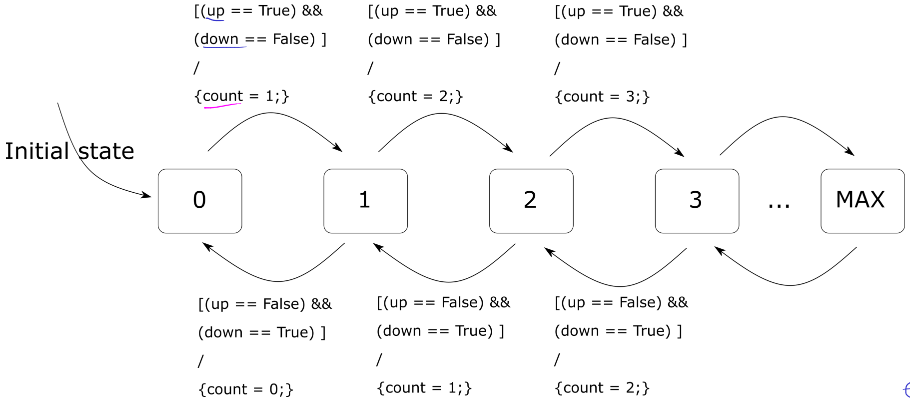
Embedded System Design and Modeling

III. Extended FSMs and Timed Automata

FSM example

- Recall the previous FSM example



count++;

count--;

Figure 1: Parking system FSM

- Can we make it is simpler to draw?

Extended FSMs

- **Extended FSM** = FSM with **internal variables**

```
Inputs:
  up: bool
  down: bool
Outputs:
  count: integer (0, MAX)
Variables:
  count: integer (0, MAX)
```

0 500

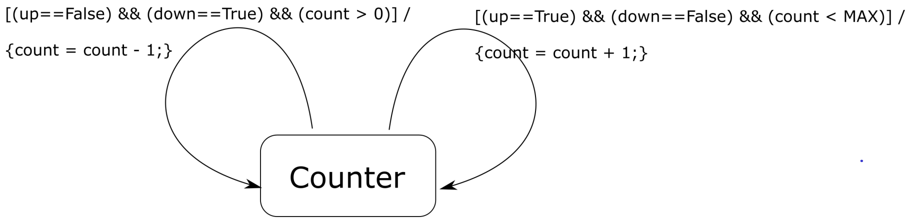


Figure 2: Extended FSM with variable “count”

Extended FSM

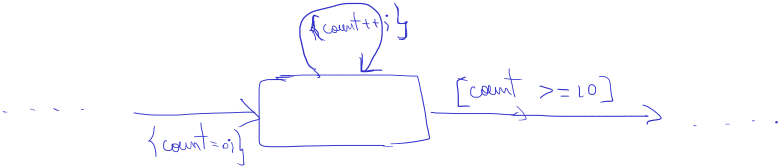
- The $\boxed{\text{state of the model}}$ = the current “bubble” and the values of **all the internal variables**
- Example: OS hibernation in Windows:
  - state of computer = all the RAM memory values
  - if all memory is written down on HDD, and reloaded tomorrow, the system effectively resumes operation from where it left off
- Number of
  States is not anymore “the number of bubbles”
  - there is only one “bubble” in our FSM
  - but there are MAX+1 states (all possible values of the count variable)

Declarations

- Always make explicit declaration of:
  - model inputs
  - model outputs
  - model internal variables
  - and their data types

