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## B.Tech. Degree III Semester Examination November 2017

CE/CS/EC/EE/IT/ME/SE

**AS 15-1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES**  
(2015 Scheme)

Time : 3 Hours

Maximum Marks : 60

**PART A**  
(Answer *ALL* questions)

(10 × 2 = 20)

- I. (a) Define rank of a matrix and hence find the rank of  $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$ .
- (b) Find the eigen values and eigen vector of  $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ .
- (c) Check whether the vectors  $(1, 3, 2)$ ,  $(5, -2, 1)$ ,  $(-7, 13, 4)$  are linearly dependent if so find the relation connecting them.
- (d) Verify whether the collection of all polynomials of degree  $n$  form a vector space or not under usual addition and scalar multiplication of Polynomials. Justify.
- (e) Show that the Fourier series for  $f(x) = x, -\pi < x < \pi$  is given by
- $$f(x) = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin nx}{n}$$
- (f) Find the Fourier cosine transform of  $5e^{-2x} + 2e^{-5x}$ .
- (g) Obtain the half range sine series of the function  $f(x) = kx(x-l)$  in  $0 \leq x \leq l$ .
- (h) Find the Laplace transform of  $t \cos^3 t$ .
- (i) Find the inverse Laplace transform of  $\log \left[ \frac{s^2 + a^2}{s^2 - b^2} \right]$ .
- (j) Using convolution theorem, find the inverse Laplace transform of  $\frac{s}{(s^2 + a^2)^2}$ .

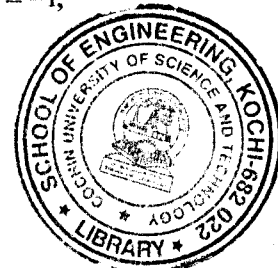
**PART B**

(4 × 10 = 40)

- II. (a) Test for consistency and solve  $4x - 2y + 6z = 8$ ;  $x + y - 3z = -1$ ;  
 $15x - 3y + 9z = 21$ .

- (b) Diagonalize the matrix  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$ .

OR



(P.T.O.)

- III. (a) Using Cayley-Hamilton theorem find the inverse of  $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ .
- (b) Reduce the quadratic form  $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$  to Canonical form.
- IV. (a) Suppose the vectors  $u, v, w$  are linearly independent. Show that the vectors  $u + v, u - v, u - 2v + w$  are also linearly independent.
- (b) Let  $V$  be the vector space of  $2 \times 2$  matrices. Let  $W$  be the subspace of symmetric matrices. Show that  $\dim W = 3$ , by finding a basis of  $W$ .

**OR**

- V. (a) Suppose the mapping  $F: R^2 \rightarrow R^2$  is defined by  $F(x, y) = (x + y, x)$ . Show that  $F$  is a linear transformation.
- (b) Let  $S$  be a subset of an inner product space  $V$ . Then prove that the orthogonal complement of  $S$  is a subspace of  $V$ .
- VI. (a) Find the Fourier series for  $f(x) = x^2$  in  $-1 < x < 1$ .
- (b) Express the function  $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$  as a Fourier integral. Hence

evaluate  $\int_0^\infty \frac{\sin \lambda \cos \lambda x \, d\lambda}{\lambda}$ .

**OR**

- VII. (a) Find the Fourier sine transform of  $e^{-x}$ ,  $x \geq 0$ . Hence evaluate  $\int_0^\infty \frac{x \sin mx}{1 + x^2} dx$ .
- (b) Show the Fourier transform of  $f(x) = e^{-\frac{x^2}{2}}$  is  $e^{-\frac{s^2}{2}}$ .

- VIII. (a) Solve (using Laplace transform)  
 $y'' + y' = t^2 + 2t$ ,  $y(0) = 4$ ,  $y'(0) = -2$ .
- (b) Find the Laplace transform of periodic function.

**OR**

- IX. (a) Solve  $\frac{dy}{dt} + 3y + 2 \int_0^t y \, dt = t$  for which  $y(0) = 0$ .
- (b) Using Laplace transform evaluate  $\int_0^\infty \frac{e^{-2t} - e^{-3t}}{t} dt$ .

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