

Embedded System Design and Modeling

III. Extended FSMs and Timed Automata

FSM example

► Recall the previous FSM example

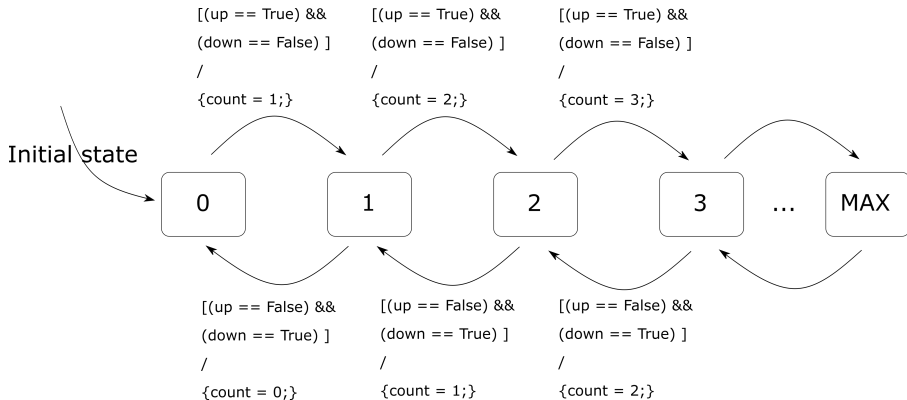


Figure 1: Parking system FSM

► Can we make it is simpler to draw?

Extended FSMs

► Extended FSM = FSM with **internal variables**

Inputs:

up: bool

down: bool

Outputs:

count: integer (0, MAX)

Variables:

count: integer (0, MAX)

$[(up == False) \ \&\& \ (down == True) \ \&\& \ (count > 0)] /$

$\{count = count - 1;\}$

$[(up == True) \ \&\& \ (down == False) \ \&\& \ (count < MAX)] /$

$\{count = count + 1;\}$

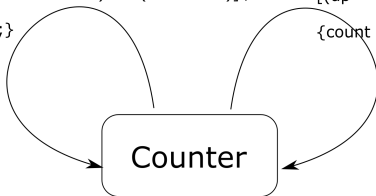


Figure 2: Extended FSM with variable “count”

Extended FSM

- ▶ The state of the model = the current “bubble” and the values of **all the internal variables**
- ▶ Example: OS hibernation in Windows:
 - ▶ state of computer = all the RAM memory values
 - ▶ if all memory is written down on HDD, and reloaded tomorrow, the system effectively resumes operation from where it left off
- ▶ State is not anymore “the number of bubbles”
 - ▶ there is only one “bubble” in our FSM
 - ▶ but there are $\text{MAX}+1$ states (all possible values of the count variable)

Declarations

- ▶ Always make explicit declaration of:
 - ▶ model inputs
 - ▶ model outputs
 - ▶ model internal variables
 - ▶ and their data types

Measure time

- ▶ Extended FSM are useful for modeling **time-based** conditions:
 - ▶ measure passage of time: increment a variable every *tick*
 - ▶ only works if the FSM is time-triggered

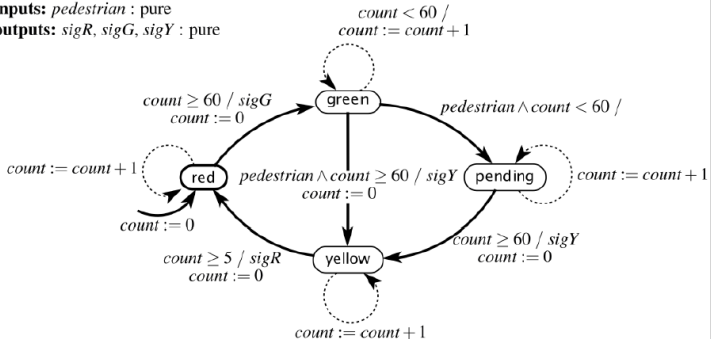
Example: pedestrian crossing light

- ▶ How is time measured in the model below?
- ▶ How many states does the model below have?

