BTS–IV–04.19–0646 Reg. No. [ ] C

B.Tech. Degree IV Semester Examination April 2019

CE/CS/EC/EE/IT/ME/SE AS 15-1401 COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS

(2015 Scheme)

Time : 3 Hours Maximum Marks : 60

PART A

(Answer *ALL* questions)

$(10 \times 2 = 20)$

I. (a) If f(z) is an analytic function with constant modulus, show that f(z) is constant.

(b) Find the orthogonal trajectories of the family of curves $r^2 \cos 2\theta = c$.

(c) Show that the transformation $w = \frac{1}{z}$ transforms all circles and straight lines into circles and straight lines in the w-plane. Which circles in the z-plane become straight lines in the w-plane and which circles are transformed into other circles?

(d) Evaluate $\int_c \frac{z}{z^2 - 3z + 2} dz$ where c is the circle $|z+1| = 1$.

(e) Determine the poles and the residue at each pole of $\frac{z^2+1}{z^2 - 2z}$.

(f) Find the Taylor's series expansion of $f(z) = \frac{1}{z^2 - z - 6}$ about $z = 1$.

(g) Form the partial differential equation by eliminating the arbitrary function from $xyz = f(x+y+z)$.

(h) Solve $yp + xq + pq = 0$.

(i) Find the solution of Laplace equation over a rectangular region by the method, of separation of variables.

(j) Using D'Alembert's method find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection f (x)=a(x-x^2).

PART B

$(4 \times 10 = 40)$

II. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not regular at the origion, although C-R equations are satisfied.

(b) Find the analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$.

OR

III. (a) Determine the linear fractional transformation that sends the points $z = 0, -1, 2i$ into the points $w = 5i, \infty, -\frac{i}{3}$ respectively. Find the image of $|z| < 1$ under this transformation.

(b) Under the transformation $w = \frac{z-i}{1-iz}$, find the map of the circle $|z| = 1$ in the w plane.

(P.T.O.)

IV. (a) Evaluate $\int_c \frac{e^z}{(z-i)^2(z^2+4)}$ dz where $c:|z-1|=1/2$, using Cauchy's integral formula.

(b) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\, d\theta$.

OR

V. (a) Find the Laurent expansion of $\frac{7z^2+9z-18}{z^3-9z}$ in the region (i) $0<|z|<3$ (ii) $|z|>3$.

(b) Evaluate $\int_c \frac{dz}{(z^2+4)^2} c:|z-i|=2$, using residue theorem.

VI. (a) Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$.

(b) Solve $z^2(p^2+q^2)=x^2+y^2$.

OR

VII. (a) Solve $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2 \partial y}=2e^{2x}+3x^2 y$.

(b) Solve $\left(D^2+DD'-6D'^2\right)z=\cos(2x+y)$.

VIII. (a) Derive one dimensional heat equation.

(b) A tightly stretched string of length 1 with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $V_0 \sin^3(\frac{\pi x}{\ell})$. Find the displacement y(x,t).

OR

IX. (a) A bar 100 cm long, with insulated sides, has its ends kept of 0°C and 100°C until steady state conditions prevail. The two ends are then suddenly insulated and kept so. Find the temperature distribution.

(b) Solve $u_{xx}+u_{yy}=0$ which satisfies the conditions $u(0,y)=u(l,y)=u(x,0)=0$ and $u(x,a)=\sin\frac{n\pi x}{l}$.

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