Embedded System Design and Modeling

III. Extended FSMs and Timed Automata

FSM example

- Recall the previous FSM example

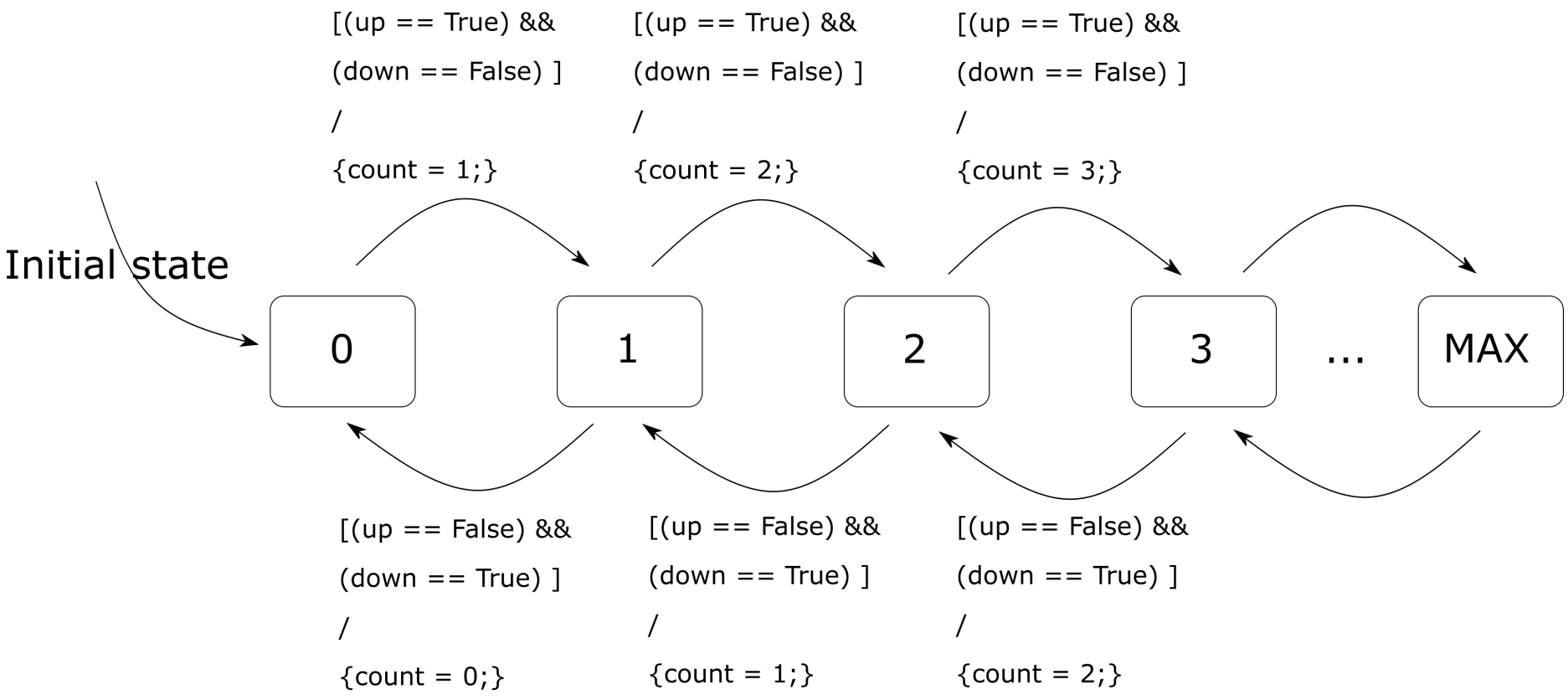


Figure 1: Parking system FSM

- Can we make it is simpler to draw?

Extended FSMs

- **Extended FSM** = FSM with **internal variables**

Inputs:
  up: bool
  down: bool
Outputs:
  count: integer (0, MAX)
Variables:
  count: integer (0, MAX)