variable: *count*: $\{0, \dots, 60\}$

inputs: *pedestrian*: pure

outputs: *sigR*, *sigG*, *sigY*: pure



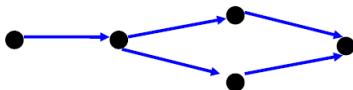
This model assumes one reaction per second
(a *time-triggered* model)

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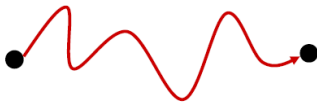
Figure 3: Extended FSM with time measuring (image from Seshia' slides)

Hybrid systems

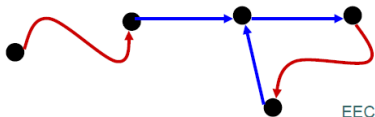
Discrete System (FSM)



Continuous System



Hybrid System



—→ jump

—→ flow

Figure 4: Hybrid systems (image from Seshia' slides)

Hybrid systems

- ▶ **Hybrid systems** = system with mixes discrete and continuous behavior
- ▶ Example: a PID controller with different modes:
 - ▶ a set of distinct functioning model (e.g. Startup / Normal / Idle)
 - ▶ each state is a sub-system implemented with continuous dynamics
- ▶ State **refinement** = a lower-level implementation of a state

Types of hybrid systems

- ▶ **Timed automata** = hybrid system where every state refinement just measures passage of time (differential equation of degree 1)
- ▶ **Higher-order systems** = hybrid system where every state refinement uses higher-order differential equation (2 or more)
- ▶ **Two-level control systems** = complex controllers with two levels of operation
 - ▶ high-level discrete modes of operation (e.g. ECU Power Modes: Normal / Startup / Sleep Mode 1 / Sleep Mode 2)
 - ▶ low-level refinements with continuous dynamics

Timed automata

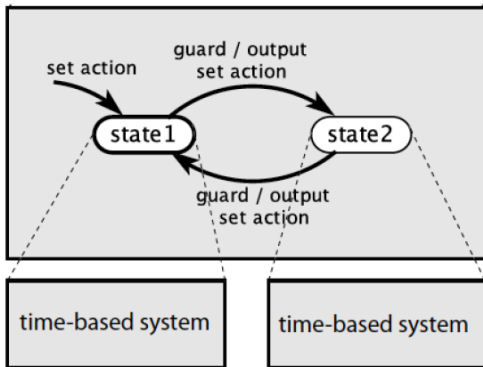


Figure 5: Timed automaton example (image from Seshia's slides)

Example

► Mouse Double-click detector model

continuous variable: $x(t) \in \mathbb{R}$

inputs: $click \in \{present, absent\}$

outputs: $single, double \in \{present, absent\}$

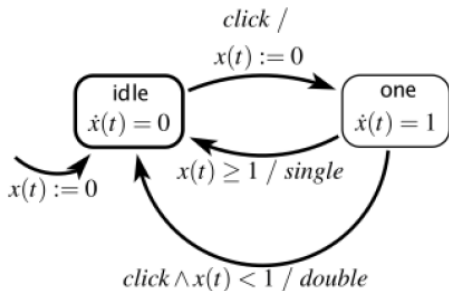


Figure 6: (image from Seshia's slides)

- Here $\dot{x}(t) = 1$ means “ $x(t)$ increases linearly with time”, so it measures time

