Embedded System Design and Modeling

III. Extended FSMs and Timed Automata

FSM example

- Recall the previous FSM example

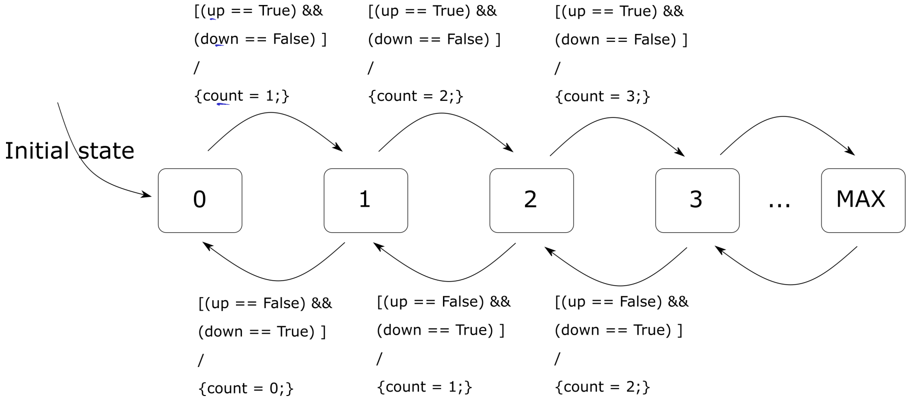
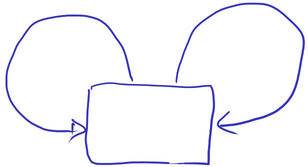


Figure 1: Parking system FSM

count = count $\pm$ 1



- Can we make it is simpler to draw?

Extended FSMs

- **Extended FSM** = FSM with **internal variables**

Inputs:
up: bool
down: bool
Outputs:
count: integer (0, MAX)
Variables:
count: integer (0, MAX)

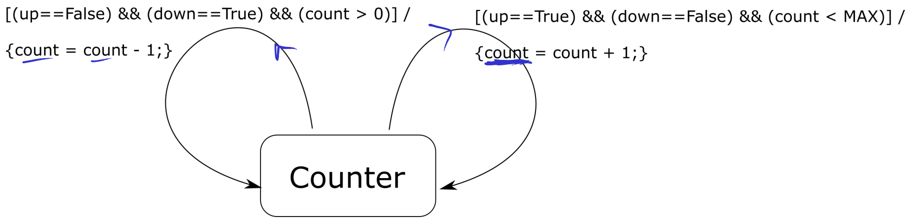


Figure 2: Extended FSM with variable "count"

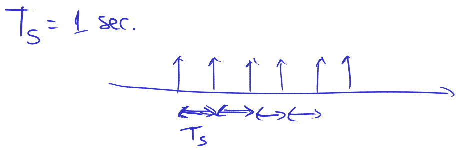
Extended FSM

- The state of the model = the current "bubble" and the values of **all the internal variables**
- Example: OS hibernation in Windows:
  - state of computer = all the RAM memory values
  - if all memory is written down on HDD, and reloaded tomorrow, the system effectively resumes operation from where it left off
- State is not anymore "the number of bubbles"
  - there is only one "bubble" in our FSM
  - but there are MAX+1 states (all possible values of the count variable)

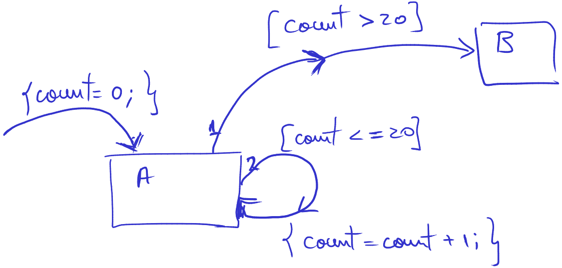
Declarations

- Always make explicit declaration of:
  - model inputs
  - model outputs
  - model internal variables
  - and their data types

Measure time



- Extended FSM are useful for modeling **time-based** conditions:
  - measure passage of time: increment a variable every *tick*
  - only works if the FSM is time-triggered



Example: pedestrian crossing light

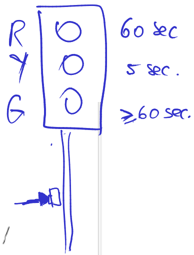
- How is time measured in the model below?
- How many states does the model below have?

