Summary of tight lower bounds for divergences

P, Q: Probability distributions on real line with given means and variances.

$$R_P := \frac{\sqrt{\mathrm{Var}[X]_P}}{\left|\mathrm{E}[X]_P - \mathrm{E}[Y]_Q\right|}, \qquad R_Q := \frac{\sqrt{\mathrm{Var}[Y]_Q}}{\left|\mathrm{E}[X]_P - \mathrm{E}[Y]_Q\right|}, \qquad \text{where } X \sim P \text{ and } Y \sim Q.$$

Divergence	Lower bound	Equality condition
KL-divergence[1] $\sum p_i \ln \frac{p_i}{q_i}$	$\int_0^1 \frac{\lambda}{(1-\lambda)R_P^2 + \lambda R_Q^2 + \lambda(1-\lambda)} d\lambda$	2-point distributions
χ^2-divergence $\frac{\sum (p_i - q_i)^2}{q_i}$	$\frac{1}{R_Q^2}$ (**Hammersley–Chapman–Robbins bound**)	2-point distributions
Squared Hellinger distance[2] $\frac{1}{2}\sum(\sqrt{p_i} - \sqrt{q_i})^2$	$1 - (R_P + R_Q)\sqrt{\frac{1}{(R_P + R_Q)^2 + 1}}$	2-point distributions
Total variation distance[3] $\frac{1}{2}\sum\lvert p_i - q_i\rvert$	$\frac{1}{(R_P + R_Q)^2 + 1}$	3-point distributions

[1] "**On Relations Between the Relative Entropy and χ^2-Divergence, Generalizations and Applications.**"
[2] **arXiv [2105.12972]**
[3] **arXiv [2212.05820]**