BTS–III(S)–04.18–0782 Reg. No. | | | | | | | | |

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B.Tech. Degree III Semester Supplementary Examination April 2018

CE/CS/EC/EE/IT/ME/SE
AS 15 -1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES
(2015 Scheme)

Time : 3 Hours Maximum Marks : 60

PART A
(Answer *ALL* questions)

$(10 \times 2 = 20)$

I. (a) Prove that the eigen values of an idempotent matrix are either zero or unity.

(b) Determine the values of λ for which the following set of equations may possess non trival solution $3x_1 + x_2 - \lambda x_3 = 0$, $4x_1 - 2x_2 - 3x_3 = 0$, $2\lambda x_1 + 4x_2 + \lambda x_3 = 0$.

(c) Find a non zero vector **w** that is orthogonal to $u = (1,2,1)$ and v = (2,5,4) in R^3.

(d) V=R^n and W=$\{(x_1, x_2 ......x_n) | x_1 x_2 = 0\}$. Is W a subspace of V?

(e) Define half range sine and cosine series.

(f) Using Fourier sine integral show that

$$\int_0^\infty \frac{1-\cos \pi\omega}{\omega} \sin \omega x \, d\omega = \begin{cases} \frac{\pi}{2}, 0 < x < \pi \\ 0, x > \pi \end{cases}.$$

(g) Express $f(x) = \frac{x}{2}$ as a Fourier series in the interval $-\pi < x < \pi$.

(h) Find inverse Laplace transform of $\cot^{-1}(s/2)$.

(i) Find Laplace transform of $\frac{1-e^t}{t}$.

(j) Define unit step function and find Laplace transform of unit step function.

PART B

$(4 \times 10 = 40)$

II. (a) Test for consistency and solve $2x + y - z = 0$, $2x + 5y + 7z = 52$, $x + y + z = 9$.

(b) Find eigen values and eigen vectors of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

OR

III. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and hence find A^{-1}.

(b) Diagonalize $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and find the modal matrix.

(P.T.O.)

IV. (a) Define linear transformation and check whether $T:R^2 \to R$ defined by $T(x,y) = xy$ is linear.

(b) Let V be the vector space of polynomials f(t) with inner product $< f, g >= \int_{-1}^{1} f(t)g(t)dt$. Apply the Gram Schmidt orthogonalization process to $\{1, t, t^2, t^3\}$ to find an orthogonal basis.

OR

V. (a) Define basis and dimension. Also find the value of λ for which the set of vectors $(1, \lambda, 5), (1, 0, 1)$ and $(2, 0, 1)$ of R^3 form a basis.

(b) Show that (1,1,2), (1,2,5), (5,3,4) are linearly dependent and also express the relation connecting them.

VI. (a) Find a Fourier series to represent $x\text{-}x^2$ from $x = -\pi$ *to* $x = \pi$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \pi^2 / 12$.

(b) Find the Fourier Sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m > 0$.

OR

VII. (a) Obtain the Fourier cosine series of $f(x) = \begin{cases} x & when\ 0 < x < \frac{\pi}{2} \\ 0 & when\ \frac{\pi}{2} < x < \pi \end{cases}$.

(b) Find the Fourier series expansion of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2$.

VIII. (a) Define gamma function. Also find $\Gamma\frac{1}{2}$.

(b) Find Laplace transform of (i) t sin3t cos2t (ii) $\int_0^\infty t^3 e^{-t} \sin t\ dt$.

OR

IX. (a) Solve $y'' + 5y' + 6y = 5e^t, y(0) = 2, y'(0) = 1$.

(b) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$.

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