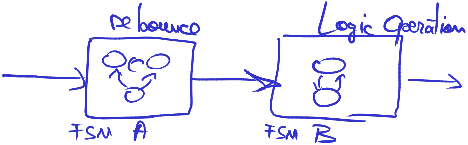
Embedded System Design and Modeling

VI. Composition of State Machines

Composition of state machines



- How to combine multiple smaller FSMs into a bigger one?
- What problems arise?
- Two types of compositions:
  1. **Spatial** composition: how are the components connected?
  2. **Temporal** composition: how do the components react in time?

Spatial composition

Spatial composition = how are two components connected, how does the information flow between the components

- **Side-by-side** composition = no common inputs/outputs, no shared data
- **Cascade** composition = Outputs of one FSM are inputs to another one
- **Feedback** composition = (Some) outputs of a FSM are inputs to the same FSM, or to some other component which is in front

Side-by-side composition

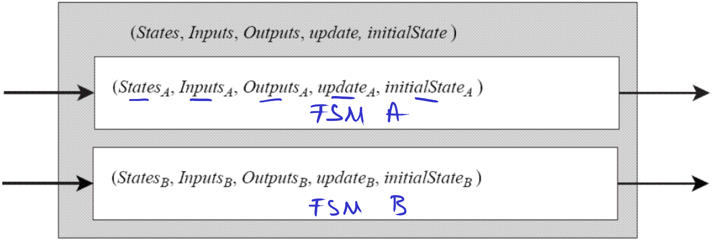


Figure 1: Side-by-side composition

Cascade composition

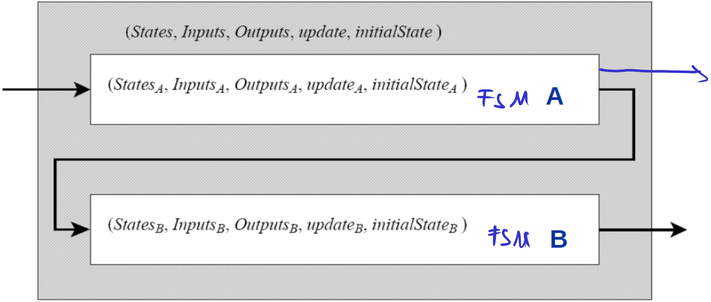


Figure 2: Cascade composition

- Outputs of FSM A are inputs to FSM B

Feedback composition

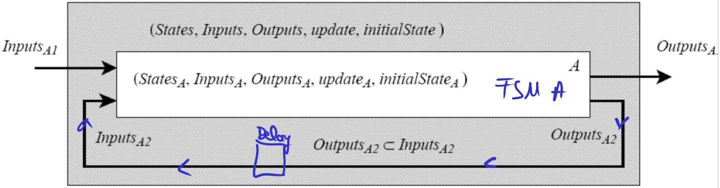


Figure 3: Feedback composition

- Some outputs of the FSM are coming back as inputs

Temporal composition

Temp. → Seq.
Temp. → Parallel
Parallel → Synch.
Parallel → Asynch.
Asynch. → Interleaved
Asynch. → Non-interleaved

Temporal composition = when do two components react?

- **Sequential vs Parallel** composition:
  - **Sequential** = the two FSM do not work at the same time
  - **Parallel** = the two FSM work at the same time
- **Asynchonous vs Synchronous** composition = only for parallel composition
  - **Synchronous** = transitions are taken at the same time in both FSMs
  - **Asynchronous** = transitions are taken at independent times in the FSMs

