

Embedded System Design and Modeling

VI. Composition of State Machines

Composition of state machines

- ▶ How to combine multiple smaller FSMs into a bigger one?
- ▶ What problems arise?
- ▶ Two types of compositions:
 1. **Spatial** composition: how are the components connected?
 2. **Temporal** composition: how do the components react in time?

Spatial composition

Spatial composition = how are two components connected, how does the information flow between the components

- ▶ **Side-by-side** composition = no common inputs/outputs, no shared data
- ▶ **Cascade** composition = Outputs of one FSM are inputs to another one
- ▶ **Feedback** composition = (Some) outputs of a FSM are inputs to the same FSM, or to some other component which is in front

Side-by-side composition

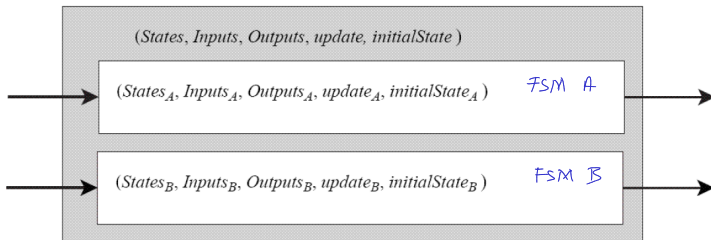


Figure 1: Side-by-side composition

Cascade composition

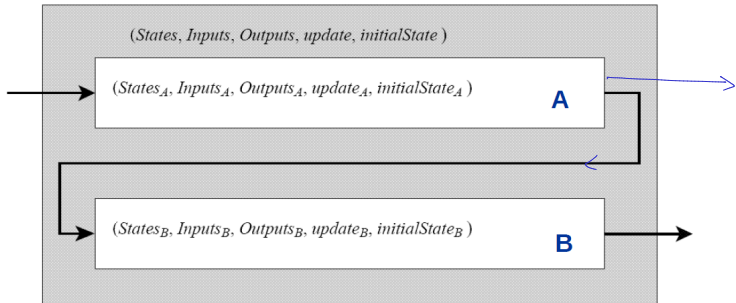


Figure 2: Cascade composition

- Outputs of FSM A are inputs to FSM B

Feedback composition

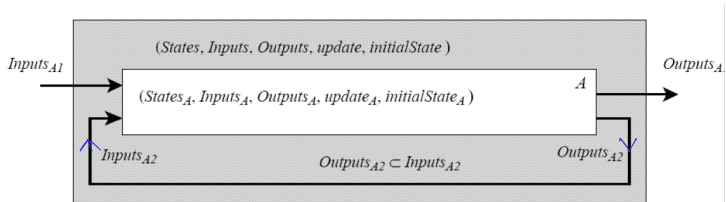


Figure 3: Feedback composition

- Some outputs of the FSM are coming back as inputs

Temporal composition

Temporal composition = when do two components react?

▶ Sequential vs Parallel composition:

- ▶ **Sequential** = the two FSM do not work at the same time
- ▶ **Parallel** = the two FSM work at the same time

▶ Asynchronous vs Synchronous composition = only for parallel composition

- ▶ **Synchronous** = transitions are taken at the same time in both FSMs
- ▶ **Asynchronous** = transitions are taken at independent times in the FSMs

Sequential composition

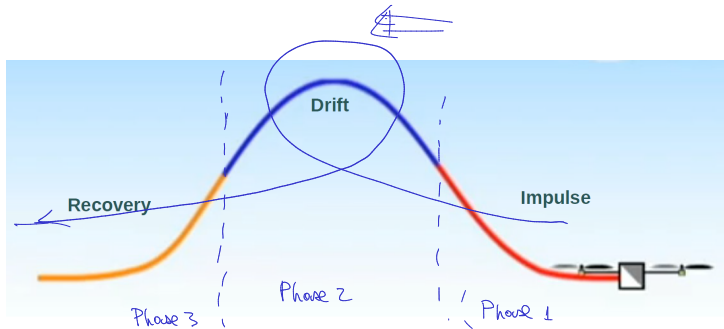


Figure 4: Example of Sequential composition

- ▶ 13579/:<https://www.youtube.com/watch?v=D3QgGpzzIM>
- ▶ The drone has three modes of operation, working **in sequence**