Input: button : bool

Output: R : bool
Y : bool
G : bool

Var: count : integer;

$T_s = 1$ sec.



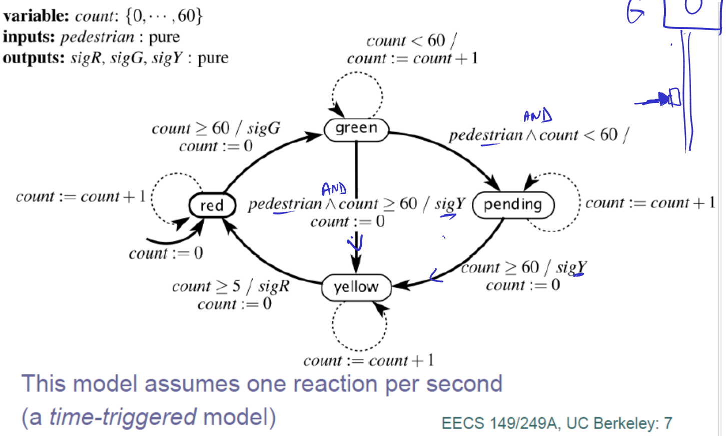
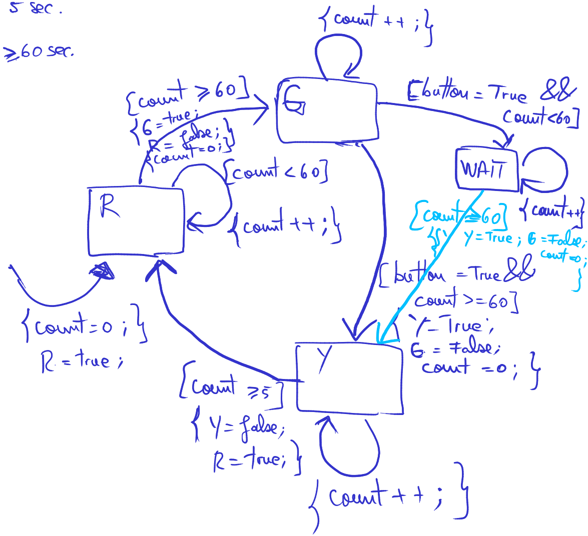


Figure 3: Extended FSM with time measuring [1]



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[1] image from Seshia' slides

Hybrid systems

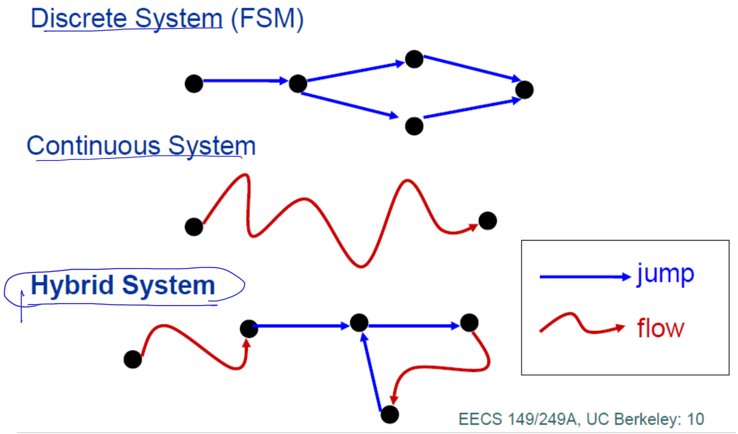
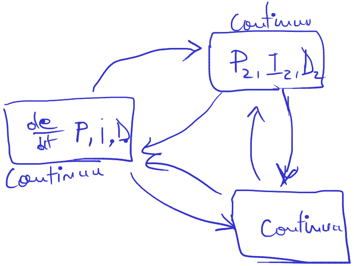


