

B. Tech. Degree IV Semester Special Supplementary Examination February 2020

CE/CS/EC/EE/IT/ME/SE

AS 15-1401 COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS (2015 Scheme)

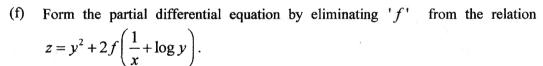
Time: 3 Hours

Maximum Marks: 60

PART A (Answer ALL questions)

 $(10 \times 2 = 20)$

- I. (a) Define analytic function. Give an example of a function which is analytic and an example which is not analytic.
 - (b) Show that an analytic function with constant modulus is constant.
 - (c) Find the Taylor series expansion of $f(z) = \frac{z-1}{z+1}$ about the point z = 1.
 - (d) Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at each of the poles.
 - (e) Evaluate $\int_{C} \frac{dz}{z^2 e^z}$ where C is |z| = 1.



- (g) Solve $z = p^2 + q^2$.
- (h) Solve $q^2 p = y x$.
- (i) Using the method of separation of variables solve $\frac{\partial u}{\partial x} + 4 = \frac{\partial u}{\partial t}$ with $u = 4e^{-3x}$ when t = 0.
- (j) Derive one dimensional wave equation.

PART B

 $(4 \times 10 = 40)$

- II. (a) Find the image of the circle |z-1|=1 in the complex plane under the mapping $w=\frac{1}{z}$.
 - (b) Find the bilinear transformation which maps the points z = 2, i, -2 into the points w = 1, i, -1:

OR

- III. (a) Find the analytic function f(z) = u + iv if $u + v = \frac{x}{x^2 + v^2}$ and f(1) = 1.
 - (b) Show that $u = e^x \cos y + xy$ is harmonic and find its harmonic conjugate.

- Expand $\frac{1}{(z-1)(z-2)}$ in Laurent's series valid for |z| < 1 and 1 < |z| < 2. IV.
 - Evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$. (b)

OR

- Evaluate $\int_{c} \frac{dz}{(z^2+4)^2}$ where C is the circle |z-i|=2 using residue theorem. V. (a)
 - State and prove Cauchy's integral formula. (b)
- Find the partial differential equation of all spheres of fixed radius having their VI. (a) centers in the xy plane.
 - Solve $x^2p^2 + y^2q^2 = z^2$. (b)

OR

VII. Solve the following:

- $r+s-6t=\cos(2x+y).$ (a)
- x(y-z)p+y(z-x)q=z(x-y).(b)
- Obtain D'Alembert's solution of wave equation. VIII. (a)
 - Derive one dimensional heat equation. (b)

- Solve the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ subject to the boundary IX. (a) u(0,t) = 0, $u(\ell, t) = 0$, u(x,0) = x.
 - Obtain the solution of Laplace's equation by the method of separation of variables. (b)