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# ELE 503 Homework 7

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## Problem 1a

```
A = [-30 0 0 0; 0 0 1 0; 1225 -1225 -21 0; 0 1 0 0];
B = [30; 0; 0; 0];
C = [0 0 0 1];
D = zeros;

Ts = 0.35; %define settling time of system

load sroots; %load tabulated bessel poles

plantPoles = eig(A);
adp = s1/Ts + imag(plantPoles(2))*i;
sPoles1 = s4/Ts; %Normalized Bessel Poles only
sPoles2 = [plantPoles(4) adp conj(adp) s1/Ts]; % SDPP,ADP,NBP

K1 = place(A,B,sPoles1);
K2 = place(A,B,sPoles2);

% Classical Stability Margins, Robustness Bounds

%Normalized Bessel Poles only
fprintf('----- Bessel Poles -----\n\n');
asm(A,B,K1)
[del1_1,del2_1] = rb_regsf(A,B,K1,0)

% SDPP,ADP,SBP
fprintf('----- SDPP/ADP Poles -----\n\n');
asm(A,B,K2)
[del1_2, del2_2] = rb_regsf(A,B,K2,0)
```

----- Bessel Poles -----

Upper gain margin for input #1 is 3.29 dB

Lower gain margin for input #1 is -30.1 dB

Phase margin for input #1 is 49 degrees

del1\_1 =

0.4565

del2\_1 =

0.3147

----- SDPP/ADP Poles -----

Upper gain margin for input #1 is 30.1 dB

Lower gain margin for input #1 is -30.1 dB

Phase margin for input #1 is 98 degrees

del1\_2 =

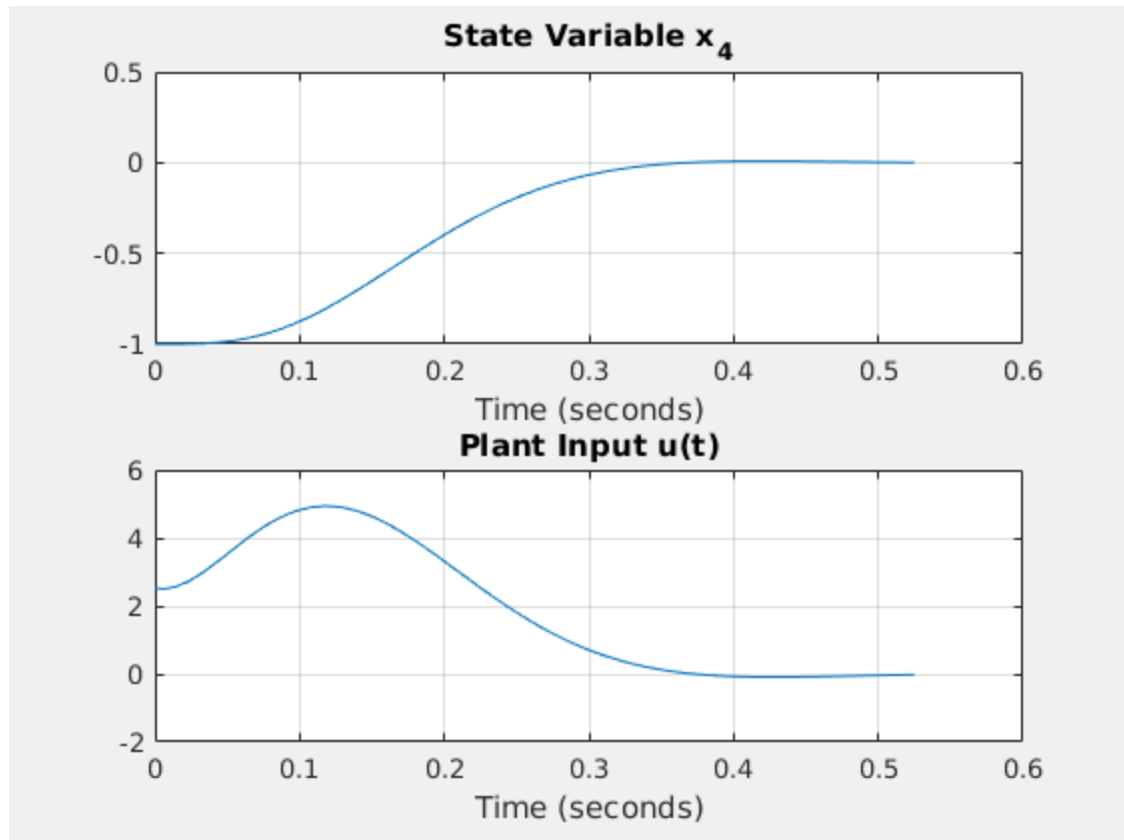
