Investigation of the parameters of an optimised biconvex lens

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December 14, 2015

Abstract

The optimal performance of a general simple lens consisting of two refracting surfaces was simulated. The biconvex lens was found to perform best in focusing a collimated beam of light rays at a particular point. The resolution of the lens was quantified by the root mean square radius of the rays from the optical axis at the focus. The curvatures of the lens were optimised with respect to variations in the lens's focal length and the wavelength and diameter of the incident beam. Decreasing the focal length or the wavelength or increasing the diameter of the beam led to an increase in spherical aberration so that the rays could be resolved beyond the diffraction limit at the focus.

1 Introduction and Theory

Rays form a model of light which proves to be very useful in calculating the path of light waves in most of the typical experimental set-ups. Rays are vertical vectors to the wavefronts of light waves and point to the direction of energy propagation. [1] The dimensions of all typical optical elements used in experiments are orders of magnitudes larger than the wavelengths of light waves. Therefore, it is possible to use ray tracing techniques as a valid approximation to calculate the light paths in a particular set-up. [2]

Typical optical elements interacting with rays include simple lenses which transmit light rays by affecting their focus. They are made by a single material and essentially consist of two consecutive spherical refracting surfaces. An incident ray is refracted by both surfaces before exiting the element. Generally, the refractive index n of the lens depends on the wavelength of the ray according to the empirical Sellmeier equation [4] which is monotonically decreasing in the visible spectrum ranging between $1.531 - 1.513$ which suggests only slight variations with different wavelengths. Spherical surfaces are inexpensive to construct, but cannot focus incident rays perfectly due to the geometrical phenomenon of spherical aberration. [3] Hence, they are combined in forming a lens, so that one surface corrects the other. Figure 1 illustrates a biconvex lens which focuses all incident rays almost at a specific point.

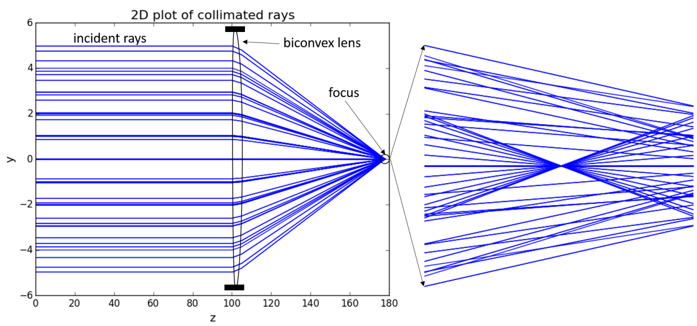

Figure 1: An optimised biconvex lens performing at wavelength $\lambda = 588\ nm$ corresponding to a refractive index $n \approx 1.517$, with an incident cylindrical beam of diameter $D = 10\ mm$. z is the optical axis. There is still some aberration at the focus. Distances are in mm.

It is easy to calculate paths for a single pair of a ray and a lens, but for larger systems consisting of a beam of many rays, computational methods are necessary. Simple optical elements and their interactions with ray objects were simulated using Python. The elements represent lenses made by borosilicate crown glass which is a common material [4]. The aim of the simulation was to examine the performance of lenses constructed from two refracting surfaces. The performance of the lenses was optimised and further investigated by varying relevant parameters including the wavelength and the diameter of the incident beam of rays and the focal length of the lens.

2 Method

Rays and optical elements were represented by classes with methods that allows physically meaningful interaction between the two. The optical elements most relevant to the optimisation are objects of lenses with two refracting surfaces which were allowed to be either planar or spherical with any positive or negative curvature C_1 and C_2 respectively for the left and the right surface. The sign convention indicates that $C_1 > 0 > C_2$ in figure 1 where a biconvex lens is depicted.

