COL 864 - AI for Cognitive Robot Intelligence
Homework 1

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Question 1

State Space:

$$\mathbf{X} = \{X_1....X_{25}\}$$

Where X_i is defined as the $i^{th} grid$ starting from $(0,0)$ as shown in the Figure.

Observation Space:

$$\mathbf{E} = \{bump, rotor\}$$

Where $bump = 1$ implies the robot hearing a bump sound.

Transition Model:

$$\mathbf{P}(X_t|X_{0:t-1}) = \mathbf{P}(X_t|X_{t-1}) = \begin{cases} 0.25 & \text{if } X_t, X_{t-1} neighbours \\ 0.25 & \text{if } X_t = X_{t-1}, X_t edge \\ 0.5 & \text{if } X_t = X_{t-1}, X_t corner \\ 0 & \text{otherwise} \end{cases}$$

Observation Model:

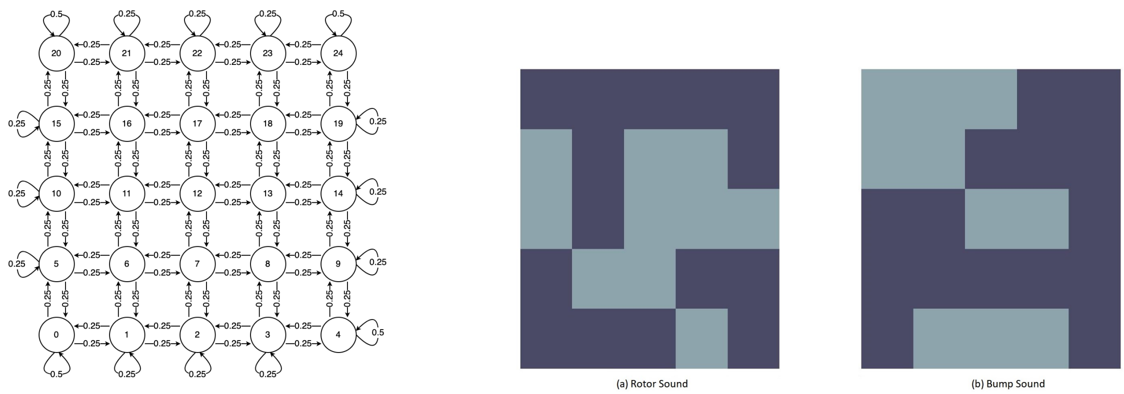
$$\mathbf{P}(E_t|X_{0:t}E_{1:t-1}) = \mathbf{P}(E_t|X_t)$$

Joint Distribution:

$$P(X_{0:t}E_{1:t}) = P(X_0)\prod_{i=1}^{t} P(X_i|X_{i-1})P(E_i|X_i)$$

Assumptions:

1. The agent has a uniformly random probability of moving in any of the four directions (up, down, left, right). If it is at the end of the grid, further movement in that direction makes the agent stay at the same position.
2. The probability of observing a bump or a rotor sound is independent of each other, as well as dependant only on the current position of the agent.
3. We assume the likelihood of the agents initial position X_0 is uniform over the initial starting states



(a) Transition Model (b) Observation Model

Figure 1: Graphical Model

In Figure 1(a) the arrows give the transition probability between states. We assume the agent moves at every time step. In (b) each square indicates probability of hearing bump/rotor when the agent is at position. The light squares represent 0:9 and dark square 0:1.

