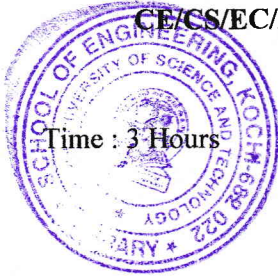


## B.Tech. Degree IV Semester Examination April 2019



**CE/CS/EC/EE/IT/ME/SE AS 15-1401 COMPLEX VARIABLES AND PARTIAL  
DIFFERENTIAL EQUATIONS  
(2015 Scheme)**

Time : 3 Hours

Maximum Marks : 60

**PART A**  
(Answer *ALL* questions)

(10 × 2 = 20)

- I. (a) If  $f(z)$  is an analytic function with constant modulus, show that  $f(z)$  is constant.
- (b) Find the orthogonal trajectories of the family of curves  $r^2 \cos 2\theta = c$ .
- (c) Show that the transformation  $w = \frac{1}{z}$  transforms all circles and straight lines into circles and straight lines in the  $w$ -plane. Which circles in the  $z$ -plane become straight lines in the  $w$ -plane and which circles are transformed into other circles?
- (d) Evaluate  $\int_c \frac{z}{z^2 - 3z + 2} dz$  where  $c$  is the circle  $|z+1| = 1$ .
- (e) Determine the poles and the residue at each pole of  $\frac{z^2 + 1}{z^2 - 2z}$ .
- (f) Find the Taylor's series expansion of  $f(z) = \frac{1}{z^2 - z - 6}$  about  $z = 1$ .
- (g) Form the partial differential equation by eliminating the arbitrary function from  $xyz = f(x+y+z)$ .
- (h) Solve  $yp + xq + pq = 0$ .
- (i) Find the solution of Laplace equation over a rectangular region by the method, of separation of variables.
- (j) Using D'Alembert's method find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection  $f(x) = a(x-x^2)$ .

**PART B**

(4 × 10 = 40)

- II. (a) Show that the function  $f(z) = \sqrt{xy}$  is not regular at the origin, although C-R equations are satisfied.
- (b) Find the analytic function whose real part is  $e^{-x}(x \sin y - y \cos y)$ .

**OR**

- III. (a) Determine the linear fractional transformation that sends the points  $z = 0, -1, 2i$  into the points  $w = 5i, \infty, -\frac{i}{3}$  respectively. Find the image of  $|z| < 1$  under this transformation.
- (b) Under the transformation  $w = \frac{z-i}{1-iz}$ , find the map of the circle  $|z| = 1$  in the  $w$  plane.

(P.T.O.)

IV. (a) Evaluate  $\int_c \frac{e^z}{(z-i)^2(z^2+4)} dz$  where  $c: |z-1|=1/2$ , using Cauchy's integral formula.

(b) Evaluate  $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos \theta} d\theta$ .

OR

V. (a) Find the Laurent expansion of  $\frac{7z^2+9z-18}{z^3-9z}$  in the region (i)  $0 < |z| < 3$   
(ii)  $|z| > 3$ .

(b) Evaluate  $\int_c \frac{dz}{(z^2+4)^2}$  where  $c: |z-i|=2$ , using residue theorem.

VI. (a) Solve  $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ .

(b) Solve  $z^2(p^2+q^2) = x^2+y^2$ .

OR

VII. (a) Solve  $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$ .

(b) Solve  $(D^2 + DD' - 6D'^2)z = \cos(2x+y)$ .

VIII. (a) Derive one dimensional heat equation.

(b) A tightly stretched string of length 1 with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity  $V_0 \sin^3(\frac{\pi x}{\ell})$ . Find the displacement  $y(x,t)$ .

OR

IX. (a) A bar 100 cm long, with insulated sides, has its ends kept at  $0^\circ\text{C}$  and  $100^\circ\text{C}$  until steady state conditions prevail. The two ends are then suddenly insulated and kept so. Find the temperature distribution.

(b) Solve  $u_{xx} + u_{yy} = 0$  which satisfies the conditions  $u(0,y) = u(l,y) = u(x,0) = 0$  and  $u(x,a) = \sin \frac{n\pi x}{l}$ .

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