As seen in figure 1, the general approach involved generating a bundle of collimated light rays at the input plane $z = 0$. They then travelled parallel to the optical axis and were propagated by the lens so as to focus on a fixed focal point, which in the figure corresponds at the output plane $z \approx 178.1$ mm and thus to a focal length $f = 75.6$ mm. Since there is still spherical aberration at the focal point, the root mean square distance on the output plane (rms radius) of the bundle from the optical axis was used as a quantitative metric for the spread. The positions of the surfaces on the optical axis and therefore their separation $s = 5$ mm remained constant so that the two curvatures of the lens can be optimised to give the smallest rms radius of the bundle on the output plane. This method establishes the optimal lens with a particular focal length.

In order to ensure that the optimising function would find a pair of curvatures that minimises the rms radius, a particular technique was used. Firstly, an initial estimate of the order of $C_1 = 0.005$ mm^{-1} was given and the other curvature was calculated by the lensmaker's equation [3, 4]

$$f^{-1} = (n-1)[C_1 - C_2 + (1 - n^{-1})sC_1C_2] \tag{1}$$

where the symbols are consistent with above definitions. C_1 was then optimised with respect to the rms radius to give a pair of curvatures which are close to optimal. The next step was to use this pair as an initial guess for the optimisation of both curvatures so as to get the final optimised pair. It is worth noting, that the third term in the equation is very small for small curvature values.

It is important to check if a lens can focus beyond the corresponding diffraction scale defined as $\frac{\lambda f}{D}$, where λ is the wavelength of the incident light, f is the focal length of the surface and D is the diameter of the beam. Therefore, variations in any of these parameters were investigated so as to compare the relative change between the rms radius and the diffraction scale.

3 Results

The optimisation of the lens is primarily important because a lens with a specific focal length is required for an experiment. Therefore, figure 2 illustrates the trend of the optimisation for a reasonable range of focal lengths ($10 - 140$ mm) which may be useful in laboratory set-ups.

Looking at the upper graphs of both subfigures, a consistent trend in the curvatures can be observed. Probably the most important note is that the optimal lens is biconvex at any focal length. At low focal length values the magnitudes of the curvatures drop relatively fast, but then, after about 30 mm they tend to 0 asymptotically. Therefore, at very high focal lengths, the optical lens can be approximated as a planoconvex with a convex left surface and a planar right surface, since C_1 drops less fast. The