Parallel composition

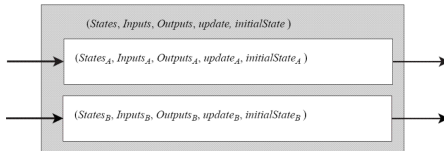


Figure 5: Side-by-side composition

- ▶ The two FSMs form an **equivalent model**
- ▶ When do the transitions in these FSM take place?
 - ▶ Synchronous: simultaneously
 - ▶ Asynchronous: independently

Synchronous composition

- ▶ Consider the two FSM on the left (A and B)
- ▶ The equivalent model is on the right

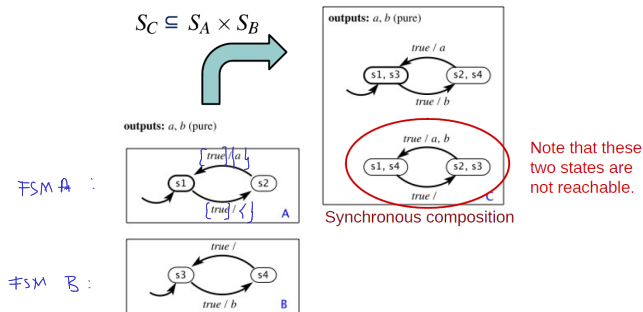
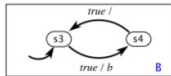
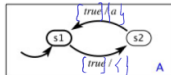
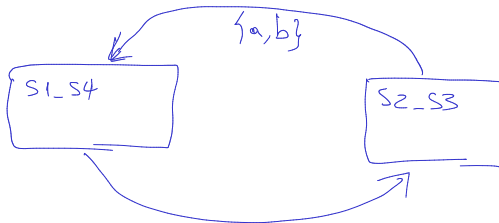
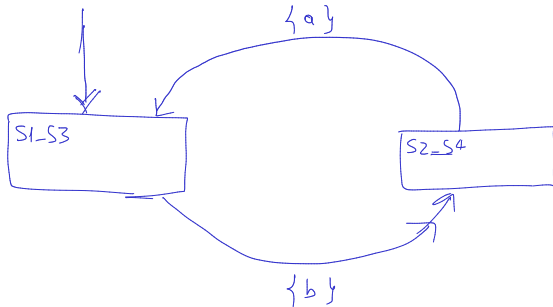


Figure 6: Synchronous composition

Synchronous composition



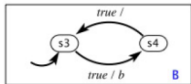
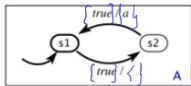
Redraw here



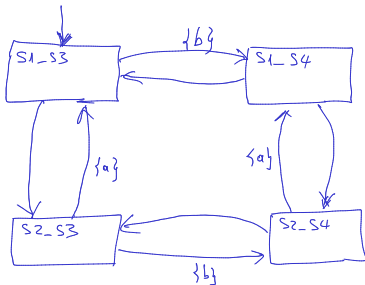
Synchronous composition

- ▶ In the equivalent model:
 - ▶ States = combination of states of the two FSMs
 - ▶ Transition = transition in FSM A and FSM B, happening simultaneously.
 - ▶ There might exist unreachable states in the equivalent model (states that will never be reached)