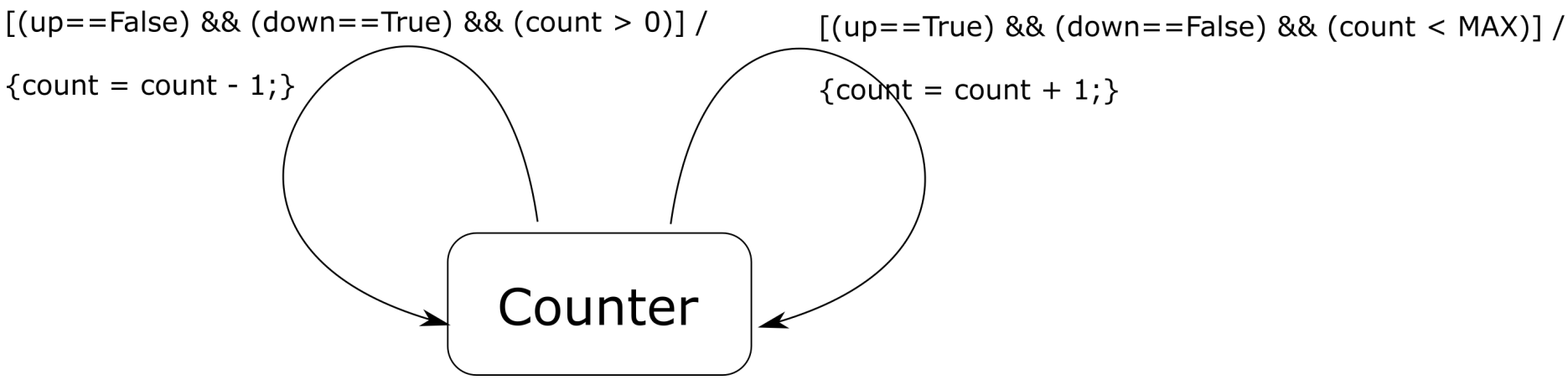


Figure 2: Extended FSM with variable “count”

Extended FSM

- The state of the model = the current "bubble" and the values of **all the internal variables**
- Example: OS hibernation in Windows:
  - state of computer = all the RAM memory values
  - if all memory is written down on HDD, and reloaded tomorrow, the system effectively resumes operation from where it left off
- State is not anymore "the number of bubbles"
  - there is only one "bubble" in our FSM
  - but there are MAX+1 states (all possible values of the count variable)

Declarations

- Always make explicit declaration of:
  - model inputs
  - model outputs
  - model internal variables
  - and their data types

Measure time

- Extended FSM are useful for modeling **time-based** conditions:
  - measure passage of time: increment a variable every *tick*
  - only works if the FSM is time-triggered

Example: pedestrian crossing light

- How is time measured in the model below?
- How many states does the model below have?

