

# Quantum teleportation

## 1) Initial state

$$\hookrightarrow |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|P_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\hookrightarrow |\psi\rangle P_{00} = (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

2) Alice apply a CNOT gate with 1<sup>st</sup> qubit as control and her half of ~~EPR~~ EPR as target

$$\hookrightarrow \text{CNOT}(0,1)(|\psi\rangle P_{00}) = \text{Initial state}$$

$$\text{CNOT}(0,1) \left[ \frac{1}{\sqrt{2}} (\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)) \right]$$

$$\hookrightarrow \frac{1}{\sqrt{2}} [\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)]$$

3) Alice apply a H in her first qubit

$$\hookrightarrow \frac{1}{2} [\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)] \quad \text{Eq(1)}$$

$$\hookrightarrow \frac{1}{2} \left[ \begin{aligned} &|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\beta|0\rangle + \alpha|1\rangle) + \\ &|10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle) \end{aligned} \right] \quad \text{Eq(2)}$$

$$\alpha(|0\rangle + |1\rangle)(|0\rangle|0\rangle + |1\rangle|1\rangle) \rightarrow \text{part 1 of Eq. 1}$$

$$\alpha|0\rangle|0\rangle \otimes |0\rangle + \alpha|0\rangle|1\rangle \otimes |1\rangle \\ + \alpha|1\rangle|0\rangle \otimes |0\rangle + \alpha|1\rangle|1\rangle \otimes |1\rangle$$

$$\beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle) \rightarrow \text{part 2 of Eq. 1}$$

$$\beta|0\rangle|1\rangle \otimes |0\rangle + \beta|0\rangle|0\rangle \otimes |1\rangle$$

$$\bullet - \beta|1\rangle|1\rangle \otimes |0\rangle - \beta|1\rangle|0\rangle \otimes |1\rangle$$



$$|0\rangle \otimes (\alpha|0\rangle|0\rangle + \alpha|1\rangle|0\rangle + \beta|0\rangle|1\rangle - \beta|1\rangle|1\rangle)$$

$$|1\rangle \otimes (\alpha|0\rangle|1\rangle + \alpha|1\rangle|1\rangle + \beta|0\rangle|0\rangle - \beta|1\rangle|0\rangle)$$

$$\cancel{|0\rangle|0\rangle} \otimes \alpha|0\rangle + \cancel{|1\rangle|0\rangle} \otimes \alpha|0\rangle + \cancel{|0\rangle|1\rangle} \otimes \beta|0\rangle \\ + |1\rangle|1\rangle \otimes [-\beta|0\rangle]$$

$$\cancel{|0\rangle|1\rangle} \otimes \alpha|1\rangle + |1\rangle|1\rangle \otimes \alpha|1\rangle + \cancel{|0\rangle|0\rangle} \otimes \beta|1\rangle + \cancel{|1\rangle|0\rangle} \otimes -\beta|1\rangle$$

$$|0\rangle|0\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) + |0\rangle|1\rangle \otimes (\beta|0\rangle + \alpha|1\rangle)$$

$$+ |1\rangle|0\rangle \otimes (\alpha|0\rangle - \beta|1\rangle) + |1\rangle|1\rangle \otimes (-\beta|0\rangle + \alpha|1\rangle)$$

Eq. (2)