Exercise 2
3D Computer Vision

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1 Properties of Rotation Matrices

1.1

A general rotation matrix is orthogonal. Let A denote a $n \times n$ matrix. A is orthogonal if its column and row vectors are orthonomal vectors. This means for column vectors $v_1, v_2, ..., v_n$ of A:

$$v_i^T v_j = \begin{cases} 1 \ if \ i = j \\ 0 \ if \ i \neq j \end{cases} \tag{1}$$

Therefore $A^T A = I$.
A^{-1} is defined as $AA^{-1} = I$.
$(A^T A)A^{-1} = IA^{-1} = A^{-1}$. Matrix multiplication is associative, so $(A^T A)A^{-1} = A^T(AA^{-1}) = A^T I = A^T$. Therefore we have $A^T = A^{-1}$.

1.2

The determinant describes how the volume of a region is changed when transformed by that matrix. Since a rotation won't change volume or orientation the determinant is 1.

2 Transformation Chain

To link coordinates of points in 3D external space with their coordinates in the image, a few transformations have to be applied. The location and orientation of the camera reference frame in regard to a known "world" reference frame are known as extrinsics R and T. To transform coordinates from the world coordinate frame to the camera coordinate frame the following has to be applied:

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R & T \\ 0_3^T & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \tag{2}$$

Since these coordinates are in the camera coordinate frame, they need to be transformed into pixel coordinates. This is done with the help of the intrinsics K:

$$K[I_3|0_3] = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3}$$

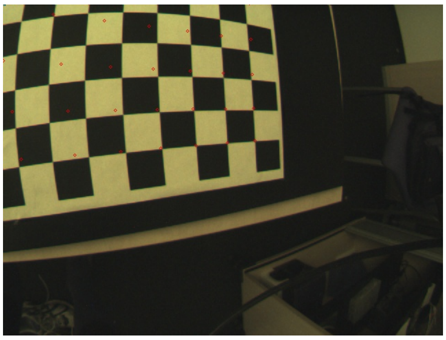
K, R and T can be combined to P:

$$P = K[I_3|0_3] \begin{bmatrix} R & T \\ 0_3^T & 1 \end{bmatrix} \tag{4}$$

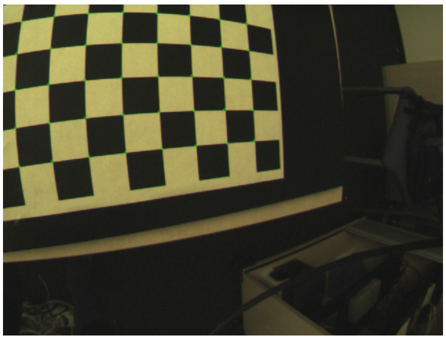
So we get $X_p = PX_w$
Homogenous coordinates allow to represent translations, rotations, scaling with matrices.

2.1 Implementation

The result of projecting the points can be seen in Figure 1. The image without correction can be seen in Figure 1a, and in Figure 1b after compensating for distortion.



(a)



(b)

Figure 1: Projected Points with and without distortion correction