Image restoration

→ Periodic Noise :-

→ Band reject filters :-

① Ideal

$$H_{ibr}(u,v) = \begin{cases} 1 & \text{if } D(u,v) < D_0 - \frac{W}{2} \\ 0 & \text{if } D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 1 & \text{if } D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

② Butterworth

$$H_{Bbr}(u,v) = \frac{1}{1 + \left[\frac{D(u,v)\,W}{D^2(u,v) - D_0^2}\right]^{2n}}$$

$2n \to$ order

③ Gaussian

$$H_{Gbr}(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)\,W}\right]^2}$$

→ Bandpass filter :-

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

→ Notch reject filter :-

$$H(u,v) = \begin{cases} 0 & \text{if } D_1(u,v) \le D_0 \\ & \text{or } D_2(u,v) \le D_0 \\ 1 & \text{otherwise} \end{cases}$$

where $D_0 \to$ centre of the notch

$$D_1(u,v) = \left[\left\{\left(u - \frac{M}{2}\right) - u_0\right\}^2 + \left\{\left(v - \frac{N}{2}\right) - v_0\right\}^2\right]^{1/2}$$

$$\Rightarrow H(u,v) = \frac{T}{\pi(ua+vb)} \sin(\pi(ua+vb)) \times e^{-j\pi(ua+vb)}$$

for $x_0(t) = \frac{at}{T}$ & $y_0(t) = \frac{bt}{T}$

12/04/14

Tutorial

Image Restoration
↳ objective
↳ Enhancement
↳ Subjective

SSIM – standard static Index measurement

PSNR

imnoise(A, type of noise)

Gaussian, salt & pepper, speckle

Eg: $F(z) = \begin{cases} 1 - e^{-(z-a)^2/b} & ; \text{ for } z \geq a \\ 0 & ; \quad z < a \end{cases} \qquad b > 0$

let w be random variable (use rand fn)

$F(z) = w \quad \Rightarrow 1 - e^{-(z-a)^2/b} = w$

$\Rightarrow e^{-(z-a)^2/b} = 1 - w$

$\Rightarrow -(z-a)^2/b = \ln(1-w)$

$\Rightarrow (z-a)^2 = -b\ln(1-w)$

$\Rightarrow z = a + \sqrt{-b\ln(1-w)}$

Eg: $F(z) = \begin{cases} 1 - e^{-az} & ; \quad z \geq 0 \\ 0 & ; \quad z < 0 \end{cases}$

$\therefore F(z) = w$ (rand variable)

$$1 - e^{-az} = w$$

$$\Rightarrow e^{-az} = 1-w \Rightarrow -az = \ln(1-w)$$

$$\Rightarrow z = -\frac{1}{a}\ln(1-w)$$

<u>spatial filtering</u>

nargin → no. of input arguments.

<u>16/04/14</u>

→ <u>Inverse filtering</u> :-

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$

$\Rightarrow G(u,v) = H(u,v) \cdot F(u,v) + N(u,v)$

$$= \frac{H(u,v)\,F(u,v) + N(u,v)}{H(u,v)}$$

$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$

$$H(u,v) = e^{-k\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right]^{5/6}}$$

→ Const. (turbulence)

→ <u>Weiner filtering</u> :-

$$\hat{F}(u,v) = \left[\frac{H^*(u,v) \cdot S_f(u,v)}{S_f(u,v)\,|H(u,v)|^2 + S_\eta(u,v)}\right] G(u,v)$$

where, $H(u,v)$ → degradation fn $|H(u,v)|^2 = H^*(u,v) \cdot H(u$

$S_\eta(u,v) = |N(u,v)|^2$ → Noise power

$S_f(u,v) = |F(u,v)|^2$ — Undegradable image pow

Based on minimum mean square error (MMSE) i.e.

