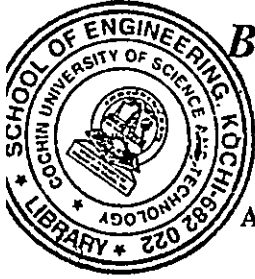


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B.Tech. Degree IV Semester Supplementary Examination April 2021

CE/CS/EC/EE/IT/ME/SE-

AS 15-1401 COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS
(2015 Scheme)

Time: 3 Hours

Maximum Marks: 60

PART A (Answer ALL questions)

(10 × 2 = 20)

- I.
 - (a) Verify Cauchy Riemann equations for an analytic function.
 - (b) Prove that real and imaginary parts of an analytic function are harmonic.
 - (c) Test the analyticity of the function $f(z) = \bar{z}$.
 - (d) Evaluate $\int_C \frac{\sin(3z)}{z + \frac{\pi}{2}} dz$ if C is the circle $|z| = 5$.
 - (e) Show that $\frac{1}{z^2} = \frac{1}{4} \sum_{n=0}^{\infty} (-1)^n (n+1) \left(\frac{z-2}{2}\right)^n$ when $|z-2| < 2$.
 - (f) Form the partial differential equation by eliminating the arbitrary function g from the relation $g(x+y+z, x^2+y^2+z^2) = 0$.
 - (g) Solve $z = p^2 + q^2$.
 - (h) Solve $q^2 - p = y - x$.
 - (i) Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.
 - (j) Derive one dimensional wave equation.

PART B

(4 × 10 = 40)

- II.
 - (a) Find an analytic function whose imaginary part is $3x^2y - y^3$ also find its harmonic conjugate.
 - (b) Find the image of the circle $|z-1|=1$ in the complex plane under the mapping $w = \frac{1}{z}$.

OR

- III.
 - (a) Show that $w = \sin z$ transform the semi infinite strip $0 \leq x \leq \frac{\pi}{2}, y \geq 0$ ONTO the first quadrant of the w -plane.
 - (b) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i$ and $z_3 = -2$ into the points $w_1 = 1, w_2 = i$ and $w_3 = -1$.

(P.T.O.)

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IV.

(a) Using Cauchy's integral formula, find the value of $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle $|z+1-i|=2$.

(b) Find the residue of $f(z) = \frac{\sin(z)}{z \cos(z)}$ at each of its poles inside the circle $|z|=2$.

OR

V.

(a) Using residue theorem, evaluate $\int_C \frac{dz}{(z^2+4)^2}$ where C is the circle $|z-i|=2$.

(b) Using contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos(\theta)}$.

VI.

Solve

(i) $x^2 p^2 + y^2 q^2 = z^2$

(ii) $(x-y)p + (y-x-z)q = z$

OR

VII.

Solve

(i) $(D^2 - 2DD')(z) = e^{2x} + x^3 y$

(ii) $(D^2 + DD' - 6D^2)(z) = y \cos(x)$

VIII.

(a) Derive one dimensional heat equation.

(b) Obtain the solution of the Laplace equation by the method of separation of the variables.

OR

IX.

(a) A string is stretched and fastened to two points $x=0$ and $x=l$ apart. Motion is started by displacing the string in to the form $y=k(lx-x^2)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance x from one end at time in the form of Fourier series.
