BTS – III(15)–11.16-1508 Reg. No. | | | | | | | | |

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B.Tech. Degree III Semester Examination November 2016

CE/CS/EC/EE/IT/ME/SE
AS 15–1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES
(2015 Scheme)

Time : 3 Hours Maximum Marks : 60

PART A
(Answer *ALL* questions)

$10 \times 2 = 20$

I. (a) Prove that every square matrix and its transpose have same eigen values.

(b) Explain when a linear system of equations has unique solution, no solution and more than one solution.

(c) If x, y, z are linearly independent vectors of a vector space V prove that $x,\ x+y,\ x+y+z$ are also linearly independent.

(d) Define basis and dimension of a vector space with example.

(e) Express $f(x) = x^2,\ -\pi < x < \pi$ as a Fourier series.

(f) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases}$

(g) Find the half range cosine series for $f(x) = x.$ is $(0, \pi)$ and deduce the sum $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$

(h) Find the Laplace transform of tSin3tCos2t.

(i) Find the Laplace transform of $\log\left[\frac{S(S+1)}{S^2+1}\right]$.

(j) Find the Laplace transform of periodic function.

PART B

$4 \times 10 = 40)$

II. (a) State Cayley-Hamilton theorem and verify it for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}.

(b) Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ hence find A^4.

OR

III. (a) Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to a Canonical form by orthogonal reduction.

(b) Find the value of λ *and* μ for which the equations $2x+3y+5z=9$; $7x+3y-2z=8$; $2x+3y+\lambda z=\mu$ have (i) no solution (ii) unique solution (iii) more than one solution.

IV. (a) Define vector space, subspace and give an example of a vector space having a subset which is a subspace and a subset which is not a subspace. Justify your answer.

(b) Let W be any plane is R^3 passing through the origin. Verify whether W is a subspace of R^3.

OR

V. (a) Apply Gram-Schmidt orthogonalization process to find an orthogonal basis and an orthonormal basis for the subspace U of R^4 spanned by $V_1=(1,1,1,1)$, $V_2=(1,2,4,5)$, $V_3=(1,{}^-3,{}^-4,{}^-2)$

(b) Let T be a map from R^3 to R defined by $T(x,y,z)=x^2+y^2+z^2$. Verify whether T is a linear transformation.

VI (a) Find the Fourier series for $f(x)$, given $f(x)=\begin{cases}0 \; in -1<x<0\\ 1 \; in \; 0<x<1\end{cases}$ and $f(x+2)=f(x)$ for all x.

(b) Using Fourier integral for $f(x)=e^{-ax}(a>0)$ show that $\int_o^\infty \frac{\lambda \, Sin\,\lambda x}{\lambda^2+a^2}d\lambda=\frac{\pi}{2}e^{-ax}$.

OR

VII. (a) Find the Fourier transform of $f(x)=\begin{cases}1-x^2 \; in|x|\le l\\ 0 \quad in|x|>l\end{cases}$. Hence prove that

$$\int_o^\infty \frac{SinS-S\cos S}{S^3}\cos\frac{s}{2}\,ds=\frac{3\pi}{16}.$$

(b) Find the half range sine series for the function $f(x)=\begin{cases}x-1 & 0\le x\le 1\\ 1-x, & 1\le x\le 2\end{cases}$

VIII. (a) State convolution theorem and use it to find the inverse Laplace transform of $\frac{1}{\left(S^2+4\right)^2}$.

(b) Solve (using transform) $y''+2y'-3y=\sin t,$ *given* $y=0,\; y'(0)=0$ *when* $t=0$.

OR

IX. (a) Determine y which satisfies the equation $\frac{dx}{dt}+2y+\int_o^t y\,dt=2\cos t,\; y(0)=1$.

(b) Prove that $\beta(m,n)=\frac{\overline{|m}\,\overline{|n}}{\overline{|(m+n)}}$.

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