BTS-III(R/S)-02.22-0010 Reg. No. [ ]

B

B.Tech. Degree III Semester Regular/Supplementary Examination February 2022

CS/IT 19-202-0303/19-204-0303 DISCRETE COMPUTATIONAL STRUCTURES

(2019 Scheme)

Time: 3 Hours Maximum Marks: 60

PART A

(Answer *ALL* questions)

$(8 \times 3 = 24)$

I. (a) Determine whether the compound propositions $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent or not using truth table.

(b) If functions $f, g, h : R \to R$ defined by $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$. Find compositions $(fo(goh))(x)$ and $((fog)oh)(x)$.

(c) Write an algorithm to find the maximum element in a finite set of numbers.

(d) State generalised pigeon hole principle. What is the minimum number of students required in a class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

(e) Define complete bipartite graph. Draw complete bipartite graph $K_{3,4}$ and $K_{2,3}$.

(f) Draw the ordered rooted tree corresponding to the arithmetic expression written in prefix notation $\uparrow + 2\ 3 - 5\ 1$

(g) What is a semigroup? Let A={0,1} and binary operator * is multiplication. Determine whether the algebraic system (A, *) is a semigroup or not?

(h) Define bounded lattice, with an example. What are the properties of a bounded lattice?

PART B

$(4 \times 12 = 48)$

II. (a) Prove the following logical equivalences using laws of equivalences. (6)

(i) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

(ii) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

(b) Prove the validity of the following arguments using rules of inference. (6)

(i) If I practice, then I will win the competition. If I do not go to picnic then I will practice. But I did not win the competition. Therefore I went to picnic.

(ii) A student in this class has not read the book. Everyone in this class passed the first exam. Therefore someone who passed the first exam has not read the book.

OR

III. (a) Prove by mathematical induction, $1^2 + 2^2 + 3^2 + \ldots + n^2 = (n(n+1)(2n+1))/6$ for all positive integers n. (6)

(b) What is an equivalence relation? Let R be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation. (6)

(P.T.O.)

IV. (a) What is time complexity and space complexity of an algorithm? Define the three asymptotic notations used for algorithm analysis. (6)

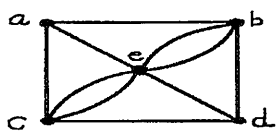
(b) Solve recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2$, with initial conditions $a_0 = 2$ and $a_1 = 1$. (6)

OR

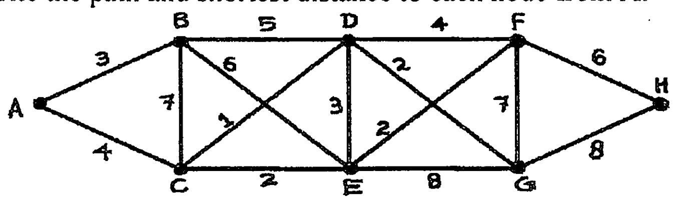
V. (a) Among the first 500 positive integers, determine how many integers are (i) not divisible by 2 nor by 3 nor by 5? (ii) divisible by 2 only? (6)

(b) Solve recurrence relation $a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2$, with initial conditions $a_0 = 3$ and $a_1 = -3$. (6)

VI. (a) What is an Euler circuit and Euler path? Determine whether the given graph has an Euler circuit or Euler path. Construct Euler circuit or Euler path if one exists (5)

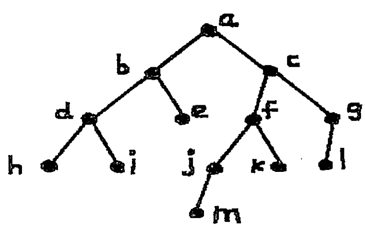


(b) Find the shorted path from node A to H for the following graph using Dijkstra's algorithm. Write the path and shortest distance to each node from A. (7)

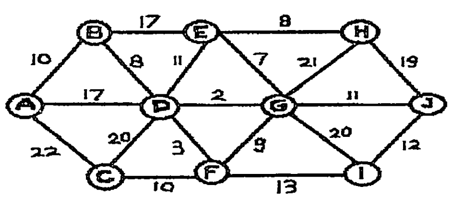


OR

VII. (a) What is a binary tree? Perform in order, pre order and post order traversal of the following tree. (5)



(b) What is the Minimal Spanning Tree (MST) of a graph? Find the Minimal Spanning Tree of the following graph using Kruskal's algorithm. Write the total weight of MST. (7)



VIII. (a) Define semigroup, monoid and group. Determine whether the algebraic systems (O,*) and (E,*) form a semigroup, monoid or group where O is the set of odd integers and E is set of even integers and * is multiplication. (6)

(b) Define field. Let A be the set of integers from 0 to 6. Check whether the algebraic system (A, $+_7$, $*_7$) form a field if $+_7$ is addition modulo 7 and $*_7$ multiplication modulo 7. (6)

OR

IX. Let D be the set of divisors of 60 and divisibility / be the partial ordering. Draw the Hasse diagram. Find the following: (12)

(i) Upperbound, lowerbound, LUB and GLB of S where S = { 12,30}

(ii) Upperbound, lowerbound, LUB and GLB of S where S = { 10,15}

(iii) Determine whether (D, /) a lattice or not. Justify your answer.