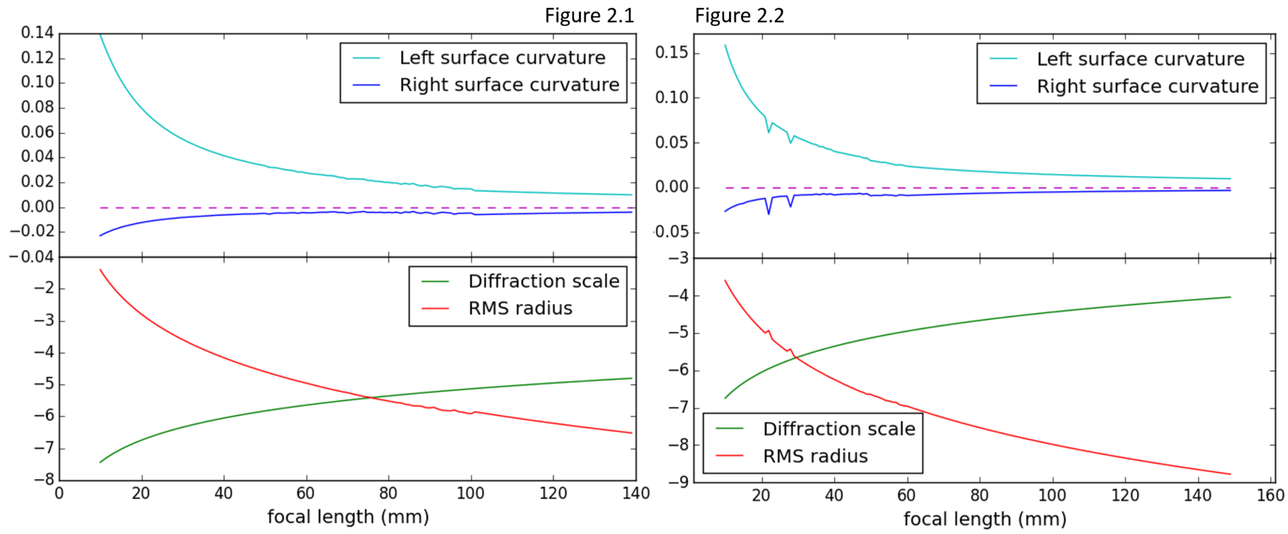

Figure 2: The diameter of the incident beam is $D = 10\ mm$ and $5\ mm$ in subfigures 2.1 and 2.2 respectively. The wavelength is constant at $\lambda = 588\ nm$. The trend of the curvatures and the trend of the logarithm of the rms radius at the focal point is seen in the upper and lower graphs respectively.

only difference between a beam with lower diameter, $5\ mm$ instead of $10\ mm$, in terms of the curvature trend, is that at very low focal lengths the lens is more curved as seen by the slightly different scale of the vertical axes, but this is smoothed out immediately in the graph. The two spikes in subfigure 2.2 mean that the optimisation function did not manage to find minima that are consistent with the neighbouring points, which is a consequence of the limitations of the numerical approximations used to investigate the system.

The lower graphs illustrate the relation between the diffraction scale and the rms radius for varying focal length while all other parameters are kept constant. The natural logarithms of these quantities are plotted here for greater readability of the graphs, but this does not affect the trends since the logarithmic function is monotonic. At low focal lengths the rms radius is higher than the diffraction scale meaning that the rays falling on the output plane can be resolved from one another. At high focal lengths the rays cannot be resolved, since the diffraction scale always increases linearly with focal length, while the rms drops lower. This is a similar situation with two stars in space that are too close to one another that cannot be resolved on Earth. The critical focal length at which the rays can marginally be resolved changes according to the beam diameter. For diameters of $10\ mm$ and $5\ mm$, the critical focal length is $75.6\ mm$ and $24.0\ mm$ respectively. This is sensible since a thinner beam is more easily focused on a point.

In order to investigate the impact of the beam diameter in more depth, the focal length can now be kept constant, so that the optimisation runs for a varying beam diameter. Figure 3 depicts the trends for a reasonable range of $2 - 16\ mm$.

The upper graphs clearly suggest that the difference between the curvatures of the two lens surfaces is always the same for a constant focal length. The difference is lower in subfigure 3.2 which corresponds to a higher focal length. Both facts imply that the optimisation agrees with the lensmaker's equation. The curvatures remain within a small range of values while they have a minimum after the point at which the rms radius equals to the diffraction scale which may be due to the geometrical properties of a biconvex lens. The lower plots indicate that at a smaller focal length, the critical value of beam diameter for the rays to be marginally resolved on the output plane is lower. Therefore, if the focal point needs to be close to the lens, a thinner beam is required so that the rays can focus well. The critical diameter value of $5\ mm$ at $f = 24.0\ mm$ in subfigure 3.1 agrees with subfigure 2.2 which has a critical focal length value of $24.0\ mm$ at $D = 5\ mm$.

