Working Title

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Animals can identify and discriminate a myriad of odors using a surprisingly limited class of distinct olfactory receptor genes. It was recently noted that a critical feature of our odor environment may resolve this inconsistency: most naturally-occurring odors are composed of only a tiny subset of these many volatile molecular constituents [1]. In principle, the high-dimensional odor signal could be accurately decompressed from a limited number of receptor measurements, provided (i) the odor signal is sufficiently sparse in odorant space and (ii) the responses are sufficiently dispersed. The latter requires that odor receptors are not odorant-specific but bind many odorants with distinct affinities, and that a given odorant elicits responses in several such receptors. Odor identity is therefore coded in the unique combination of responses the odor elicits in the sensing repertoire – a combinatorial coding strategy.

Transformations of ORN response in the subsequent neural circuitry lends further credence to a combinatorial coding strategy. In *Drosophila*, ORNs in the periphery synapse to neuropil compartments in the antennal lobe called glomeruli (ORNs expressing a given OR synapse to the same glomerulus), where they then divergently and randomly synapse to Kenyon cells in the mushroom body (MB) [2, 3]. In the MB, the responses are decoded and sent to higher order brain centers where they are translated to behavioral response [4, 5]. It has been suggested that in a combinatorial coding strategy, this dispersion of ORN response along the olfactory pathway from AL to MB may further facilitate flexible learning and robust odor discrimination [1]. Indeed, this response diffusivity has been observed in olfactory receptor neurons (ORNs) in the *Drosophila melanogaster* sensing periphery [6].

Combinatorial coding relies fundamentally on randomness in the matrix of response, and one potential complication in this coding paradigm is response saturation. Maximum ORN firing rates increase with single odorant concentration rather inhomogenously, potentially jeopardizing response breadth as odor concentrations change. ORNs expressing the receptor Or35a, for example, fire at varying rates spanning 30 to 300 Hz in response to one of eleven distinct odorants at dilutions of 10^{-4} [6]. But at dilutions of 10^{-2}, these maximum rates have all elevated to a narrow range between 200 and 250 Hz . Similar apparent saturation occurs for ORNs expressing

Or85b, Or22a, and OR67a in response to the same panel of odorants. Discuss natural statistics odors fluctuating. Discuss the statistics changings

On the other hand, it has been shown that ORN local field potentials and firing rates adapt to odor stimuli in time [7–9]. Specifically, it has been shown that in response to single odorant pulses, *Drosophila* ORN firing rate gain scales inversely with mean odor concentration, according to the Weber-Fechner law observed in vision [8–10]. This observed scaling law was obeyed by all the odorant-ORN pairs tested, pointing to a mechanistic origin involving the universal co-receptor Orco. Further implicating Orco in gain modulation is the observation that Orco dephosphorylizes following prolonged exposure to odors, leading to receptor desensitization [11, 12].

Here, we incorporate these findings into a biophysical model that combines odor transduction and receptor activity in the *Drosophila* olfactory periphery with a neural framework for decoding distributed ORN responses. We find that front-end adaptation enacted through the Weber-Fechner Law preserves odor identification across wide variations in odor intensity, and maintains odor discrimination accuracy in conflicting environments. Further, we find that this adaptive mechanism promotes active odor perception in temporally dynamic odor landscapes. Together, our findings suggest a fundamental role for input gain control in maintaining olfactory sensitivity within naturalistic odor environments.

I.   RESULTS

A.   A model of odor encoding that preserves the diversity of ORN response and adaptive scaling

The theoretical framework of odor discrimination consists of two stages: a biophysical model of odor transduction and subsequent ORN activity (encoding), and a computational paradigm for then inferring odor identity and intensity from this repertoire of ORN response (decoding). The encoding framework is a generalization of one recently used to model observed gain control in individual *Drosophila* ORNs [8]. The apparent generality of Weber's Law (it was observed robustly in xx odorant/receptor pairs) implicates structural modi-

fication of the universal co-receptor Orco, rather than odorant-specific Ors alone. Thus, Or/Orco complexes were modeled as a single functional unit at which the gain modulation occurs. These complexes reside in an active (open ion channels) or inactive conformation, the relative likelihood of which is assumed to be a Boltzmann distribution with Or-specific free energies ϵ_a. Importantly, Weber's Law can be satisfied by feeding back the Or/Orco activity onto ϵ_a in a prescribed way.

Here, we generalize this model to a repertoire of a distinct ORNs, each housing a collection of Or/Orco complexes. In the presence of an odor consisting of a few molecular constituents, ORN spiking activity is dictated by the combined effect of Or/Orco activation energies ϵ_a and odor identity. For simplicity, odorant binding and unbinding is assumed to be in equilibrium, whereby a given odorant-receptor pair (i-a) depends on two disassociation constants, K_{ia}^* and K_{ia}, for odorant binding in the active and inactive conformations, respectively. Together, these comprise a coupled stochastic system that translates the binding of odors of varying identities and concentrations into a repertoire of ORN response (Fig. 1a). In steady state, the active fraction $\bar{A}_a$ of Or/Orco complexes in ORN a is:

$$\bar{A}_a = \left(1 + e^{\epsilon_a}\frac{1 + \sum_i^N s_i/K_{ia}}{1 + \sum_i^N s_i/K_{ia}^*}\right)^{-1}, \qquad (1)$$

where s_i is the concentration of the odorant i, $i = 1, ..., N$. Odors bind to active Or/Orco complexes much faster than to inactive complexes, whereby $K_{ia}^* \ll K_{ia}$. In the case of a mono-molecular odor ($N = 1$), Weber's Law can be satisfied exactly if the complex activation energy scales logarithmically with the odor concentration, $\epsilon_a \sim \ln s_i$. For an odor with several molecular constituents at distinct concentrations, we instead scale the energy with the mean odorant concentration, $\epsilon_a \sim \ln\langle s_i\rangle$, the average being taken over all non-zero concentrations s_i . Finally, we assume that the free energies modulate only within a finite range, $\epsilon_\mathrm{L} < \epsilon_a < \epsilon_\mathrm{H}$.