$$e^2 = E\left\{(F-\hat{F})^2\right\}$$

$$\Rightarrow \quad \hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \times \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

where $K = \dfrac{S_\eta(u,v)}{S_f(u,v)} = (SNR)^{-1}$

→ If $S_\eta(u,v) = 0$, then

$$\hat{F}(u,v) = \frac{1}{H(u,v)} \cdot G(u,v) = H^{-1}(u,v) \cdot G(u,v).$$

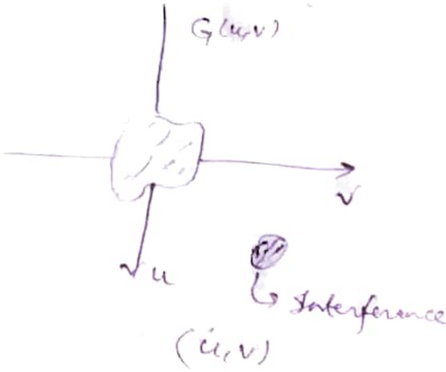
$$D_1(u,v) = \left[\left\{\left(u-\frac{M}{2}\right)+u_0\right\}^2 + \left\{\left(v-\frac{N}{2}\right)+v_0\right\}^2\right]^{1/2}$$

→ Notch pass filter :-

→ optimum notch filtering :-

We need to isolate particular pattern and then subtract a variable which is ~~waited~~ weighted for of pattern from the

It minimizes the local variance restore



$$g(x,y) = f(x,y) + \eta(x,y)$$

$\downarrow \mathcal{F}(\ )$ $G(u,v)$ $\qquad$ $\downarrow \mathcal{F}(\ )$ $N(u,v)$

$$N(u,v) = \underbrace{H(u,v)}_{\text{Notch pass}} \cdot G(u,v)$$

$$\underbrace{\hat{f}(x,y)}_{\text{erroneous}} = g(x,y) - \eta(x,y)$$

For optimum

$$\hat{f}(x,y) = g(x,y) - \overset{\text{weighted}}{\underbrace{w(x,y)}_{\text{To find}}}\, \eta(x,y)$$

Local variance

$$\sigma_L^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \left\{ \hat{f}(x+s, y+t) - \underbrace{\bar{\hat{f}}(x,y)}_{\text{mean}} \right\}^2$$

$$\bar{\hat{f}}(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \hat{f}(x+s, y+t)$$

$$\Rightarrow \sigma_L^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_s \sum_t \Big\{ \big[g(x+s, y+t) - w(x+s, y+t)\, \eta(x+s, y+t) \big] - \big[\bar{g}(x,y) - \overline{w(x,y) \cdot \eta(x,y)} \big] \Big\}^2$$

Assumption,

$$w(x+s, y+t) = w(x,y)$$

$$\Rightarrow \sigma_L^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_s \sum_t \Big\{ \big[g(x+s, y+t) - w(x,y)\, \eta(x+s, y+t) \big] - \big[\bar{g}(x,y) - w(x,y)\, \bar{\eta}(x,y) \big] \Big\}^2$$

for optimization or to achieve minn value of σ_L^2

$$\frac{\partial \sigma_L^2}{\partial w(x,y)} = \frac{\partial}{\partial w(x,y)} [\quad] = [\quad] = 0$$

$$\Rightarrow \boxed{ w_{opt}(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y) \cdot \bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - (\overline{\eta(x,y)})^2} }$$

$$\hat{f}_{opt}(x,y) = g(x,y) - w_{opt}(x,y) \cdot \eta(x,y)$$

04/04/14

Assignment → (2) (submit to 513EC1007@nitrkl.ac.in by 12/04/14)

(1) Take any gray image where spatial dependant noise is visible and name it Fig-a.

(2) Present the spectrum corresponding to Fig-a in fig-b

(3) comment on fig-b, particularly about the actual image information and possible noise.

(4) extract the noise using a suitable bandpass/notchpass filter. Present its spatial and spectral appearance

in gray image naming Fig-c and Fig-d respectively

(5) Find the histogram of Fig-c.

(6) Find the local variance and mean.