$\boxed{\boxed{A} \longrightarrow \boxed{B}}$

Sequential composition

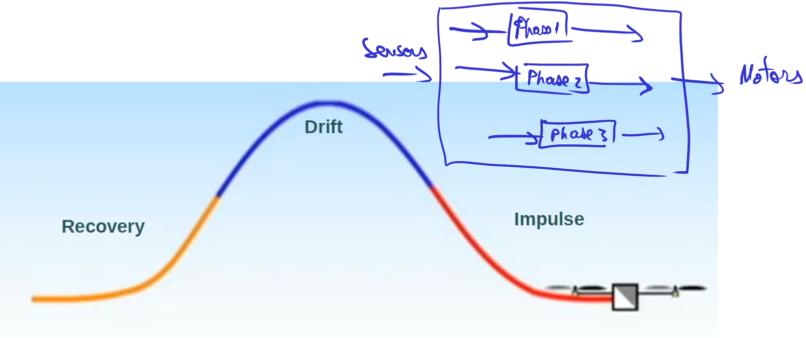


Figure 4: Example of Sequential composition

- 13579/:https://wwwoutubeom/watch?vD3QgGpzzIM
- The drone has three modes of operation, working **in sequence**

Parallel composition

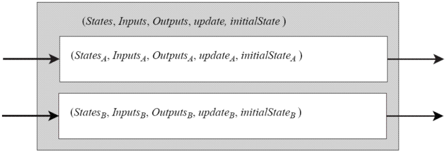


Figure 5: Side-by-side composition

- The two FSMs form an **equivalent model**
- When do the transitions in these FSM take place?
  - Synchronous: simultaneously
  - Asynchronous: independently

Synchronous composition

- Consider the two FSM on the left (A and B)
- The equivalent model is on the right

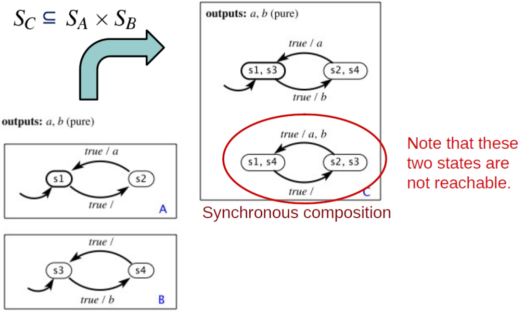
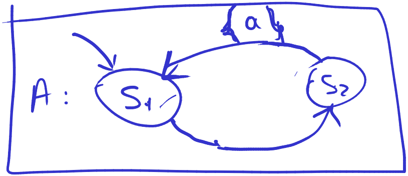
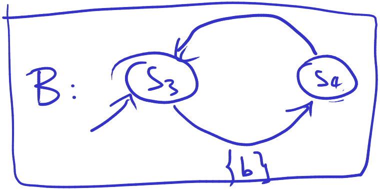


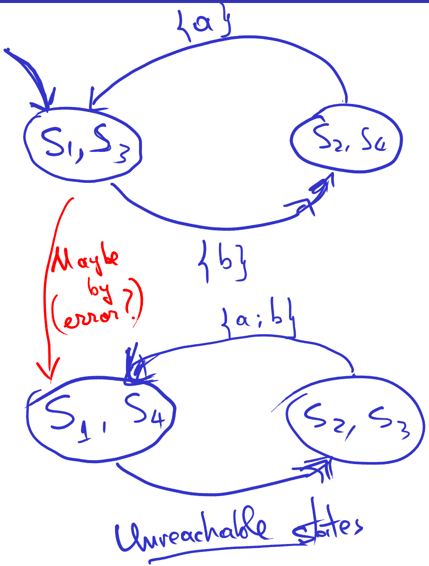
Figure 6: Synchronous composition

Synchronous composition



Redraw here

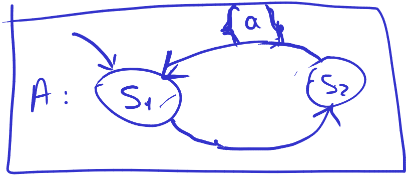




Synchronous composition

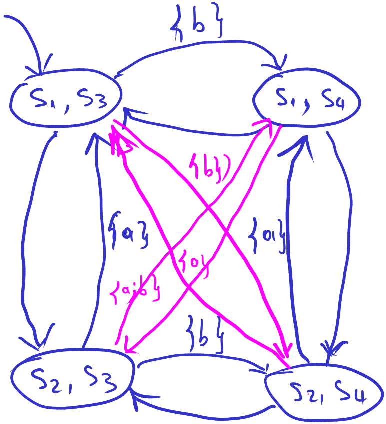
- In the equivalent model:
  - States = combination of states of the two FSMs
  - Transition = transition in FSM A and FSM B, happening simultaneously.
  - There might exist unreachable states in the equivalent model (states that will never be reached)

