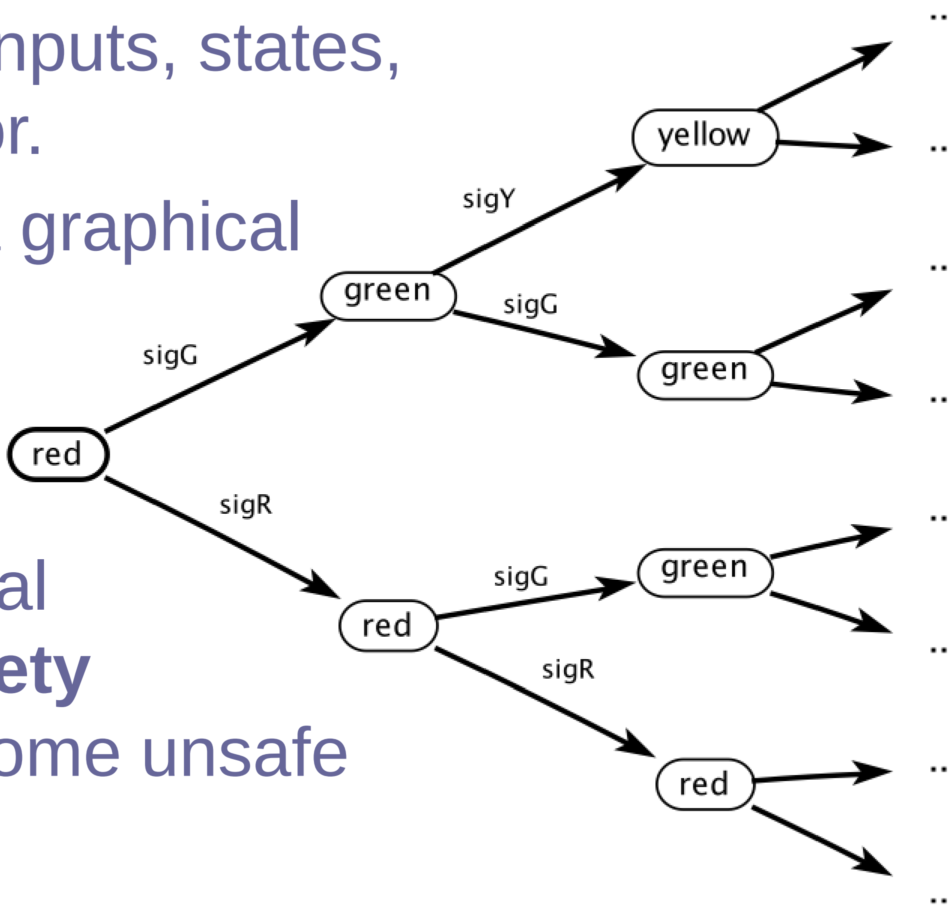
variable: $count$: $\{0, \cdots, 60\}$
inputs: $pedestrian$: pure
outputs: $sigR$, $sigG$, $sigY$: pure



Behaviors and Traces

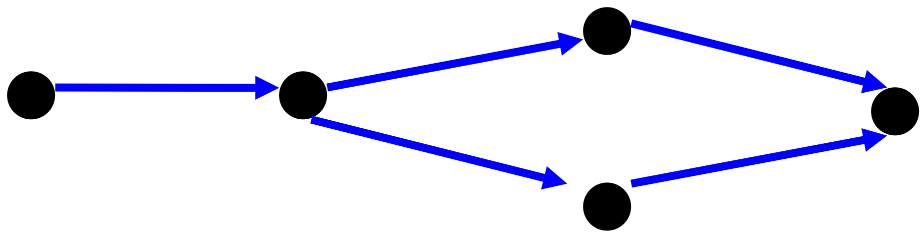
- FSM **behavior** is a sequence of (non-stuttering) steps.
- A **trace** is the record of inputs, states, and outputs in a behavior.
- A **computation tree** is a graphical representation of all possible traces.

FSMs are suitable for formal analysis. For example, **safety** analysis might show that some unsafe state is not reachable.

