Solutions to Sec 1.2 Pre-lab Questions

The mark of each question is 1.

1. A-1, A-2 In Section 1.1.1.3 we obtained an equation (1.1.26) that described the dynamic behavior of the load shaft speed as a function of the motor input voltage. Starting from this equation, find the transfer function $\frac{\Omega_l(s)}{V_m(s)}$.

Answer 1.2.1

Outcome	Solution
A-1	Taking the Laplace transform of the equations and assuming $\omega_l(0^-) = 0$ gives $$J_{eq} s \Omega_l(s) + B_{eq,v} \Omega_l(s) = A_m V_m(s) \quad \text{(Ans.1.2.1)}$$
A-2	Solving for $\Omega_l(s)/V_m(s)$ gives the plant transfer function $$\frac{\Omega_l(s)}{V_m(s)} = \frac{A_m}{J_{eq} s + B_{eq,v}} \quad \text{(Ans.1.2.2)}$$

2. A-1, A-2 Express the steady-state gain (K) and the time constant (τ) of the process model (Equation (1.1.1)) in terms of the J_{eq}, $B_{eq,v}$, and A_m parameters.

Answer 1.2.2

Outcome	Solution
A-1	We need to match the coefficients of the transfer function found in (Ans.1.2.2) to the coefficients of the transfer function in equation 1.1.1.
A-2	The time constant parameter is $$\tau = \frac{J_{eq}}{B_{eq,v}} \quad \text{(Ans.1.2.3)}$$ and the steady-state gain is $$K = \frac{A_m}{B_{eq,v}} \quad \text{(Ans.1.2.4)}$$

9. A-1, A-2 Referring to Section 1.1.2.1, find the expression representing the time constant τ of the frequency response model given in Equation 1.1.31. Begin by evaluating the magnitude of the transfer function at the cutoff frequency ω_c.

Answer 1.2.9

Outcome	Solution
A-1	By definition, the DC gain drops 3 dB (or $\frac{1}{\sqrt{2}}$) at this frequency. Therefore, $$\lvert G_{wl,v}(w_c)\rvert = \frac{1}{2} \lvert G_{wk,v}(0)\rvert \sqrt{2} \quad \text{(Ans.1.2.20)}$$
A-2	Applying this to the SRV02 frequency response magnitude in 1.1.31 above gives: $$\frac{1}{2} \lvert G_{wl,v}(0)\rvert \sqrt{2} = \frac{G_{wl,v}(0)}{\sqrt{1 + \tau_{e,f}^2 \, \omega_c^2}} \quad \text{(Ans.1.2.21)}$$ We can then solve for the time constant as: $$\tau_{e,f} = \frac{1}{\lvert \omega_c \rvert} \quad \text{(Ans.1.2.22)}$$

10. A-2, A-3 Referring to Section 1.1.2.2, find the steady-state gain of the step response and compare it with Equation 1.1.34. **Hint:** The the steady-state value of the load shaft speed can be defined as $\omega_{l,ss} = \lim_{t \to \infty} \omega_l(t)$.

Answer 1.2.10

Outcome	Solution
A-2	Using the definition of the steady-state value of the load shaft

$$\omega_{l,ss} = \lim_{t\to\infty} \omega_l(t) \qquad \text{(Ans.1.2.23)}$$

The limit of the servo step response given in (1.1.40) is

$$\omega_{l,ss} = K\,A_v + \omega_l(t_0) \qquad \text{(Ans.1.2.24)}$$

and the steady-state gain is

$$K = \frac{\omega_{l,ss} - \omega_l(t_0)}{A_v} \qquad \text{(Ans.1.2.25)}$$

A-3 This is consistent with the $\Delta y/\Delta u$ relationship in Equation 1.1.34.

11. A-2, A-3 Evaluate the step response given in equation 1.1.40 at $t = t_0 + \tau$ and compare it with Equation 1.1.34.

Answer 1.2.11

Outcome	Solution
A-2	Substituting $t = t_0 + \tau$ in equation 1.1.40 gives the load shaft rate

$$\omega_l(t_0+\tau) = K\,A_v\,\left(1 - e^{-1}\right) + \omega_l(t_0) \qquad \text{(Ans.1.2.26)}$$

A-3 This is consistent with the $y(t_1)$ expression in equation 1.1.34.

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