Ignoring for the moment the slower effects of adaptation (i.e., assuming ϵ_a is fixed), this model represents the instantaneous change in the activity as a function of ligand concentration, i.e. similar to the maximum firing rate measured in [6]. We first show that the steady state response reproduce the diversity of observed ORN tuning curves. Olfactory receptors in *Drosophila* can range from narrowly tuned, responding to a single odorant, to quite broad, responding to various distinct odorants spanning multiple functional groups. We incorporate this diversity of response into our framework by treating K_{ia}^* and K_{ia} as random variables with pre-defined statistics. Since $K_{ia}^* \ll K_{ia}$, the model dynamics are well dictated only by $K_{i,a}^*$ alone, so we only consider variations among these. Figures 1b-1d shows how a simple choice of statistics on $K_{i,a}^*$ can naturally produce a diverse repertoire of response closely mimicking observed *Drosophila* ORN tun-

ing curves. The tuning curves, of which some are narrowly peaked, some are broad, and some are weakly responding, are produced by sampling at two stages. For a given receptor a, $K_{i,a}^*$ are chosen uniformly in some range (this dictates how receptor a responds to distinct odorants), while diversity among receptors is incorporated by sampling the bounds of each range from a hyperdistribution, also chosen uniform. Receptors with narrow ranges produce peaked tuning curves (the orange ORN), while and those with broader ranges produce more disperse tuning curves (blue ORN).

The enforcement of Weber's Law can maintain this distributed response across concentration changes (Fig. 1e). Fig. 1f shows the response of three ORNs to distinct complex but sparse odors at varying mean odor concentrations. Without adaptive feedback, the breadth of the firing rate distributions narrow, homogenizing the responses across odor identity. If the ORN repertoire produces the same pattern of activity in response to distinct odors, odor identity information is lost. Conversely, by scaling $\epsilon_a \sim \ln\langle s_i\rangle$, distributed responses are maintained through a large range of concentrations. As we will see, the maintenance of this disperse response is central to reliable odor decoding in fluctuating environments.

B. A combinatorial decoding process for nonlinear neural response

In *Drosophila*, odorant identity is inferred from spatiotemporal patterns of neural activity in Kenyon cells housed in the mushroom body. As these activity patterns result from a combination of ORN response and downstream neural processing, front-end gain control can play a crucial role in helping to preserve neural representations of odor identity. The complicating factor in this decoding process is the disparity between measurement dimension and stimulus dimension: while *Drosophila* only express 60 olfactory receptor genes, the space of aromatic odorants is 10^3 or more, appearing to suggest that odor decoding is a fundamentally under-determined problem. However, naturally-occurring odors are comprised of only a small subset of this large space of volatile compounds. As previously noted [1], this is quite suggestive, as rigorous mathematical results show that sparse signals passed through linear sensors of sufficiently random response can be reliably reconstructed, despite measurement paucity [13–15]. This reconstruction technique, known in computer vision and elsewhere as compressed sensing, is naturally suited to the observed features of olfactory circuitry.

In compressed sensing, successful decoding relies on a sufficiently dispersed response. But large fluctuations in intensity characteristic of naturalistic environments could markedly affect response combinatorics or quench activity dispersedness (as in Fig. 1e-1f), confounding decoding fidelity. Conversely, we expect that since imposing the Weber-Fechner scaling relation maintains the recep-
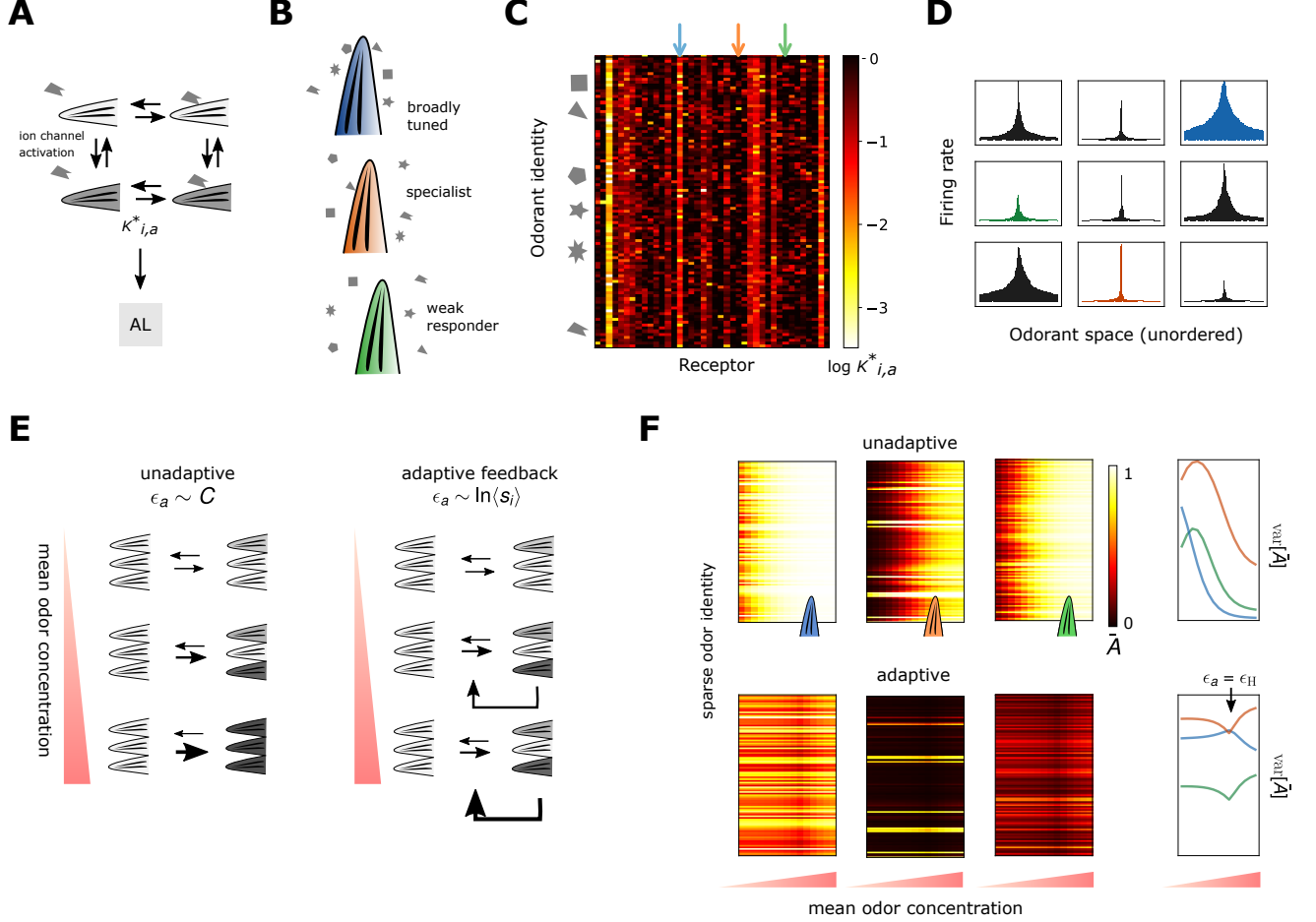
FIG. 1: **A** Odor binding model. Odorants bind to Or/Orco complexes, which may be quiescent (light gray) or active (dark gray). The likelihood that a complex is in an active state depends on its binding affinity to constituent odor molecules (through $K_{i,a}^*$) and the energetic cost of receptor activation. **B** Or classes exhibit distinct binding distributions. **C** Example instantiation of the active binding disassociation matrix $K_{i,a}^*$ (see Methods). **D** Sample Or/Orco activity of 9 of the 40 ORNs represented by the matrix in (C). The tuning curves, individually ordered by odorant for visualization, exhibit broad, specialized, and weak responses. **E** Schematic of Weber Law feedback at signal transduction. When Weber Law is enforced, activity distributions of active complexes (shaded darkly) remain invariant with increasing signal concentrations. **F** Steady state Or/Orco activity for the three sample ORNs in (B), (C), and (D), in an unadaptive system (top row) and adaptive system (bottom row) in response to 100 different sparse odors; each matrix row corresponds to a distinct odor identity. The variance of Or/Orco activity across odor identity can rapidly diminish in the absence of front-end adaptation (final column).

