

--	--	--	--	--	--	--	--

**B.Tech. Degree III Semester Regular/Supplementary
Examination February 2022**

**CE/CS/EC/EE/IT/ME/SE 19-200-0301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES
(2019 Scheme)**

Time: 3 Hours

Maximum Marks: 60

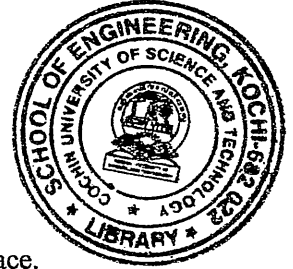
PART A
(Answer *ALL* questions)

(8 × 3 = 24)

- I. (a) Explain rank of a matrix and hence find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

- (b) Find the Eigen values and Eigen vector of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
- (c) Verify whether the set V of all n^{th} degree polynomial is a vector space. Justify your answer.
- (d) Let V be the set of all positive real numbers with addition defined by $x + y = xy$ and scalar multiplication defined by $\alpha x = x$. Examine whether V is a vector space.
- (e) Obtain the half range cosine series for $f(x) = x$ in $(0, \pi)$ and hence deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
- (f) Express $f(x) = |x|$, $-\pi < x < \pi$ as a Fourier series.
- (g) Find the Laplace transform of $te^{-t} \sin t$.
- (h) Find the inverse Laplace transform of $\log\left(\frac{1+s}{s^2}\right)$.



PART B

(4 × 12 = 48)

- II. (a) Find the values of a and b for which the equations $x + y + z = 3$; $x + 2y + 2z = 6$; $x + ay + 3z = b$ have (i) no solution (ii) unique solution.
- (b) State Cayley-Hamilton theorem and use it to find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

OR

(P.T.O.)

BTS-III(R/S)-02.22-0001

III. (a) Diagonalize $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ hence find A^5 .

(b) Find the Eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and hence reduce

$6x^2 + 3y^2 + 3z^2 - 2yz + 4zx - 4xy$ to canonical form (sum of squares).

IV. (a) Explain basis and dimension of a vector space and find a basis for the vector space of all 2×2 matrices. Justify your answer.

(b) Determine whether the following set of vectors form a basis in R^3 . Justify your answer $V_1 = (2, 2, 0), V_2 = (0, 0, 3), V_3 = (3, 0, 2)$.

OR

V. (a) Explain inner product, inner product space with examples.

(b) Find a non zero vector w that is orthogonal to $u_1 = (1, 2, 1)$ and $u_2 = (2, 5, 4)$ in R^3 .

VI. (a) Obtain the Fourier series to represent x^2 from $x = -l$ to $x = l$.

(b) Using Fourier sine integral for $f(x) = e^{-ax}, (a > 0)$ show

$$\text{that } \int_0^{\infty} \frac{\lambda \sin \lambda x}{\lambda^2 + a^2} d\lambda = \frac{\pi}{2} e^{-ax}.$$

OR

VII. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{in } |x| \leq 1 \\ 0 & \text{in } |x| > 1 \end{cases}$ hence prove that

$$\int_0^{\infty} \frac{\sin s - s \cos s}{s^3} \cos \frac{s}{2} ds = \frac{3\pi}{16}.$$

(b) Find the Fourier cosine transform of $5e^{-2x} + 2e^{-5x}$.

VIII. (a) State convolution theorem and use it. Find $L^{-1} \left[\frac{1}{s(s^2 + 1)} \right]$.

(b) Use Laplace transform to solve $\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} - 5y = 5$ given that $y = 0,$

$$\frac{dy}{dt} = 2 \text{ when } t = 0.$$

OR

IX. (a) Find y which satisfies the equation $\frac{dy}{dt} + 4y + 5 \int_0^t y dt = e^{-t}$ when

$$y(0) = 0.$$

(b) Evaluate $\int_0^{\infty} \frac{e^{-2t} - e^{-3t}}{t} dt.$