Simulation:

Taking seed value as 5. We get the following simulated sequences:

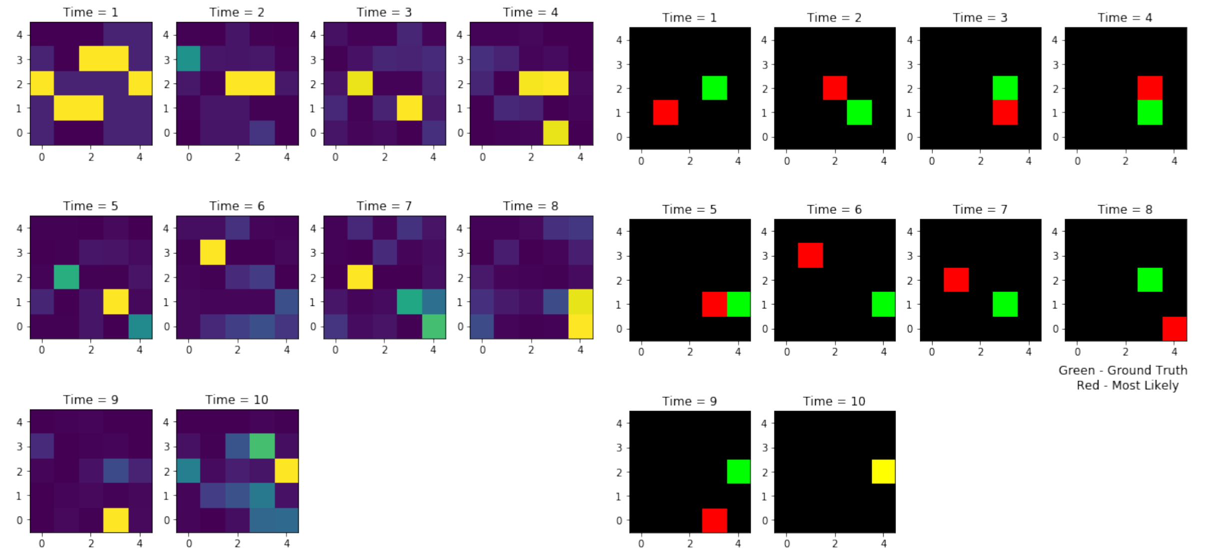
1. Ground truth positions $(X_0...X_t)$ = [19, 19, 14, 9, 9, 14, 13, 18, 23, 22, 23]
2. Bump Observations ($E_1....E_t$ = [0, 0, 0, 0, 0, 1, 1, 0, 0, 1]
3. Rotor Observations ($E_1....E_t$ = [0, 0, 1, 0, 0, 1, 1, 1, 0, 0]

Question 2

Filtering Task:

$$P(X_{t+1}|e_{1:t+1}) = P(e_{t+1}|X_{t+1}) \sum_{x_t} P(X_{t+1}|x_t)(x_t|e_{1:t})$$

Here, the first term comes from the observation model, the first term within the summation comes from the transition model. The last term is a recursive call to the filtering task for the previous time step.



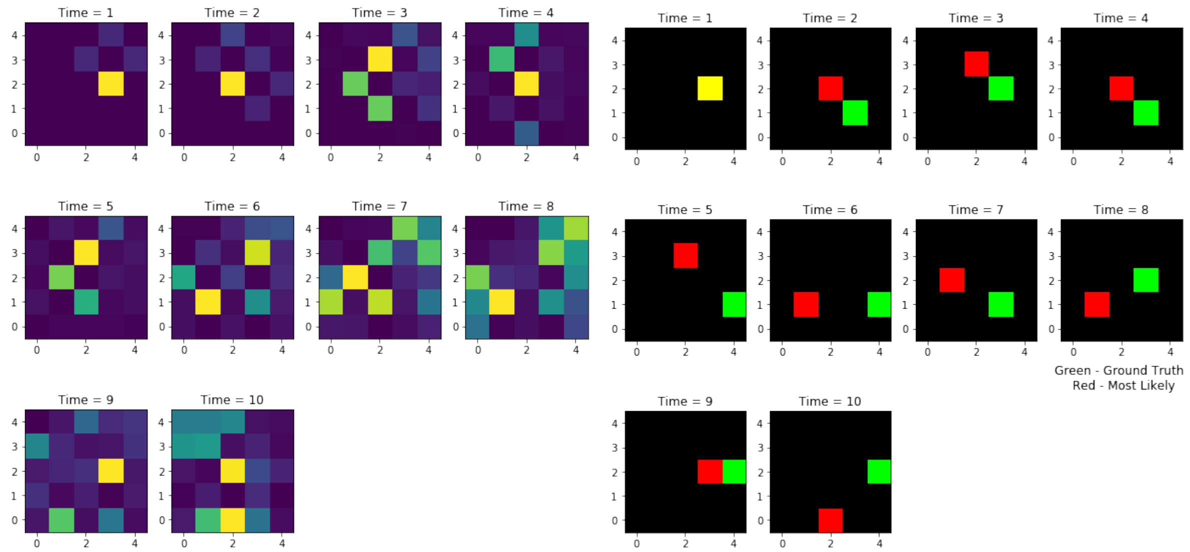
(a) Probability Likelihood (b) Most Likely vs Ground Truth