tor activity distribution over the dynamic range of ϵ_a, odor representations can be preserved naturally through a range of concentrations.

To incorporate the linear framework of compressed sensing into our nonlinear receptor and activation model, we treat the odor encoding process exactly, while approximating the decoding process to first order. The latter assumption allows the compressed sensing reconstruction – a constrained optimization problem – to remain convex, whereby the global minimum is unique (see Methods). Specifically, we represent the nonzero components s_k of the sparse odor signal $\mathbf{s}$ as $s_k = s_0 + \Delta s_k$, where s_0 is the center of the linearization. The target of the decoding process are the identities and intensities of the 'excess' signals Δs_i. The 'excess' Or/Orco activity is then defined as:

$$\Delta \bar{A}_a \equiv \bar{A}_a(\mathbf{s}) - \bar{A}_a(\mathbf{s}_0), \qquad (2)$$

where we have assumed that the neural system has access to a baseline odor signal $\mathbf{s}_0$, but must infer exact odor concentrations s_i. The decoding process minimizes the the L_1 norm of Δs_i, equivalent to enforcing signal sparsity, while enforcing the linear constraints arising from the excess activities, i.e. the ORN responses. To assess the decoding performance, we denote an odor signal as accurately decoded if the sparse odorant components are all estimated to within 25% of their correct value and the components absent in the original signal, $s_j = 0$, are all estimated as less than 10% of the mean excess concentration, $\hat{s}_j \leq \langle \Delta s_i \rangle$. The former constraint is a measure of accurately inferring signal *intensity*; the latter of signal *identity*.
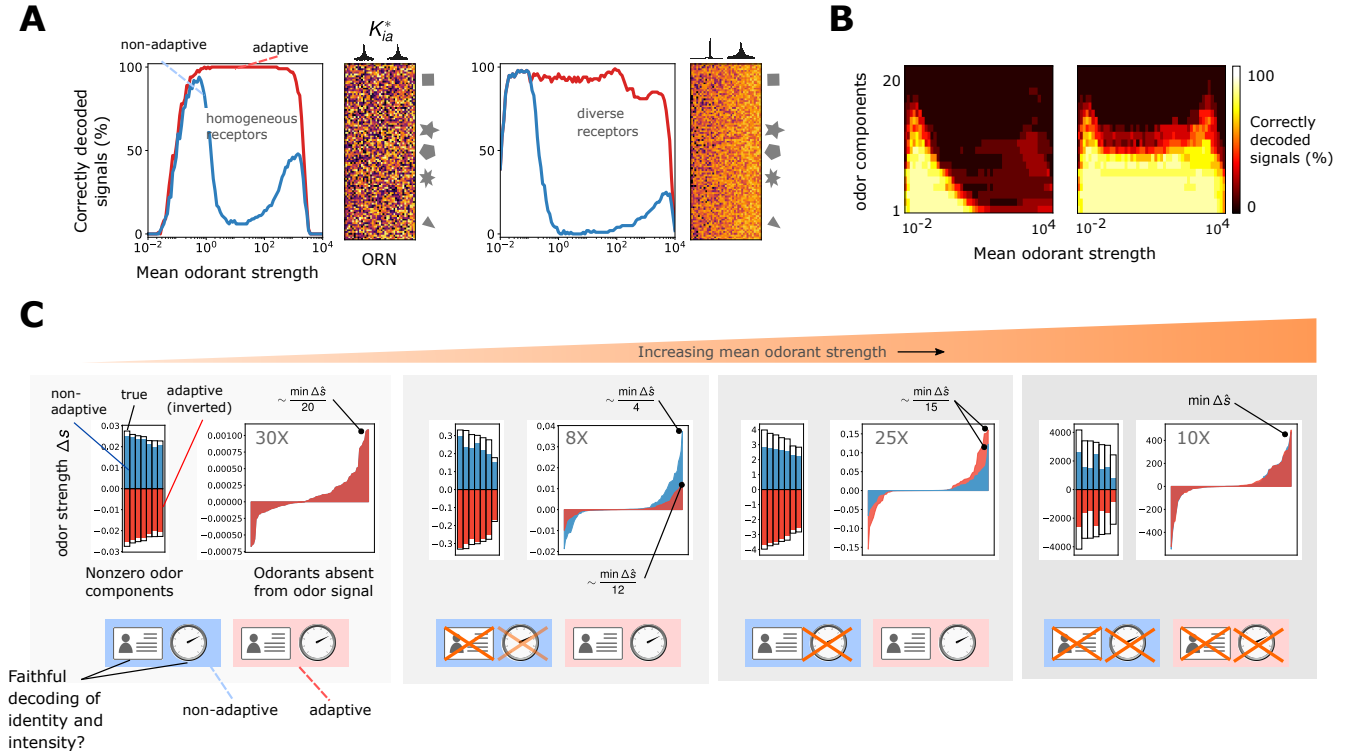
FIG. 2: **A** Percent correctly decoded odors, with (red) and without (blue) Weber Law adaptation. For both a homogeneous and diverse receptor repertoire, odor decoding is preserved over background changes in the adaptive system. Heatmaps of K_{ia}^* matrices illustrate receptor homogeneity for the two systems. **B** Dependence of decoding accuracy on odor sparsity. At low concentrations, both systems can decode relatively complex signals; as concentration increases, complex signals are mis-identified in the absence of adaptive feedback. **C** Separation of intensity and identity decoding. At low concentrations, both signal identity and intensity are correctly inferred in a non-adaptive system. Increases in signal concentrations can lead to errors in odor intensity, identity, or both. When Weber Law is enforced, the representation of odor intensity and its identity are maintained through a wide regime of odor concentrations.