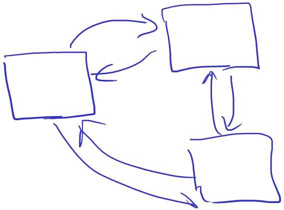
Figure 4: Hybrid systems [2]

[2] image from Seshia' slides

Hybrid systems

- **Hybrid systems** = system with mixes discrete and continuous behavior
- Example: a PID controller with different modes:
  - a set of distinct functioning model (e.g. Startup / Normal / Idle)
  - each state is a sub-system implemented with continuous dynamics
- State **refinement** = a lower-level implementation of a state

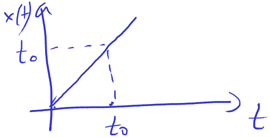


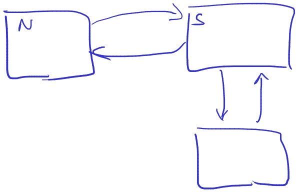


Types of hybrid systems

- **Timed automata** = hybrid system where every state refinement just measures passage of time (differential equation of degree 1)
- **Higher-order systems** = hybrid system where every state refinement uses higher-order differential equation (2 or more)
- **Two-level control systems** = complex controllers with two levels of operation
  - high-level discrete modes of operation (e.g. ECU Power Modes: Normal / Startup / Sleep Mode 1 / Sleep Mode 2)
  - low-level refinements with continuous dynamics

$x'(t) = 1$





Timed automata

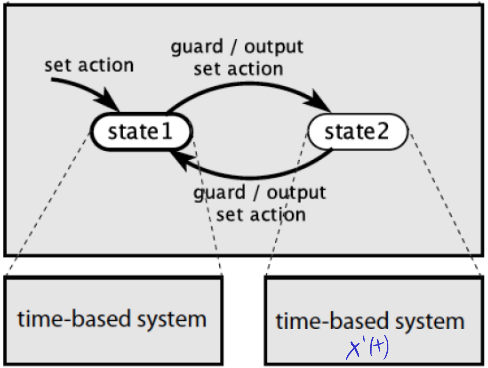


Figure 5: Timed automaton example [3]

[3] image from Seshia' slides

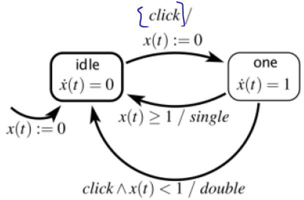
Example

Timed automaton

$\dot{x}(t) = x'(t) = \frac{dx}{dt}$

first derivative

continuous variable: $x(t) \in \mathbb{R}$ = internal variable
inputs: *click* $\in \{present, absent\}$ = bool
outputs: *single*, *double* $\in \{present, absent\}$



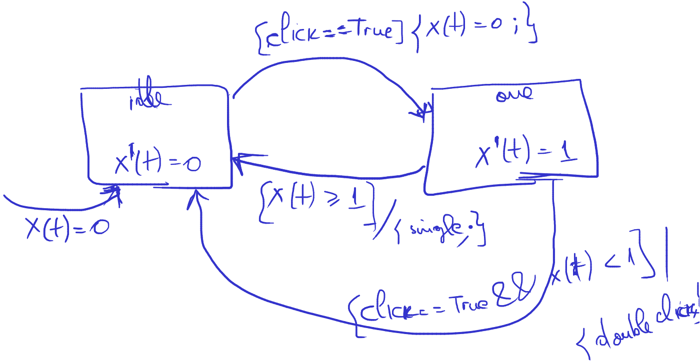
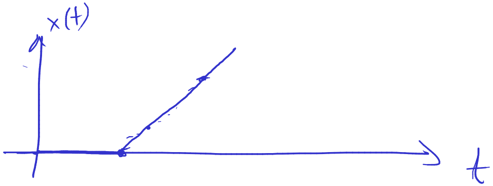


Figure 6: Mouse Double-click detector model [4]



- Here $\boxed{\dot{x}(t) = 1}$ means "x(t) increases linearly with time", so it measures time
- How many states does this model have?