Figure 2: Filtering Task

In the left figure, the yellow signifies regions of high probability density and blue signifies low regions. Similarly, in the right figure, green area signify ground truth position, red signifies most likely position as per filtering task.

Question 3

Filtering Task (Single Modality):



(a) Probability Likelihood (b) Most Likely vs Ground Truth

Figure 3: Filtering Task (Rotor only)

As we see, the likelihood of possible states is more spread out, with multiple peaks in the state space. Furthermore, the most likely location is further apart from the ground truth. In conclusion, only one observation (rotor) is unable to perform as well.

Future Locations:

$$P(X_{t+1}|e_{1:t}) = \sum_{x_t} P(X_{t+1}|x_t)(x_t|e_{1:t})$$

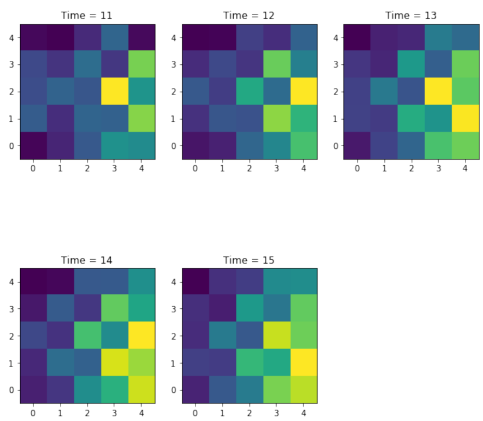


Figure 4: Future Likelihoods (5 timesteps)

Question 4

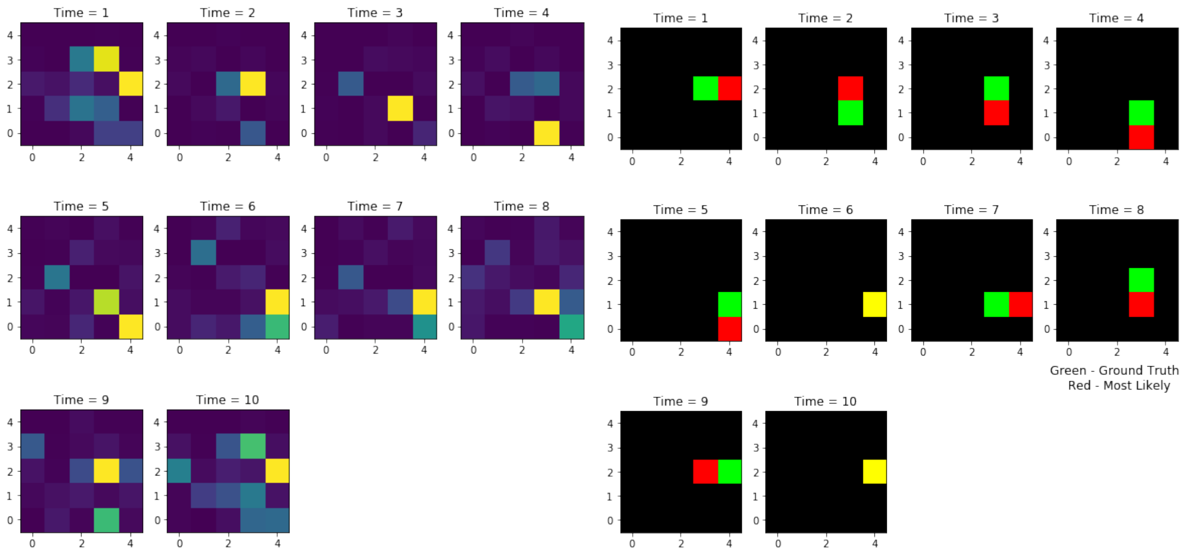
Smoothing Task:

$$\mathbf{P}(X_k|e_{1:t}) = \mathbf{P}(X_k|e_{1:k})\mathbf{P}(e_{k+1:t}|X_k)$$

Here the first term is just a filtering task until k. The second term is defined as follows:

$$\mathbf{P}(e_{k+1:t}|X_k) = \sum_{x_{k+1}} (e_{k+1}|x_{k+1})(e_{k+2:t}|x_{k+1})\mathbf{P}(x_{k+1}|X_k)$$

Here the first time is from the observation model and the third term is from the transition model. The second term can be recursively calculated by the same formula.



(a) Probability Likelihood

(b) Most Likely vs Ground Truth

Figure 5: Smoothing Task

Most Likely Path:

$$\max_{x_1,\ldots x_t} P(x_1 \ldots xtX_{t+1}|e_{1:t+1}) = P(e_{t+1}|X_{t+1}) \max_{x_t} P(X_{t+1}|x_t)(\max_{x_1,\ldots x_{t-1}} (x_1 \ldots xt-1x_t|e_{1:t}))$$

Here the first term is from the observation model. The second term can be maximised over the transition model. And the third term is recursively defined by this equation.

Ground Truth $(X_0 : X_t) = [19 \rightarrow 19 \rightarrow 19 \rightarrow 19 \rightarrow 24 \rightarrow 24 \rightarrow 24 \rightarrow 24 \rightarrow 19 \rightarrow 18 \rightarrow 13]$

Most Likely $(X_0 : X_t) = [19 \rightarrow 24 \rightarrow 24 \rightarrow 23 \rightarrow 22 \rightarrow 22 \rightarrow 23 \rightarrow 18 \rightarrow 19 \rightarrow 14 \rightarrow 9]$

Manhattan Distance between paths = 15

Question 5

Lake Size 25x25:

