BTS-III(R/S)-02.22-0001 Reg. No. [ ]

A

B.Tech. Degree III Semester Regular/Supplementary Examination February 2022

CE/CS/EC/EE/IT/ME/SE 19-200-0301 LINEAR ALGEBRA AND TRANSFORM TECHNIQUES

(*2019 Scheme*)

Time: 3 Hours Maximum Marks: 60

PART A

(Answer *ALL* questions)

$(8 \times 3 = 24)$

I. (a) Explain rank of a matrix and hence find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

(b) Find the Eigen values and Eigen vector of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(c) Verify whether the set V of all n^{th} degree polynomial is a vector space. Justify your answer.

(d) Let V be the set of all positive real numbers with addition defined by $x+y=xy$ and scalar multiplication defined by $\alpha x = x$. Examine whether V is a vector space.

(e) Obtain the half range cosine series for $f(x)=x$ in $(0,\pi)$ and hence deduce the sum of the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots$

(f) Express $f(x)=|x|,\ -\pi < x < \pi$ as a Fourier series.

(g) Find the Laplace transform of $te^{-t}\sin t$.

(h) Find the inverse Laplace transform of $\log\left(\frac{1+s}{s^2}\right)$.

PART B

$(4 \times 12 = 48)$

II. (a) Find the values of a and b for which the equations $x+y+z=3$; $x+2y+2z=6; x+ay+3z=b$ have (i) no solution (ii) unique solution.

(b) State Cayley-Hamilton theorem and use it to find the inverse of the matrix

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$$

OR

(P.T.O.)

III. (a) Diagonalize $A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ hence find A^5.

(b) Find the Eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and hence reduce $6x^2+3y^2+3z^2-2yz+4zx-4xy$ to canonical form (sum of squares).

IV. (a) Explain basis and dimension of a vector space and find a basis for the vector space of all 2×2 matrices. Justify your answer.

(b) Determine whether the following set of vectors form a basis in R^3. Justify your answer $V_1=(2,2,0), V_2=(0,0,3), V_3=(3,0,2)$.

OR

V. (a) Explain inner product, inner product space with examples.

(b) Find a non zero vector w that is orthogonal to $u_1=(1,2,1)$ and $u_2=(2,5,4)$ in R^3.

VI. (a) Obtain the Fourier series to represent x^2 from $x=-l$ to $x=l$.

(b) Using Fourier sine integral for $f(x)=e^{-ax},(a>0)$ show that $\int_0^\infty \frac{\lambda \sin \lambda x}{\lambda^2+a^2}d\lambda=\frac{\pi}{2}e^{-ax}$.

OR

VII. (a) Find the Fourier transform of $f(x)=\begin{cases} 1-x^2 & \text{in } |x|\le 1 \\ 0 & \text{in } |x|>1 \end{cases}$ hence prove that $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}ds=\frac{3\pi}{16}$.

(b) Find the Fourier cosine transform of $5e^{-2x}+2e^{-5x}$.

VIII. (a) State convolution theorem and use it. Find $L^{-1}\left[\frac{1}{s\left(s^2+1\right)}\right]$.

(b) Use Laplace transform to solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}-5y=5$ given that $y=0$, $\frac{dy}{dt}=2$ when $t=0$.

OR

IX. (a) Find y which satisfies the equation $\frac{dy}{dt}+4y+5\int_0^t ydt=e^{-t}$ when $y(0)=0$.

(b) Evaluate $\int_0^\infty \frac{e^{-2t}-e^{-3t}}{t}dt$.

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