1.0000

del2\_2 =

1

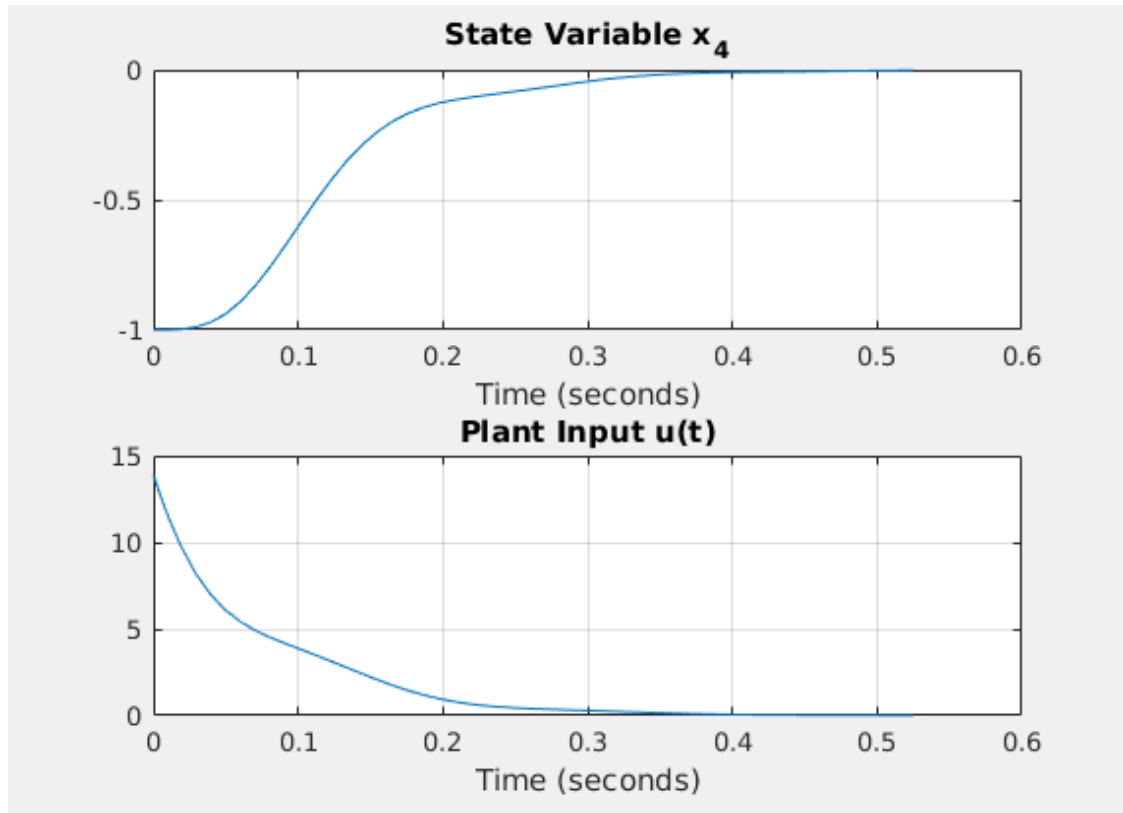
## Problem 1b.1 (Bessel Poles Regulator)

```
x0 = [0;0;0;-1];
K = K1;
results = sim("reg_asf.slx", 'StopTime', '0.525');
tout = results.tout;
u = results.u;
x = results.x;
%run reg_asfp script, open result
open('BesselPoles.fig')
```



## Problem 1b.2 (Selected Poles Regulator)

```
x0 = [0;0;0;-1];
K = K2;% The first set of bessle poles gave decent classical stability
      bounds,
results = sim("reg_asf.slx", 'StopTime', '0.525');
tout = results.tout;
u = results.u;
x = results.x;
%run reg_asfp script, open result
open('SelectedPoles.fig')
```



## Problem 1c (Analysis)

The first set of bessell poles gave decent classical stability bounds, however the delta values are unacceptable. There is also a very slight overshoot in the position, as seen on the graph of  $x_4$ .

The second set of poles gave extremely good delta bounds, however the graphs look abnormal to me. After checking my work I can find no error. The only concern with the second regulator design is that the plant input sharply spikes to 15 volts (assuming the units are indeed volts), which would saturate the cart pendulum system and likely cause issues.

