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B.Tech. Degree III Semester Examination November 2016

CE/CS/EC/EE/IT/ME/SE AS 15-1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES (2015 Scheme)

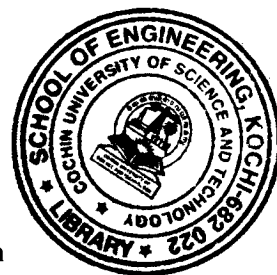
Time : 3 Hours

Maximum Marks : 60

PART A (Answer *ALL* questions)

10 × 2 = 20

- I. (a) Prove that every square matrix and its transpose have same eigen values.
- (b) Explain when a linear system of equations has unique solution, no solution and more than one solution.
- (c) If x, y, z are linearly independent vectors of a vector space V prove that $x, x+y, x+y+z$ are also linearly independent.
- (d) Define basis and dimension of a vector space with example.
- (e) Express $f(x) = x^2, -\pi < x < \pi$ as a Fourier series.
- (f) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases}$
- (g) Find the half range cosine series for $f(x) = x$ in $(0, \pi)$ and deduce the sum $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
- (h) Find the Laplace transform of $t \sin 3t \cos 2t$.
- (i) Find the Laplace transform of $\log \left[\frac{S(S+1)}{S^2+1} \right]$.
- (j) Find the Laplace transform of periodic function.



PART B

4 × 10 = 40)

- II. (a) State Cayley-Hamilton theorem and verify it for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and hence find } A^{-1}.$$

- (b) Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ hence find A^4 .

OR**(P.T.O.)**

- III. (a) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to a Canonical form by orthogonal reduction.
- (b) Find the value of λ and μ for which the equations $2x + 3y + 5z = 9$; $7x + 3y - 2z = 8$; $2x + 3y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) more than one solution.

- IV. (a) Define vector space, subspace and give an example of a vector space having a subset which is a subspace and a subset which is not a subspace. Justify your answer.
- (b) Let W be any plane in R^3 passing through the origin. Verify whether W is a subspace of R^3 .

OR

- V. (a) Apply Gram-Schmidt orthogonalization process to find an orthogonal basis and an orthonormal basis for the subspace U of R^4 spanned by $V_1 = (1, 1, 1, 1)$, $V_2 = (1, 2, 4, 5)$, $V_3 = (1, -3, 4, -2)$
- (b) Let T be a map from R^3 to R defined by $T(x, y, z) = x^2 + y^2 + z^2$. Verify whether T is a linear transformation.

- VI. (a) Find the Fourier series for $f(x)$, given $f(x) = \begin{cases} 0 & \text{in } -1 < x < 0 \\ 1 & \text{in } 0 < x < 1 \end{cases}$ and $f(x+2) = f(x)$ for all x .
- (b) Using Fourier integral for $f(x) = e^{-ax}$ ($a > 0$) show that $\int_0^{\infty} \frac{\lambda \sin \lambda x}{\lambda^2 + a^2} d\lambda = \frac{\pi}{2} e^{-ax}$.

OR

- VII. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{in } |x| \leq 1 \\ 0 & \text{in } |x| > 1 \end{cases}$. Hence prove that $\int_0^{\infty} \frac{S \sin S - S \cos S}{S^3} \cos \frac{s}{2} ds = \frac{3\pi}{16}$.
- (b) Find the half range sine series for the function $f(x) = \begin{cases} x-1 & 0 \leq x \leq 1 \\ 1-x & 1 \leq x \leq 2 \end{cases}$

- VIII. (a) State convolution theorem and use it to find the inverse Laplace transform of $\frac{1}{(S^2 + 4)^2}$.
- (b) Solve (using transform) $y'' + 2y' - 3y = \sin t$, given $y = 0$, $y'(0) = 0$ when $t = 0$.

OR

- IX. (a) Determine y which satisfies the equation $\frac{dx}{dt} + 2y + \int_0^t y dt = 2 \cos t$, $y(0) = 1$.
- (b) Prove that $\beta(m, n) = \frac{[m]n}{[m+n]}$.