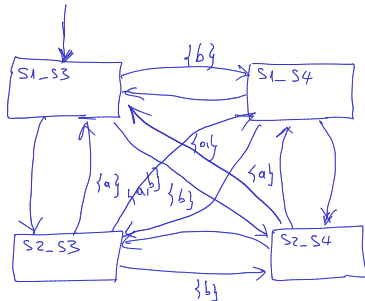
Asynchronous composition



Draw here



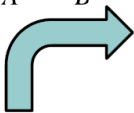
Interleaving
semantics



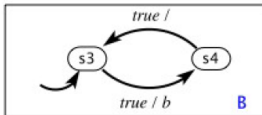
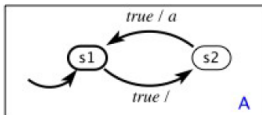
Simultaneous
semantics

Asynchronous composition

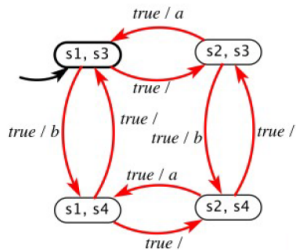
$$S_C \subseteq S_A \times S_B$$



outputs: a, b (pure)



outputs: a, b (pure)



Note that now
all states are
reachable.

Asynchronous composition
using interleaving semantics

Figure 7: Asynchronous composition

Asynchronous composition

- ▶ In the equivalent model:
 - ▶ States = combination of states of the two FSMs
 - ▶ Transitions in the two FSMs can take place at irregular and independent (not synchronized) times
 - ▶ All states are reachable
 - ▶ because one model can be much faster than the other

Asynchronous composition

Flavors of asynchronous composition

- ▶ How are simultaneous transitions handled?
- ▶ **Interleaving** semantics:
 - ▶ simultaneous transition in models A and B is not allowed (we may have either a transition in model A, or a transition in B)
 - ▶ i.e. transition from A takes place first, then transition from B takes place after a non-zero time delay (or vice-versa)
- ▶ **Simultaneous** semantics:
 - ▶ simultaneous transition in models A and B is allowed
 - ▶ for example, we may have either
 - ▶ transition only in model A
 - ▶ transition only in model B
 - ▶ Simultaneous transition in models A and B

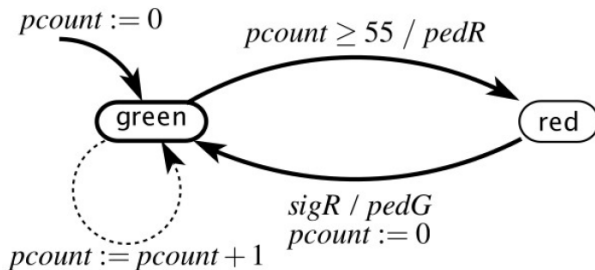
Example

variable: $pcount: \{0, \dots, 55\}$

input: sigR : pure

outputs: pedG, pedR : pure

*pedestrian
Traffic
Light*



This light stays green for 55 seconds, then goes red.
Upon receiving a sigR input, it repeats the cycle.

Figure 8: Composition - Pedestrian Light

Example

variable: $count: \{0, \dots, 60\}$

inputs: $pedestrian: \text{pure}$

outputs: $sigR, sigG, sigY: \text{pure}$

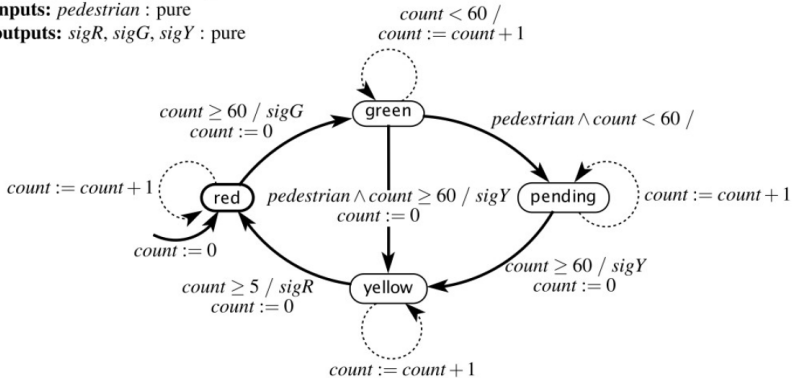
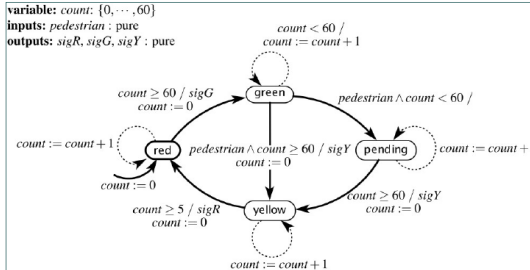