C. Identity and intensity preservation in sparse decoding with adaptive feedback

We apply this scheme to receptor systems consisting of 50 Or/Orco complexes interacting with a 100-dimensional odorant space. Without adaptive feedback, nearly all 100 random sparse odor signals (each odor has 7 nonzero odorants) are still correctly inferred in a particular regime of mean odor concentration (Fig. 2a; blue curves). We find that this holds true for two distinct neural systems, one of which contains homogeneously but all broadly responding receptors (for each a, K_{ia}^* are sampled from the same distribution), the second of which is more indicative of *Drosophila* physiology and exhibits a diverse repertoire ranging from broad to highly specialized (as in Fig. 1). In both cases, however, decoding fidelity is not concentration invariant, dropping sharply outside this regime of faithful signal decoding.

Conversely, we hypothesize that by stabilizing the excess activity levels through Weber-Fechner adaptive feedback, such sensitivity can be mitigated. Enforcing this scaling law above a mean odor concentration of $\langle s_i \rangle = 10^{-1}$, and matching ϵ_a to the unadaptive system

otherwise, we find that coding fidelity is now maintained over a five-fold change in odor concentration (Fig. 2a; red curves). We further illustrate this behavior for systems with odorant binding distributions that are chosen exponentially and normally (Supplementary; to be added). This invariance holds across differing levels of signal sparsity (Fig. 2b). In the adaptive system, signals with complexity as high as 10 odorants are robustly decoded over several orders of odor concentration. In the non-adaptive system even mono-molecular signals are mis-identified at increased concentrations, a consequence of response homogenization across the ORN repertoire.

A critical feature of olfactory systems is the ability to simultaneously decode odor intensity and identity, aspects which can in principle overlap [16, 17]. Compressed decoding conflates these two aspects into a single computation by inferring not only the exact component magnitudes of an odor signal (intensity), but also which molecular components constitute the high-dimensional signal in the first place (identity). Despite the conflation of these in practice, it is possible that in this framework, one aspect may be preserved while the other is violated.

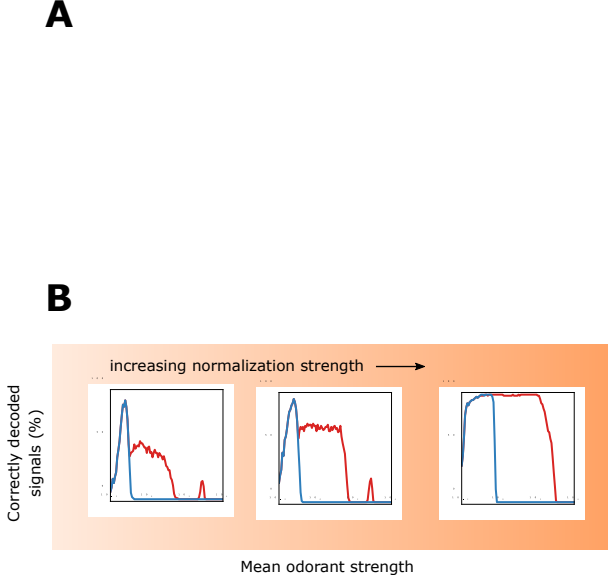Separating errors in odor intensity from those in error

A

B

FIG. 3: **A** Schematic of divisive normalization / lateral inhibition **B** Increasing divisive normalization w and w/o Weber Law

identity, we find that for an un-adaptive system, either or both may contribute depending on odor concentration. For moderate concentrations, the inferred zero components reach a substantial fraction of the mean odor concentration, while the estimates of the sparse components are largely even with their true values – the identity of the odor is compromised (Fig. 2c). As concentrations increase further, identity is now preserved (zero components are estimated well below the mean), but errors in odor intensity have magnified. This illustrates that in the absence of front-end gain control, errors both in identity and intensity can confound odor representations, while Weber Law feedback can mitigate these conflicts.