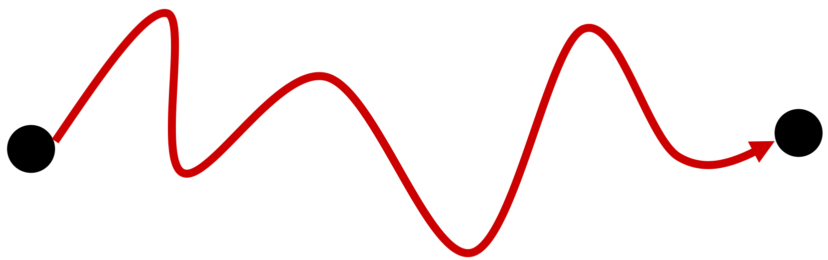


Evolution of states for discrete/continous/hybrid systems

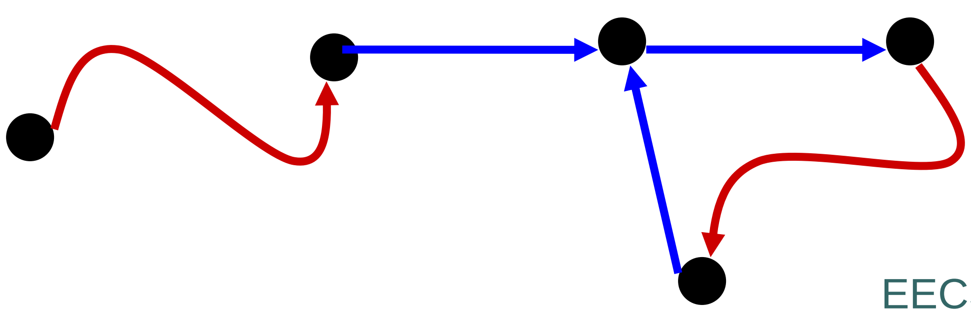
Discrete System (FSM)

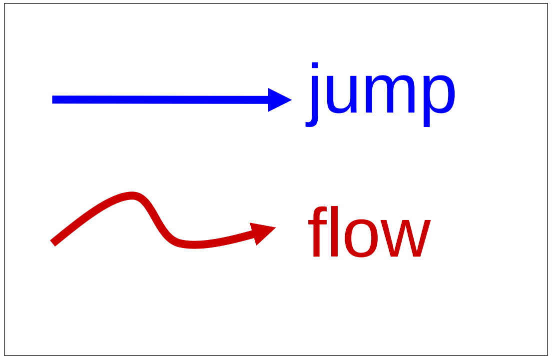


Continuous System



Hybrid System





Hybrid systems

- **Hybrid systems** = systems mixing discrete and continuous behavior
- Example: a complex controller may have:
  - A set of discrete functioning modes (discrete states): e.g. Startup / Operation / Idle
  - Each discrete state is implemented as a separate continuous-time system (state refinement)
- State **refinement** = a lower-level implementation of a state