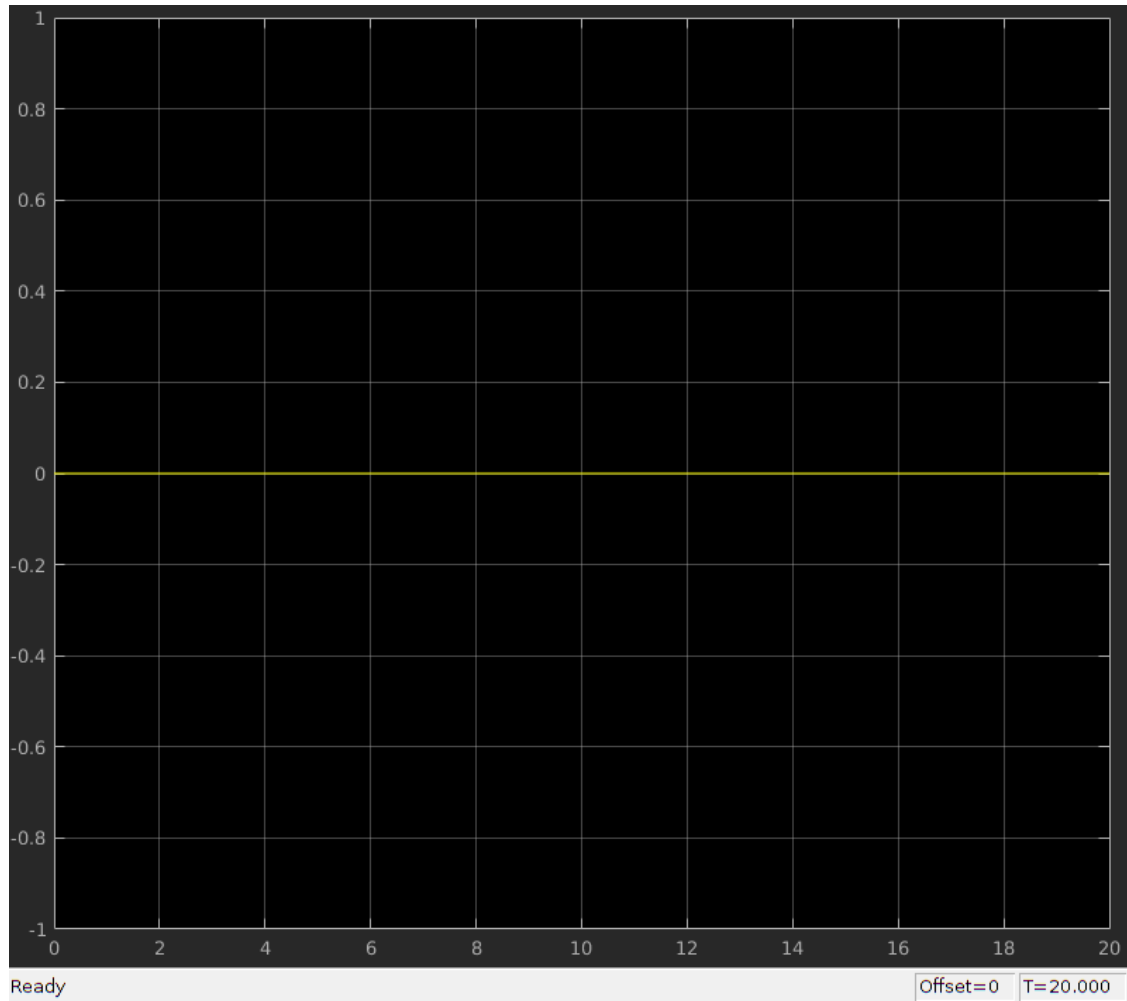
## Problem 2a (6.10a)