(7) Finally, find the optimum filtering weight fn $W_{opt}(x,y)$

→ Linear, position-invarient degradation :-

$$g(x,y) = H\left[f(x,y)\right] + \eta(x,y)$$

degradation fn. (operator)

(1) Linearity :-

$$H\left[af_1(x,y) + bf_2(x,y)\right] = aH\left[f_1(x,y)\right] + bH\left[f_2(x,y)\right]$$

(a) Say $a=1$, $b=1$

⇒ addivity property

(b) say, $f_2(x,y) = 0$

$$H\left[af_1(x,y) + b\cancel{f_2(x,y)}^{\to 0}\right] = aH\left[f_1(x,y)\right]$$

⇒ Homogenity property

(2) position/space invarient :-

$g(x,y) = H\left[f(x,y)\right]$, is said to be p/s invarient

if $g(x-\alpha, y-\beta) = H\left[f(x-\alpha, y-\beta)\right]$

→ Now we take

$g(x,y) = H\left[f(x,y)\right]$ → for simplicity

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\,\delta(x-\alpha, y-\beta)\,d\alpha\,d\beta$$

$$g(x,y) = H[f(x,y)]$$

$$= H \iint f(\alpha,\beta)\, \delta(x-\alpha, y-\beta)\, d\alpha\, d\beta$$

$$= \iint H[f(\alpha,\beta)\, \delta(x-\alpha, y-\beta)]\, d\alpha\, d\beta \quad \text{[linearity}$$

$$= \iint f(\alpha,\beta)\, H[\delta(x-\alpha, y-\beta)]\, d\alpha\, d\beta \quad \text{(space invari}$$

Let $h(x,\alpha,y,\beta) = H[\delta(x-\alpha, y-\beta)] = h(x-\alpha, y-\beta)$

→ "point spreading function"

$$\therefore g(x,y) = \iint f(\alpha,\beta)\, h(x,\alpha,y,\beta)\, d\alpha\, d\beta$$

$$= \iint f(\alpha,\beta)\, h(x-\alpha, y-\beta)\, d\alpha\, d\beta \quad [\because \text{space invariant}$$

$$= \int_{-\infty}^{\infty}\!\!\int f(\alpha,\beta)\, h(x-\alpha, y-\beta)\, d\alpha\, d\beta + \eta(x,y)$$

$$\boxed{g(x,y) = f(x,y) * h(x,y) + \eta(x,y)}$$

$$\Rightarrow G(u,v) = F(u,v) \cdot H(u,v) + N(u,v)$$

$$\Rightarrow G(u,v)\, H^{-1}(u,v) = F(u,v) \{ \overline{H(u,v)\, H^{-1}(u,v)} \} + N(u,v)\, H^{-1}(u,v)$$

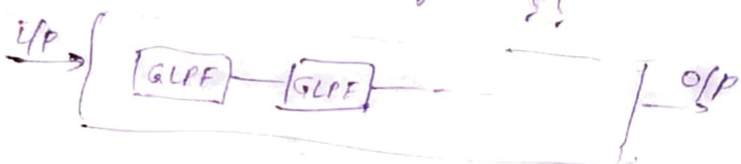
Identity fn

07/04/14

Tutorial

1). Given an image of size $m \times n$. You are asked to perform an exp that consist of a repeatedly low pass filtering the image using one Gaussian LPF. cutoff freq. v_0, let K_{min} be the smallest positive no. representable in the machi you are going to perform the exp.

Let K denotes the no. of application of the filter. Can you predict without doing the exp., what is the result of the system ??



