

--	--	--	--	--	--	--	--

B.Tech. Degree III Semester Supplementary Examination April 2018

CE/CS/EC/EE/IT/ME/SE
AS 15 -1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES
(2015 Scheme)

Time : 3 Hours

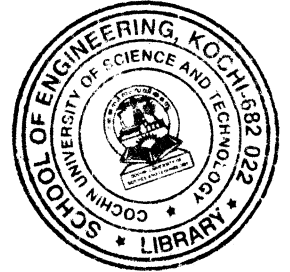
Maximum Marks : 60

PART A
(Answer *ALL* questions)

(10 × 2 = 20)

- I. (a) Prove that the eigen values of an idempotent matrix are either zero or unity.
 (b) Determine the values of λ for which the following set of equations may possess non trivial solution $3x_1 + x_2 - \lambda x_3 = 0$, $4x_1 - 2x_2 - 3x_3 = 0$, $2\lambda x_1 + 4x_2 + \lambda x_3 = 0$.
 (c) Find a non zero vector \mathbf{w} that is orthogonal to $\mathbf{u} = (1, 2, 1)$ and $\mathbf{v} = (2, 5, 4)$ in \mathbb{R}^3 .
 (d) $V = \mathbb{R}^n$ and $W = \{(x_1, x_2, \dots, x_n) | x_1 x_2 = 0\}$. Is W a subspace of V ?
 (e) Define half range sine and cosine series.
 (f) Using Fourier sine integral show that
$$\int_0^\infty \frac{1 - \cos \pi \omega}{\omega} \sin \omega x \, d\omega = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

 (g) Express $f(x) = \frac{x}{2}$ as a Fourier series in the interval $-\pi < x < \pi$.
 (h) Find inverse Laplace transform of $\cot^{-1}(s/2)$.
 (i) Find Laplace transform of $\frac{1 - e^t}{t}$.
 (j) Define unit step function and find Laplace transform of unit step function.



PART B

(4 × 10 = 40)

- II. (a) Test for consistency and solve $2x + y - z = 0$, $2x + 5y + 7z = 52$, $x + y + z = 9$.
 (b) Find eigen values and eigen vectors of
$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$
.

OR

- III. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and hence find A^{-1} .

- (b) Diagonalize $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and find the modal matrix.

(P.T.O.)

- IV. (a) Define linear transformation and check whether $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $T(x, y) = xy$ is linear.
 (b) Let V be the vector space of polynomials $f(t)$ with inner product $\langle f, g \rangle = \int_{-1}^1 f(t)g(t)dt$. Apply the Gram Schmidt orthogonalization process to $\{1, t, t^2, t^3\}$ to find an orthogonal basis.

OR

- V. (a) Define basis and dimension. Also find the value of λ for which the set of vectors $(1, \lambda, 5), (1, 0, 1)$ and $(2, 0, 1)$ of \mathbb{R}^3 form a basis.
 (b) Show that $(1, 1, 2), (1, 2, 5), (5, 3, 4)$ are linearly dependent and also express the relation connecting them.
- VI. (a) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \pi^2/12$.
 (b) Find the Fourier Sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m > 0$.

OR

- VII. (a) Obtain the Fourier cosine series of $f(x) = \begin{cases} x & \text{when } 0 < x < \frac{\pi}{2} \\ 0 & \text{when } \frac{\pi}{2} < x < \pi \end{cases}$.
 (b) Find the Fourier series expansion of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2$.
- VIII. (a) Define gamma function. Also find $\Gamma(\frac{1}{2})$.
 (b) Find Laplace transform of (i) $t \sin 3t \cos 2t$ (ii) $\int_0^\infty t^3 e^{-t} \sin t dt$.
- OR**
- IX. (a) Solve $y'' + 5y' + 6y = 5e^t, y(0) = 2, y'(0) = 1$.
 (b) Apply convolution theorem to evaluate $L^{-1} \left\{ \frac{1}{(s^2 + 1)(s^2 + 9)} \right\}$.
