level pool routing method. As before, the peak inflow of 360 cfs at 60 min is reduced to 270 cfs occurring at 80 minutes.

8.4 HYDROLOGIC RIVER ROUTING

The *Muskingum method* is a commonly used hydrologic routing method for handling a variable discharge-storage relationship. This method models the storage volume of flooding in a river channel by a combination of wedge and prism storages (Fig. 8.4.1). During the advance of a flood wave, inflow exceeds outflow, producing a *wedge* of storage. During the recession, outflow exceeds inflow, resulting in a negative wedge. In addition, there is a *prism* of storage which is formed by a volume of constant cross section along the length of prismatic channel.

Assuming that the cross-sectional area of the flood flow is directly proportional to the discharge at the section, the volume of prism storage is equal to KQ where K is a proportionality coefficient, and the volume of wedge storage is equal to $KX(I-Q)$, where X is a weighting factor having the range $0 \leq X \leq 0.5$. The total storage is therefore the sum of two components,

$$S = KQ + KX(I - Q) \tag{8.4.1}$$

which can be rearranged to give the storage function for the Muskingum method

$$S = K[XI + (1 - X)Q] \tag{8.4.2}$$

and represents a linear model for routing flow in streams.

The value of X depends on the shape of the modeled wedge storage. The value of X ranges from 0 for reservoir-type storage to 0.5 for a full wedge. When $X = 0$, there is no wedge and hence no backwater; this is the case for a level-pool reservoir. In this case, Eq. (8.4.2) results in a linear-reservoir model, $S = KQ$. In natural streams, X is between 0 and 0.3 with a mean value near 0.2. Great accuracy in determining X may not be necessary because the results of the method are relatively insensitive to the value of this parameter. The parameter K is the time of travel of the flood wave through the channel reach. A procedure called the *Muskingum-Cunge* method is described in Chapter 9 for determining the values of K and X on the basis of channel characteristics and flow rate in the channel. For hydrologic routing, the values of K and X are assumed to be specified and constant throughout the range of flow.

The values of storage at time j and $j + 1$ can be written, respectively, as

$$S_j = K[XI_j + (1 - X)Q_j] \tag{8.4.3}$$

and

$$S_{j+1} = K[XI_{j+1} + (1 - X)Q_{j+1}] \tag{8.4.4}$$

Using Eqs. (8.4.3) and (8.4.4), the change in storage over time interval Δt (Fig. 8.2.1) is

$$S_{j+1} - S_j = K\{[XI_{j+1} + (1 - X)Q_{j+1}] - [XI_j + (1 - X)Q_j]\} \tag{8.4.5}$$

The change in storage can also be expressed, using Eq. (8.2.2), as

$$S_{j+1} - S_j = \frac{(I_j + I_{j+1})}{2}\Delta t - \frac{(Q_j + Q_{j+1})}{2}\Delta t \tag{8.4.6}$$

Combining (8.4.5) and (8.4.6) and simplifying gives

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \tag{8.4.7}$$

which is the routing equation for the Muskingum method where

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X) + \Delta t} \tag{8.4.8}$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t} \tag{8.4.9}$$

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t} \tag{8.4.10}$$

Note that $C_1 + C_2 + C_3 = 1$.

If observed inflow and outflow hydrographs are available for a river reach, the values of K and X can be determined. Assuming various values of X and using known values of the inflow and outflow, successive values of the numerator and denominator of the following expression for K, derived from (8.4.5) and (8.4.6), can be computed.

$$K = \frac{0.5\,\Delta t[(I_{j+1} + I_j) - (Q_{j+1} + Q_j)]}{X(I_{j+1} - I_j) + (1 - X)(Q_{j+1} - Q_j)} \tag{8.4.11}$$

The computed values of the numerator and denominator are plotted for each time interval, with the numerator on the vertical axis and the denominator on the horizontal axis. This usually produces a graph in the form of a loop. The value of X that produces a loop closest to a single line is taken to be the correct value for the reach, and K, according to Eq. (8.4.11), is equal to the slope of the line. Since K is the time required for the incremental flood wave to traverse the reach, its value may also be estimated as the observed time of travel of peak flow through the reach.

If observed inflow and outflow hydrographs are not available for determining K and X, their values may be estimated using the Muskingum-Cunge method described in Sec. 9.7.

Example 8.4.1. The inflow hydrograph to a river reach is given in columns 1 and 2 of Table 8.4.1. Determine the outflow hydrograph from this reach if $K = 2.3$ h, $X = 0.15$, and $\Delta t = 1$ h. The initial outflow is 85 ft^3/s.

Solution. Determine the coefficients C_1, C_2, and C_3 using Eqs. (8.4.8) – (8.4.10):

$$C_1 = \frac{1 - 2(2.3)(0.15)}{2(2.3)(1 - 0.15) + 1} = \frac{0.31}{4.91} = 0.0631$$

$$C_2 = \frac{1 + 2(2.3)(0.15)}{4.91} = \frac{1.69}{4.91} = 0.3442$$

$$C_3 = \frac{2(2.3)(1 - 0.15) - 1}{4.91} = \frac{2.91}{4.91} = 0.5927$$

Check to see that the sum of the coefficients C_1, C_2, and C_3 is equal to 1.

$$C_1 + C_2 + C_3 = 0.0631 + 0.3442 + 0.5927 = 1.0000$$

For the first time interval, the outflow is determined using values for I_1 and I_2 from Table 8.4.1, the initial outflow $Q_1 = 85$ cfs, and Eq. (8.4.7) with $j = 1$.