$\rightarrow$ $G(u,v) = H(u,v) \cdot F(u,v)$

$$= e^{-D^2(u,v)/2D_0^2} \cdot F(u,v)$$

$$G_K(u,v) = e^{-KD^2(u,v)/2D_0^2} \cdot F(u,v)$$

$$H_K(u,v) = e^{-KD^2(u,v)/2D_0^2}$$

Yes, we can predict the result

for $K \rightarrow$ large

$$H_K(u,v) = e^{-KD^2(u,v)/2D_0^2}$$

$$= \begin{cases} 1 & \text{if } (u,v) = (0,0) \\ 0 & \text{otherwise} \end{cases}$$

2) Derive an expression for the min of K that will guaranty ~~that~~ given

$$H_K(u,v) = \begin{cases} 1 & ; (u,v) = (0,0) \\ 0 & \text{otherwise} \end{cases}$$

$$e^{-K D^2(u,v)/2D_0^2} < K_{min}.$$

$$\Rightarrow K > -\frac{\ln(K_{min})}{D^2(u,v)/2D_0^2}$$

$$\Rightarrow K > \frac{2D_0^2 \ln(K_{min})}{D^2(u,v)}$$

3). Consider a set of images generated by an exp in dealing with steller event. In this set each image contains a set of bright wildly scattered dots corresponding to These dots are barely visible due to super-impose illumination resulting from atmospheric. If these images are noted as product of const illum with a set of impulses then given enhancement procedure based on homomorphi design to bring component due to the

→ For one star

$$K_1 \delta_1(x_0, y_0) \quad K_1 \delta_2(x_1, y_1)$$

$$f(x,y) = K\,\delta(x_0, y_0)$$

$$\ln f(x,y) = \ln K + \ln\{\delta(x_0, y_0)\}$$

$$= K' + \delta'(x_0, y_0)$$

$$F[\ln f(x,y)] = K'\, 2\pi\, \delta(0,0) \pm e^{-j2\pi}(\quad)$$

4).

i/p $\boxed{\text{D.I.P}}$ (i) o/p $\boxed{\text{9.I.Q}}$ (ii)

11/04/15

→ Estimation of degradation function :

1. Estimation by image observation
2. " " Experimentation
3. " " mathematical modeling

1. 

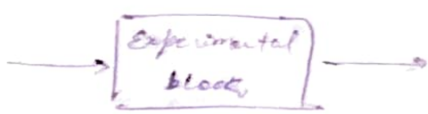
$\rightarrow g_s(x,y) = h_s(x,y) * \hat{f}(x,y)$

M×N $\Rightarrow G(u,v) = H_s(u,v) \cdot \hat{F}(u,v)$

$$\Rightarrow H_s(u,v) = \frac{G_s(u,v)}{\hat{F}(u,v)}$$

degradation fn

2. Design a system for the exp.
collect reference image of same type



$g_d(x,y)$

$$H(u,v) = \frac{G(u,v)}{\mathcal{F}[A\,\delta(x,y)]}$$

impulse of light

$$\Rightarrow H(u,v) = \frac{G(u,v)}{A}$$

3. const.

$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$ → Hufnagel & Stanley [1964]

Atmospheric turbulence
It is almost like a Gaussian LPF eqn.

① Example : Modeling an occurrance

Let us consider an image has been blurred by uniform linear motion betn the image & sensor during acquisition

$x_0(t)$ & $y_0(t)$ → spatial movement for T seconds.

$$g(x,y) = \int_0^T \left[f(x - x_0(t), y - y_0(t)) \right] dt$$

↳ degraded image

$$G(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[\int_0^T f(x - x_0(t), y - y_0(t)) \right] e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$= \int_0^T \left[\iint f(x - x_0(t), y - y_0(t))\, e^{-j2\pi(ux+vy)}\, dx\, dy \right] dt$$

$$= \int_0^T \left[\iint f(x - x_0(t), y - y_0(t))\, e^{-j2\pi\{u(x - x_0(t) + x_0(t)) + v(y - y_0(t) + y_0(t))\}}\, dx\, dy \right] dt$$

$$= \int_0^T \left[F(u,v)\, e^{-j2\pi\{ux_0(t) + vy_0(t)\}} \right]$$

$$= F(u,v) \left[\int_0^T e^{-j2\pi(ux_0(t) + vy_0(t))}\, dt \right]$$

$$= F(u,v) \cdot H(u,v)$$

where, $H(u,v) = \int_0^T e^{-j2\pi(ux_0(t) + vy_0(t))}\, dt$