Example: Another Thermostat

- ▶ Another thermostat model as a Timed Automaton

Temperature threshold is 20 with minimum times T_c and T_h in each mode

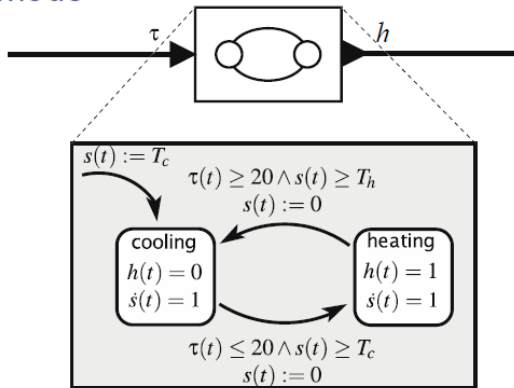


Figure 7: (image from Seshia's slides)

Example: Another Thermostat

- ▶ Another thermostat model as a Timed Automaton

Temperature threshold is 20 with minimum times T_c and T_h in each mode

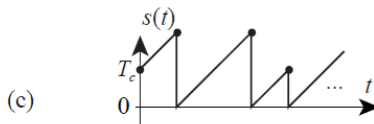
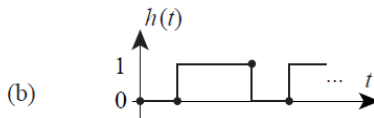
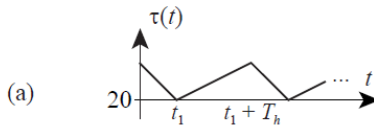
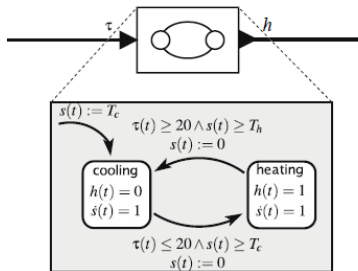


Figure 8: (image from Seshia's slides)

Example: Another Traffic Light

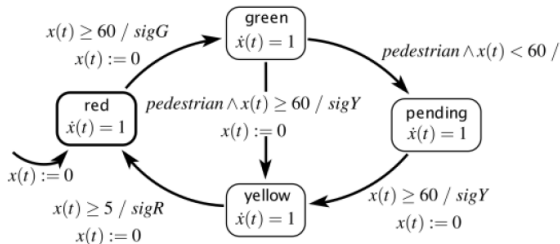
► Traffic Light controller Timed Automaton

Timed automaton model of a traffic light controller

continuous variable: $x(t): \mathbb{R}$

inputs: *pedestrian*: pure

outputs: *sigR*, *sigG*, *sigY*: pure



This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

Figure 9: (image from Seshia's slides)

Example: Tick generator

- ▶ Timed Automaton to generate a *tick* every T seconds

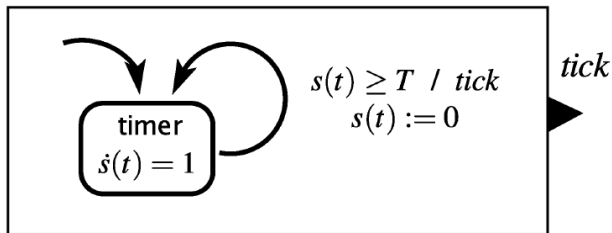
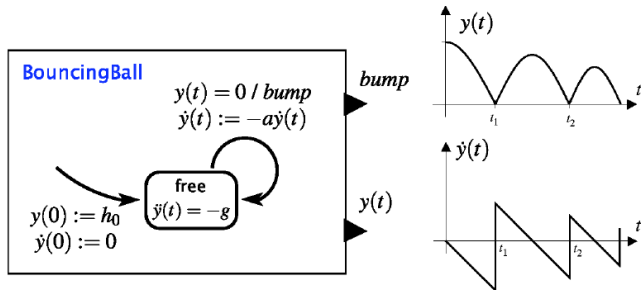


Figure 10: (image from Seshia's slides)

Example: Bouncing Ball

- Timed Automaton to simulate a bouncing ball movements

Hybrid Automaton for Bouncing Ball



y – vertical distance from ground (position)

a – coefficient of restitution, $0 \leq a \leq 1$



FSM simulation software

- ▶ FSM simulation software
- ▶ Used in this class: Stateflow (Simulink / Matlab)
- ▶ Features:
 - ▶ State Actions
 - ▶ Temporal Logic
 - ▶ Other events
 - ▶ ... other ...