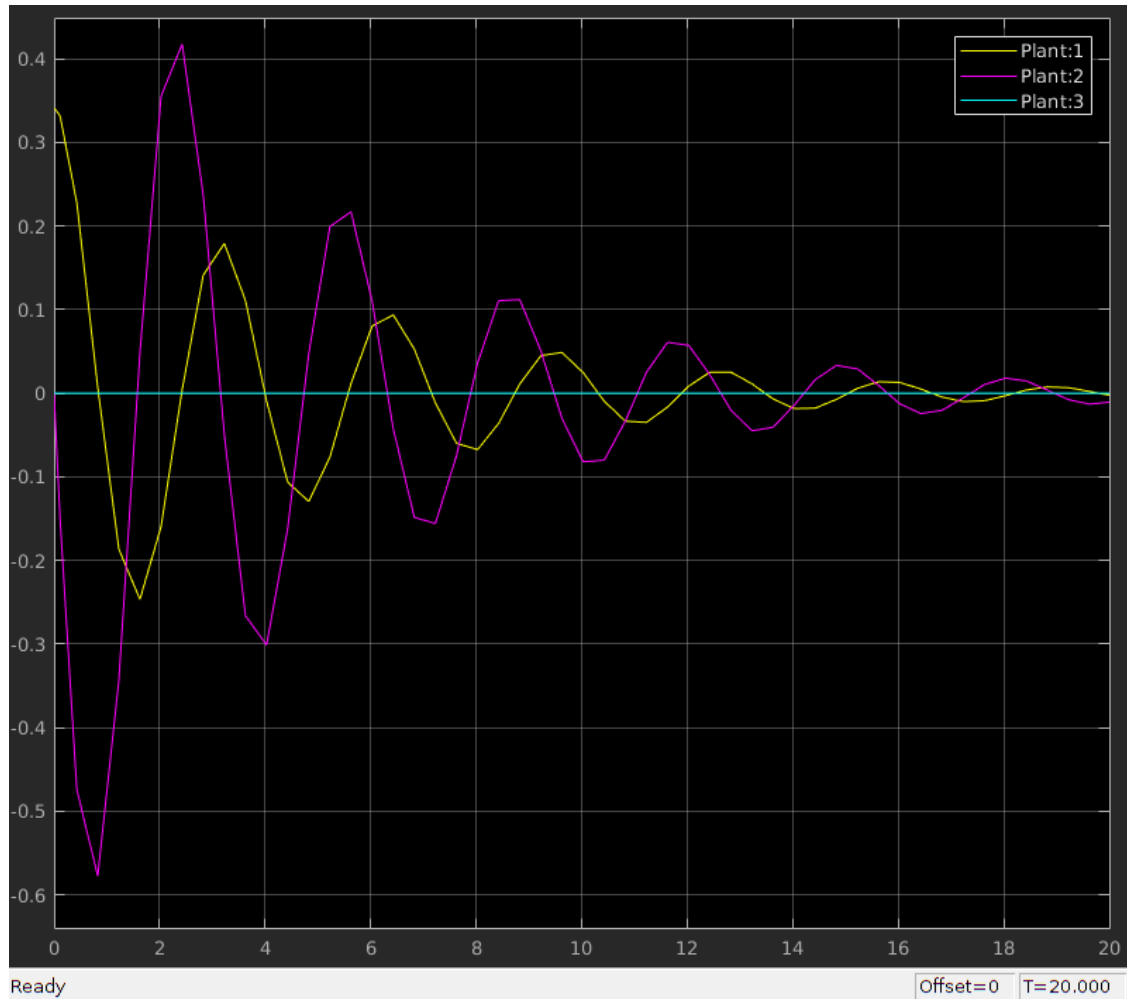
Note that the following constants result in a system that decays in ~ 17.5 seconds

```
A = [0 1 0; -4 -.4 40; 0 0 -4];
B = [0;0;2];
x0 = [0.34;0;0];
K = zeros(1,3);
open_system('reg_asf/u')
open_system('reg_asf/x')
results = sim("reg_asf.slx",'StopTime', '20');

tout = results.tout;
```

```
u = results.u;  
x = results.x;  
  
snapnow  
close_system('reg_asf');
```





## Problem 2b.1 (Bessel Pole Regulator)

```

plantPoles = eig(A)
Ts = 4.5;
spoles1 = s3/Ts;
K1 = place(A,B,spoles1);
K = K1;
fprintf('----- Bessel Poles -----\n\n');
asm(A,B,K1)
[del1_1,del2_1] = rb_regsf(A,B,K1,0)

results = sim("reg_asf.slx", 'StopTime', '6.75');
tout = results.tout;
u = results.u;
x = results.x;

plantPoles =

    -0.2000 + 1.9900i
    -0.2000 - 1.9900i

```

```
-4.0000 + 0.0000i

----- Bessel Poles -----

Upper gain margin for input #1 is 0.95 dB

Lower gain margin for input #1 is -30.1 dB

Phase margin for input #1 is 11 degrees

del1_1 =

    0.1152

del2_1 =

    0.1033
```

## Problem 2b.2 (SDPP/ADP Regulator)

Here we choose 1 SDPP and 2 ADP type regulator poles

```
adp = s1/Ts + imag(plantPoles(1))*1i;
spoles2 = [plantPoles(3) adp conj(adp)];
K2 = place(A,B,spoles2);
K = K2;
fprintf('----- SDPP/ADP Poles -----\n\n');
asm(A,B,K2)
[del1_2, del2_2] = rb_regsf(A,B,K2,0)

results = sim("reg_asf.slx", 'StopTime', '6.75');
tout = results.tout;
u = results.u;
x = results.x;

----- SDPP/ADP Poles -----

Upper gain margin for input #1 is 30.1 dB

Lower gain margin for input #1 is -30.1 dB

Phase margin for input #1 is 93 degrees

del1_2 =

    1.1978

del2_2 =
```