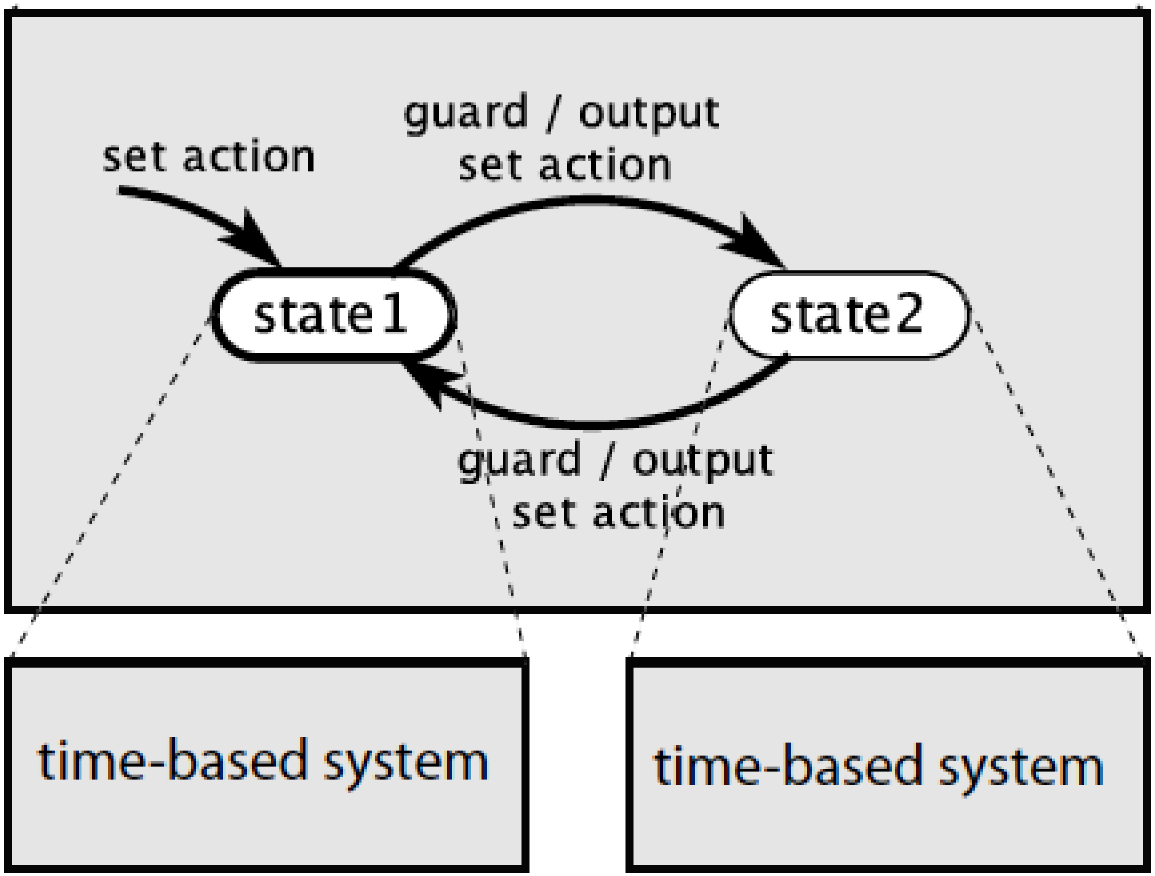
Classes/Examples of hybrid systems

- **Timed automata** = time-based hybrid system where every state refinement just measures passage of time
- **Higher-order systems** = time-based hybrid system where state refinements use second order (or higher) derivatives
  - e.g. like timed automata, but with more complicated behavior with respect to time
- **Two-level control systems** = complex controllers based on two levels of operation:
  - Supervisory control: contains the higher-level discrete modes of operation (e.g.: ECU Power modes: Normal / Sleep Mode 1 / Sleep Mode 2)
  - Low-level control: contains the implementation of each operation mode, as a continuous-time model

Where do Hybrid Systems arise?

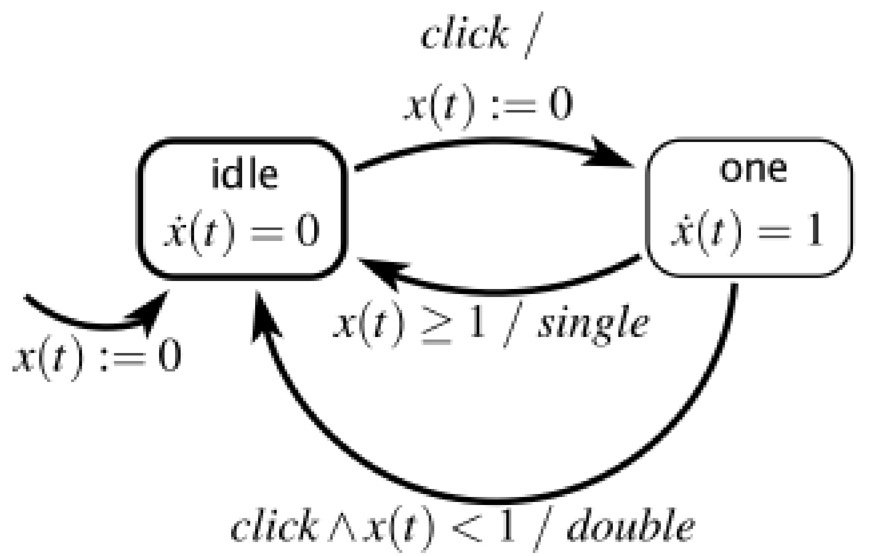
- Digital controller of physical “plant”
  - thermostat
  - intelligent cruise/powertrain control in cars
  - aircraft auto pilot
- Phased operation of natural phenomena
  - bouncing ball
  - biological cell growth
- Multi-agent systems
  - ground and air transportation systems
  - interacting robots

An alternative to FSMs that is explicit about the passage of time: ***Timed automata***, a special case of hybrid systems.



Example: Mouse Double Click Detector

continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$

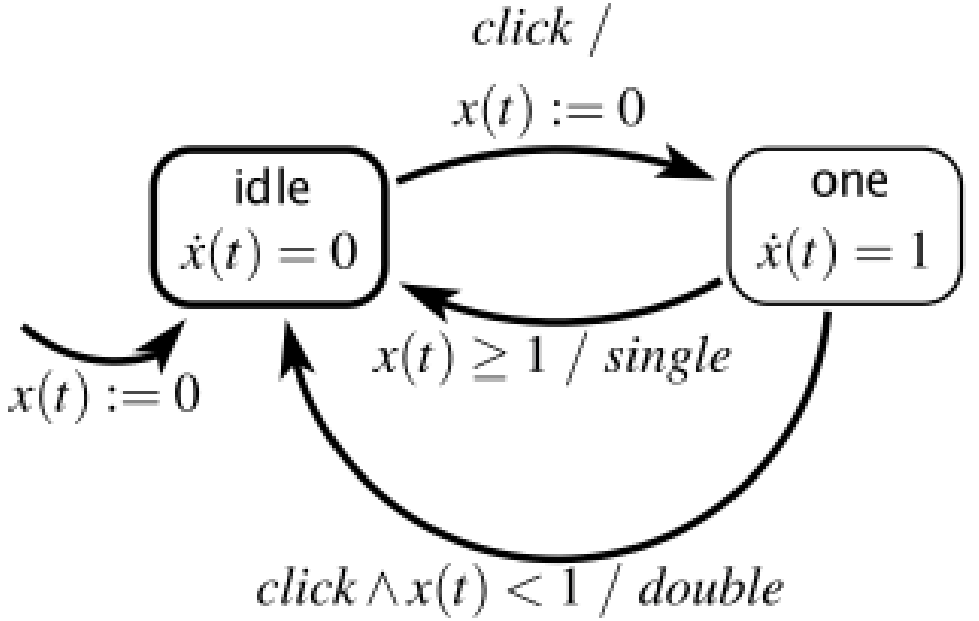


$\dot{x}(t) = 1$ <=> "start measuring time"

This simple form of hybrid system is called a timed automaton, where the dynamics is just passage of time.