[4] image from Seshia' slides

Example: Another Thermostat

Temperature threshold is 20 with minimum times T_c and T_h in each mode

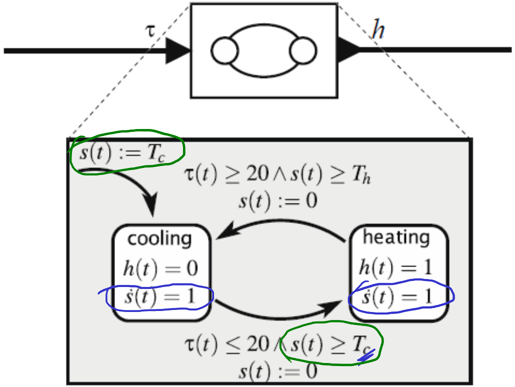
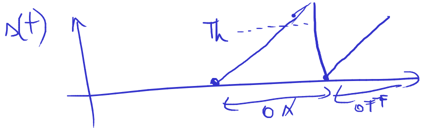


Figure 7: - Another thermostat model as a Timed Automaton [5]

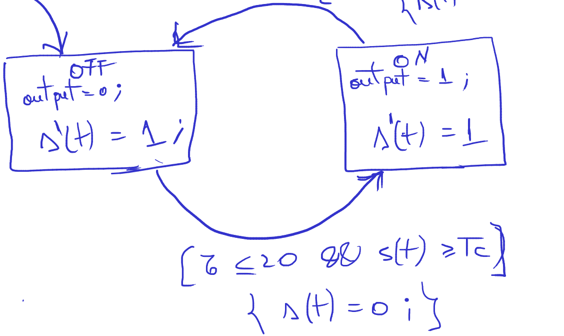
[5] image from Seshia' slides



$h(t)$ = the output variable
$s(t)$ = internal
$\tau(t)$ = input (temperature)

$s(t) = \cancel{0}\ T_c$

$[\tau \geq 20 \ \&\& \ s(t) \geq T_h]$
$\{s(t) = 0;\}$



Example: Another Thermostat

Temperature threshold is 20 with minimum times T_c and T_h in each mode

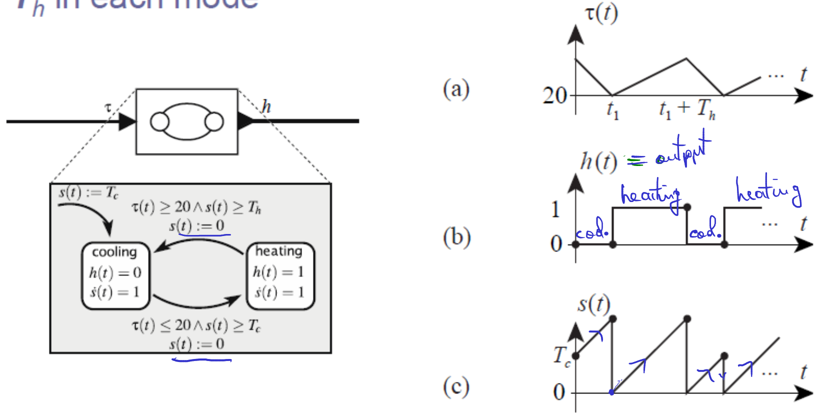


Figure 8: Input and output signals [6]

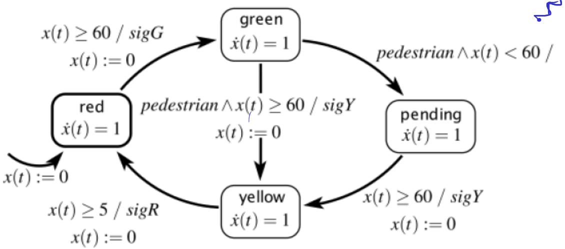
[6] image from Seshia' slides

Timed automaton model of a traffic light controller

continuous variable: $x(t) : \mathbb{R}$
inputs: *pedestrian* : pure
outputs: *sigR*, *sigG*, *sigY* : pure