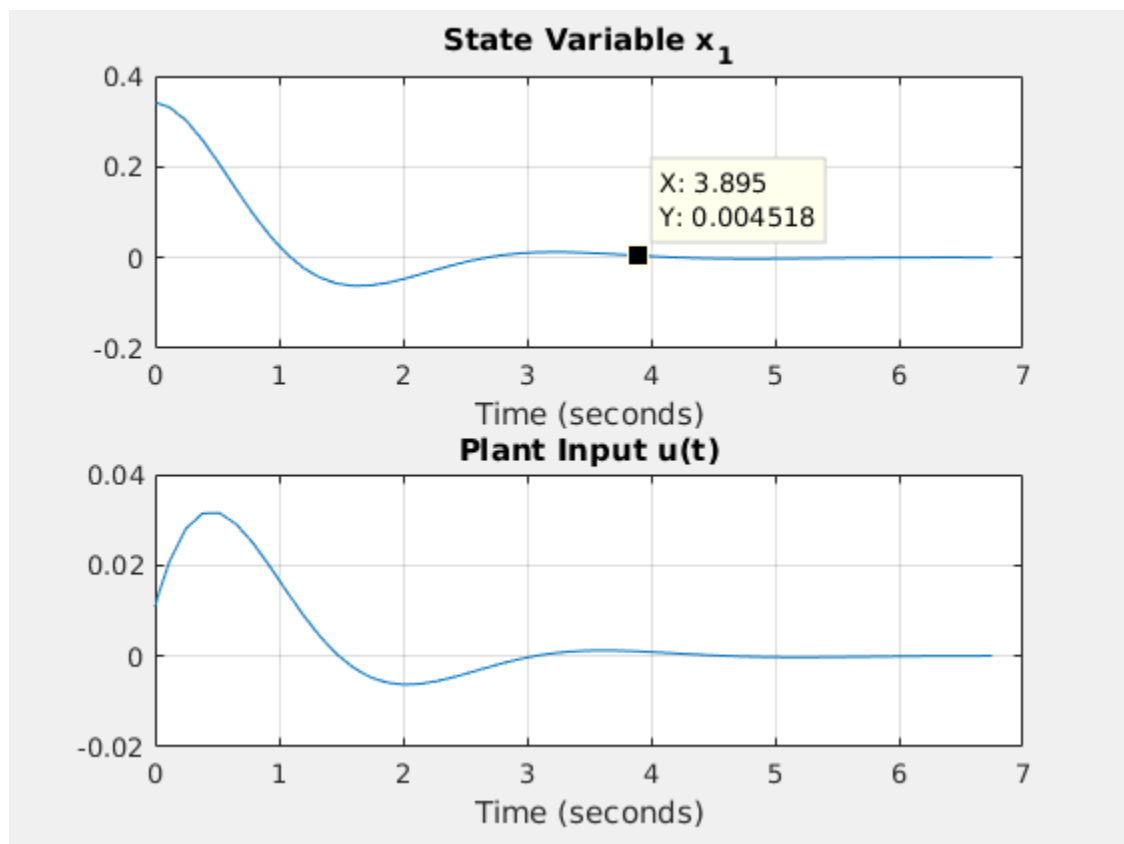
1

## Problem 2c (Graphing)

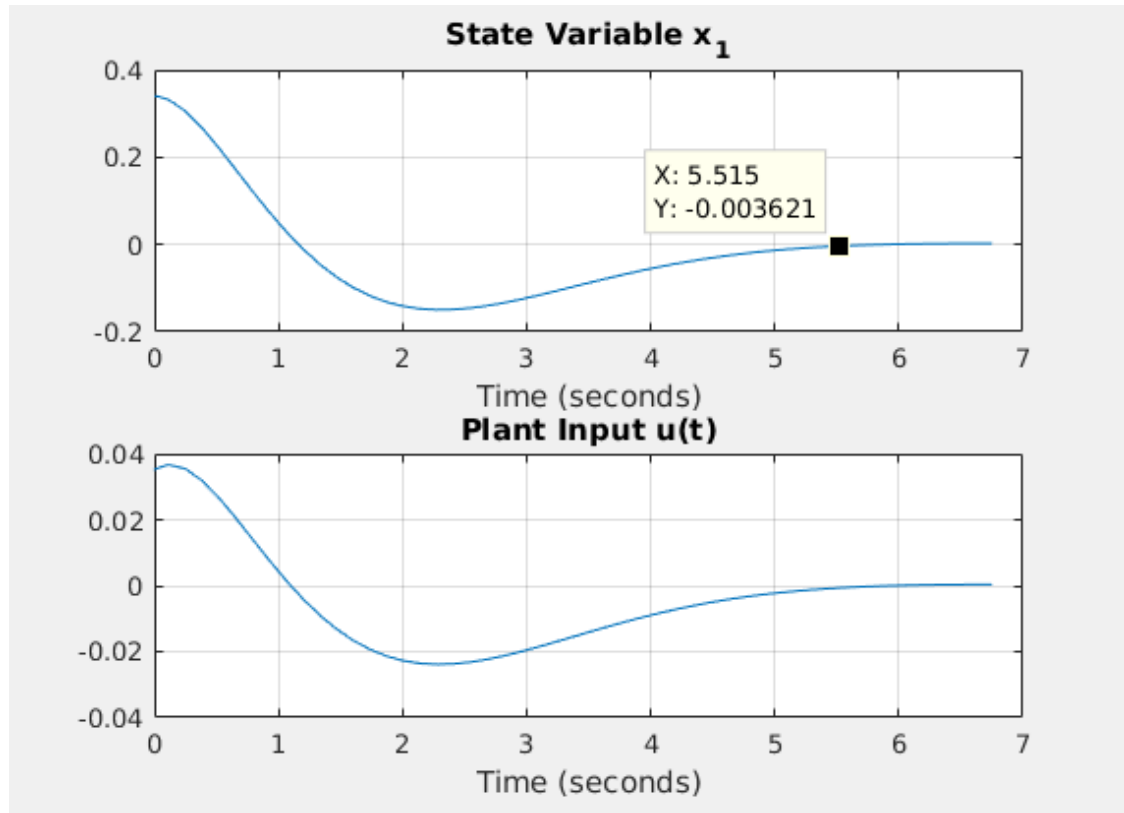
The Scaled Bessel Poles yield a system which decays to zero in approximately 5.5 seconds (1% Settling time)