Figure 9: Composition - Car Light

Example

Pedestrian Light with Car Light



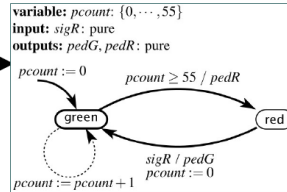
$sigY$

$sigG$

$sigR$

What is the size of
the state space of the
composite machine?

$sigR$



$pedG$

$pedR$

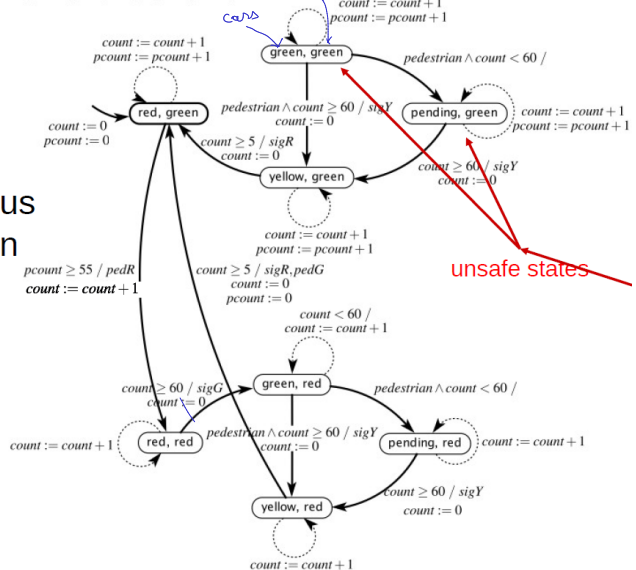
Figure 10: Cascade Composition - Both Lights

Example

variables: $count: \{0, \dots, 60\}, pcount: \{0, \dots, 55\}$

input: $pedestrian: pure$

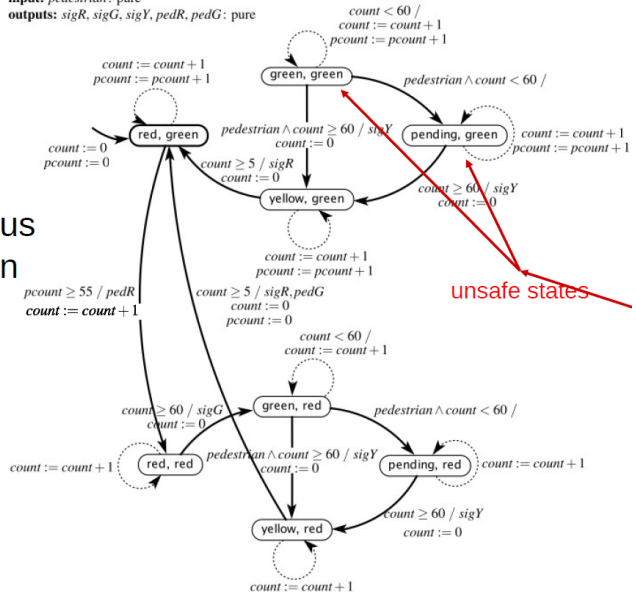
outputs: $sigR, sigG, sigY, pedR, pedG: pure$



Synchronous
composition

Example

variables: $count: \{0, \dots, 60\}, pcount: \{0, \dots, 55\}$
input: $pedestrian: \text{pure}$
outputs: $sigR, sigG, sigY, pedR, pedG: \text{pure}$



Synchronous
composition