

## Embedded System Design and Modeling

## II. Modeling of discrete systems

# Actor model of systems

A system can be decomposed as inter-connected building blocks, called “actors”

- ▶ Each actor has:
  - ▶ 0, 1 or more input ports
  - ▶ 0, 1 or more output ports
  - ▶ an internal computation / function / what it does
- ▶ Connections = Signals

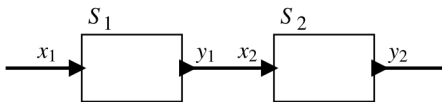


Figure 1: Actor model of systems<sup>1</sup>

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<sup>1</sup>(Image from Lee & Seshia (2017))

How to describe what a component does?

- ▶ Continuous dynamics (previous lecture)
- ▶ Discrete dynamics (from now on)

Ancient philosophy debate: Heraclitus (continuous) vs Parmenides (discrete)

- ▶ **Dynamical system** = system whose state evolves in time
- ▶ **Discrete dynamics** = the system operates in a sequence of discrete steps
  - ▶ there are no continuous changes (no continuous signals)
  - ▶ like digital circuits (values change only on clock edge, e.g., rising/falling edge)
- ▶ It's more a mathematical model (real-life is continuous), but still extremely useful

# Sample discrete system

Example of a discrete system model:

- Sense the cars that enter and leave a parking area (e.g., at barriers), and display the current number of cars in the parking lot on a display.

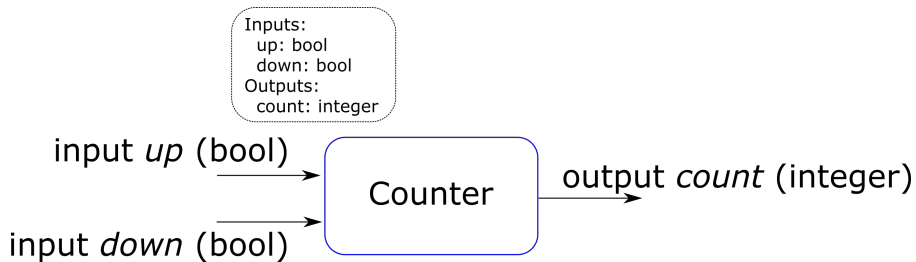


Figure 2: Parking system

- ▶ **State** of the system = condition of the system at a particular point in time
  - ▶ The state encompasses everything in the past that has any influence at the current moment
- ▶ When a transition's guard evaluates to true, the system **reacts**
- ▶ A **reaction** reads inputs, updates the internal state, may produce outputs, and enters a new state
- ▶ Moving from one state to the next is a **transition**.

# Finite State Machine representation

- ▶ Finite State Machine = a system whose operation is described as a set of states and transitions

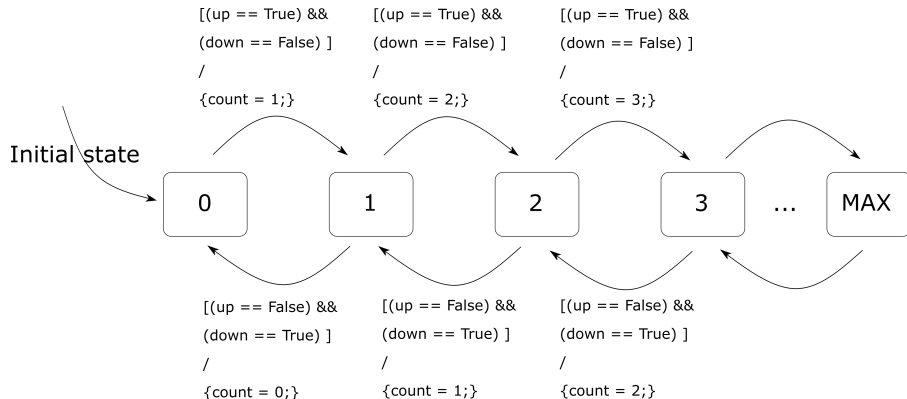


Figure 3: Parking system FSM



# Components of a FSM representation

- ▶ States = the “bubbles”
- ▶ Transitions = the arrows
- ▶ Conditions (guards) = the conditions under which transitions are taken (inside “[ ]”)
- ▶ Actions = the instructions executed when a transition is taken (after “/”, inside “{ }”)

# FSM notations

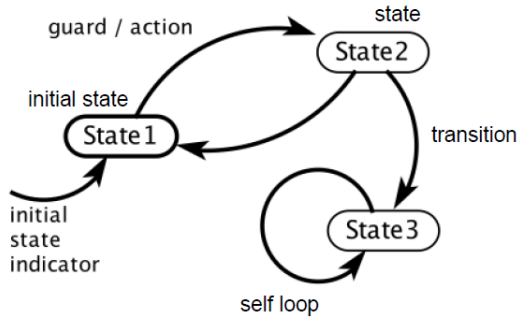


Figure 4: FSM Notations <sup>2</sup>

<sup>2</sup>Image from Seshia's slides

## Conditions and actions

- ▶ A transition is taken when its guard evaluates to true
- ▶ When a transition is taken, the actions execute
- ▶ It is possible that no transition is taken, so the system preserves its state (“default transition”)
- ▶ The **initial transition** indicates the starting state

