Embedded System Design and Modeling

II. Modeling of discrete systems

Actor model of systems

A system can be decomposed as inter-connected building blocks, called "actors"

- Each actor has:
 - ▶ 0, 1 or more input ports
 - ▶ 0, 1 or more output ports
 - ▶ an internal computation / function / what it does
- ► Connections = Signals

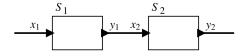


Figure 1: Actor model of systems¹

¹(Image from Lee & Seshia 2017)

Actor dynamics

How to describe what a component does?

- ► Continuous dynamics
- ▶ Discrete dynamics

Discrete dynamics

- ▶ **Dynamic system** = system whose state evolves in time
- ▶ **Discrete dynamics** = the system operates in a sequence of discrete steps
 - there are no continuous changes (no continuous signals)
 - ▶ like digital circuits (values change only on clock front)
- ▶ It's more a mathematical model (real-life is continuous), but still extremely useful

Sample discrete system

Example of discrete system model:

Sense the cars which enter and leave a parking area (e.g. at barriers), and display the current number of cars inside the parking on a display.

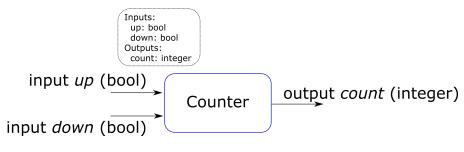


Figure 2: Parking system

State, reaction, transitions

- ► State of the system = condition of system at a particular point in time
 - ► The state encompasses everything in the past that has any influence at the current moment
- ▶ When any input is True, the system **reacts**
- ► **Reaction** means that the system changes its internal state, and enters a new state
- Moving from one state to the next state means a **transition**.

Finite State Machine representation

► Finite State Machine = a system whose operation is described as a set of states and transitions

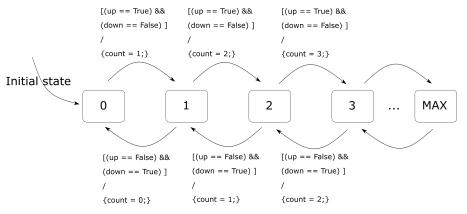


Figure 3: Parking system FSM

Components of a FSM representation

- ► States = the "bubbles"
- ► Transitions = the arrows
- Conditions (guards) = the conditions on the transitions are taken (inside "[]")
- ▶ Actions = the instructions executed when a transition is taken(after "/", inside "{}")

FSM notations

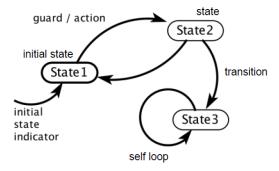


Figure 4: FSM Notations ²

²image from Seshia's slides

Conditions and actions

- A transition is taken when its condition becomes True
- ▶ When a transition is taken, the actions are executed
- ▶ It is possible that no transition is taken, so the system preserves its state ("default transition")
- ▶ The **initial transition** indicates which is the starting states

FSM mathematical model

A FSM is a tuple (States, Inputs, Outputs, update, initialState) consisting of the following:

- ightharpoonup States = a set 0, 1, ...M
- ▶ Inputs = a set of variables with their data types
- Outputs = a set of variables with their data types
- ▶ update = a function f : $States \times Inputs \rightarrow States \times Outputs$
 - ▶ the function takes as inputs = old state + current input values
 - ▶ the function outputs = new state + current output values
- ▶ initialState = the initial state

If all of the above is known, everything is known about the model.

Conditions and transitions

- Conditions and transitions can be written in many ways
- ► Here we use a simple C / Matlab instructions:
 - ▶ use == to check equality
 - ▶ ! means negation
 - ► True, False = boolean values
- **Examples**:
 - ▶ [a == True]
 - ▶ [!a == True]
 - >= [x >= 3]
 - ▶ [x < b]
 - etc . . .

Thermostat

 $Model\ example:\ thermostat$

[To draw]

When does a reaction occur?

- When are transitions checked? (when do the reactions happen)?
- Two variants:
 - **Event-triggered** model
 - ► Time-triggered model
- Event-triggered model:
 - ► The reaction can take time any time.
 - ► The environment triggers the transition, via an **event**
 - ► Works like an **interrupt** in microcontrollers
- ► Time-triggered model:
 - The reaction occurs periodically, on the global tick of an external clock
 - e.g. everything runs at $T_s = 10$ ms, 20ms etc.

Time-triggered models

- ► Simplest case = time-triggered models
- ► How it works:
 - ▶ the clock ticks, the FSM "wakes up" in a certain state
 - ► the inputs are read
 - the outgoing transitions from the current state are verified
 - ▶ if a transition is true, it is executed, the system enters a new state
 - the system "goes to sleep" until the next tick

Event vs time-triggered models

Advantages/disadvantages of time-triggered models:

- ▶ Bad: if a input changes very fast, within a T_s interval, the model may not see it
- ► Good: all inputs are read simultaneously
- Good: simple to understand

Advantages/disadvantages of event-triggered models:

- ▶ Bad: the inputs are not synchronized (in a condition a > b, perhaps a changes 1ms faster than b, and this leads to a wrong result
- ► Good: no risk that values are lost
- Bad: difficult to analyze, difficult to understand

Properties of discrete models

Properties of discrete models

- ▶ **Determinism**: In every state, for all possible input values, at most one transition is enabled
 - if you know the initial state and all the inputs' evolution, you know the complete behavior of the system
- ▶ **Non-determinism**: Models unknown behavior (unknown inputs), or random transitions

Determinism computation tree

For a fixed input sequence and initial state:

- ► A deterministic system exhibits a single behavior
- ► A non-deterministic system exhibits a set of behaviors, visualized as a computation tree

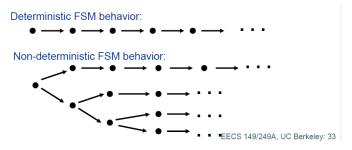


Figure 5: Computation tree ³

³image from Seshia's slides