The Selected Poles yield a system which decays to zero in approximately 3.85 seconds (1% Settling time)

```
open('BesselPoles2.fig')  
open('SelectedPoles2.fig')
```







## Problem 2d (Analysis)

The second set of poles yields far better results than the first set of tabulated poles. In fact, the robustness of the first system is completely unacceptable, see 2b.1, while the second system is very stable.

## Problem 3.1 (Tabulated Bessel Poles)

This system seems stable when we only look at the classical bounds. However when we look at the robustness bounds, the system is unusable.

```
A = [0 1 0 0; 23.1 0 0 -0.1189; 0 0 0 1; 0 0 0 -25];
B = [0; 12.5; 0; 26.33];
x0 = [0; 0.17; 0; 0];
```

```
Ts = 0.95;
```

```
plantPoles = eig(A);
```

```
sPoles1 = s4/Ts;
```

```
K1 = place(A,B,sPoles1);
```

```
K = K1;
```

```

load_system('reg_asf')
results = sim("reg_asf.slx", 'StopTime', '1.425');
tout = results.tout;
u = results.u;
x = results.x;

%Normalized Bessel Poles only
fprintf('----- Bessel Poles -----\n\n');
asm(A,B,K1)
[del1_1,del2_1] = rb_regsf(A,B,K1,0)

open('BesselPoles3x1.fig');
open('BesselPoles3x2.fig');

----- Bessel Poles -----

Upper gain margin for input #1 is 11.53 dB

Lower gain margin for input #1 is -4.64 dB

Phase margin for input #1 is 22 degrees

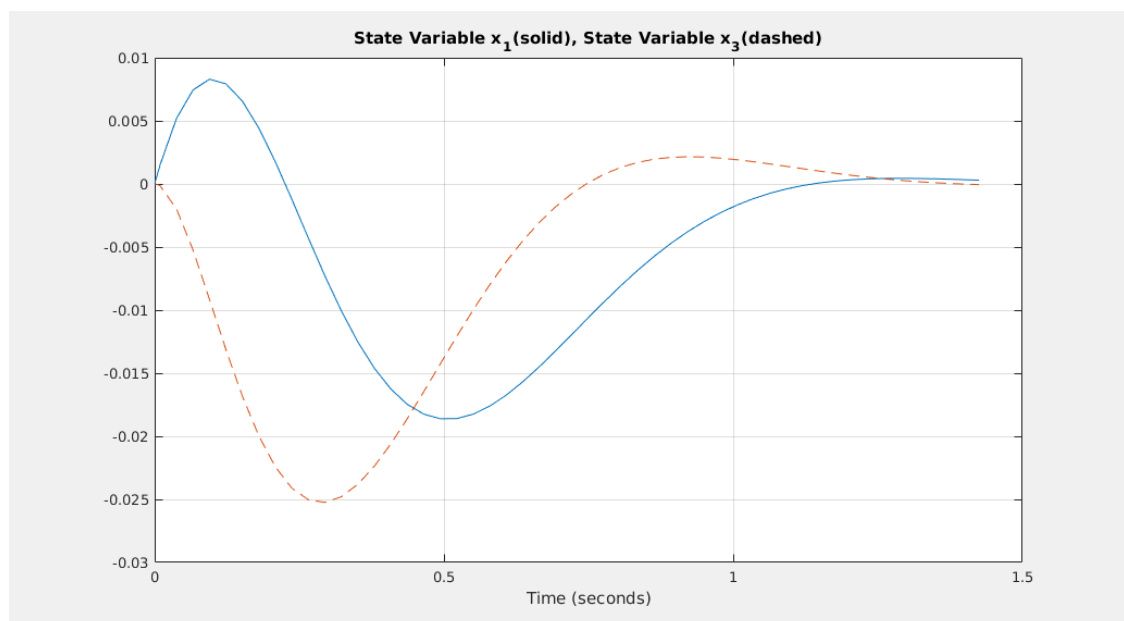
del1_1 =

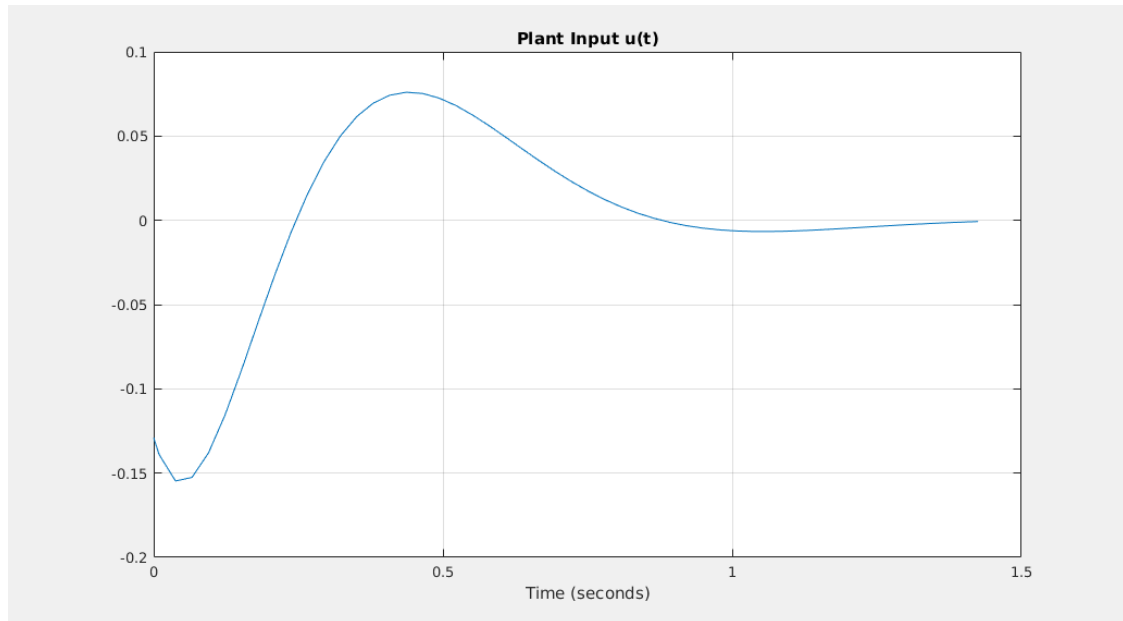
    0.3240

del2_1 =

    0.3693