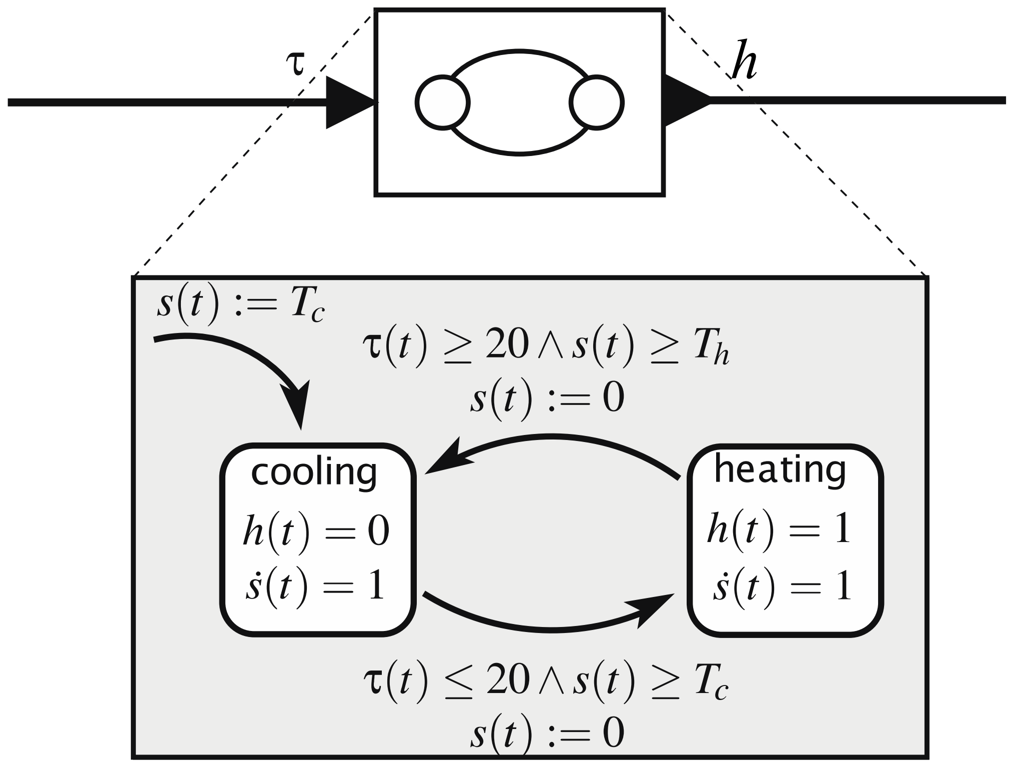
Quiz: How many states does this automaton have?

continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$



Example: Timed Automaton to Prevent Thermostat Chattering

Temperature threshold is 20 with minimum times $\boldsymbol{T_c}$ and $\boldsymbol{T_h}$ in each mode