{count++;}

$\dot{x}(t) = 1;$

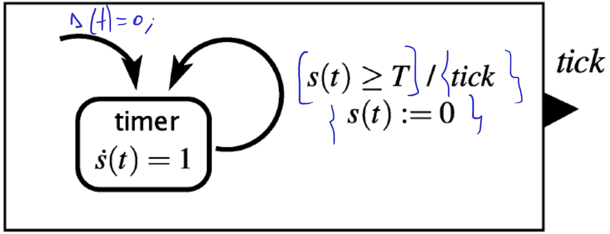


This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

Figure 9: Traffic Light controller Timed Automaton [7]

[7] image from Seshia' slides

Example: Tick generator



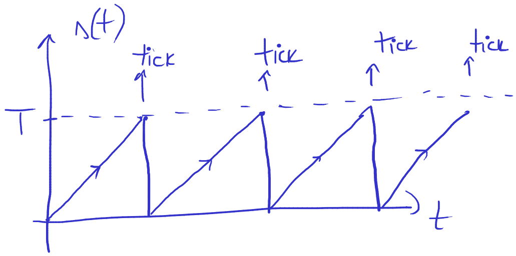


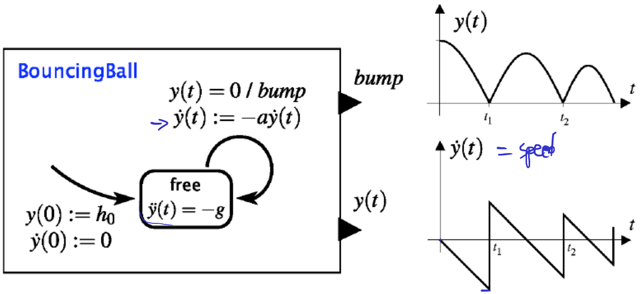
Figure 10: Timed Automaton to generate a *tick* event every T seconds [8]

[8] image from Seshia' slides

Example: Bouncing Ball

Hybrid Automaton for Bouncing Ball

$a = 0.9$

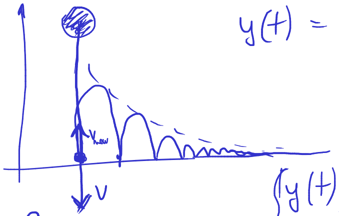


y – vertical distance from ground (position)
a – coefficient of restitution, $0 < a < 1$

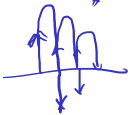
Figure 11: Timed Automaton to simulate a bouncing ball movements [9]

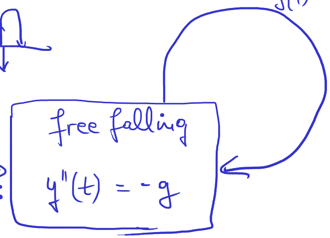
[9] image from Seshia' slides

$y(t)$ = height of the ball



$[y(t) = 0]$
$\dot{y}(t) = -a \cdot \dot{y}(t)$





$y(0) = \text{height}$;
$\dot{y}(0) = 0$

g

a

$y'' = a = -g$

FSM simulation software

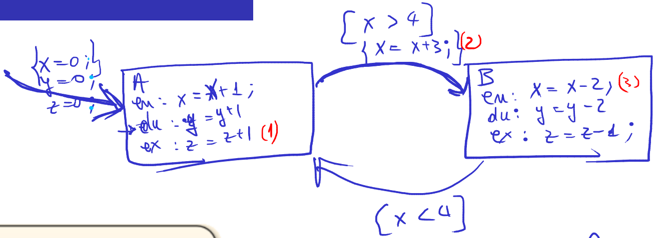
- FSM simulation software
- Used in this class: Stateflow (Simulink / Matlab)
- Features:
  - State Actions
  - Temporal Logic
  - Other events
  - . . . other . . .

