

B. Tech. Degree III Semester Examination November 2019

CE/CS/EC/EE/IT/ME/SE AS 15-1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES

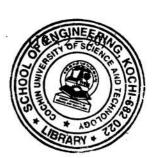
(2015 Scheme)

Time: 3 Hours

Maximum Marks: 60

PART A (Answer ALL questions)

- I. (a) By reducing to Echelon-form find the rank of the matrix



- (b) Prove that every square matrix and its transpose have same eigen values.
- (c) Show that the vectors (1,2,3), (3,-2,1), (1,-6,-5) are linearly dependent and find the relation connecting them.
- (d) Explain basis and dimension of a vector space with example.
- (e) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
- (f) Express $f(x) = |x|, -\pi < x < \pi$ as a Fourier series.
- (g) Explain periodic function with two examples.
- (h) Find $L [te^{-t} \cos ht]$.
- (i) Find $L^{-1} \left[\log \frac{s(s+1)}{s^2+1} \right]$.
- (j) Prove that $\beta(m,n) = \beta(n,m)$.

PART B

 $(4 \times 10 = 40)$

- II. (a) Test for consistency and hence solve x-2y+3z=2; 2x-3z=3; x+y+z=0.
 - (b) State Cayley-Hamilton and verify it for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

OF

- III. (a) Diagonalize $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ hence find A^6 .
 - (b) Reduce the quadratic form 2xy + 2yz + 2zx into canonical form.

(P.T.O.)

- IV. (a) Give an example of a vector space with dimension n. Justify your answer.
 - (b) Explain inner product space with an example.

OR

- V. (a) Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis for the subspace V of \mathbb{R}^3 spanned by $V_1 = (0,1,2)$ $V_2 = (1,1,2)$ $V_3 = (1,0,1)$.
 - (b) Find k so that u = (1, 2, k, 3) and v = (3, k, 7, -5) in \mathbb{R}^4 are orthogonal.
- VI. (a) Obtain the half range sine series of the function f(x) = kx(x-l) in $0 \le x \le l$.
 - (b) Using Fourier sine integral for $f(x) = e^{-ax}(a > 0)$ show that $\int_{0}^{\infty} \frac{\lambda \sin \lambda x}{\lambda^{2} + a^{2}} d\lambda = \frac{\pi}{2} e^{-ax}.$

OR

- VII. (a) Obtain a Fourier expansion for $\sqrt{1-\cos x}$ in the interval $-\pi < x < \pi$.
 - (b) Find the Fourier sine transform of $f(x) = \begin{cases} x & \text{in } 0 < x < 1 \\ 2 x & \text{in } 1 < x < 2 \\ 0 & \text{in } x > 2 \end{cases}$
- VIII. (a) State convolution theorem and hence find $L^{-1} \left[\frac{s^2}{\left[s^2 + a^2 \right] \left[s^2 + b^2 \right]} \right]$.
 - (b) Solve (using Laplace transform) $y'' + 2y' - 3y = \sin t$ given y = 0, y'(0) = 0 when t = 0.

OR

- IX. (a) Prove that $\beta(m,n) = \frac{\overline{|m|n}}{\overline{|(m+n)}}$.
 - (b) Evaluate $\int_{0}^{\infty} \frac{e^{-2t} e^{-3t}}{t} dt$.