D. Inhibitory normalization

forthcoming

E. Weber Law scaling permits odor discrimination amid confounding backgrounds

Olfactory sensing in naturalistic settings relies on the ability to discriminate multiple odors, which may differ or overlap in chemical makeup and intensity. Though Weber Law adaptation can preserve decoding accuracy over changes in odor intensity, a system which adapts to mean concentrations alone may well fail in the presence of distinct odors of largely differing concentrations. To test this, we consider two sparse odors. The first we call

the "foreground" odor, and hold its component concentrations fixed at s_i. The second, which can span intensities a few orders below or above Δs_0, we refer to as the "background" odor, investigating how its intensity and identity can affect the discrimination of the foreground odorants s_i.

When the foreground odor is molecularly less complex than the background odor, the regime over which it is accurately decoded is largely identical in the adaptive and non-adaptive cases (Fig. 4). We attribute this to the complexity of the background odor, which heavily determines the strength of adaptive feedback, irrespective of the foreground. On the other hand, the identity of the background is only correctly inferred in a particular intensity regime, which is markedly narrower in the non-adaptive case.

When the odor complexities are matched, the regime in which the foreground is correctly decoded has reduced substantially. Further, in the non-adaptive system, in only in a small range of background concentrations can both odors be simultaneously decoded accurately. This behavior magnifies as the foreground becomes more complex (Fig. 4). We conclude that adaptive feedback, when incorporated in a combinatorial coding scheme, aids the robust odor discrimination despite disparities in molecular complexity and intensity.

F. Dynamic adaptation preserves active odor perception in fluctuating odor environments

We have seen that in a combinatorial coding framework, front-end adaptive gain can maintain odor decoding accuracy over large concentration ranges, and can preserve discrimination capabilities amid odor backgrounds. So far, we have assumed that odor signals are static in time, and that adaptation from the neural circuitry feeds back onto the receptor sensitivity instantly and perfectly. But realistic odor environments are highly intermittent and widely fluctuating, with encounter intensities that can span several orders [18]. Further, energetic constraints can the rate of adaptation [19].

To account for temporal aspects in both the odor environment and sensing periphery, we first relax the assumption that adaptation is instantaneous, instead letting the activity of each Or/Orco complex decay linearly to baseline levels $\bar{A}_{a,0}$ through accompanying modulation of the complex free energies. Specifically, in response to a dynamic odor signal $s(t)$, the activity of complex a is still given by Eq. 1, but now with time-dependent free energies, $\epsilon_a \to \epsilon_a(t)$ obeying the dynamics

$$\frac{d\epsilon_a(t)}{dt} = \frac{1}{\tau_a} \left[\bar{A}_a - \bar{A}_{a,0} \right], \qquad (3)$$

where $\bar{A}_a = \bar{A}(s(t), \epsilon_a(t))$ is the time-dependent activity level and τ_m sets the timescale of adaptation. We compare the results for several timescales between 10 ms and several seconds.
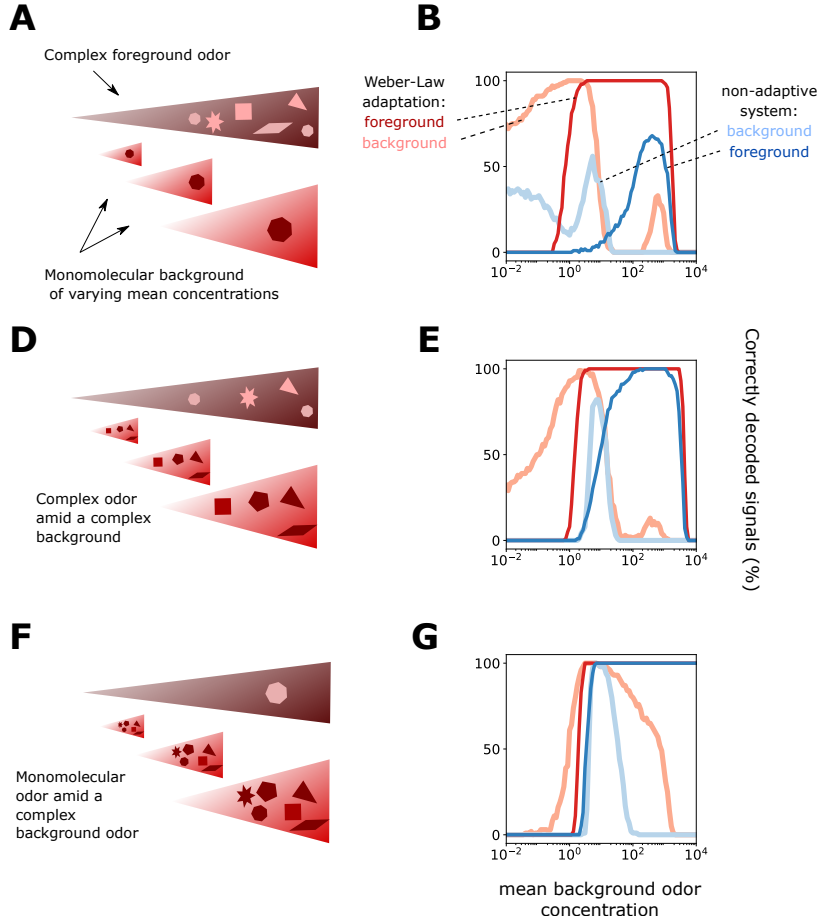
FIG. 4: Signal discrimination figure; additions forthcoming

Next, to mimic a naturalistic odor signal, we used previously-recorded voltage traces from a photo-ionization detector whose statistics were verified to match those of natural odor plumes. This data was scaled to concentrations applicable to our model framework (which is in arbitrary units). To mimic a complex odor, this time trace was set as the background odor level for the nonzero components of an odor signal, s_0, while individual odorant concentrations were chosen as this value plus some deviation Δs_k.