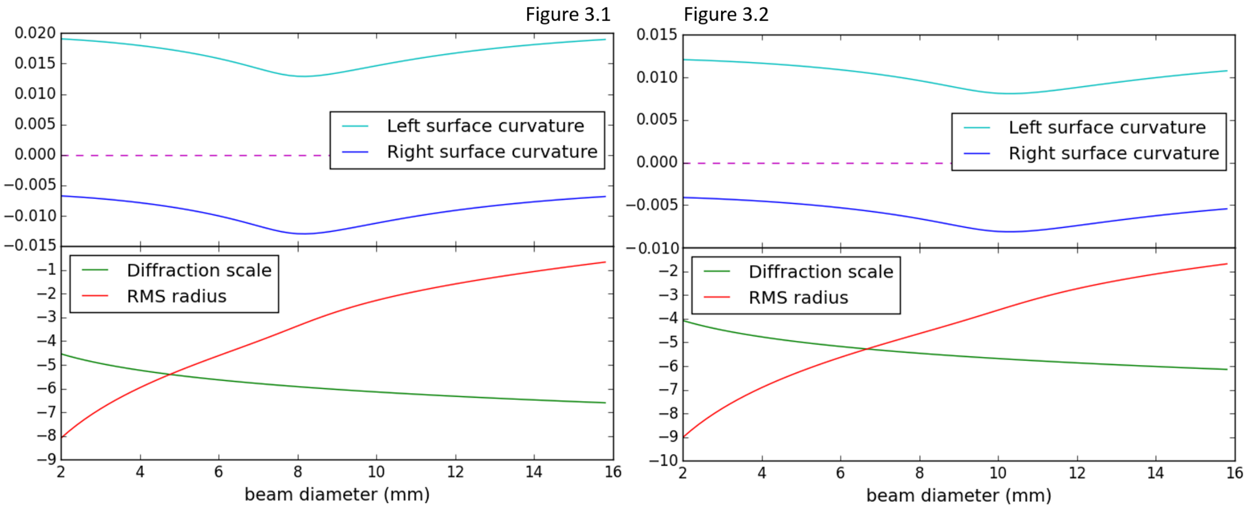

Figure 3: The focal length of the lens is $f = 24.0\ mm$ and $120\ mm$ in subfigures 3.1 and 3.2 respectively. The wavelength is constant at $\lambda = 588\ nm$. The trend of the curvatures and the trend of the rms radius at the focal point is seen in the upper and lower graphs respectively.

Finally, figure 4 illustrates the behaviour of the optical system when the wavelength of the incident beam varies. The range of wavelengths studied corresponds to the visible spectrum. The diffraction scale is proportional to the wavelength and this is the main reason why the resolution of the lens is not expected to be constant with respect to wavelength. However, it is more interesting to observe the increasing trend of the rms radius as light colour changes from blue to red. The change is small compared to the changes caused by variations of the beam diameter or the focal length. This can be seen by the scale of the rms axis and it is because the volatility of the refractive index of the lens due to wavelength variations is small as stated in the discussion of the Sellmeier equation. The critical wavelength value of 588 nm at $f = 75.6\ mm$ in figure 4 agrees with subfigure 2.1 which has a critical focal length value of $75.6\ mm$ at $\lambda = 588\ nm$.

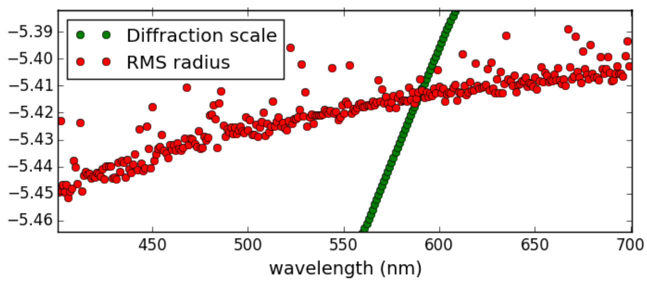

Figure 4: This is a scatter plot of the rms radius varying with wavelength. The focal length of the lens is $f = 75.6\ mm$ and the beam diameter is $D = 10\ mm$.

4 Conclusion

The aim of the simulation was to find the optimal curvature combination of a lens consisting of two refracting surfaces and explore the dependence of the lens's resolution on relevant parameters. The trend of the curvatures at varying focal length as well as the effect of the incident beam diameter and wavelength to the resolution of the system was examined.

A biconvex lens was found to generally be better at focusing a collimated beam of light at a particular point, although it can be approximated by a planoconvex at large distances. The incident beam diameter seemed to substantially affect the resolution of the lens, while the wavelength affected the resolution by a less significant amount. The numerical approximations used for the optimisation raise some outliers, especially in the wavelength graph. This can be improved by dynamically setting correct bounds in the optimisation process so that all optimised points are consistent with the neighbouring points. The optical system can also be investigated with regard to the thickness s of the lens and in the case of incident rays that do not travel parallel to the optical axis.

References

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