Measure time

$T_s = 5\ ms$



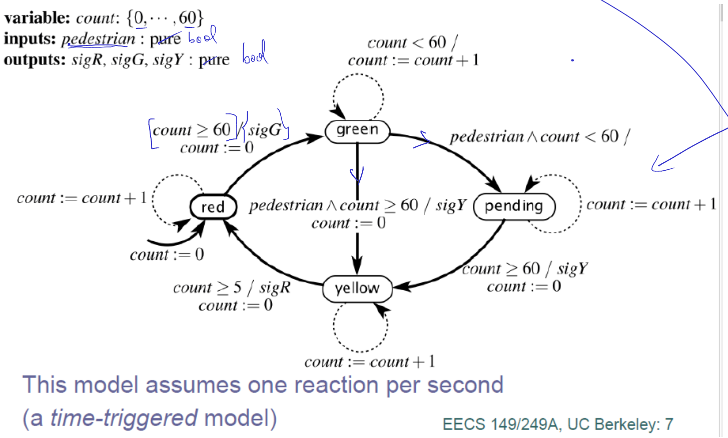
- Extended FSM are useful for modeling **time-based** conditions:
  - measure passage of time: increment a variable every "*tick*"
  - only works if the FSM is time-triggered

Example: pedestrian crossing light

- How is time measured in the model below?
- How many states does the model below have?

$\wedge$ = "AND"

Once we have green, green stays for at least 60 sec.



$$N = \underbrace{60 \text{ states for count}}_{\text{RED}}^{0\ldots59} + \underbrace{61 \text{ ~~states~~}}_{\text{Green}}^{0\ldots60 \text{ count values}} + \underbrace{60 \text{ states}}_{\text{pending}}^{0\ldots59} + \underbrace{5 \text{ states}}_{\text{yellow}} = \boxed{186}$$

Figure 3: Extended FSM with time measuring (image from Seshia' slides)

Hybrid systems

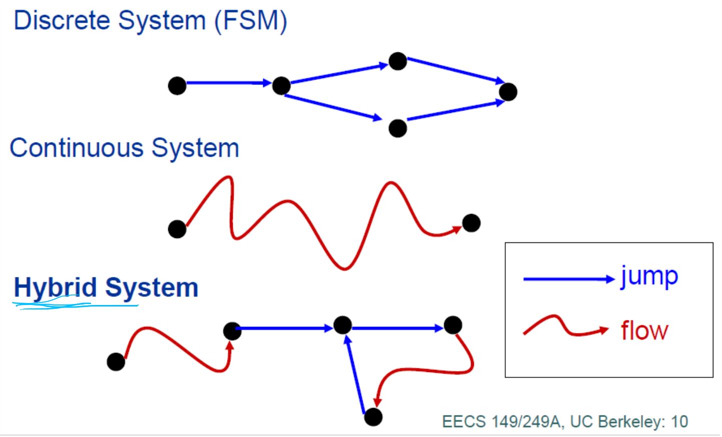
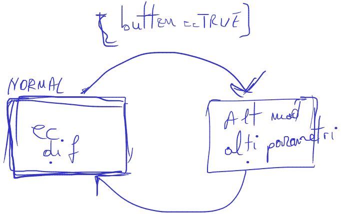


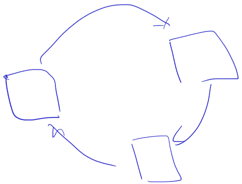
Figure 4: Hybrid systems (image from Seshia' slides)

Hybrid systems

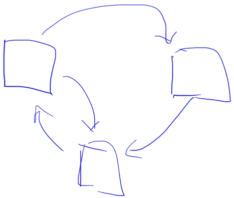
- **Hybrid systems** = system with mixes discrete and continuous behavior
- Example: a PID controller with different modes:
  - a set of distinct functioning model (e.g. Startup / Normal / Idle)
  - each state is a sub-system implemented with continuous dynamics
- State **refinement** = a lower-level implementation of a state



Types of hybrid systems



- **Timed automata** = hybrid system where every state refinement just measures passage of time (differential equation of degree 1)
- **Higher-order systems** = hybrid system where every state refinement uses higher-order differential equation (2 or more)
- **Two-level control systems** = complex controllers with two levels of operation
  - high-level discrete modes of operation (e.g. ECU Power Modes: Normal / Startup / Sleep Mode 1 / Sleep Mode 2)
  - low-level refinements with continuous dynamics