Considering first a set of 100 randomly chosen sparse odors of unique identities, we plot decoding errors arising from either mis-identification of odor identity, or that of odor intensity, in two windows containing odor whiffs. Specifically, we plot, over time, the percentage of correctly estimated nonzero component intensities (Fig. 5), as well as the the percentage of correctly identified zero components, for several distinct adaptation timescales. For slow adaptation, with timescales greater than a few hundred ms (lighter curves in Fig. 5), we see definite trends in both intensity and identity coding. Errors in odor intensity are quite sensitive to the faster fluctuations

in the odor signal, even if the signal remains appreciable throughout the whiff; for example the whiff demarcated by the purple and green markers in Fig.. Conversely, the majority of the odor identity is perceived as soon as the signal appears above a given whiff threshold, remaining at that level of accuracy throughout the whiff. As adaptation speed increases, the coding of odor intensity is nearly perfected within the first 100 ms of the whiff onset, remaining insensitive to further fluctuations throughout the whiff. Interestingly, odor intensity coding improves steadily as the whiff endures, though it takes several times the adaptation timescale to minimize the errors.

Piecing this out in a bit more detail, we find that the latter of these – the failure of odor identification rather than intensity – is most acutely affected by the slower adaptation dynamics: the difference between zero component estimates at whiff onset and at whiff closure are nearly indistinguishable for long timescales, but can reduce by an order of magnitude for faster ones. Finally, we note that, while the number of mis-identified zero components may appear minor (20% or so), this corresponds to
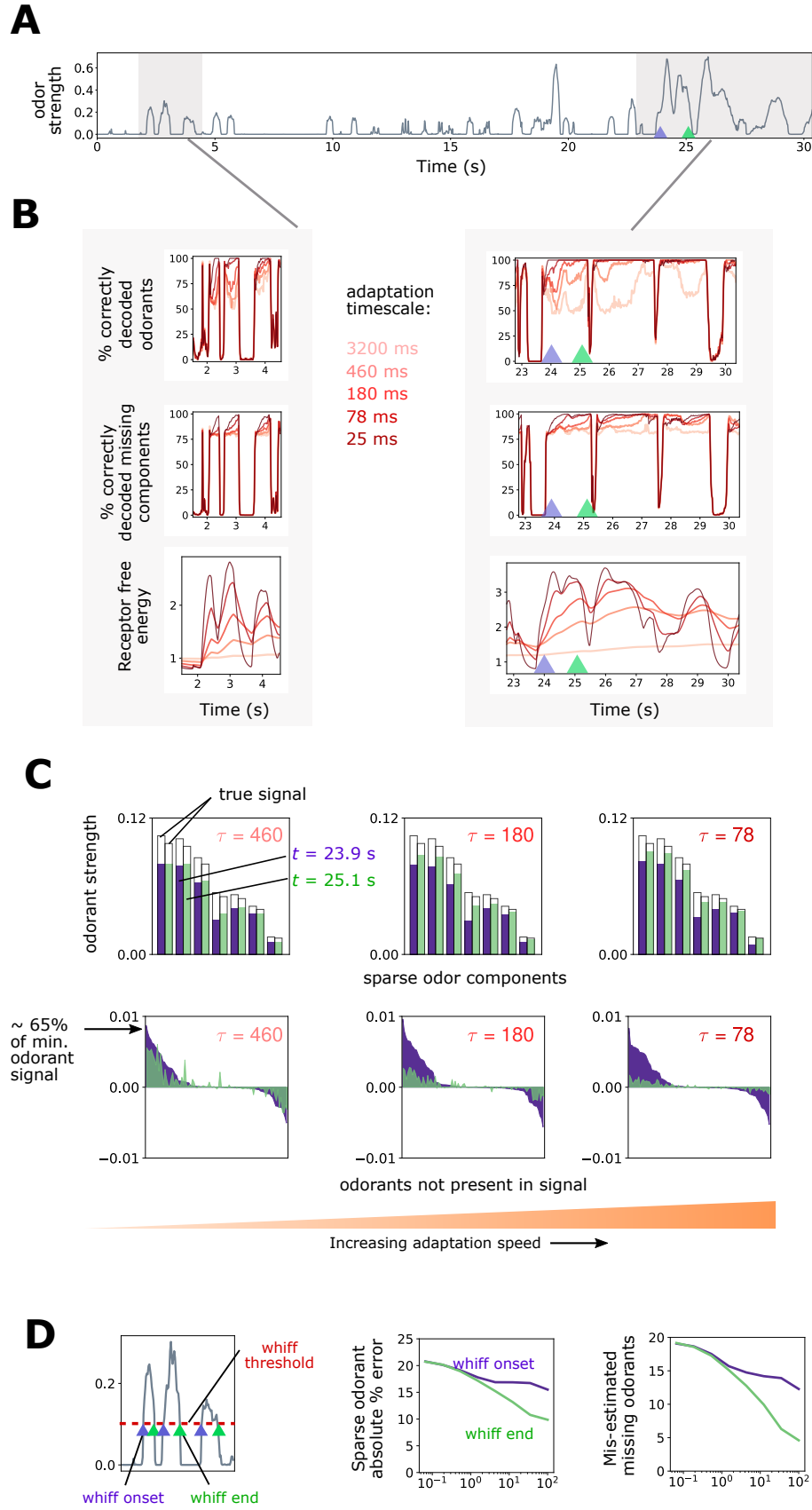
FIG. 5: A test figure with
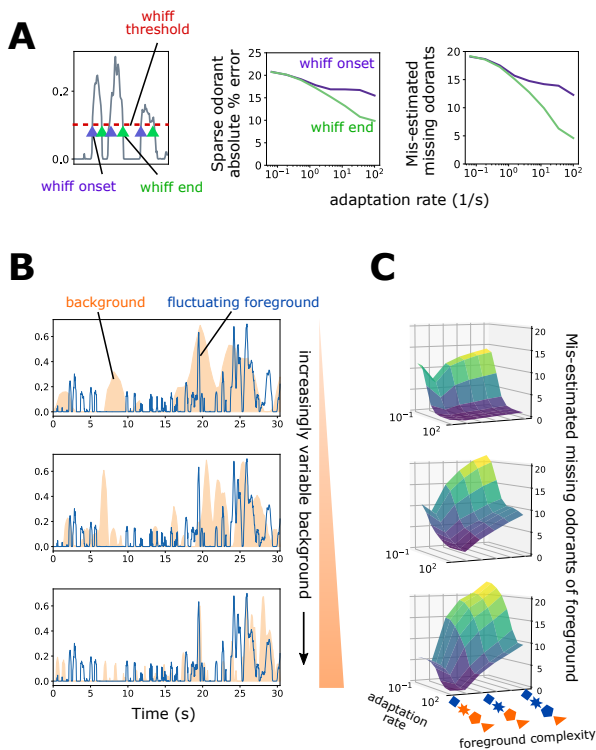its caption side by side