State actions

- ▶ Actions can exist not only on transitions, but also **inside states**
- ▶ Three main types of **State Actions**:
 - ▶ **entry (en)**: executed only when a **state is entered**
 - ▶ **exit (ex)**: executed only when a **state is exited**
 - ▶ **during (du)**: executed when we are in state which is neither entered, not exited

State actions

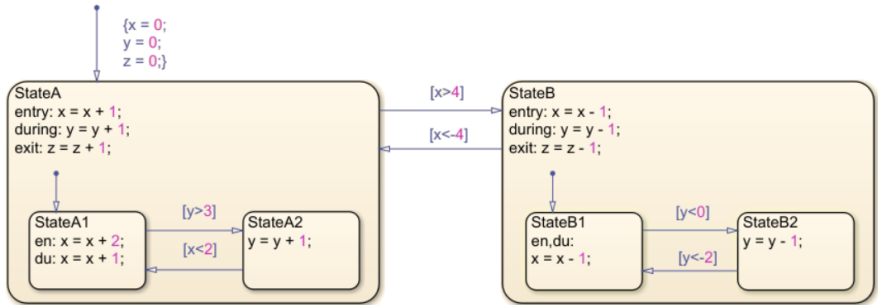


Figure 11: State Actions example (image from Matlab docs)

State actions

- ▶ State actions can be avoided (use only transitions actions), but sometimes one or the other are more convenient

Temporal logic

- ▶ For time-based conditions, states certain predefined variables, which can be used to **measure time spent in a state**
 - ▶ ***tick***: measures time steps
 - ▶ is incremented at **every time step**
 - ▶ is reset to 0 every time a state is exited or entered
 - ▶ actual duration **depends** on model step size
 - ▶ ***sec* / *msec***: measures seconds or milliseconds
 - ▶ is incremented every second / millisecond
 - ▶ is reset to 0 every time a state is exited or entered
 - ▶ actual duration is **independent** on model step size

Temporal logic

- ▶ Temporal operators **after()**, **on()**, **every()** can generate events which can be used in conditions
- ▶ Examples:
 - ▶ **after(10, tick)**:
 - ▶ event is fired after 10 time steps spent in a state
 - ▶ evaluates to FALSE for the first 9 steps, is TRUE every time after that
 - ▶ **on(x, tick)**:
 - ▶ event is fired only **once**, exactly after x time steps spent in a state
 - ▶ evaluates to FALSE for the first $x - 1$ time moments, is TRUE only once at the x -th moment, is FALSE after that
 - ▶ **every(x, tick)**:
 - ▶ event is fired periodically after x time steps
 - ▶ evaluates to FALSE for the first $x - 1$ time moments, is TRUE once at the x -th moment, then FALSE for the next $x - 1$ time moments, then TRUE again, and so on

Temporal logic

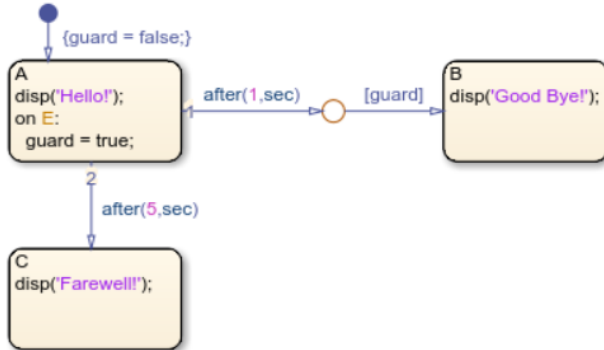


Figure 12: Temporal Logic example (image from Matlab docs)