# C



# B.Tech. Degree IV Semester Examination April 2017

**CE/CS/EC/EE/IT/ME/SE**

**AS 15-1401 COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS**

*(2015 Scheme)*

Time: 3 Hours Maximum Marks: 60

## PART A

(Answer *ALL* questions)

$(10 \times 2 = 20)$

I. (a) Test whether the function $f(z) = xy + iy$ analytic or not.

(b) Find an analytic function whose imaginary part is $3x^2y - y^3$.

(c) Find the image of the circle $|z-1| = 1$ in the complex plane under the mapping $\omega = \frac{1}{z}$.

(d) Show that cross ratio remains invariant under a bilinear transformation.

(e) Show that (i) $\frac{1}{z^2} = \sum_{n=0}^{\infty}(n+1)(z+1)^n$ when $|z+1| < 1$ and

(ii) $\frac{1}{z^2} = \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$ when $|z-2| < 2$

(f) Form the partial differential equation of all spheres having centres on the z-axis.

(g) Solve $3p^2 - 2q^2 = 4pq$.

(h) Solve $z = p^2 + q^2$.

(i) Derive one dimensional heat equation.

(j) Solve the equation $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ where $u(0, y) = 8e^{-3y}$ by the method of separation of variables.

## PART B

$(4 \times 10 = 40)$

II. (a) Evaluate using Cauchy's integral formula $\int_c \frac{\sin 3z}{z + \frac{\pi}{2}} dz$ if $c$ is the circle $|z| = 5$.

(b) Find Laurent's series expansion of $\frac{1}{(z+1)(z+3)}$ in powers of $(z+1)$ for the range $0 < |z+1| < 2$.

**OR**

III. (a) Evaluate using residue theorem $\int_c \frac{z \sec z}{(1-z^2)} dz$ where $c$ is the ellipse $4x^2 + 9y^2 = 9$.

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

IV. (a) Show that the polar form of Cauchy-Riemann equations are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.

(b) Discuss the mapping properties of $\omega = z^2$.

**OR**

V. (a) Find the bilinear transformation which maps the points $z = 1, i, -1$ ONTO the points $\omega = i, 0, -i$.

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied at the origin.

VI. (a) Form the partial differential equation by eliminating the arbitrary function from $\phi(x^2 + y^2 + z^2, xyz) = 0$.

(b) Find the general solution of $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - z^2)$.

**OR**

VII. (a) Solve $(D^2 - 5DD' + 6D'^2)z = e^{x+y}$.

(b) Solve $q^2 - p = y - x$.

VIII. (a) Solve one dimensional wave equation.

(b) Obtain solution of Laplaces equation over a rectangular region by the method of separation of variables.

**OR**

IX. (a) Obtain $D'$ Alembert's solution of wave equation.

(b) An insulated rod of length $l$ has its ends $A$ and $B$ maintained at 0°c and 100°c respectively until steady state condition prevail. If B is suddenly reduced to 0°c and maintained at 0°c, find the temperature at a distance $x$ from $A$ at time $t$.

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