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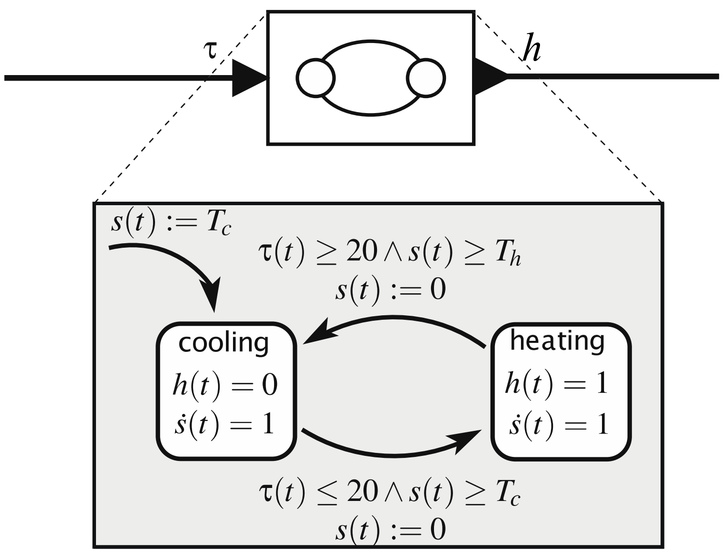
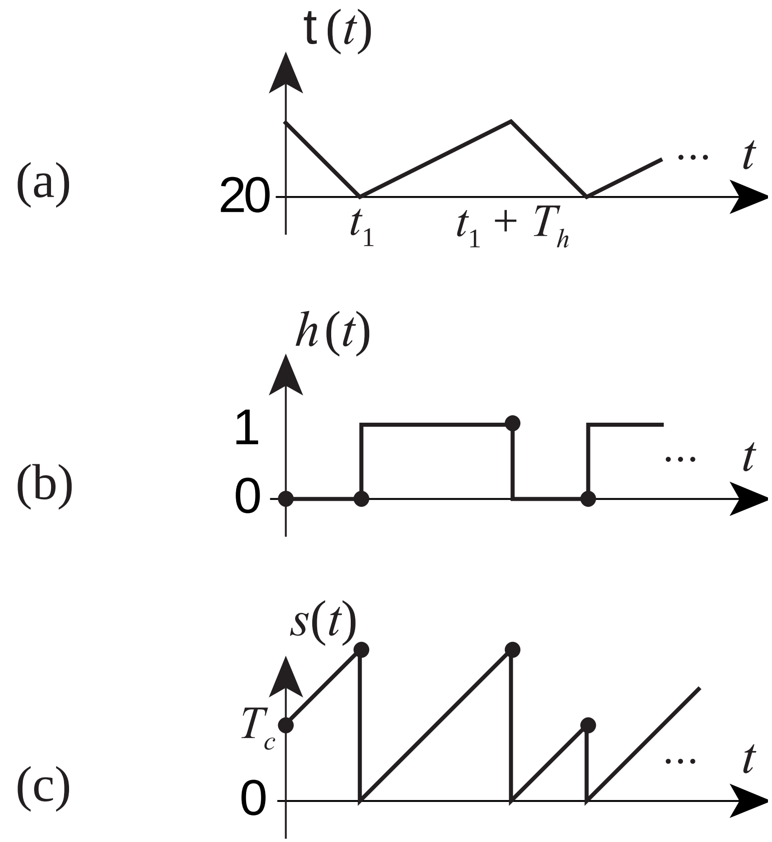
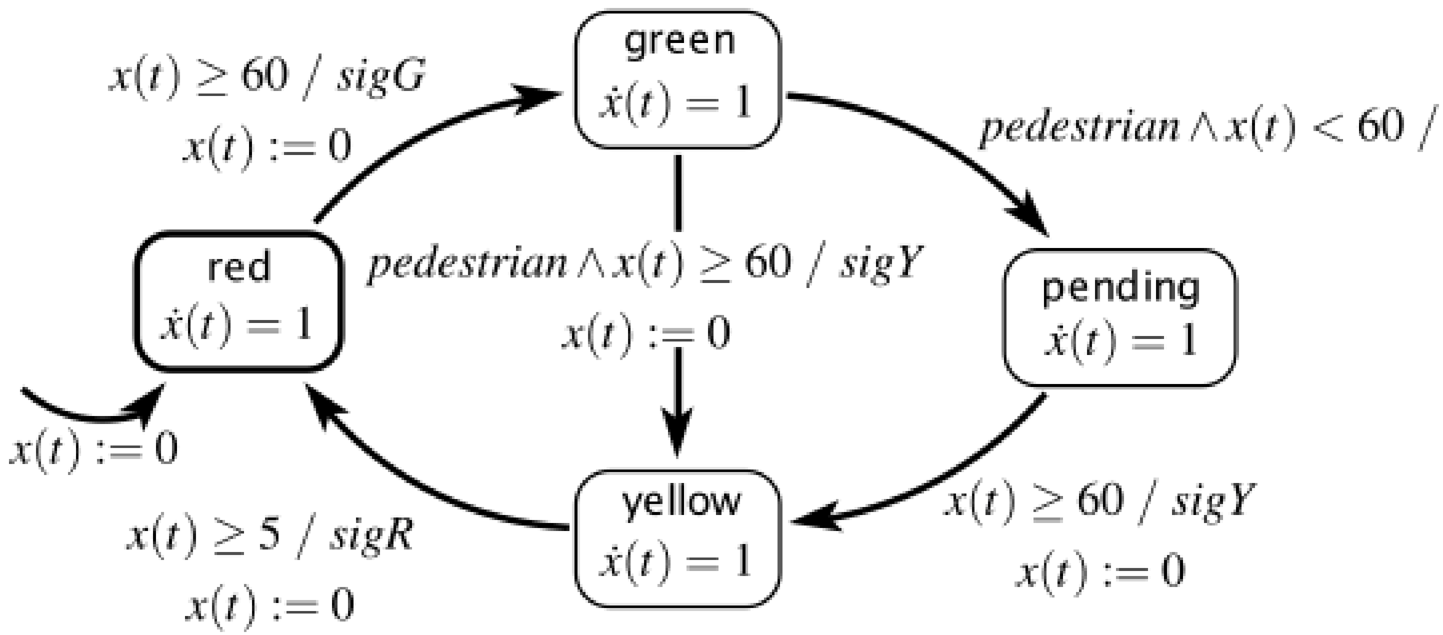




