BTS-IV(S)-04.21-0246 Reg. No. [ ][ ][ ][ ][ ][ ][ ][ ]

B

B.Tech. Degree IV Semester Supplementary Examination April 2021

CE/CS/EC/EE/IT/ME/SE-
AS 15-1401 COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS
(*2015 Scheme*)

Time: 3 Hours Maximum Marks: 60

PART A
(Answer *ALL* questions)

(10 × 2 = 20)

I. (a) Verify Cauchy Riemann equations for an analytic function.

(b) Prove that real and imaginary parts of an analytic function are harmonic.

(c) Test the analyticity of the function $f(z)=\bar{z}$.

(d) Evaluate $\int_C \frac{\sin(3z)}{z+\frac{\pi}{2}}dz$ if C is the circle $|z|=5$.

(e) Show that $\frac{1}{z^2}=\frac{1}{4}\sum_{n=0}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$ $when|z-2|<2.$

(f) Form the partial differential equation by eliminating the arbitrary function g from the relation $g(x+y+z,x^2+y^2+z^2)=0$.

(g) Solve $z=p^2+q^2$.

(h) Solve $q^2-p=y-x$.

(i) Using the method of separation of variables solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$ where $u(x,0)=6e^{-3x}$.

(j) Derive one dimensional wave equation.

PART B

(4 × 10 = 40)

II. (a) Find an analytic function whose imaginary part is $3x^2y-y^3$ also find its harmonic conjugate.

(b) Find the image of the circle $|z-1|=1$ in the complex plane under the mapping $\omega=\frac{1}{z}$.

OR

III. (a) Show that $\omega=\sin z$ transform the semi infinite strip $0\le x\le\frac{\pi}{2}, y\ge 0$ ONTO the first quadrant of the ω – plane.

(b) Find the bilinear transformation which maps the points $z_1=2, z_2=i$ and $z_3=-2$ into the points $\omega_1=1, \omega_2=i$ and $\omega_3=-1$.

(P.T.O.)

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IV. (a) Using cauchy's integral formula, find the value of $\int_C \frac{z+4}{z^2+2z+5}dz$ where C is the circle $|z+1-i|=2$.

(b) Find the residue of $f(z)=\frac{\sin(z)}{z\cos(z)}$ at each of its poles inside the circle $|z|=2$.

OR

V. (a) Using residue theorem, evaluate $\int_C \frac{dz}{(z^2+4)^2}$ where C is the circle $|z-i|=2$.

(b) Using contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos(\theta)}$.

VI. Solve

(i) $x^2p^2+y^2q^2=z^2$

(ii) $(x-y)p+(y-x-z)q=z$

OR

VII. Solve

(i) $(D^2-2DD')(z)=e^{2x}+x^3y$

(ii) $(D^2+DD'-6D'^2)(z)=y\cos(x)$

VIII. (a) Derive one dimensional heat equation.

(b) Obtain the solution of the Laplace equation by the method of separation of the variables.

OR

IX. (a) A string is stretched and fastened to two points $x=0$ and $x=l$ apart. Motion is started by displacing the string in to the form $y=k(lx-x^2)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance x from one end at time in the form of fourier series.

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