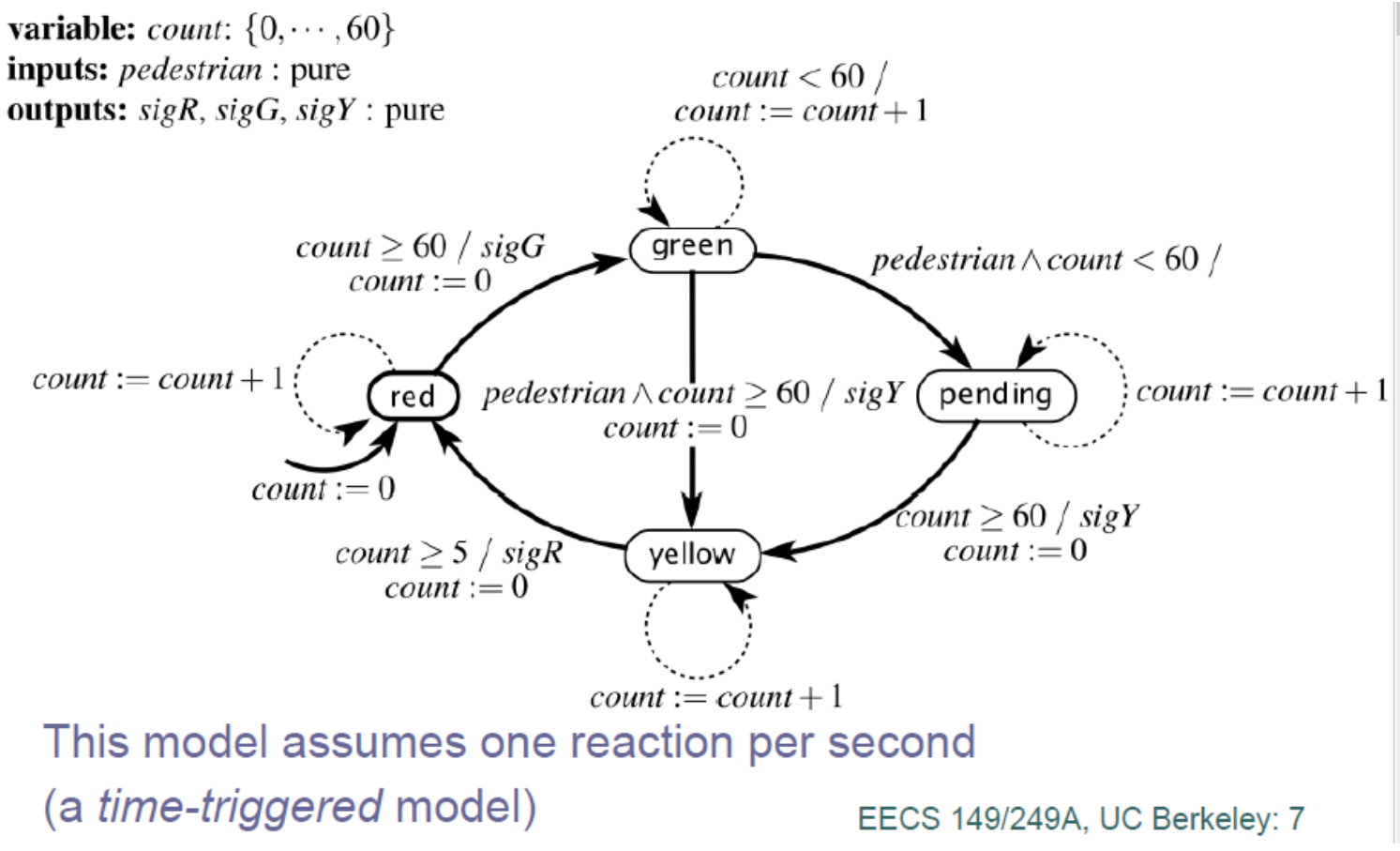


Figure 3: Extended FSM with time measuring (image from Seshia' slides)

