Embedded System Design and Modeling

II. Modeling continuous systems

Continuous systems

- Physical processes are usually in continuous domain
  - e.g. electrical, mechanical
- Processes in continuous domain are described by **differential equations**
  - i.e. with unknown functions + their derivatives + their second derivatives + ...
- Simple (ideal) case: differential equations are linear:
  - only operations allowed: sum, multiplication by a constant
- Every electrical/mechanical part defines a certain relation between the unknowns

Electrical systems

Electrical systems:

- Unknown functions $=$ voltage $+$ current in all branches
- Electrical (ideal) elements:
  - resistance: $u(i) = R \cdot i(t)$
  - capacity: $i(t) = C \cdot \frac{d}{dt} u(t)$
  - etc.
- One big system of linear differential equations (SCS course, basically)
  - Kirchhoff equations $<=>$ equations between currents and voltages $<=>$ linear differential equation system
- Example: an RC system (solve at blackboard)

Mechanical systems

Mechanical systems:

- Unknown functions = coordinates x(t), y(t), z(t)
  - speeds = derivatives of the positions
  - acceleration = derivative of speed = second derivative of positions
  - (forces: $F = m \cdot a = m \cdot \frac{d^2}{dt^2}x(t)$)
- Mechanical (ideal) elements:
  - (Consider just a single dimension $x(t)$, is easier)
  - inertial force: $F = m \cdot a = m \cdot \frac{d^2}{dt^2}x(t)$
  - friction force:
    - sliding friction: $\vec{F_f} = -\mu \vec{N} = -\mu \cdot m \cdot \frac{d^2}{dt^2}x(t)$
    - viscous friction: $\vec{F_v} = -C_v \cdot \vec{v} = -C_v \cdot \frac{d}{dt}x(t)$
  - etc. . .

Mechanical systems

- Mechanical elements are described by linear differential equations, just like electrical ones
  - they are just idealizations, physical processes can be highly nonlinear (more complex)
  - but wait, so are electrical devices actually, and this hasn't stopped us. . .
- Example: oscillations after releasing of a loaded spring
  - (solve at blackboard)

Electrical - mechanical analogies

- Multiple ways to define analogies between electrical and mechanical characteristics
- Here is the one we will use:

Electr.	Mech. (linear)	Mech. (rotational)
Current [A]	Force [N]	Torque ("cuplu") [N.m]
Voltage [V]	Speed [m/s]	Angular speed [rad/s]

Simple model of a DC motor

- Motor: gateway between the two electrical and mechanical domains
  - converts electric energy to mechanical energy, and vice-versa
- (Simple) model of a DC motor:

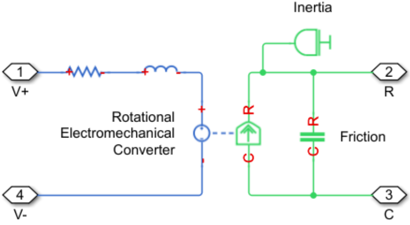

Figure 1: Simple model of a DC motor

Image from Mathworks Simulink (`ssc_dcmotor` example model)

DC motor model: electrical side

Electrical circuit of the DC motor model:

- Resistance: models the resistance of the windings
  - $u(t) = R \cdot i(t)$
- Inductance: models the inductive behavior of the windings
  - $u(t) = L \cdot \frac{d}{dt} i(t)$
- Controlled voltage source:
  - Voltage ("back electro-magnetic force voltage") is proportional to motor angular speed $S(t)$ on the mechanical side (think of a dynamo)
  - $u(t) = K_e \cdot S(t)$

DC motor model: mechanical side

Mechanical circuit of the DC motor model (no load):

- Controlled force/torque source
  - Generates force/torque proportional to the current $i(t)$ on the electrical side
  - $T = K_t \cdot i(t)$
- Inertia: models the inertial force of the moving part of the motor
  - $T_i = -m \cdot acceleration = -m \cdot \frac{d}{dt} S(t)$
- Friction: models the (viscous) friction force of the moving part of the motor, proportional to speed
  - $T_f = -C_v \cdot S(t)$
- Inertia and Friction forces (torques) oppose the force(torque) of the motor, hence they have minus sign.

Laplace transform

- Both electrical and mechanical sides are described by linear differential equations
- The Laplace transform is a useful tool (remember SCS)
  - derivation = multiplication by s
  - integration = multiplication by $1/s$
  - transform function H(s) = output(s)/input(s)
- Exercise: write the equations of all electrical and mechanical elements in Laplace transform

Full electrical model

- All the mechanical elements can be modeled in the electrical domain
  - since they are all just differential equations, basically
  - obtain a full model in the electrical domain only

The controlled voltage source

- How to model the controlled voltage source?
- Like this:
  - voltage is proportional to speed: $U(s) = K_e \cdot S(s)$
  - speed = integral of acceleration: $S(s) = S_0 + 1/s \cdot A$
  - acceleration is proportional to force (force(torque) / mass) = $C_{const} \cdot T(s)$
  - force/torque = proportional to current: $T(s) = K_t \cdot I(s)$
- Result: $U(s) = K_e \cdot (S_0 + 1/s \cdot C_{const} \cdot K_t I(s))$
- $U(s) = Constant_1 + Constant_2 \cdot 1/s \cdot I(s)$
  - voltage depends on integral of current (plus a constant)
  - what kind of electrical element acts like this?

The controlled voltage source

- The controlled voltage source can be modeled as a **capacity**
  - on the electrical side
- The constant term added in front = the initial voltage on the capacity
- The capacitance value depends on all the motor parameters

The inertial force

- Inertia = a force which opposes (i.e. reduces) the motor force, and is proportional to acceleration
- Use the analogy:
  - force = current
  - speed = voltage
  - acceleration = derivative of speed = derivative of voltage
- Inertia = a current which opposes (i.e. reduces) the motor current, and is proportional to derivative of voltage
  - current proportional to derivative voltage <=> a capacity
  - reduces motor current <=> in parallel with the controlled voltage source (steals some current)
- Inertia model = a **capacity in parallel** with the controlled voltage source
  - it steals current from the motor (i.e. reduces motor force)
  - amount of current stolen = proportional to derivative of voltage

The friction force

- (Viscous) friction = a force which opposes (i.e. reduces) the motor force, and is proportional to speed
- Use the analogy:
  - force = current
  - speed = voltage
- (Viscous) friction = a current which opposes (i.e. reduces) the motor current, and is proportional to voltage
- (Viscous) friction model = a **resistance in parallel** with the controlled voltage source
  - it steals current from the motor (i.e. reduces motor force)
  - amount of current stolen = proportional to voltage (speed)

The sliding friction force

- There can also exist a sliding friction force = friction force which does not depend on speed, but is a constant
  - that's the friction force you encountered in high-school physics, likely
- Exercise: how is this force modeled in electrical domain?

The full electrical model

- Draw picture at blackboard
- This is a second order model (1L, 1C)
  - the two capacities are in parallel, so they can be added into a single one
- Derive a transfer function (input = voltage on motor input, output = motor speed)
- Take home message: simple DC motor no-load model = a second order RLC model
  - this is a no-load model (motor doesn't move anything heavy)

Motor under load

- Exercise: suppose our motor drags/lifts a constant weight
  - i.e. like a crane lifting a big weight from the ground
- How to model the load?
  - think ...

Motor under load

- Exercise: suppose our motor drags/lifts a constant weight
  - i.e. like a crane lifting a big weight from the ground
- How to model the load?
  - like a constant force/torque opposing the motor force/torque

Simulink model

- Simulink has a DC motor model already integrated
- You will use it in the lab