Timed automaton model of a traffic light controller

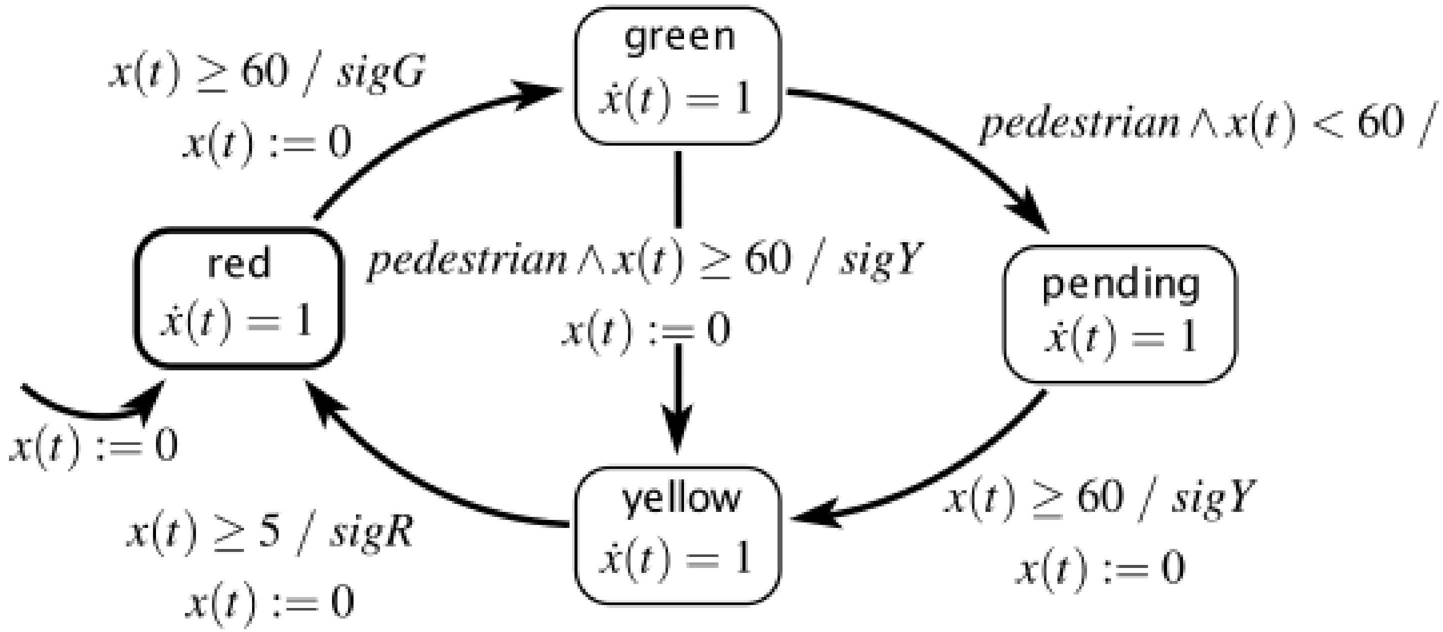
continuous variable: $x(t)\colon \mathbb{R}$
inputs: $pedestrian\colon$ pure
outputs: $sigR, sigG, sigY\colon$ pure



This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

When do reactions occur in a hybrid automaton?

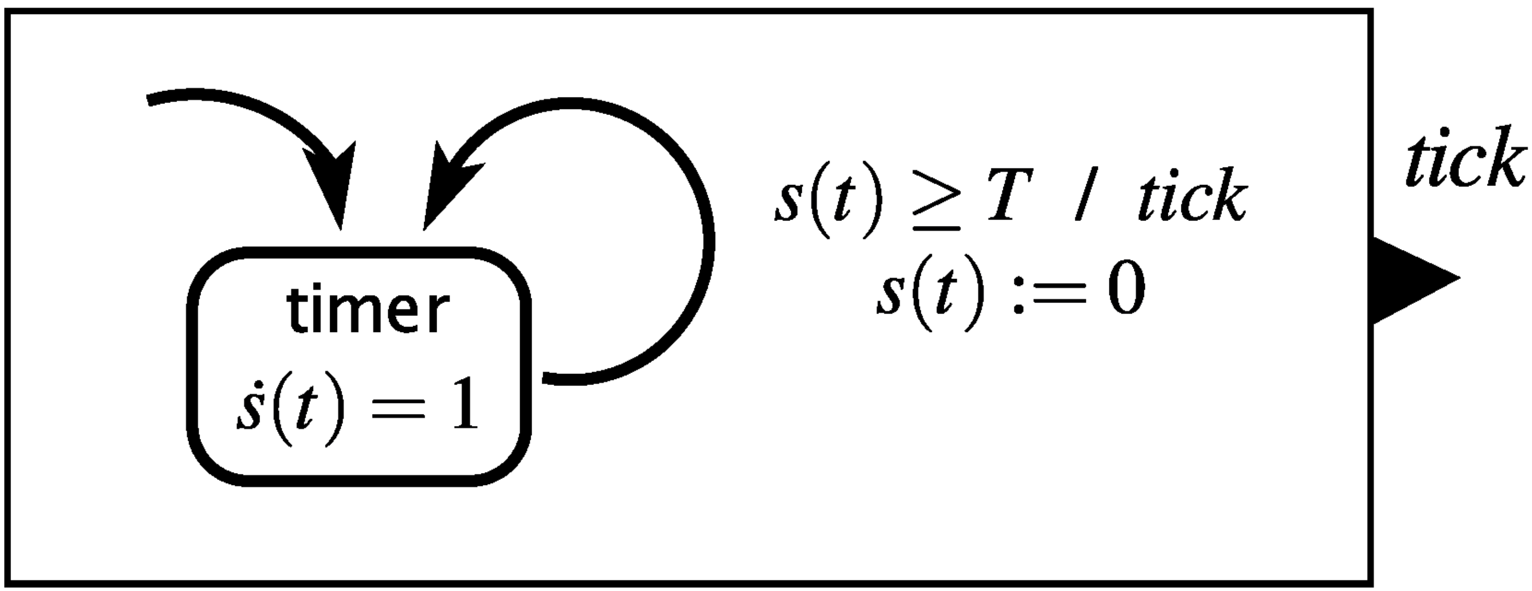
continuous variable: $x(t) \colon \mathbb{R}$
inputs: $pedestrian \colon$ pure
outputs: $sigR, sigG, sigY \colon$ pure



