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B.Tech. Degree IV Semester Examination April 2018

CE/CS/EC/EE/IT/ME/SE

AS 15-1401 COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS
(2015 Scheme)

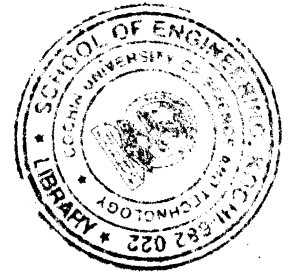
Time : 3 Hours

Maximum Marks : 60

PART A
(Answer *ALL* questions)

(10 × 2 = 20)

- I. (a) Prove the polar form of Cauchy Riemann Equations, $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$.
- (b) Prove that $v = \tan^{-1}(y/x)$ is harmonic.
- (c) Find the bilinear transform which maps the points $z: 1, i, -1$ into the points $w: 0, 1, \infty$.
- (d) Evaluate $\int_0^{2+3i} (x^2 - iy) dz$ along the path $y = x^2$.
- (e) State and prove Cauchy's Integral formula.
- (f) Evaluate $\int_C \frac{e^{2z} dz}{(z+1)^4}$, $C: |z| = 2$.
- (g) Form partial differential equation from $z = f(x+ct) + \phi(x-ct)$.
- (h) Solve $p(1+q) = qz$.
- (i) Using method of separation of variables solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$.
- (j) Find the solution of one dimensional heat equation.



PART B

(4 × 10 = 40)

- II. (a) Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied at the origin.
- (b) Let $w = u + iv$ be an analytic function. Then prove that the family of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ cut orthogonally.

OR

- III. (a) Show that the function $f(z) = \sinh z$ is an analytic function and find its derivative.
- (b) Find the image of $|z - 2i| = 2$ under the mapping $w = \frac{1}{z}$.

(P.T.O.)

IV. (a) Using Cauchy's Integral Formula, evaluate $\int_C \frac{z}{(z-1)(z-2)^2} dz$, $C: |z-2| = \frac{1}{2}$.

(b) Expand using Laurent's series :

(i) $f(z) = \frac{1}{z(z-1)}$, $0 < |z| < 1$

(ii) $f(z) = \frac{1}{z^2 - 3z - 2}$, $0 < |z-1| < 1$

OR

V. (a) Using Cauchy's Residue theorem, evaluate $\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz$, $C: |z| = 2$.

(b) Using Contour integration, evaluate $\int_0^{2\pi} \frac{\cos \theta}{3 + \sin \theta} d\theta$.

VI. Solve

(i) $z^2(p^2 + q^2 + 1) = b^2$

(ii) $p^2 + q^2 = z^2(x + y)$

OR

VII. Solve

(i) $x^2(y-z)p + y^2(z-x)q - z^2(x-y) = 0$

(ii) $(D^3 - 3D^2D^1 + 4D^1^3)z = e^{x+2y}$

VIII. (a) Derive one dimensional wave equation.

(b) Obtain solution of Laplace equation by the method of separation of variables.

OR

IX. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in the position $y=f(x)$. It is vibrating by giving each of its points a velocity

$\frac{\partial y}{\partial t} = g(x)$ at $t=0$. Find $y(x, t)$ in the form of Fourier Series.