```





## Problem 3.2 (Selected Poles)

This system is only slightly better when only the classical margins are considered, but it is the only acceptable one when we consider robustness bounds. The settling time is comparable.

```
sPoles2 = [plantPoles(4) s3/Ts];

K2 = place(A,B,sPoles2);

K = K2;

load_system('reg_asf')
results = sim("reg_asf.slx", 'StopTime', '1.425');
tout = results.tout;
u = results.u;
x = results.x;

% SDPP,ADP,SBP
fprintf('----- SDPP/ADP Poles -----\n\n');
asm(A,B,K2)
[del1_2, del2_2] = rb_regsf(A,B,K2,0)

open('SelectedPoles3x1.fig');
open('SelectedPoles3x2.fig');

----- SDPP/ADP Poles -----

Upper gain margin for input #1 is 30.1 dB

Lower gain margin for input #1 is -8.91 dB

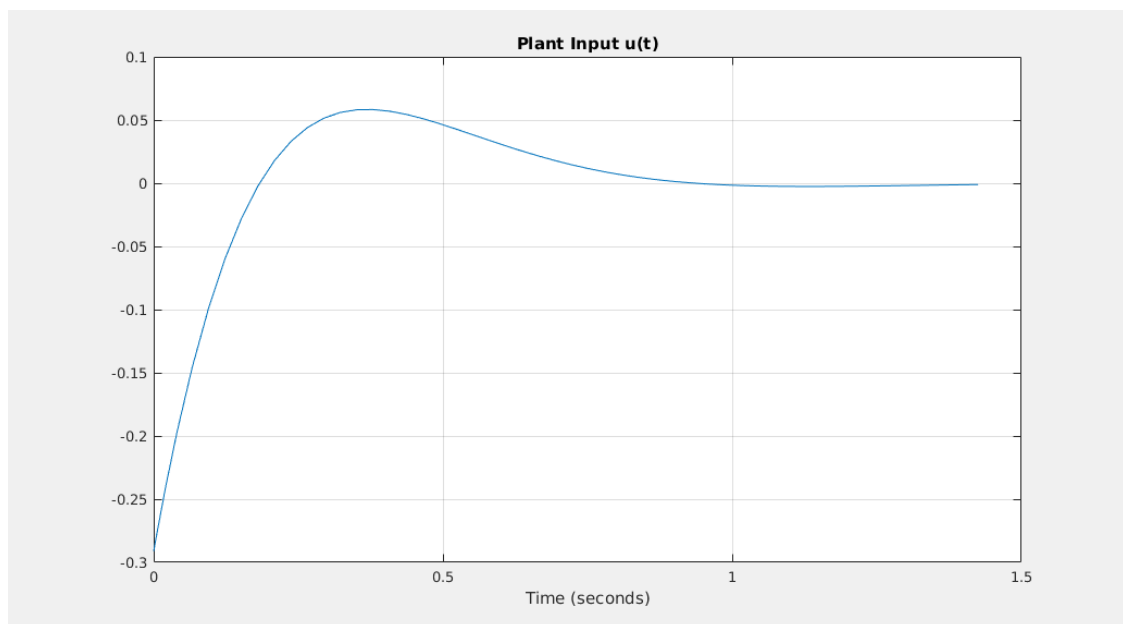
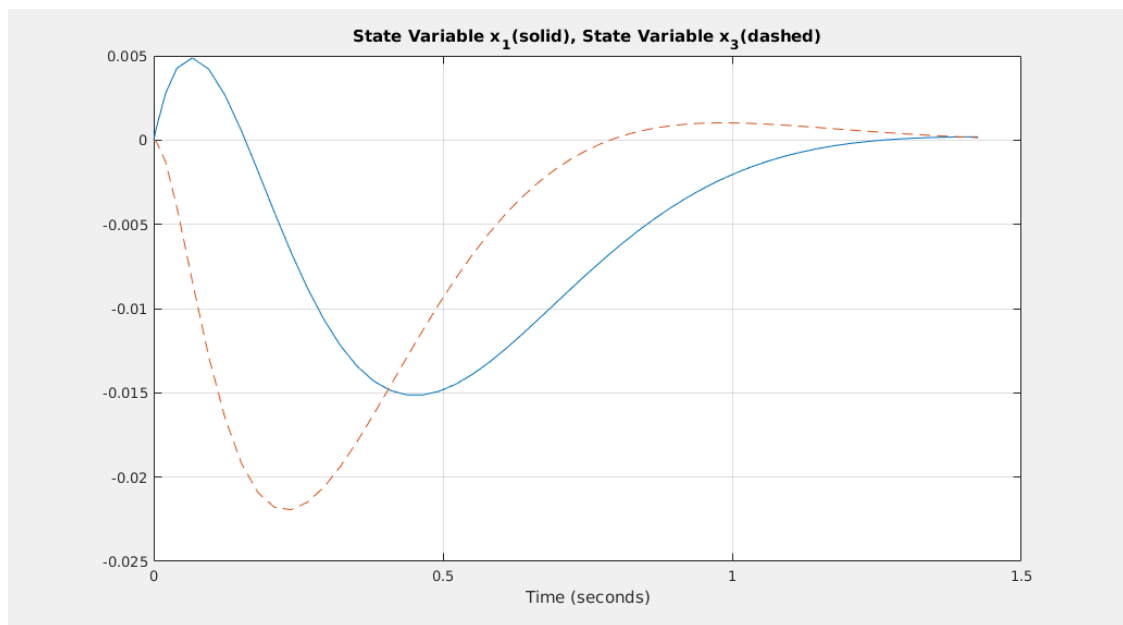
Phase margin for input #1 is 59 degrees
```

$\delta e11\_2 =$ 

0.5834

 $\delta e12\_2 =$ 

0.9761



## Problem 4a/b (MIMO System Regulation)

```
A = [1.38 -0.2077 6.715 -5.676; -0.5814 -4.29 0 0.675; 1.067 4.273
      -6.654 5.893; 0.048 4.273 1.343 -2.104];
B = [0 0; 5.679 0; 1.136 -3.146; 1.136 0];

Ts = 20;

sPoles = [-8.6635 -5.0572 -1.991 s1/Ts];

K1 = place(A,B,sPoles);