Hybrid systems

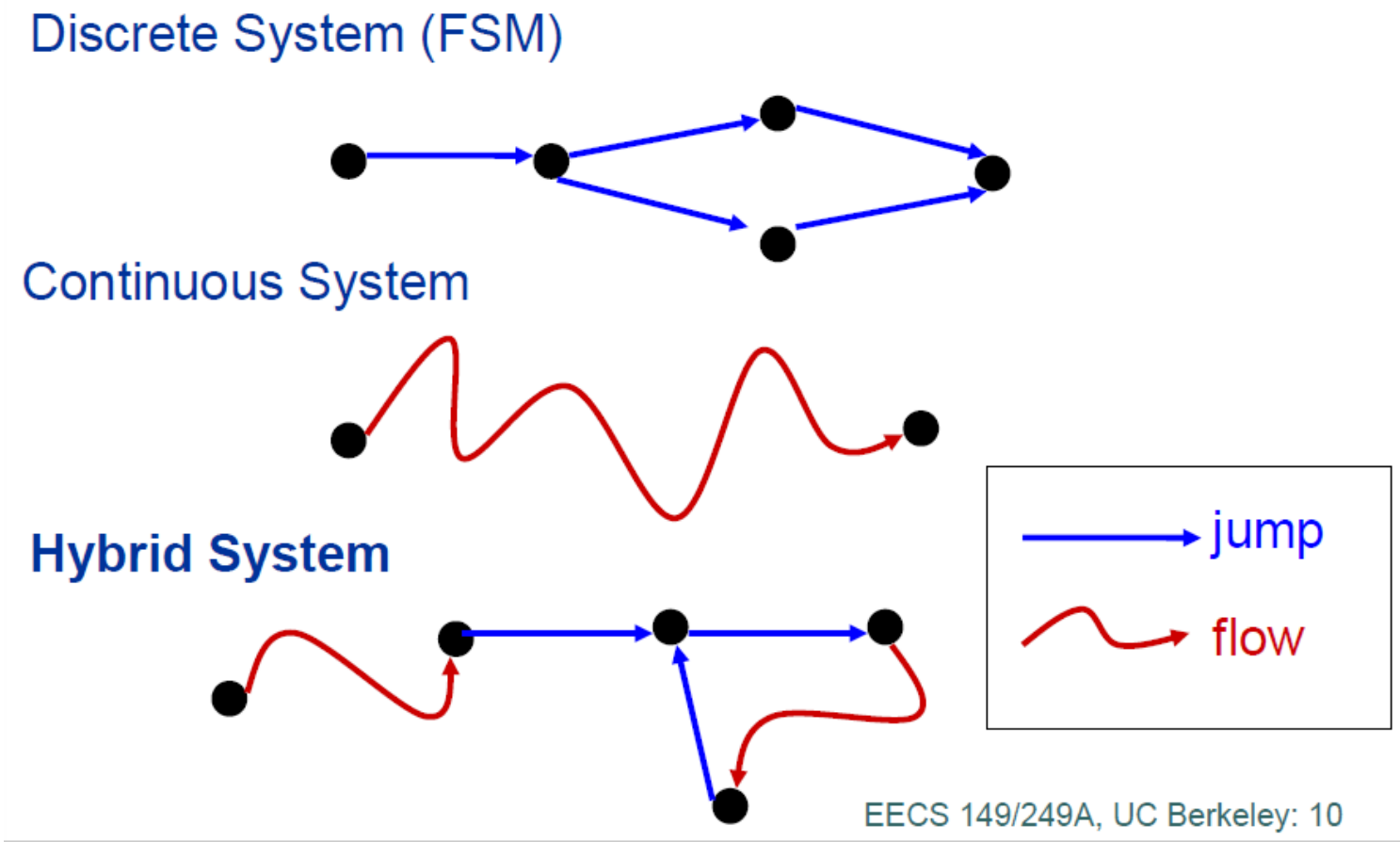


Figure 4: Hybrid systems (image from Seshia' slides)

Hybrid systems

- **Hybrid systems** = system with mixes discrete and continuous behavior
- Example: a PID controller with different modes:
  - a set of distinct functioning model (e.g. Startup / Normal / Idle)
  - each state is a sub-system implemented with continuous dynamics
- State **refinement** = a lower-level implementation of a state

Types of hybrid systems

- **Timed automata** = hybrid system where every state refinement just measures passage of time (differential equation of degree 1)
- **Higher-order systems** = hybrid system where every state refinement uses higher-order differential equation (2 or more)
- **Two-level control systems** = complex controllers with two levels of operation
  - high-level discrete modes of operation (e.g. ECU Power Modes: Normal / Startup / Sleep Mode 1 / Sleep Mode 2)
  - low-level refinements with continuous dynamics

Timed automata

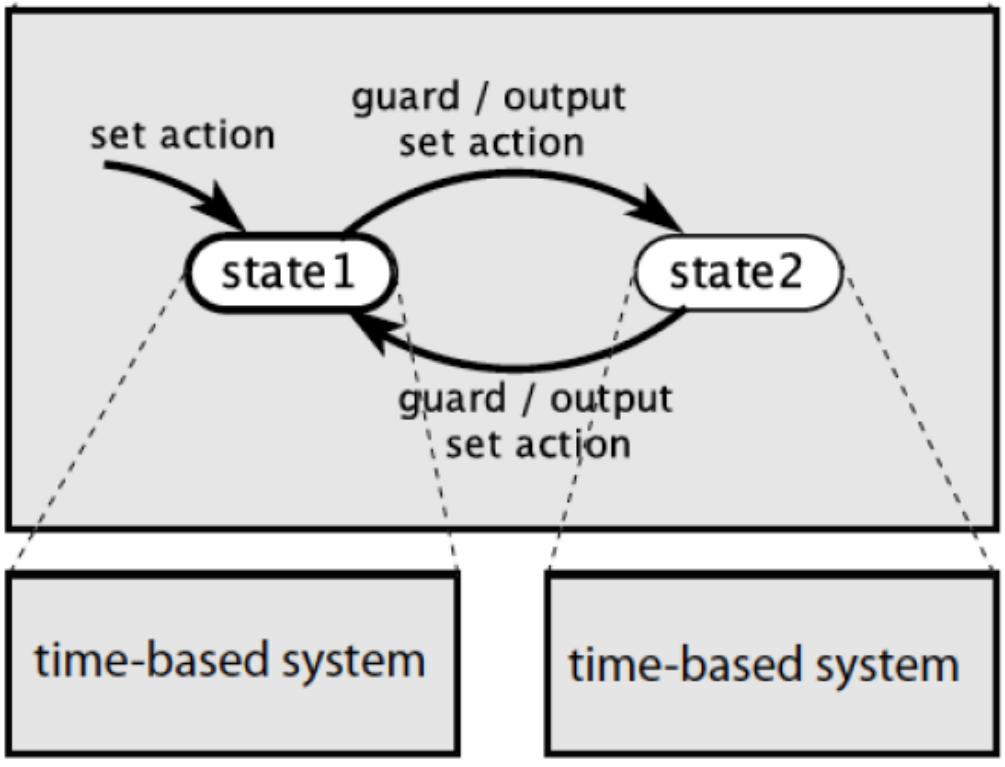


Figure 5: Timed automaton example (image from Seshia's slides)