Timed automata

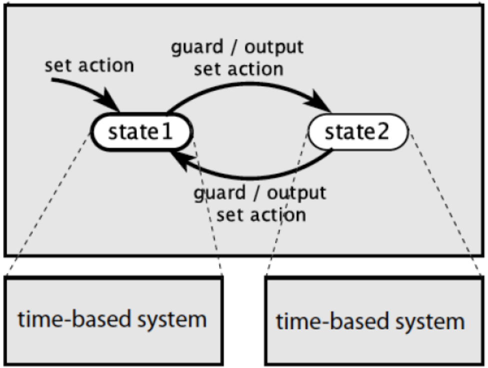


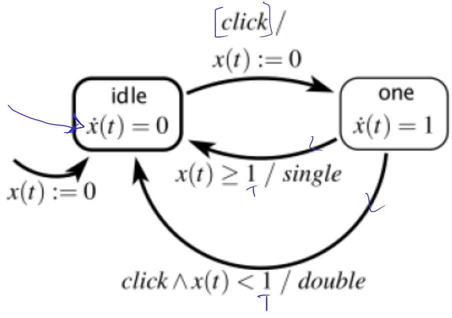
Figure 5: Timed automaton example (image from Seshia's slides)

Example

- Mouse Double-click detector model

continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$

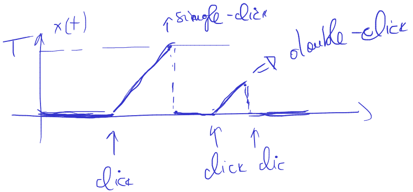
differential equation of order 1



x = count

$\dot{x}(t) = 0$

$\dot{x}(t) = 0 \iff x'(t) = 0$



$x'(t) = 1$

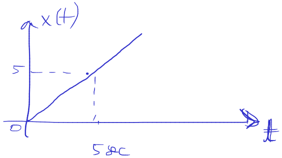


Figure 6: (image from Seshia's slides)

- Here $\dot{x}(t) = 1$ means "x(t) increases linearly with time", so it measures time

Example: Another Thermostat

- Another thermostat model as a Timed Automaton

Temperature threshold is 20 with minimum times T_c and T_h in each mode

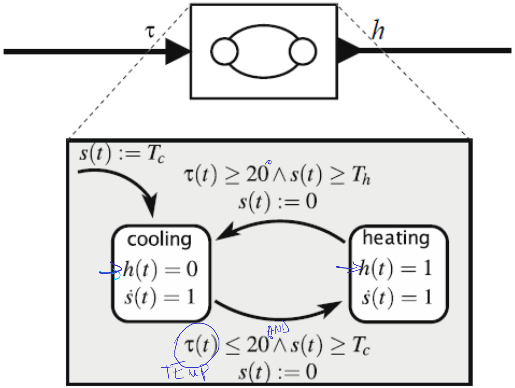
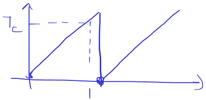


Figure 7: (image from Seshia's slides)

$h(t)$ = ON / OFF

$s(t)$ = measures time

$s'(t) = 1$



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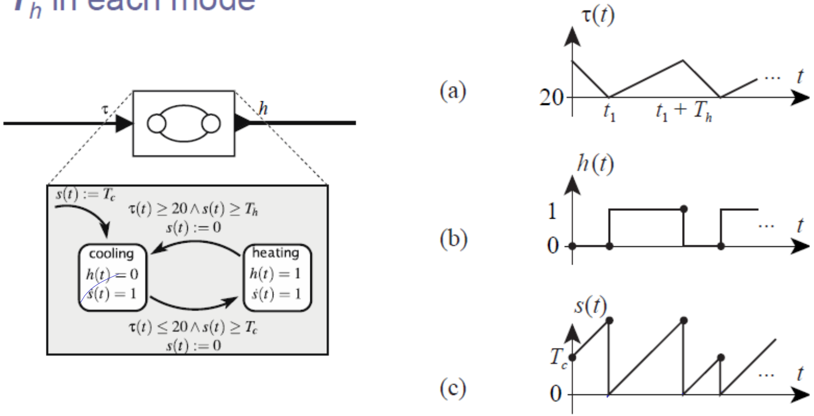


