

Exercises Week 13

Information Theory

1. Consider a communication process defined by the following **joint probability matrix**:

$$P(x_i \cap y_j) = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

- a. compute the marginal probabilities and the marginal entropies $H(X)$ and $H(Y)$;
 - b. Find the channel matrix $P(Y|X)$ and draw the graph of the channel;
 - c. compute the mutual information $I(X, Y)$, and draw the geometrical representation.
2. At the input of a binary symmetric channel with the following channel matrix

$$P(y_j|x_i) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

we apply two inputs x_1 and x_2 with probabilities $p(x_1) = \frac{3}{4}$ and $p(x_2) = \frac{1}{4}$.

- a. Draw the graph of the channel
 - b. Find $H(X)$, $H(Y)$ and $I(X, Y)$
 - c. Compute the uncertainty remaining over the input X when output symbol y_2 is received
 - d. Compute the channel capacity, the redundancy and the efficiency of the channel.
3. Consider a communication process with 2 inputs and 3 outputs. The inputs and output events have equal probabilities, and are independent.
- a. Write the joint probability matrix
 - b. draw the graph of the channel (together with the probabilities)
 - c. Compute the marginal entropies and the joint entropy, and verify that

$$H(X, Y) = H(X) + H(Y)$$

and that

$$I(X, Y) = 0$$

4. Give an example of a channel having 3 inputs and 3 outputs, with $H(Y|X) = 0$ (write the channel matrix).
5. Give an example of a channel with two inputs, such that $H(Y|x_1) \neq 0$ and $H(Y|x_2) = 0$ (write the channel matrix).