Example

- Mouse Double-click detector model

continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$

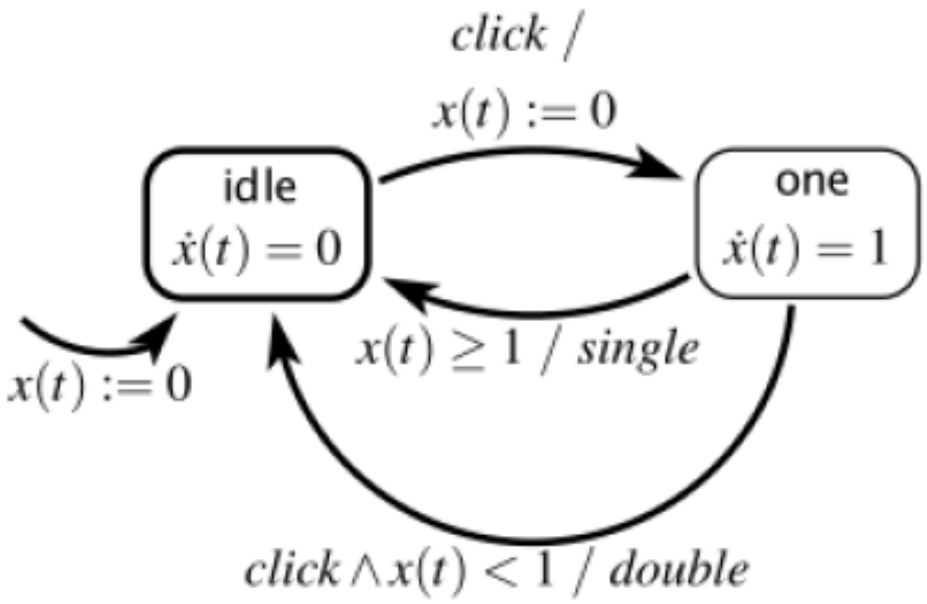


Figure 6: (image from Seshia's slides)

- Here $\dot{x}(t) = 1$ means "x(t) increases linearly with time", so it measures time

Example: Another Thermostat

- Another thermostat model as a Timed Automaton

Temperature threshold is 20 with minimum times T_c and T_h in each mode

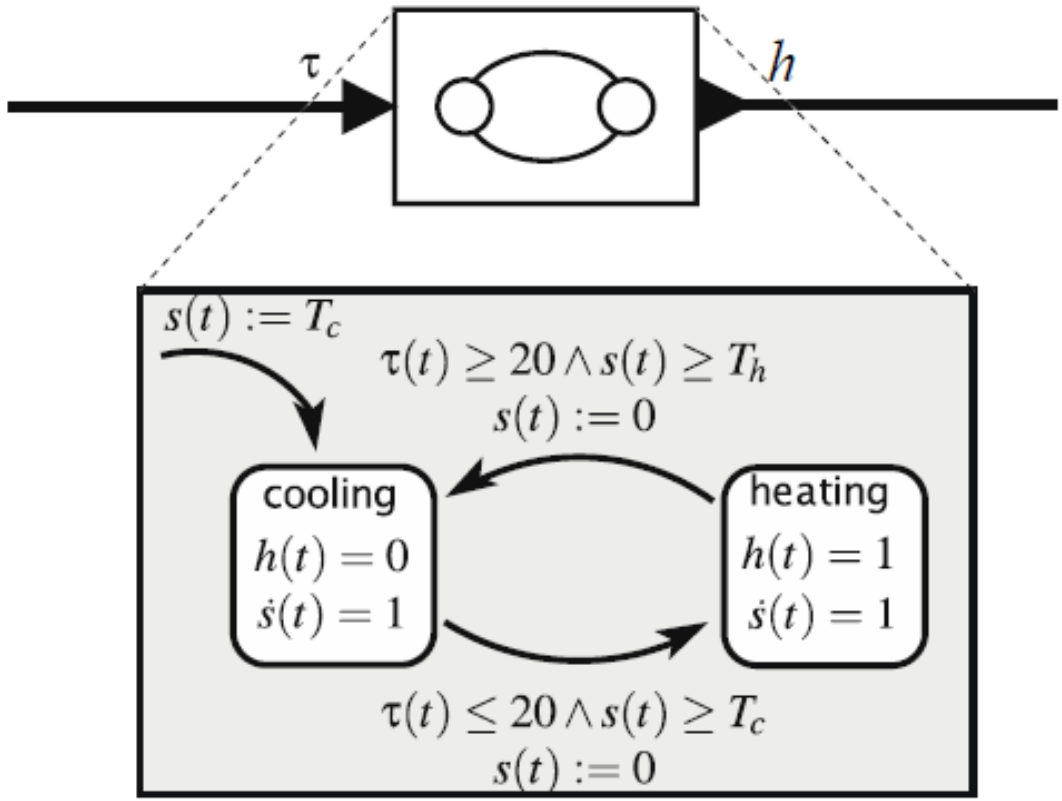
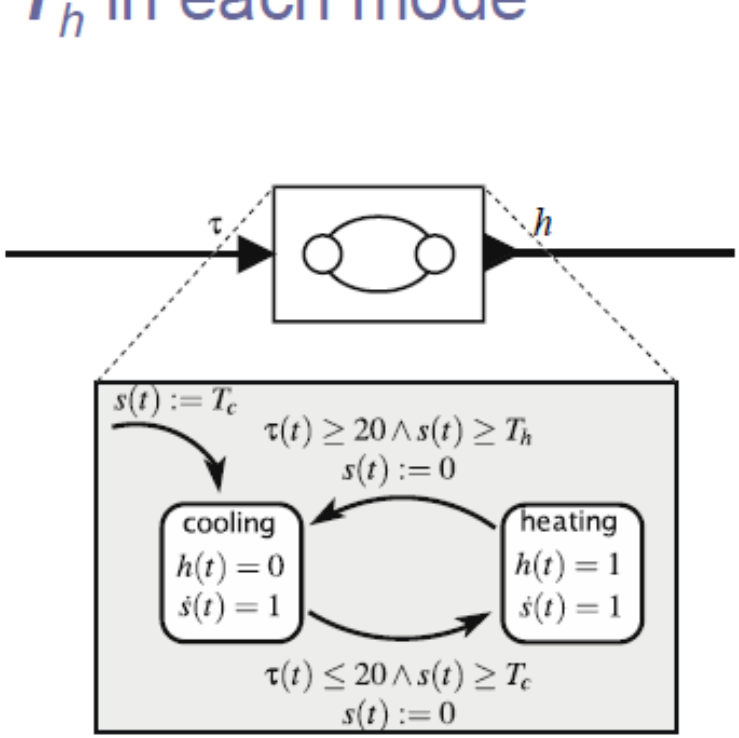