# FSM mathematical model

A FSM is a tuple (States, Inputs, Outputs, update, initialState) consisting of the following:

- ▶ States = a finite set  $\{0, 1, \dots, M\}$
- ▶ Inputs = a set of variables with their data types
- ▶ Outputs = a set of variables with their data types
- ▶ update = a function  $f : States \times Inputs \rightarrow States \times Outputs$ 
  - ▶ inputs: previous state + current input values
  - ▶ outputs: next state + current output values
- ▶ initialState  $\in States$  = the initial state

If all of the above is known, everything is known about the model.

# Mealy vs Moore machines

FSMs are often presented as either Mealy or Moore machines:

- ▶ Mealy machines: outputs depend on current state and current inputs
- ▶ Moore machines: outputs depend only on current state

# Conditions and transitions

- ▶ Conditions and transitions can be written in many ways
- ▶ Here we use simple C-style boolean expressions:
  - ▶ `==` checks equality
  - ▶ `!` means negation
  - ▶ `true`, `false` = boolean values
- ▶ Examples:
  - ▶ `[a]`
  - ▶ `[!a]`
  - ▶ `[x >= 3]`
  - ▶ `[x < b]`
  - ▶ etc.

Model example: thermostat

- ▶ States: Heating OFF, Heating ON
- ▶ Guards and actions (with hysteresis  $T_{low} < T_{high}$ ):
  - ▶  $[ T \leq T_{low} ] / \{ \text{heater} := \text{ON} \}$
  - ▶  $[ T \geq T_{high} ] / \{ \text{heater} := \text{OFF} \}$
- ▶ Hysteresis prevents rapid toggling around the setpoint.

# When does a reaction occur?

- ▶ When are transitions checked? (when do the reactions happen)?
- ▶ Two variants:
  - ▶ **Event-triggered** model
  - ▶ **Time-triggered** model
- ▶ Event-triggered model:
  - ▶ A reaction can occur at **any time**
  - ▶ The environment triggers the transition via an **event**
  - ▶ Works like an **interrupt** in microcontrollers
- ▶ Time-triggered model:
  - ▶ The reaction occurs periodically, on the global *tick* of an **external clock**
  - ▶ e.g., everything runs at  $T_s = 10\text{ ms}$ ,  $20\text{ ms}$ , etc.



# Time-triggered models

- ▶ Simplest case = time-triggered models
- ▶ How it works:
  - ▶ the clock ticks, the FSM “wakes up” in a certain state
  - ▶ the inputs are read
  - ▶ the outgoing transitions from the current state are evaluated
  - ▶ if a guard evaluates to true, that transition executes and the system enters a new state
  - ▶ the system “goes to sleep” until the next tick

# Event vs time-triggered models

Advantages/disadvantages of time-triggered models:

- ▶ Bad: if an input changes very fast, within a  $T_s$  interval, the model **may not see it**
- ▶ Good: all inputs are read simultaneously
- ▶ Good: simple to understand

Advantages/disadvantages of event-triggered models:

- ▶ Bad: the inputs are not synchronized (in a condition  $a > b$ , perhaps  $a$  changes 1 ms earlier than  $b$ , leading to a wrong result)
- ▶ Good: no risk that values are lost
- ▶ Bad: difficult to analyze, difficult to understand

# Properties of discrete models

- ▶ **Determinism:** In every state, for all possible input values, at most one transition is enabled
  - ▶ if you know the initial state and all the inputs' evolution, you know the complete behavior of the system
- ▶ **Non-determinism:** Models unknown behavior (unknown inputs), or random transitions

## Completeness and priorities

- ▶ **Completeness:** every input combination is handled (possibly via a default transition), so the model does not stall
- ▶ **Priorities:** if multiple guards can be true, define an explicit order (or disjoint guards) to keep the model deterministic

# Determinism computation tree

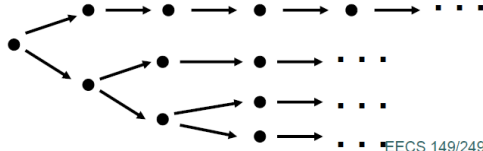
For a fixed input sequence and initial state:

- ▶ A deterministic system exhibits a single behavior
- ▶ A non-deterministic system exhibits a set of behaviors, visualized as a **computation tree**

Deterministic FSM behavior:



Non-deterministic FSM behavior:



EECS 149/249A, UC Berkeley: 33

Figure 5: Computation tree <sup>3</sup>

<sup>3</sup>Image from Seshia's slides