BTS-IV-04.18-0712 Reg. No. [ ]

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B.Tech. Degree IV Semester Examination April 2018

CE/CS/EC/EE/IT/ME/SE

AS 15–1401 COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS

(2015 Scheme)

Time : 3 Hours Maximum Marks : 60

PART A

(Answer *ALL* questions)

$(10 \times 2 = 20)$

I. (a) Prove the polar form of Cauchy Riemann Equations, $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$.

(b) Prove that $v = \tan^{-1}(y/x)$ is harmonic.

(c) Find the bilinear transform which maps the points $z: 1, i, -1$ into the points $w: 0, 1, \infty$.

(d) Evaluate $\int_0^{2+3i} (x^2 - iy)\,dz$ along the path $y = x^2$.

(e) State and prove Cauchy's Integral formula.

(f) Evaluate $\int_C \frac{e^{2z}dz}{(z+1)^4} dz,\ C: |z| = 2.$

(g) Form partial differential equation from $z = f(x+ct) + \phi(x-ct)$.

(h) Solve $p(1+q) = qz$.

(i) Using method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$.

(j) Find the solution of one dimensional heat equation.

PART B

$(4 \times 10 = 40)$

II. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied at the origin.

(b) Let $w = u + iv$ be an analytic function. Then prove that the family of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ cut orthogonally.

OR

III. (a) Show that the function $f(z) = \sinh z$ is an analytic function and find its derivative.

(b) Find the image of $|z - 2i| = 2$ under the mapping $w = \frac{1}{z}$.

(P.T.O.)

IV. (a) Using Cauchy's Integral Formula, evaluate $\int_C \frac{z}{(z-1)(z-2)^2}dz,\ C:|z-2|=\frac{1}{2}$.

(b) Expand using Laurent's series :

(i) $f(z)=\frac{1}{z(z-1)},\ 0<|z|<1$

(ii) $f(z)=\frac{1}{z^2-3z-2},\ 0<|z-1|<1$

OR

V. (a) Using Cauchy's Residue theorem, evaluate $\int_C \frac{3z^2+z-1}{(z^2-1)(z-3)}dz,\ C:|z|=2$.

(b) Using Contour integration, evaluate $\int_0^{2\pi} \frac{\cos\theta}{3+\sin\theta}d\theta$.

VI. Solve

(i) $z^2\left(p^2+q^2+1\right)=b^2$

(ii) $p^2+q^2=z^2(x+y)$

OR

VII. Solve

(i) $x^2(y-z)p+y^2(z-x)q-z^2(x-y)=0$

(ii) $\left(D^3-3D^2D^1+4D^{13}\right)z=e^{x+2y}$

VIII. (a) Derive one dimensional wave equation.

(b) Obtain solution of Laplace equation by the method of separation of variables.

OR

IX. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in the position $y=f(x)$. It is vibrating by giving each of its points a velocity $\frac{\partial y}{\partial t}=g(x)$ at $t=0$. Find $y(x,t)$ in the form of Fourier Series.

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