Figure 7: (image from Seshia's slides)

Example: Another Thermostat

- Another thermostat model as a Timed Automaton

Temperature threshold is 20 with minimum times T_c and T_h in each mode



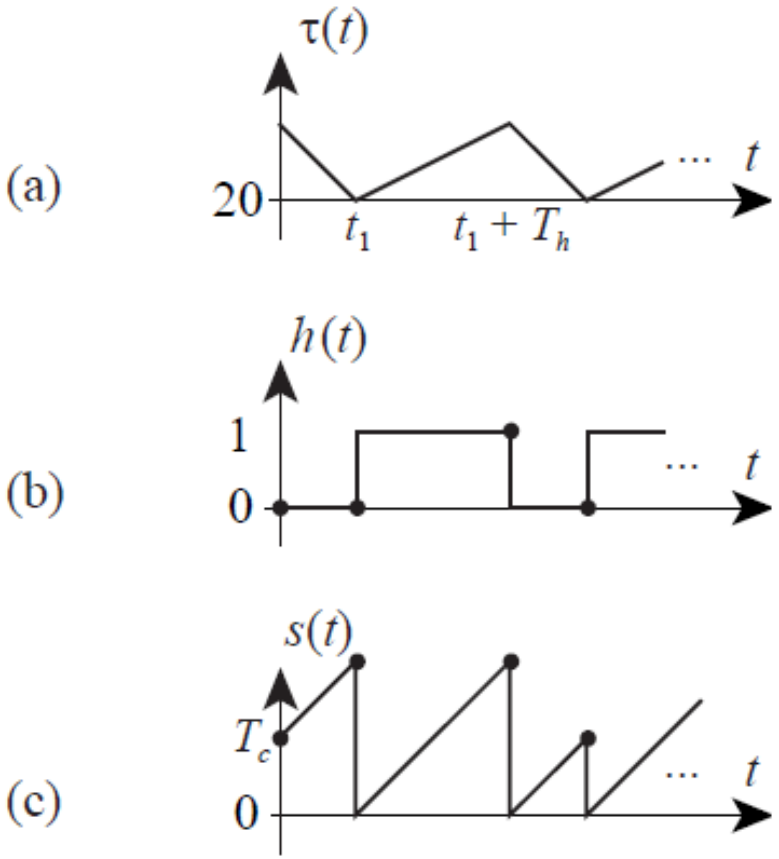
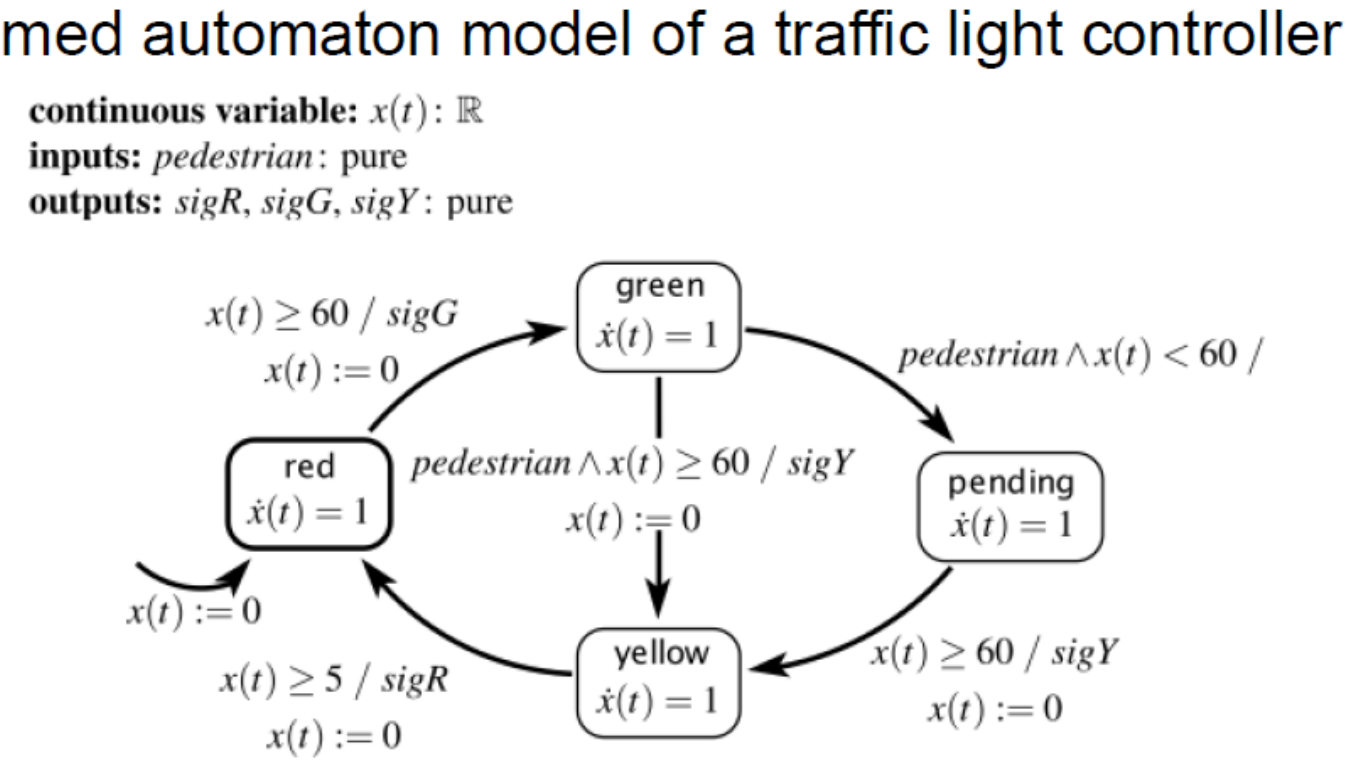


Figure 8: (image from Seshia's slides)