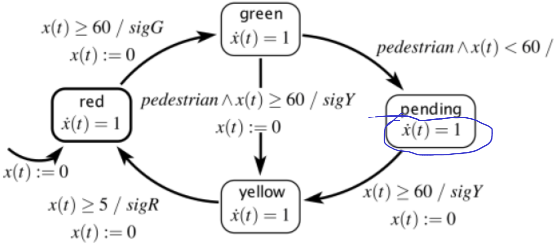
Figure 8: (image from Seshia's slides)

Example: Another Traffic Light

- Traffic Light controller Timed Automaton

Timed automaton model of a traffic light controller

continuous variable: $x(t) : \mathbb{R}$
inputs: $pedestrian$: pure
outputs: $sigR, sigG, sigY$: pure

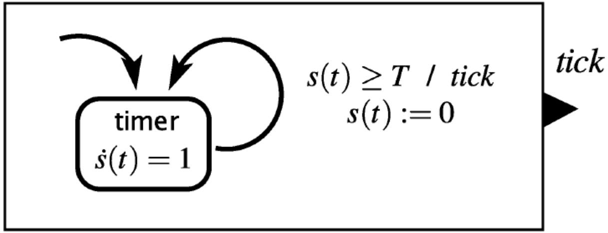


This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

Figure 9: (image from Seshia's slides)

Example: Tick generator

- Timed Automaton to generate a *tick* every T seconds



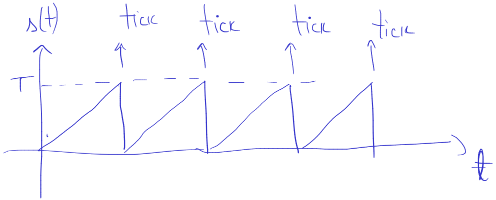


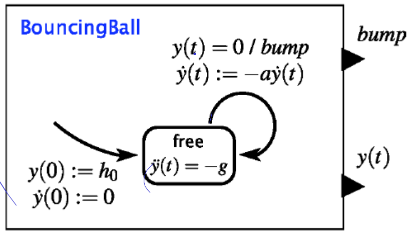
Figure 10: (image from Seshia's slides)

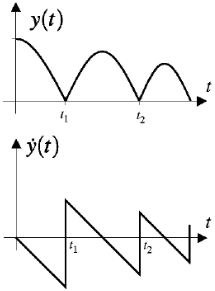
Example: Bouncing Ball

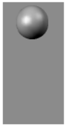
Higher-order hybrid system

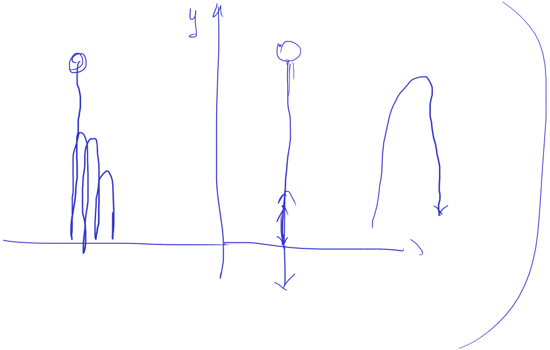
- ~~Timed Automaton~~ to simulate a bouncing ball movements

Hybrid Automaton for Bouncing Ball









y – vertical distance from ground (position)
a – coefficient of restitution, $0 \cdot a \cdot 1$

FSM simulation software

- FSM simulation software
- Used in this class: Stateflow (Simulink / Matlab)
- Features:
  - State Actions
  - Temporal Logic
  - Other events
  - . . . other . . .