Reactions are occurring continually, with the continuous state variable *x* being continually updated.

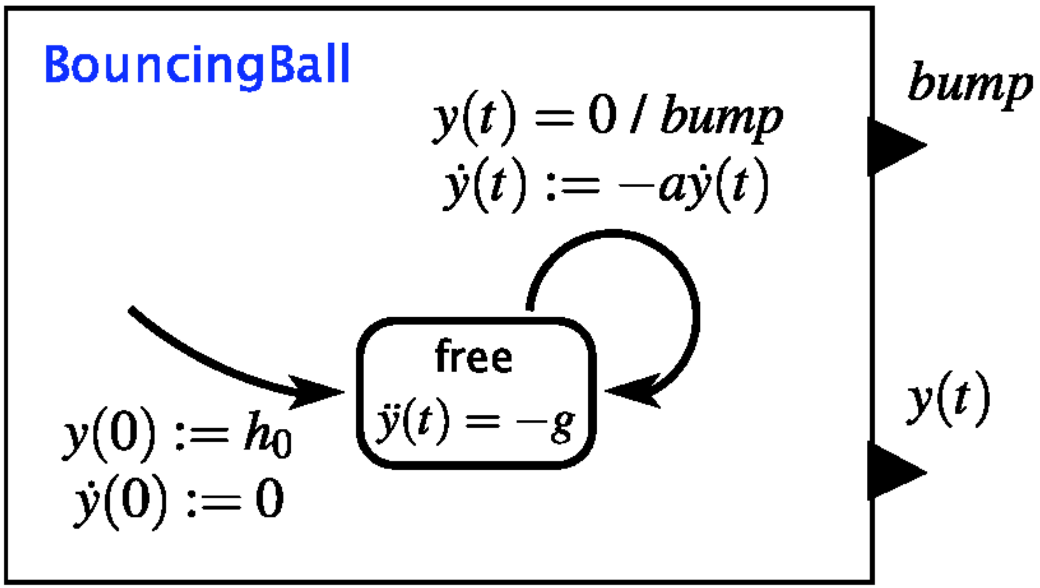
Example: “Tick” Generator (Timer)

How would you model a timer that generates a ‘tick’ each time T time units elapses?



A similar timed automaton can model a generator of a timer interrupt.