Example: Another Traffic Light

- Traffic Light controller Timed Automaton



This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

Figure 9: (image from Seshia's slides)

Example: Tick generator

- Timed Automaton to generate a *tick* every T seconds

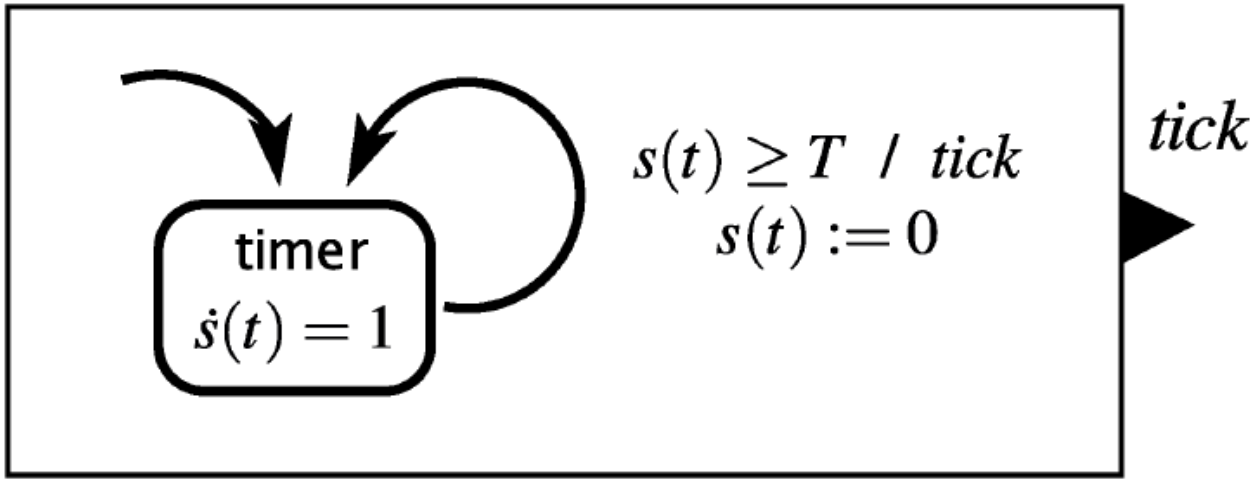


Figure 10: (image from Seshia's slides)

