Solutions to Sec 2.2 Pre-lab Questions

(Both the marks of Questions 1 and 2 are 0.5. The marks of the rest questions are 1.)

1. A-2 Calculate the maximum overshoot of the response (in radians) given a step setpoint of 45 degrees and the overshoot specification given in Section 2.1.1.3.
Hint: By substituting $y_{max} = \theta(t_p)$ and step setpoint $R_0 = \theta_d(t)$ into equation 2.1.6, we can obtain $\theta(t_p) = \theta_d(t)\ (1 + \frac{PO}{100})$. Recall that the desired response specifications include 5% overshoot.

Answer 2.2.1

Outcome	Solution
A-2	Substituting a step reference of $\theta_d(t) = 0.785$ rad and $PO = 5$ % into this equation gives the maximum overshoot as $\theta(t_p) = 0.823$ rad.

2. A-1, A-2 The SRV02 closed-loop transfer function was derived in equation 2.1.23 in Section 2.1.2.1. Find the control gains k_p and k_v in terms of ω_n and ζ. **Hint:** Remember the standard second order system equation.

Answer 2.2.2

Outcome **Solution**

A-1 The characteristic equation of the SRV02 closed-loop transfer function in 2.1.7 is

$$\tau\, s^2 + (1 + K\, k_v)\, s + K\, k_p \quad \text{(Ans.2.2.1)}$$

and can be re-structured into the form

$$s^2 + \frac{(1 + K\, k_v)\ s}{\tau} + \frac{K\, k_p}{\tau} \quad \text{(Ans.2.2.2)}$$

Equating this with the standard second order system equation gives the expressions

$$\frac{K\, k_p}{\tau} = \omega_n^2 \quad \text{(Ans.2.2.3)}$$

and

$$\frac{1 + K\, k_v}{\tau} = 2\, \zeta\, \omega_n \quad \text{(Ans.2.2.4)}$$

A-2 Solve for k_p and k_v to obtain the control gains equations

$$k_p = \frac{\omega_n^2\, \tau}{K} \quad \text{(Ans.2.2.5)}$$

and the velocity gain is

$$k_v = \frac{2\zeta\omega_n\tau - 1}{K} \quad \text{(Ans.2.2.6)}$$

3. A-2 Calculate the minimum damping ratio and natural frequency required to meet the specifications given in Section 2.1.1.3.

Answer 2.2.3

Outcome **Solution**

A-2 Substitute the percent overshoot specifications given in 2.1.19 into Equation 2.1.8 to get the required damping ratio

$$\zeta = 0.690 \quad \text{(Ans.2.2.7)}$$

Using this result and the desired peak time, given in 2.1.18, with Equation 2.1.9 gives the minimum natural frequency needed

$$\omega_n = 21.7 \text{ rad/s} \quad \text{(Ans.2.2.8)}$$

4. A-2 Based on the nominal SRV02 model parameters, K and τ, found in Laboratory 1: SRV02 Modeling, calculate the control gains needed to satisfy the time-domain response requirements given in Section 2.1.1.3.

Answer 2.2.4

Outcome	Solution
A-2	Using the model parameters

$$K = 1.53 \text{ rad/(V s)} \tag{Ans.2.2.9}$$

and

$$\tau = 0.0254 \text{ s} \tag{Ans.2.2.10}$$

as well as the desired natural frequency found in Ans.2.2.8 with Equation Ans.2.2.5, generates the proportional control gain

$$k_p = 7.82 \text{ V/rad} \tag{Ans.2.2.11}$$

Similarly, the velocity control gain is obtained by substituting the model parameters given above with the minimum damping ratio specification, in Ans.2.2.7, into Equation Ans.2.2.6

$$k_v = -0.157 \text{ V s/rad} \tag{Ans.2.2.12}$$

Thus, when these gains are used with the PV controller, the position response of the load gear on an SRV02 with a disc load will satisfy the specifications listed in 2.1.1.3.

6. A-1, A-2 For the PV controlled closed-loop system, find the steady-state error and evaluate it numerically given a ramp with a slope of $R_0 = 3.36$ rad/s. Use the control gains found in question 4.

Answer 2.2.6

Outcome	Solution
A-1	Applying the final-value theorem to the error transfer function yields the expression

$$e_{ss} = \lim_{s \to 0} \frac{R_0\,(\tau\,s + 1 + K\,k_v)}{\tau\,s^2 + s + K\,k_p + K\,k_v\,s} \tag{Ans.2.2.15}$$

A-2 When evaluated, the resulting steady-state error is

$$e_{ss} = \frac{R_0\,(1 + K\,k_v)}{K\,k_p} \tag{Ans.2.2.16}$$

The steady-state error is a constant, which is as expected since the closed-loop SRV02 position system is Type 1. Evaluating the expression with the reference slope of 3.36 rad/s, the model gain parameter $K = 1.53$, the proportional and velocity gains $k_p = 7.82$ and $k_v = 0.157$, gives the steady-state error

$$e_{ss} = 0.214\ [rad] \tag{Ans.2.2.17}$$

7. A-2 What should be the integral gain k_i so that when the SRV02 is supplied with the maximum voltage of $V_{max} = 10\,V$ it can eliminate the steady-state error calculated in question 6 in 1 second? **Hint:** Start from equation 2.1.35 and use $t_i = 1$, $V_m(t) = 10$, the k_p you found in question 4 and e_{ss} found in question 6. Remember that e_{ss} is constant.

Answer 2.2.7

Outcome A-2

Solution

Since e_{ss} is constant, evaluating the integral in Equation 2.1.35 yields

$$V_m(t) = k_p\, e_{ss} + k_i\, t_i\, e_{ss} \qquad \text{(Ans.2.2.18)}$$

Then, the integral gain is

$$k_i = \frac{V_m(t) - k_p\, e_{ss}}{t_i\, e_{ss}} \qquad \text{(Ans.2.2.19)}$$

By substituting $t_i = 1.0sec$, the maximum SRV02 voltage $V_m(t) = 10V$, $k_p = 7.82$ and the PV control steady-state error $e_{ss} = 0.214$ we find

$$k_i = 38.9 \text{ V/(rad s)} \qquad \text{(Ans.2.2.20)}$$