fprintf('----- Results using place() -----
\n\n');
[del1_1, del2_1] = rb_regf(A,B,K1,0)

clp1 = eig(A-B*K1)

fprintf('----- Results using rfbg() -----
\n\n');
[K2, del1_2, del2_2] = rfbg(A,B,sPoles,0)

clp2 = eig(A-B*K2)

----- Results using place() -----

del1_1 =

    0.0673

del2_1 =

    0.0717

clp1 =

   -8.6635
   -5.0572
   -0.2310
   -1.9910

----- Results using rfbg() -----

delta values:
0.4131    0.6834
0.5141    0.9697
0.5155    0.9801
0.5144    1.0000

K2 =
```

```
0.8563    0.2304    0.6265   -0.2464
-1.1126   -0.2568   -0.8055    0.3543
```

```
del1_2 =
```

```
0.5144
```

```
del2_2 =
```

```
1.0000
```

```
clp2 =
```

```
-0.2310
-1.9910
-5.0572
-8.6635
```

## Problem 4c (Analysis)

It is clear to see that the results given by place are not usable. The derived system is not robust, and should clearly not be tested in hardware. The second system is far more robust, due to the iterative nature of rfbg(). This system is ready for hardware testing.

It is noteworthy that the closed-loop poles for both systems are identical, but this is not enough to say anything about the feasibility of a hardware test.

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