

Exercises Week 4

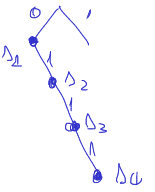
Source Coding - Basics

Message	Code A	Code B	Code C	Code D	Code E	Code F
s_1	00	0	0	0	0	0
s_2	01	10	01	100	110	10
s_3	10	110	011	<u>11</u>	10	<u>11</u>
s_4	11	1110	0111	<u>110</u>	111	<u>110</u>

Code C: Kraft: just like B

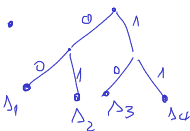
Inst: No
U.D.: Yes

$|0|011|01|01|0111$
 $\Delta_1 \Delta_3 \Delta_2 \Delta_2 \Delta_4$



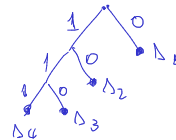
Code A: Kraft: $2^{-2} + 2^{-2} + 2^{-2} + 2^{-2} = 1 \checkmark$

Inst? Yes \Rightarrow U.D.



Code B: Kraft: $2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} < 1 \checkmark$
 $\frac{1}{2} \frac{1}{4} \frac{1}{8} \frac{1}{16}$

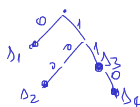
Inst? Yes \Rightarrow U.D.



Code D: Kraft: $2^{-1} + 2^{-3} + 2^{-2} + 2^{-3} = 1 \checkmark$
 $\frac{1}{2} \frac{1}{8} \frac{1}{4} \frac{1}{8}$

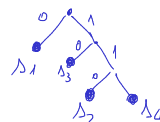
Inst: Not: (11, 110)

U.D: 110 Not!



Code E: Kraft: \checkmark

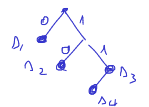
Inst? Yes \Rightarrow U.D. Yes



Code F: Kraft:

$2^{-1} + 2^{-2} + 2^{-2} + 2^{-3} > 1$
 $\frac{1}{2} \frac{1}{4} \frac{1}{4} \frac{1}{8}$

Inst? No
U.D? No.



2). $S: \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

Message	Code A	Code B
s_1	00	0
s_2	01	10
s_3	10	110
s_4	11	111

$$a). \bar{l}_A = \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 2 + \frac{1}{8} \cdot 2 = 2 \text{ bits}$$

$$\bar{l}_B = \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 3 = 1.75 \text{ bits}$$

$\frac{1}{2} \quad \frac{1}{2} \quad \frac{6}{8} = 0.75$

$$b). \eta = \frac{H(S)}{\bar{l} \cdot \log_2 D} =$$

$$\log_2 D = \log_2 2 = 1$$

$$\rho = 1 - \eta$$

$$H(S) = -\frac{1}{2} \cdot \log_2 \frac{1}{2} - \frac{1}{4} \cdot \log_2 \frac{1}{4} - 2 \cdot \frac{1}{8} \cdot \log_2 \frac{1}{8} = 1.75 \text{ bits}$$

$-1 \quad -2 \quad -3$

For code A: $\eta = \frac{1.75}{2} = 87.5 \%$

$$\rho = 1 - \eta = 12.5 \%$$

For code B: $\eta = \frac{1.75}{1.75} = 1 = 100 \%$

$$\rho = 0$$

c). $\Delta_2 \Delta_4 \Delta_3 \Delta_3 \Delta_1$

Code A: 0111101000

Code B: 101111101100

Message	Code A	Code B
s_1	00	0
s_2	01	10
s_3	10	110
s_4	11	111

d). 01101010101011110000101010...
 $\Delta_2 \Delta_3 \Delta_3 \dots$

Code A:

01101010101011110000101010
Code B : $\Delta_2 \Delta_3 \Delta_2 \Delta_2 \Delta_2 \Delta_2 \Delta_3 \Delta_2 \Delta_1 \Delta_1 \Delta_1 \Delta_2 \Delta_2 \Delta_2$

3).

Message	Codeword
s_1	??
s_2	1??
s_3	11?
s_4	0?
s_5	??

$\underline{1} \underline{0}$
 $\underline{1} \underline{1} \underline{0}$
 $\underline{1} \underline{1} \underline{1}$
 $\underline{0} \underline{0}$
 $\underline{0} \underline{1}$