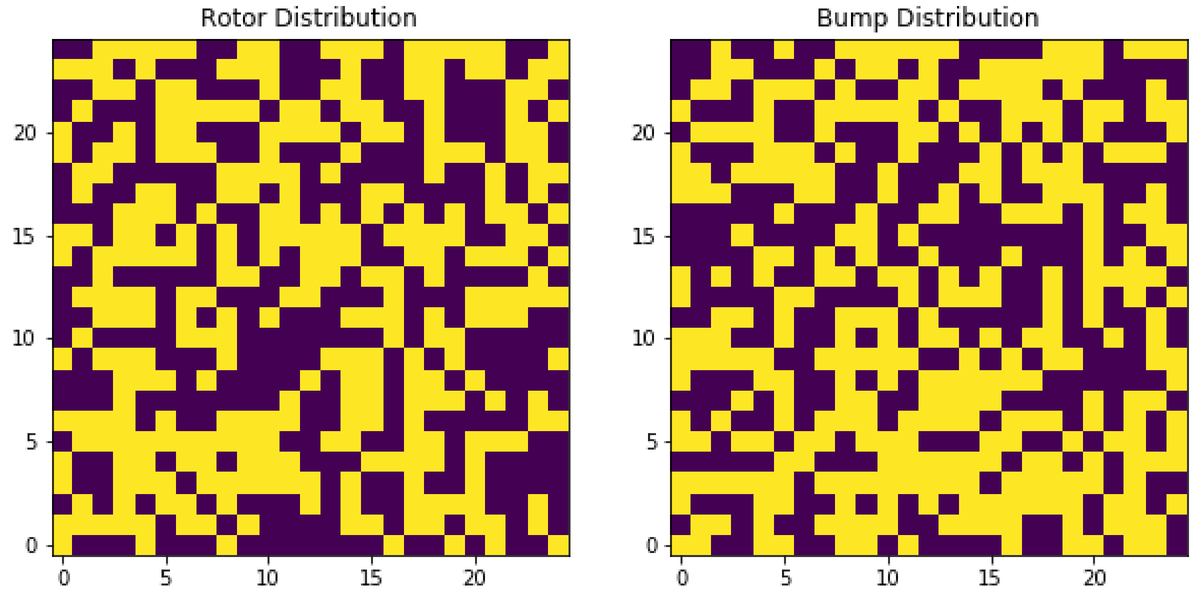
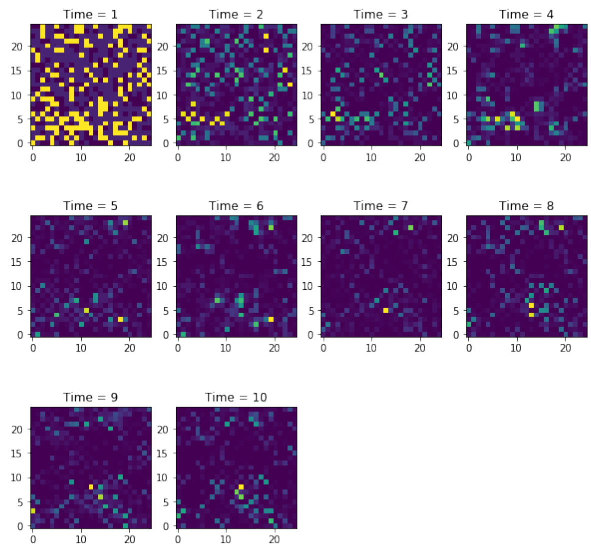
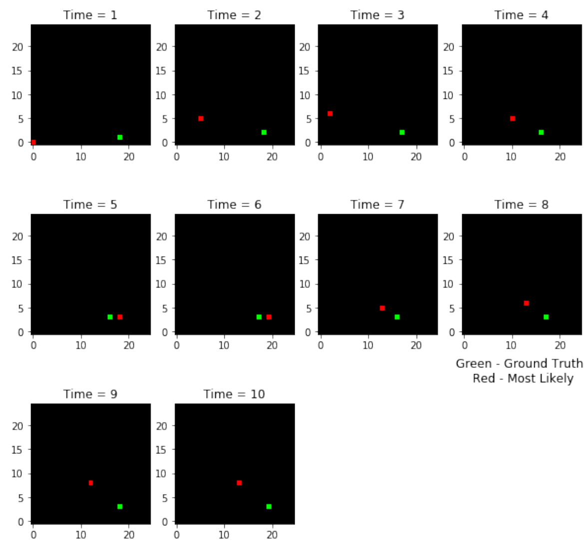


Figure 6: Rotor and Bump Distribution for 25x25 lake

Filtering Task:



(a) Probability Likelihood



(b) Most Likely vs Ground Truth

Figure 7: Filtering Task on 25x25 lake

As we can see, the distance (Manhattan) between the most likely location vs ground truth is much larger as compared to the 5x5 lake.

Most Likely Path:

Ground Truth = $[44 \rightarrow 43 \rightarrow 68 \rightarrow 67 \rightarrow 66 \rightarrow \mathbf{91} \rightarrow \mathbf{92} \rightarrow \mathbf{91} \rightarrow \mathbf{92} \rightarrow \mathbf{93} \rightarrow \mathbf{94}]$
Most Likely = $[44 \rightarrow 43 \rightarrow 68 \rightarrow 67 \rightarrow 66 \rightarrow \mathbf{41} \rightarrow \mathbf{42} \rightarrow \mathbf{41} \rightarrow \mathbf{16} \rightarrow \mathbf{41} \rightarrow \mathbf{16}]$

Manhattan Distance between paths = 20

The computations increases polynomially with increase in state space. $O(n^2)$