State actions

- Actions can exist not only on transitions, but also **inside states**
- Three main types of **State Actions**:
  - **entry (en)**: executed only when a **state is entered**
  - **exit (ex)**: executed only when a **state is exited**
  - **during (du)**: executed when we are in state which is neither entered, not exited

State actions

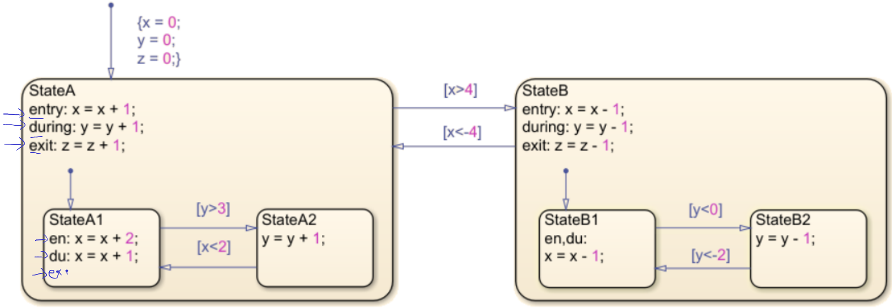
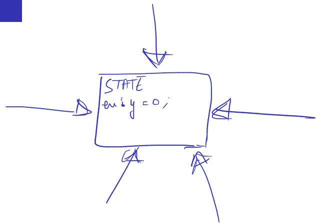


Figure 11: State Actions example (image from Matlab docs)



State actions

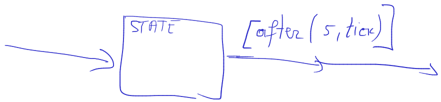
- State actions can be avoided (use only transitions actions), but sometimes one or the other are more convenient

Temporal logic

- For time-based conditions, states certain predefined variables, which can be used to **measure time spent in a state**
  - ***tick***: measures time steps
    - is incremented at **every time step**
    - is reset to 0 every time a state is exited or entered
    - actual duration **depends** on model step size
  - ***sec*** / ***msec***: measures seconds or miliseconds
    - is incremented every second / milisecond
    - is reset to 0 every time a state is exited or entered
    - actual duration is **independent** on model step size

Temporal logic

- Temporal operators **after()**, **on()**, **every()** can generate events which can be used in conditions
- Examples:
  - ***after(10, tick)***:
    - event is fired after 10 time steps spent in a state
    - evaluates to FALSE for the first 9 steps, is TRUE every time after that
  - ***on($\overset{10}{x}$, tick)***:
    - event is fired only **once**, exactly after x time steps spent in a state
    - evaluates to FALSE for the first $x - 1$ time moments, is TRUE only once at the x-th moment, is FALSE after that
  - ***every($\overset{10}{x}$, tick)***:
    - event is fired pariodically after x time steps
    - evaluates to FALSE for the first $x - 1$ time moments, is TRUE once at the x-th moment, then FALSE for the next $x - 1$ time moments, then TRUE again, and so on



Temporal logic

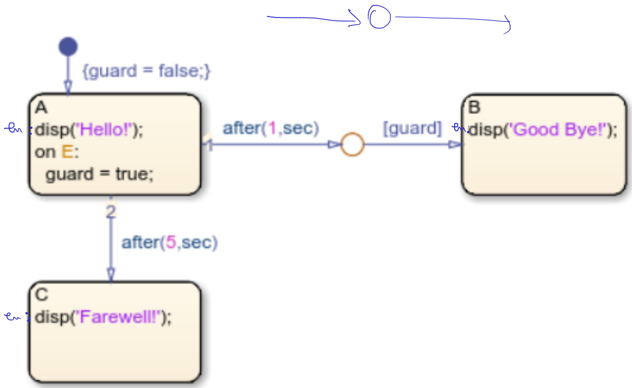


Figure 12: Temporal Logic example (image from Matlab docs)