BTS-III(S)-05.17-0727 Reg.No. [ ][ ][ ][ ][ ][ ][ ][ ]

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B.Tech. Degree III Semester Supplementary Examination May 2017

CS/IT 15-1303 DISCRETE COMPUTATIONAL STRUCTURES
(*2015 Scheme*)

Time : 3 Hours Maximum Marks : 60

PART A
(Answer *ALL* questions)

$(10 \times 2 = 20)$

I. (a) Prove that the negation of biconditional statement $\sim(p \leftrightarrow q)$ is equivalent to $(p{\sim}q)$.

(b) If f and g are functions from R to R defined by $f(x) = ax + b$
$g(x) = 1 - x + x^2$, $g \text{ o } f(x) = 9\ x^2 - 9x + 3$. Find the values of a and b.

(c) Define Recurrence Relation. The recurrence relation for 1, 1, 2, 3, 5, 8, 13 ….. is …..

(d) State Pigeon-hole principle. Seven members of a family have total ₹ 2906/- in their pockets. Show that at least one of them must have at least ₹ 416/- in his pocket.

(e) Draw a graph which contains :
 (i) an Eulerian circuit that is also a Hamiltonian circuit.
 (ii) an Eulerian circuit, not a Hamiltonian circuit.

(f) Define Bridge. Draw a graph whose every edge is a bridge.

(g) Draw a nine vertex binary tree with minimum and maximum heights. Find also the path length of both trees.

(h) Find the identity element of the algebraic system (S, *) where S is the set of integers and * is defined by $a * b = a + b + 2$ for all $a, b \in S$. Find the inverse of the element $a \in S$.

(i) If $S = N \times N$ and the binary operation * is defined by $(a, b) * (c, d) = (ac, bd)$ for all $a, b, c, d \in N$. Show that (S, *) is a semigroup. Is it a Monoid?

(j) Define a Lattice as an algebraic system.

PART B

$(4 \times 10 = 40)$

II. (a) Without constructing truth tables prove the following (2×2=4)
 (i) $\sim p \rightarrow (q \rightarrow r) \cong q \rightarrow (p \vee r)$.
 (ii) $p \rightarrow (q \rightarrow r) \cong p \rightarrow (\sim q \vee r) \cong (p \wedge q) \rightarrow r$.

(b) Use mathematical induction to prove that (6)
$1 + 3 + 5 + \ldots\ldots\ldots + (2n-1)^2 = n^2$ for $n \geq 1$.

OR

III. (a) If R is the relation on the set of integers such as $(a, b) \in R$, if and only if $3a + 4b = 7n$ for some integer n, prove that R is an Equivalence relation. (5)

(b) If f, g, h : R →R are defined by $f(x) = x^3 - 4x$, $g(x) = 1/(x^2+1)$ and $h(x) = x^4$, find $\{(f \text{ o } g) \text{ o } h\}(x)$ and $\{f \text{ o } (g \text{ o } h)\}(x)$ and check if they are equal. (5)

(P.T.O.)

IV. Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 8n^2$ with initial condition $a_0 = 4$ and $a_1 = 7$. (10)

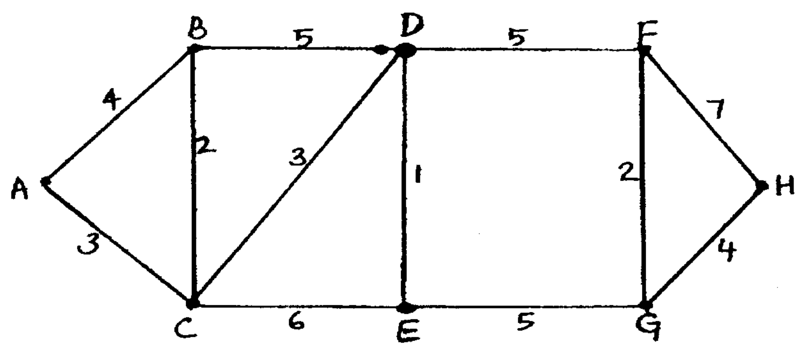
OR

V. (a) State and explain Counting principle. (5)

(b) Assuming that repetitions are not permitted, how many four-digit numbers can be formed from the six digits 1, 2, 3, 5, 7, 8 ? (5)

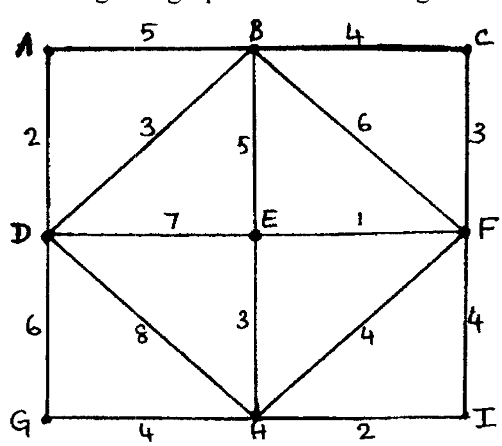
- (i) How many of these numbers are less than 4000 ?
- (ii) How many of these numbers are even ?
- (iii) How many of these numbers are odd ?
- (iv) How many of these numbers are multiples of 5 ?

VI. Using Dikstra's algorithm to find the shortest path between 'A' and 'H' vertices in the weighted graph shown in the figure. (10)



OR

VII. Define a minimum spanning tree Use Kruskal's algorithm to find Minimal Spanning Tree of the weighted graph shown in the figure. (10)



VIII. (a) If * is the binary operation of the set R of real numbers defined by $a * b = a + b + 2ab$. (3)

- (i) Find if {R, *} is a semigroup. Is it commutative?
- (ii) Find the identity element, if exists.

(b) Consider an algebraic system (G, *), where G is the set of all non-zero real numbers and * is a binary operation defined by $a * b = ab/4$. Show that (G, *) is an abelian group. (7)

OR

IX. Let $D_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$ and let the relation / (divides) be a partial ordering on D_{100}. (10)

- (i) Draw the Hasse diagram of D_{100} with relation divides.
- (ii) Determine the GLB of B, where B = {10, 20}.
- (iii) Determine the LUB of B, where B = {10, 20}.
- (iv) Determine the GLB of B, where B = {5, 10, 20, 25}.
- (v) Determine the LUB of B, where B = {5, 10, 20, 25}.

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