BTS-III-11.18-1707 Reg.No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

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B.Tech. Degree III Semester Examination November 2018

CE/CS/EC/EE/IT/SE/ME

AS 15-1301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES

(2015 Schemes)

Time : 3 Hours Maximum Marks : 60

PART A

(Answer *ALL* questions)

$(10 \times 2 = 20)$

I. (a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

(b) Explain linear system of equation and when this system has unique solution and no solution.

(c) State and prove one of the property of eigen value of a square matrix.

(d) Define subspace of a vector space with example.

(e) Explain linearly independent and dependent set of vectors in a vector space with example.

(f) Express $f(x) = |x|, -\pi < x < \pi$ as a Fourier series.

(g) Express $f(x) = \begin{cases} kx, & 0 \le x \le \frac{\pi}{2} \\ k(\pi - x), & \frac{\pi}{2} \le x \le \pi \end{cases}$ as half-range sine series.

(h) Find $L\left[t\, e^{-t} \sin t\right]$.

(i) Prove that $L\left[t^n\right] = \frac{\lceil n+1}{s^{n+1}}$.

(j) Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$.

PART B

$(4 \times 10 = 40)$

II. (a) Find the eigen values and eigen vectors of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (5)

(b) Diagonalize the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$. (5)

OR

III. (a) Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$. (5)

(b) Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the canonical form. (5)

IV. (a) Show that the set $M=M_{m\times n}$ of all $m\times n$ matrix form a vector space with usual addition and scalar multiplication. (6)

(b) Find a basis and dimension of $M_{2\times 2}$. (4)

OR

V. (a) Consider the following polynomials in p(t) with inner product (5)

$$<f,\ g> = \int_0^1 f(t)g(t)dt,\ f(t)=t+2,\ \ g(t)=3t-2,\ h(t)=t^2-2t-3$$

Find $<f,\ g>$ and $<f,\ h>$.

(b) Let S consists of the following vectors in R^4, (5)

$u_1=(1,1,0,1),\ u_2=(1,2,1,3),\ \ u_3=(1,1,-9,2),\ u_4=(16,-13,1,3)$

Show that S is orthogonal and a basis of R^4.

VI. (a) If $f(x)$ is a periodic function defined over a period $(0,\ 2\pi)$ by (6)

$f(x)=\dfrac{1}{12}(3x^2-6x\pi+2\pi^2)$. Prove that $f(x)=\sum_{n=1}^{\infty}\dfrac{\cos nx}{n^2}$ and hence show that

$$\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots\ldots.$$

(b) Find Fourier cosine and sine transform of $f(x)=e^{-ax}$. (4)

OR

VII. (a) Find the Fourier series for $f(x)$, (5)

given $f(x)=\begin{cases}0 \text{ in } -1<x<0\\ 1 \text{ in } 0<x<1\end{cases}$ and $f(x+2)=f(x)$ for all x.

(b) Find the Fourier integral of $f(x)=\begin{cases}1 \text{ for } |x|\le 1\\ 0 \text{ for } |x|>1\end{cases}$ (5)

Hence evaluate $\displaystyle\int_0^{\infty}\frac{\sin\lambda\cos\lambda x d\lambda}{\lambda}$.

VIII. (a) Using Laplace transform solve

$y''+2y'-3y=\sin t,\ y=0,\ y'(0)=0$ when $t=0$. (5)

(b) $\dfrac{dy}{dt}+2y+\int_0^t y\ dt=2\cos t,\ y(0)=1.$ (5)

OR

IX. (a) Prove that $\beta(m,n)=\dfrac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$. (4)

(b) Find the inverse Laplace transform of (6)

(i) $\dfrac{s\,e^{-s/2}+\pi e^{-s}}{s^2+\pi^2}$ (ii) $\dfrac{e^{-cs}}{s^2(s+a)}$.

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