State actions

- Actions can exist not only on transitions, but also **inside states**
- Three main types of **State Actions**:
  - **entry (en)**: executed only when a **state is entered**
  - **exit (ex)**: executed only when a **state is exited**
  - **during (du)**: executed when we are in state which is neither entered, not exited

en:
x = x + 1 ;
ex :
y = y + 2 ;

State actions



clear order

x = 0;
y = 0;
z = 0;
x = x + 1; → sleep (x = 1)

wake-up:
x > 4? NO;
y = y + 1;
sleep
wakeup :

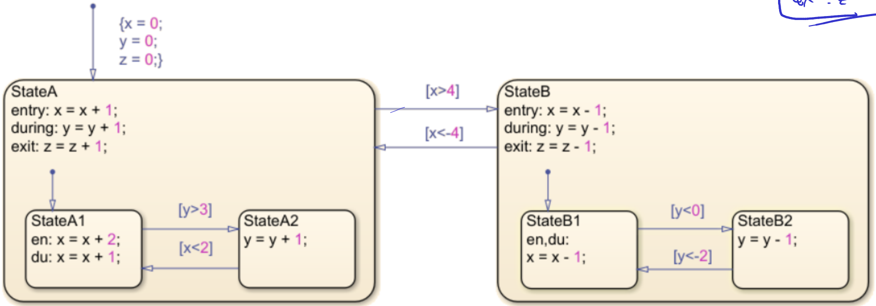
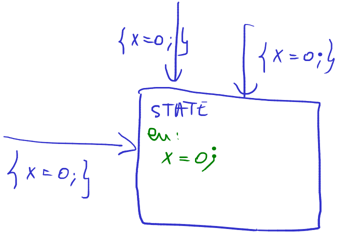


Figure 12: State Actions example [10]

[10] image from Matlab docs

State actions



- State actions can be avoided (use only transitions actions), but sometimes one or the other are more convenient

Temporal logic

- For time-based conditions, states certain predefined variables, which can be used to **measure time spent in a state**
  - ***tick***: measures time steps
    - is incremented at **every time step**
    - is reset to 0 every time a state is exited or entered
    - actual duration **depends** on model step size
  - ***sec*** / ***msec***: measures seconds or miliseconds
    - is incremented every second / milisecond
    - is reset to 0 every time a state is exited or entered
    - actual duration is **independent** on model step size

Temporal logic

- Temporal operators **after()**, **on()**, **every()** can generate events which can be used in conditions
- Examples:
  - ***after(10, tick)***:
    - event is fired after 10 time steps spent in a state
    - evaluates to FALSE for the first 9 steps, is TRUE every time after that
  - ***on(x, tick)***:
    - event is fired only **once**, exactly after x time steps spent in a state
    - evaluates to FALSE for the first $x - 1$ time moments, is TRUE only once at the x-th moment, is FALSE after that
  - ***every(x, tick)***:
    - event is fired pariodically after x time steps
    - evaluates to FALSE for the first $x - 1$ time moments, is TRUE once at the x-th moment, then FALSE for the next $x - 1$ time moments, then TRUE again, and so on

Temporal logic

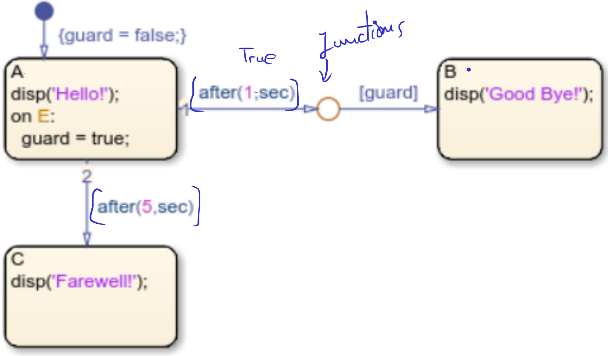


Figure 13: Temporal Logic example [11]

[11] image from Matlab docs