$$\begin{aligned} Q_2 &= C_1 I_2 + C_2 I_1 + C_3 Q_1 \\ &= 0.0631(137) + 0.3442(93) + 0.5927(85) \\ &= 8.6 + 32.0 + 50.4 \\ &= 91 \text{ cfs} \end{aligned}$$

as shown in columns (3) to (6) of Table 8.4.1. Computations for the following time intervals use the same procedure with $j = 2, 3, \ldots$ to produce the results shown in Table 8.4.1. The inflow and outflow hydrographs are plotted in Fig. 8.4.2. It can be seen that the outflow lags the inflow by approximately 2.3 h, which was the value of K used in the computations and represents the travel time in the reach.

TABLE 8.4.1
Flow routing through a river reach by the Muskingum method (Example 8.4.1).

Column:	(1) Routing period j (h)	(2) Inflow I (cfs)	(3) $C_1 I_{j+1}$ ($C_1 = 0.0631$)	(4) $C_2 I_j$ ($C_2 = 0.3442$)	(5) $C_3 Q_j$ ($C_3 = 0.5927$)	(6) Outflow Q (cfs)
	1	93				85
	2	137	8.6	32.0	50.4	91
	3	208	13.1	47.2	54.0	114
	4	320	20.2	71.6	67.7	159
	5	442	27.9	110.1	94.5	233
	6	546	34.5	152.1	137.8	324
	7	630	39.8	187.9	192.3	420
	8	678	42.8	216.8	248.9	509
	9	691	43.6	233.4	301.4	578
	10	675	42.6	237.8	342.8	623
	11	634	40.0	232.3	369.4	642
	12	571	36.0	218.2	380.4	635
	13	477	30.1	196.5	376.1	603
	14	390	24.6	164.2	357.3	546
	15	329	20.8	134.2	323.6	479
	16	247	15.6	113.2	283.7	413
	17	184	11.6	85.0	244.5	341
	18	134	8.5	63.3	202.2	274
	19	108	6.8	46.1	162.4	215
	20	90	5.7	37.2	127.6	170

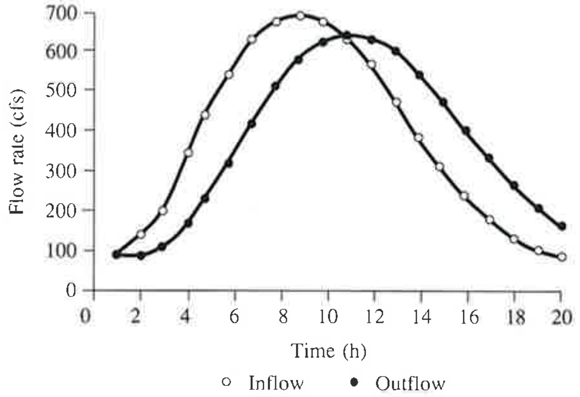

FIGURE 8.4.2
Routing of flow through a river reach by the Muskingum method (Example 8.4.1).

8.5 LINEAR RESERVOIR MODEL

A *linear reservoir* is one whose storage is linearly related to its output by a *storage constant k,* which has the dimension of time because S is a volume while Q is a flow rate.

$$S = kQ \tag{8.5.1}$$

The linear reservoir model can be derived from the general hydrologic system model [Eq. (7.1.6)] by letting $M(D) = 1$ and letting $N(D)$ have a root of $-1/k$ by making $N(D) = 1 + kD$. It can be shown further that if, in Eq. (7.1.6), $M(D) = 1$ and $N(D)$ has n real roots $-1/k_1, -1/k_2, \ldots, -1/k_n$, the system described is a cascade of n linear reservoirs in series, having storage constants $k_1, k_2, \ldots, k_n$,

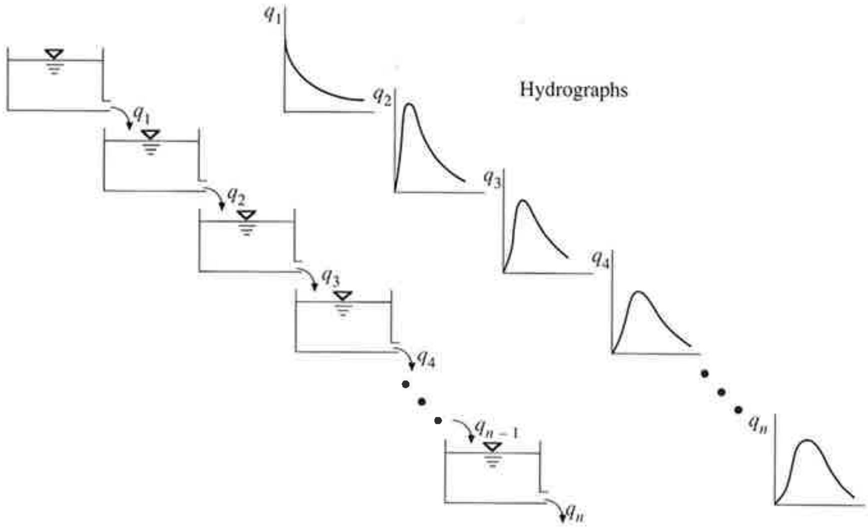

FIGURE 8.5.1
Linear reservoirs in series.