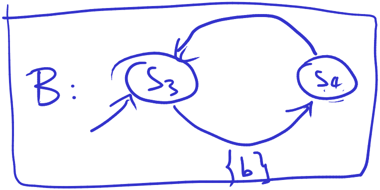
Asynchronous composition , interleaving semantics = in blue



Draw here

$\Longleftrightarrow$

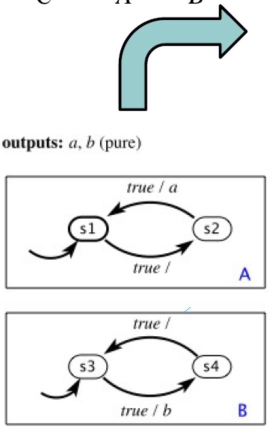


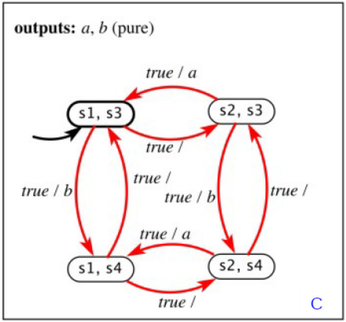


simultaneous semantics: plus the magenta ones

Asynchronous composition

$$S_C \subseteq S_A \times S_B$$





Asynchronous composition using interleaving semantics

Note that now all states are reachable.

Figure 7: Asynchronous composition

Asynchronous composition

- In the equivalent model:
  - States = combination of states of the two FSMs
  - Transitions in the two FSMs can take place at irregular and independent (not synchronized) times
  - All states are reachable
    - because one model can be much faster than the other

Asynchronous composition

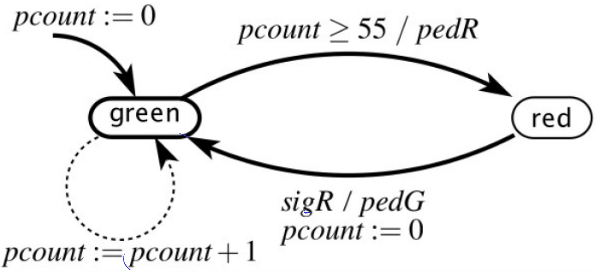
Flavors of asynchronous composition

- How are simultaneous transitions handled?
- **Interleaving** semantics:
  - simultaneous transition in models A and B is not allowed (we may have either a transition in model A, or a transition in B)
  - i.e. transition from A takes place first, then transition from B takes place after a non-zero time delay (or vice-versa)
- **Simultaneous** semantics:
  - simultaneous transition in models A and B is allowed
  - for example, we may have either
    - transition only in model A
    - transition only in model B
    - Simultaneous transition in models A and B

Example

variable: *pcount*: $\{0, \cdots, 55\}$
input: *sigR*: pure
outputs: *pedG*, *pedR*: pure

Traffic light for pedestrians



This light stays green for 55 seconds, then goes red.
Upon receiving a sigR input, it repeats the cycle.