Example: Bouncing Ball

- Timed Automaton to simulate a bouncing ball movements

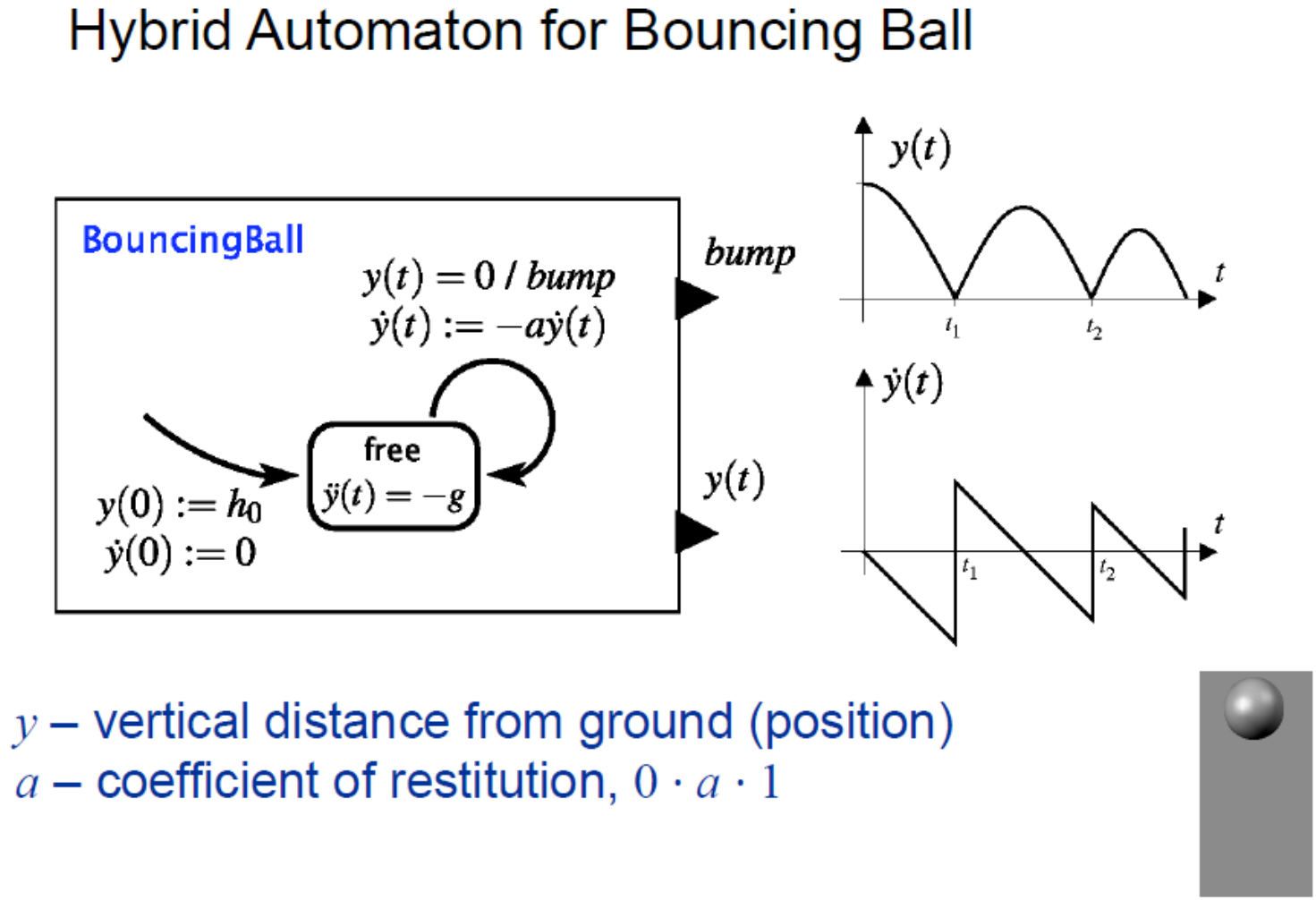


Figure 11: (image from Seshia's slides)