Hybrid Automaton for Bouncing Ball

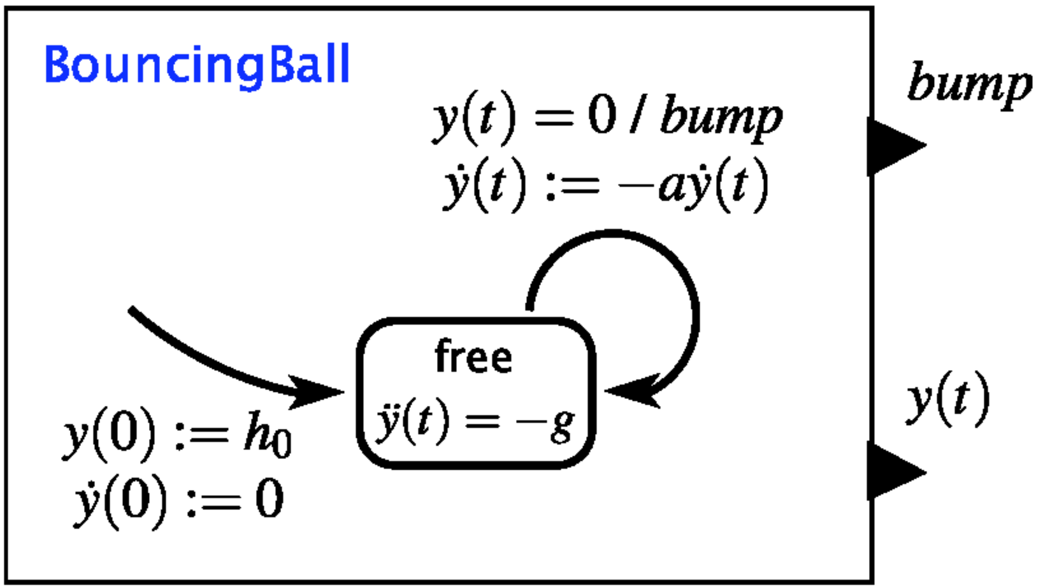


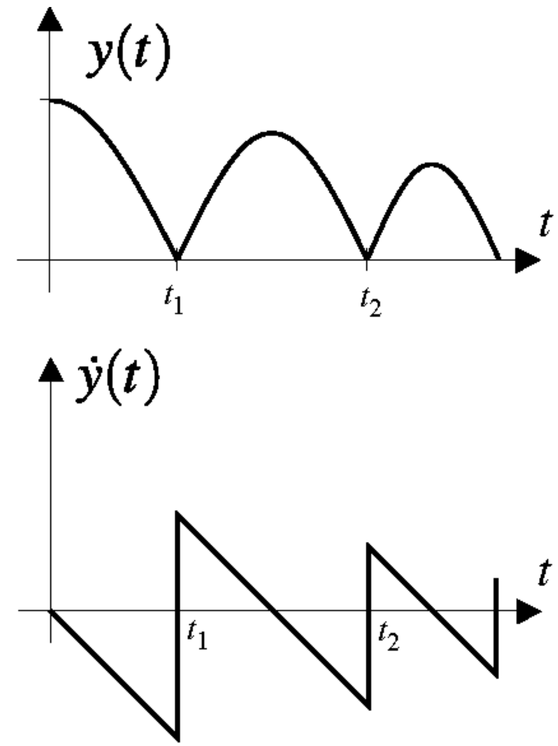
y – vertical distance from ground (position)

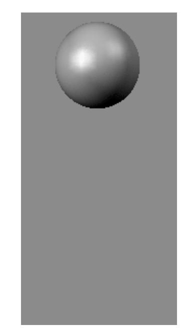
a – coefficient of restitution, $0 < a < 1$

If you plotted y(t), what would it look like?

Hybrid Automaton for Bouncing Ball



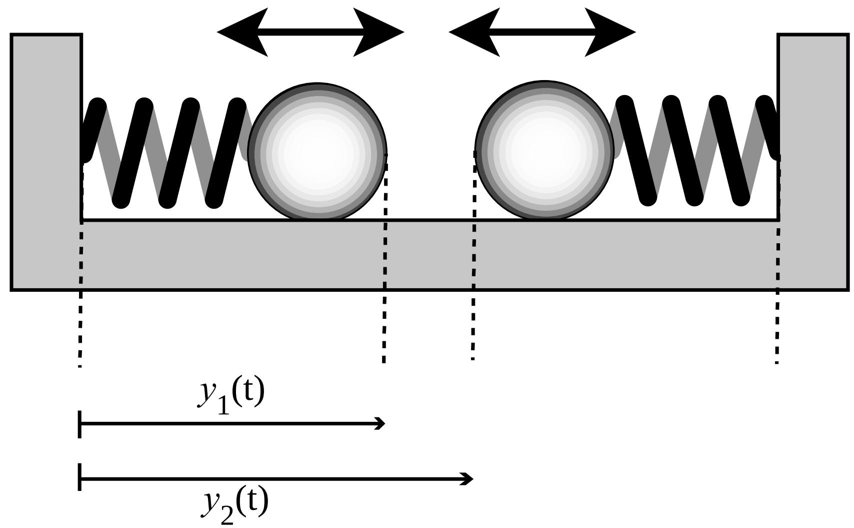


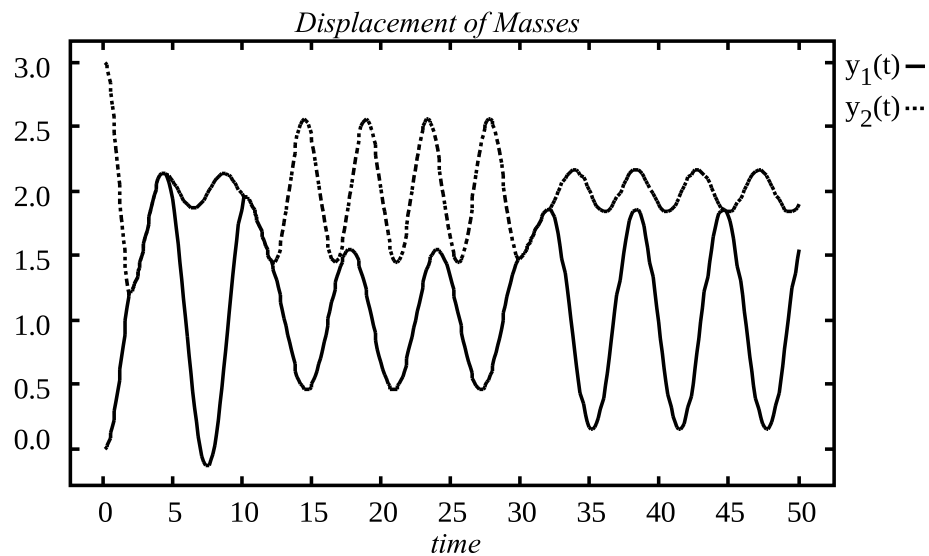


y – vertical distance from ground (position)

a – coefficient of restitution, $0 \cdot a \cdot 1$

Sticky Masses





Sticky Masses