Figure 8: Composition - Pedestrian Light

Example

Traffic light for cars

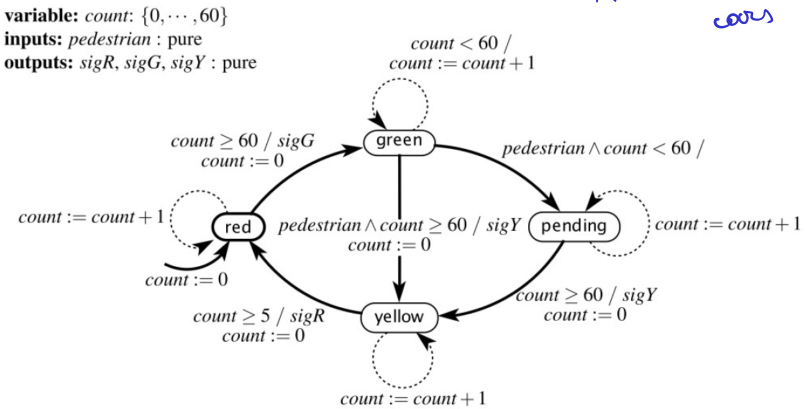


Figure 9: Composition - Car Light

Example

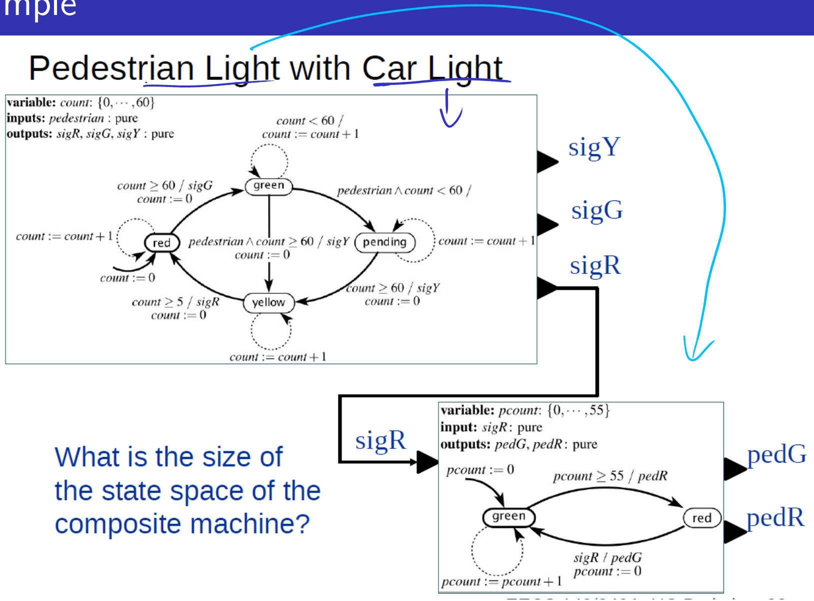
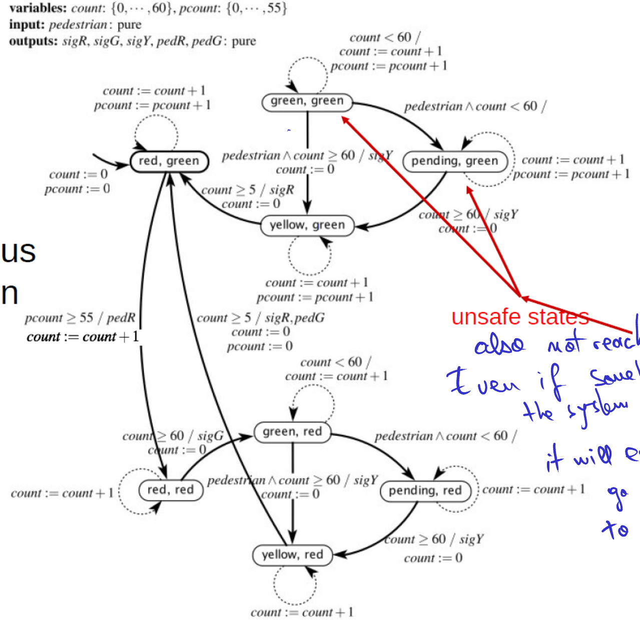


Figure 10: Cascade Composition - Both Lights

Example

Synchronous composition



also not reachable

Even if somehow the system goes there, it will eventually go back to safe states

Example

Synchronous composition