BTS-IV(SS)-02.20-0053 Reg.No. [| | | | | | | |]

A

B.Tech. Degree IV Semester Special Supplementary Examination February 2020

CE/CS/EC/EE/IT/ME/SE

AS 15–1401 COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS

(2015 Scheme)

Time : 3 Hours Maximum Marks : 60

PART A

(Answer *ALL* questions)

$(10 \times 2 = 20)$

I. (a) Define analytic function. Give an example of a function which is analytic and an example which is not analytic.

(b) Show that an analytic function with constant modulus is constant.

(c) Find the Taylor series expansion of $f(z) = \dfrac{z-1}{z+1}$ about the point $z = 1$.

(d) Find the residue of $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ at each of the poles.

(e) Evaluate $\int_c \dfrac{dz}{z^2 e^z}$ where C is $|z| = 1$.

(f) Form the partial differential equation by eliminating $'f'$ from the relation $z = y^2 + 2f\left(\dfrac{1}{x} + \log y\right)$.

(g) Solve $z = p^2 + q^2$.

(h) Solve $q^2 - p = y - x$.

(i) Using the method of separation of variables solve $\dfrac{\partial u}{\partial x} + 4 = \dfrac{\partial u}{\partial t}$ with $u = 4e^{-3x}$ when $t = 0$.

(j) Derive one dimensional wave equation.

PART B

$(4 \times 10 = 40)$

II. (a) Find the image of the circle $|z-1| = 1$ in the complex plane under the mapping $w = \dfrac{1}{z}$.

(b) Find the bilinear transformation which maps the points $z = 2, i, -2$ into the points $w = 1, i, -1$.

OR

III. (a) Find the analytic function $f(z) = u + iv$ if $u + v = \dfrac{x}{x^2 + y^2}$ and $f(1) = 1$.

(b) Show that $u = e^x \cos y + xy$ is harmonic and find its harmonic conjugate.

(P.T.O.)

IV. (a) Expand $\dfrac{1}{(z-1)(z-2)}$ in Laurent's series valid for $|z|<1$ and $1<|z|<2$.

(b) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

OR

V. (a) Evaluate $\displaystyle\int_c \frac{dz}{\left(z^2+4\right)^2}$ where C is the circle $|z-i|=2$ using residue theorem.

(b) State and prove Cauchy's integral formula.

VI. (a) Find the partial differential equation of all spheres of fixed radius having their centers in the xy plane.

(b) Solve $x^2p^2+y^2q^2=z^2$.

OR

VII. Solve the following:

(a) $r+s-6t=\cos(2x+y)$.

(b) $x(y-z)p+y(z-x)q=z(x-y)$.

VIII. (a) Obtain D'Alembert's solution of wave equation.

(b) Derive one dimensional heat equation.

OR

IX. (a) Solve the equation $\dfrac{\partial u}{\partial t}=\alpha^2\dfrac{\partial^2 u}{\partial x^2}$ subject to the boundary condition $u(0,t)=0,\ u(\ell,t)=0,\ u(x,0)=x$.

(b) Obtain the solution of Laplace's equation by the method of separation of variables.

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