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B.Tech. Degree III Semester Examination November 2019

CE/CS/EC/EE/IT/ME/SE

AS 15-1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES

(2015 Scheme)

Time : 3 Hours

Maximum Marks : 60

PART A (Answer ALL questions)

- I. (a) By reducing to Echelon-form find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

- (b) Prove that every square matrix and its transpose have same eigen values.
 (c) Show that the vectors (1,2,3), (3,-2,1), (1,-6,-5) are linearly dependent and find the relation connecting them.
 (d) Explain basis and dimension of a vector space with example.
 (e) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
 (f) Express $f(x) = |x|$, $-\pi < x < \pi$ as a Fourier series.
 (g) Explain periodic function with two examples.
 (h) Find $L[t e^{-t} \cos ht]$.
 (i) Find $L^{-1}\left[\log \frac{s(s+1)}{s^2+1}\right]$.
 (j) Prove that $\beta(m,n) = \beta(n,m)$.



PART B

(4 × 10 = 40)

- II. (a) Test for consistency and hence solve
 $x - 2y + 3z = 2$; $2x - 3z = 3$; $x + y + z = 0$.

- (b) State Cayley-Hamilton and verify it for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

OR

- III. (a) Diagonalize $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ hence find A^6 .

- (b) Reduce the quadratic form $2xy + 2yz + 2zx$ into canonical form.

(P.T.O.)

- IV. (a) Give an example of a vector space with dimension n . Justify your answer.
 (b) Explain inner product space with an example.

OR

- V. (a) Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis for the subspace V of \mathbb{R}^3 spanned by $V_1 = (0, 1, 2)$
 $V_2 = (1, 1, 2)$ $V_3 = (1, 0, 1)$.
 (b) Find k so that $u = (1, 2, k, 3)$ and $v = (3, k, 7, -5)$ in \mathbb{R}^4 are orthogonal.

- VI. (a) Obtain the half range sine series of the function
 $f(x) = kx(x-l)$ in $0 \leq x \leq l$.
 (b) Using Fourier sine integral for $f(x) = e^{-ax}$ ($a > 0$) show that

$$\int_0^{\infty} \frac{\lambda \sin \lambda x}{\lambda^2 + a^2} d\lambda = \frac{\pi}{2} e^{-ax}.$$

OR

- VII. (a) Obtain a Fourier expansion for $\sqrt{1 - \cos x}$ in the interval $-\pi < x < \pi$.
 (b) Find the Fourier sine transform of $f(x) = \begin{cases} x & \text{in } 0 < x < 1 \\ 2-x & \text{in } 1 < x < 2 \\ 0 & \text{in } x > 2 \end{cases}$

- VIII. (a) State convolution theorem and hence find $L^{-1} \left[\frac{s^2}{[s^2 + a^2][s^2 + b^2]} \right]$.
 (b) Solve (using Laplace transform)
 $y'' + 2y' - 3y = \sin t$ given $y = 0, y'(0) = 0$ when $t = 0$.

OR

- IX. (a) Prove that $\beta(m, n) = \frac{\sqrt{mn}}{\sqrt{(m+n)}}$.
 (b) Evaluate $\int_0^{\infty} \frac{e^{-2t} - e^{-3t}}{t} dt$.