FIG. 6: blah

several times the number of sparse components defining the odor signal itself, which is 7. Thus we expect that the increase from 80% to > 95% through the duration of the whiff indeed corresponds to a salient, dynamically improving perception of odor identity.

Can the dynamics of increased odor perception be retained in presence of dynamic adaptation, fluctuating odors *and* fluctuating backgrounds? We consider three cases, in which a background odor modulates on timescales roughly that of the foreground, somewhat slower, and substantially slower. To maximize potential confounds, we assume the foreground and background odors span odor intensities of roughly equal magnitude. In addition, anticipating our earlier results, we consider various levels of relative complexity among these two odors. Explain how performanec is quantified

When the timescale of background fluctuations is long, we find that performance improves markedly with adaptation speed (mis-identified zero components drop from 14 to 0 as timescale varies from 10 seconds to 0.5 seconds), but is already maximized at 500 ms, which exceeds the duration of most of the odor whiffs. In other words, contrary to the single odor case, odors are perfectly identified even when adaptive gain operates at timescales somewhat slower than the signal fluctuations. For slightly faster background fluctuations, but still slower than the foreground timescale, we see a marked degradation in performance with slower adaptation, although

again the performance largely saturates at timescales of 500 ms or so. Further, there is now a strong dependence on foreground complexity; simpler foregrounds are easier to perceive above the background (number of mis-identified zero components ~ 3), while very complex foregrounds have a larger number of misidentified components ~ 8. This pattern continues when the background fluctuates as quickly as the foreground, though the performance is only slightly degraded. Importantly, there is a monotonic gain in performance as adaptation speed increases, holding across fluctuation timescales and molecular complexity. Our key finding is that for odors that fluctuate on well-separated timescales, dynamic adaptive feedback obeying the Weber-Fechner Law and operating moderately quickly promotes the active perception of odor identity.

II. DISCUSSION

Drawing on recent evidence for the existence of the Weber-Fechner law in in *Drosophila* olfactory receptor neurons [8–10], we propose a theoretical framework for the adaptive encoding and decoding of complex, dynamic odor environments. We argue that Weber's Law, when incorporated into a combinatorial coding strategy, is central to the accurate identification and discrimination of rapidly fluctuating, potentially conflicting odor signals. Our framework relies on two steps of odor encoding and decoding, respectively: (i) a nonlinear, stochastic model of odor-receptor binding and subsequent ORN firing, and ii) decompression of the linearized steady state ORN responses. In this framework, Weber Law adaptation is enforced naturally by scaling the free energy of ORN activation logarithmically with the ORN firing response. The encoding model is a generalization of the classical model of bacterial chemotaxis [20], and is mathematically equivalent to a recently proposed competitive binding model for ORN response [21], where it was shown that inclusion of inhibitory responses increases coding capacity of a distributed system of ORNs. Regarding decoding, recent works have pointed to the importance of a distributed response in inferring high-dimensional sparse signals [1, 22, 23]. In this work, we place particular importance on the impact of intensity variations that typify odor signals in natural environments, finding that in both static and fluctuating odor landscapes, adaptive sensing at the receptor level play a central role in the simultaneous decoding of odor intensity and odor identity.

A. Maintaining a distributed response

We showed that for static odor signals, a broadly sensing but non-adaptive system can accurately estimate odor identities, though only in a limited window of concentration. In living systems, adaptation maintains information transfer by ensuring that the sensory system stays

in a regime of maximum sensitivity [24]. In compressed sensing, the fidelity of signal decoding relies also on the combinatorics of the sensor response [13–15]. Indeed, it has been noted that *diffusivity* in sensing – here incorporated through the dispersity of binding constants – underlies effective compression of high-dimensional sparse signals into a limited receptor space. Still, the nonlinearity of the steady state response, Eq., can affect the distributions of ORN activity as odor concentration increases. Thus, in the context of combinatorial coding, the central benefit conferred by the Weber Law scaling is not merely preventing ORN activities from saturating, but their distributions from distorting.

Importantly, we find that the advantages of preserving combinatorial response carry over to more complex odor environments, where multiple odors must be discriminated. The ability to recognize weak odors over strong backgrounds is particularly relevant to olfaction in nature, where signal conflicts are pervasive. Absent Weber Law scaling, odor signals are mis-identified in the presence of strong backgrounds, producing accurate estimations only beyond a minimum intensity; this minimum itself increases with odor complexity. Further, the system is largely incapable of estimating both odors accurately – true discrimination – except in limited concentration windows. In principle, the adaptive system might also be susceptible to signal conflicts: mathematically, the activity distributions are invariant only in the limit that all odorants are of equal strength (Eq), so the large deviations of the weaker odorant concentrations from this mean value could lead to sensitive distortions in the distribution of ORN activities. Nonetheless, we find that odors at least as strong as the background can be identified irrespective of odor complexities. Likewise discrimination accuracy is more robust, preserved over sizable concentration windows.

