BTS-III(15)-11.17-1554 Reg. No. [| | | | | | | |]

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B.Tech. Degree III Semester Examination November 2017

CE/CS/EC/EE/IT/ME/SE
AS 15-1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES
(2015 Scheme)

Time : 3 Hours Maximum Marks : 60

PART A
(Answer *ALL* questions)

$(10 \times 2 = 20)$

I. (a) Define rank of a matrix and hence find the rank of $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$.

(b) Find the eigen values and eigen vector of $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

(c) Check whether the vectors $(1, 3, 2)$, $(5, -2, 1)$, $(-7, 13, 4)$ are linearly dependent if so find the relation connecting them.

(d) Verify whether the collection of all polynomials of degree *n* form a vector space or not under usual addition and sealer multiplication of Polynomials. Justify.

(e) Show that the Fourier series for $f(x) = x, -\pi < x < \pi$ is given by $f(x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin nx}{n}$

(f) Find the Fourier cosine transform of $5e^{-2x} + 2e^{-5x}$.

(g) Obtain the half range sine series of the function $f(x) = kx(x-l)$ in $0 \le x \le l$.

(h) Find the Laplace transform of $t\cos^3 t$.

(i) Find the inverse Laplace transform of $\log\left[\frac{s^2 + a^2}{s^2 - b^2}\right]$.

(j) Using convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)^2}$.

PART B

$(4 \times 10 = 40)$

II. (a) Test for consistency and solve $4x - 2y + 6z = 8$; $x + y - 3z = -1$; $15x - 3y + 9z = 21$.

(b) Diagonalize the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$.

OR

(P.T.O.)

III. (a) Using Cayley-Hamilton theorem find the inverse of $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

(b) Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to Canonical form.

IV. (a) Suppose the vectors u, v, w are linearly independent. Show that the vectors $u+v,\ u-v,\ u-2v+w$ are also linearly independent.

(b) Let V be the vector space of 2×2 matrices. Let W be the subspace of symmetric matrices. Show that dim W = 3, by finding a basis of W.

OR

V. (a) Suppose the mapping $F: R^2 \to R^2$ is defined by $F(x, y) = (x+y, x)$. Show that F is a linear transformation.

(b) Let S be a subset of an inner product space V. Then prove that the orthogonal complement of S is a subspace of V.

VI. (a) Find the Fourier series for $f(x) = x^2$ in $-1 < x < 1$.

(b) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq | \\ 0 & \text{for } |x| > | \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x \, d\lambda}{\lambda}$.

OR

VII. (a) Find the Fourier sine transform of $e^{-x}, x \geq 0$. Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$.

(b) Show the Fourier transform of $f(x) = e^{\frac{-x^2}{2}}$ is $e\frac{-s^2}{2}$.

VIII. (a) Solve (using Laplace transform)
$y'' + y' = t^2 + 2t,\ y(0) = 4,\ y'(0) = -2.$

(b) Find the Laplace transform of periodic function.

OR

IX. (a) Solve $\frac{dy}{dt} + 3y + 2\int_0^t y dt = t$ for which $y(0) = 0$.

(b) Using Laplace transform evaluate $\int_0^\infty \frac{e^{-2t} - e^{-3t}}{t} dt.$

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