B. Relative abundances of ORN classes

We tested our model on a particular combination of odor sparsity, odorant dimension, and receptor dimension, for various choices of system K_D^* and odor identity. One aspect as yet unexplored is the relative distribution of different receptor types. Though ORNs of a given type project to a single glomerulus in the antennal lobe, multiplicities of one ORN class over another may still affect information transfer, assuming that responses are not noiseless (without noise, ORN multiplicities contain only redundant information). The optimality of a non-uniform receptor distribution in some environments was recently demonstrated for a linear combinatorial sensing system; a suggestive extension of this work is blah....

Say some more about future experiments or something

C. Simultaneous coding of intensity and identity

An important aspect of olfactory sensing is maintaining fidelity in encoding odor identity simultaneously with that of odor intensity [**?**]. Here we find that in some situations these aspects may decouple with variations in odor environment; often, though errors in one coincide with errors in the other. As mentioned, compressed sensing naturally conflates identity and intensity, decoding the exact strength of odor signal components while relying fundamentally on the requirement that most of components are zero. In this sense, combinatorial coding and compressed sensing decoding confronts the identity-intensity confound quite naturally, perhaps moreso than in sensory systems in which stimuli are parameterized continuously (e.g. by frequency), such as vision and audition.

An intriguing result in our framework is the distinct manner in which errors in intensity and identity diminish during the active perception of an odor whiff. Even for relatively long adaptive timescales, identity perception increases monotonically as the odor persists, insensitive to ongoing fluctuations. Further, the proportion of mis-identified odorants continues falling on timescales much longer than τ_m. Implications?

D. Divisive normalization / timescales

TODO

E. The timescales of adaptation mechanisms

TODO: discuss the temporal coding results; primacy coding; timescales of normalization

III. METHODS

A. Stochastic odor-receptor binding model

We model an odor as an N-dimensional vector $\mathbf{s} = \langle s_1, ..., s_N \rangle$, where $s_i > 0$ are the concentrations of individual volatile molecules (odorants) comprising the odor. In addition, we assume that the odors are sparse in the space of odorants, so only K components of $\mathbf{s}$ are nonzero, where $K \ll N$. The olfactory sensory system is modeled as a collection of M distinct ORNs, each of which can be bound with any one of the odorant molecules, and can be either active (firing) inactive (quiescent). We only consider competitive binding, so an ORN is bound with one odorant at most. With N possible odorants, receptor a resides in one of $2N + 2$ possible states, $\{R_a, R_a^*, R_a\text{-}s_i, R_a^*\text{-}s_i\}$, indicating receptors that are unbound/inactive, unbound/active, inactive/bound to odorant i, and active/bound to odorant i, respectively. We set $N = 100$, $K = 7$, and $M = 50$ throughout.

The system dynamics are schematized in Fig.. In the mean-field limit, the binding dynamics of these $2N + 2$ states are described by the master equations:

$$\frac{d[R_a\text{-}s_i]}{dt} = k_{ia}^+ s_i [R_a] - k_{ia}^- [R_a\text{-}s_i] \tag{4}$$

$$\frac{d[R_a^*\text{-}s_i]}{dt} = k_{ia}^{*+} s_i [R_a^*] - k_{ia}^{*-} [R_a^*\text{-}s_i], \tag{5}$$

when receptor R_a is either inactive (Eq. 4) or active (Eq. 5). Further, transitions between inactive and active states are described in the mean limit via:

$$\frac{d[R_a]}{dt} = w_a^{\mathrm{u}+} [R_a] - w_a^{\mathrm{u}-} [R_a^*] \tag{6}$$

$$\frac{d[R_a^*\text{-}s_i]}{dt} = w_{ia}^{\mathrm{b}+} [R_a\text{-}s_i] - w_{ia}^{\mathrm{b}-} [R_a^*\text{-}s_i], \tag{7}$$

when receptor R_a is either unbound (Eq. **??**) or bound (Eq. **??**). We also define corresponding disassociation constants in terms of the binding transition rates:

$$K_{ia} = \frac{k_{ia}^-}{k_{ia}^+}$$

$$K_{ia}^* = \frac{k_{ia}^{*-}}{k_{ia}^{+*}} \tag{8}$$

Following (!!), we assume that in steady state, the active firing state of an ORN is energetically suppressed from the inactive state through corresponding Boltzmann factors:

$$\frac{[R_a^*]}{[R_a]} = \frac{w_a^{\mathrm{u}+}}{w_a^{\mathrm{u}-}} \equiv e^{-\epsilon_a^{\mathrm{u}}} \tag{9}$$

$$\frac{[R_a^*\text{-}s_i]}{[R_a\text{-}s_i]} = \frac{w_{ia}^{\mathrm{b}+}}{w_{ia}^{\mathrm{b}-}} \equiv e^{-\epsilon_{ia}^{\mathrm{b}}} \quad \text{(steady state).} \tag{10}$$

However, these factors are related when detailed balance is enforced. Enforcing detailed balance upon a given 4-cycle as in Fig. (!!) gives

$$\frac{w_a^{\mathrm{u}+}}{w_a^{\mathrm{u}-}} \frac{k_{ia}^{*+}}{k_{ia}^{*-}} \frac{w_a^{\mathrm{b}-}}{w_a^{\mathrm{b}+}} \frac{k_{ia}^-}{k_{ia}^+} \equiv 1, \tag{11}$$

which in conjunction with Eqs. **??**, **??**, and **??** gives

$$\epsilon_a^{\mathrm{b}} = \epsilon_{ia}^{\mathrm{u}} + \ln\left[\frac{K_{ia}}{K_{ia}^*}\right] \tag{12}$$

B.   Odor encoding

C.   Odor decoding framework

The scaling adds the disorder mentioned in previous paper

TEMPORAL: discuss how the adaptive timescale here preserves stuff .. How to incorporate primacy coding?

Primacy coding – intensity? Not coded? Also, is it Huffman coding?

Equations can be referenced such as and other papers as well. References that only appear in the supplementary materials (figure captions, etc.) or online methods section will follow the reference numbering of the main text and appear at the end of the main reference lis.t

Important results in the supplementary data

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