

$$\vec{x} = (x, y, z); \quad c = c(\vec{x}); \quad u = u(t, \vec{x}); \quad \hat{u} = \hat{u}(\omega, \vec{x}) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t, \vec{x}) e^{-i\omega t} dt; \quad \nabla_{xy}^2 = \partial_x^2 + \partial_y^2; \quad \nabla^2 = \nabla_{xy}^2 + \partial_z^2$$

$$\partial_t^2 u = c^2 \nabla^2 u \xrightarrow{\mathcal{F}_t} -\omega^2 \hat{u} = c^2 \nabla^2 \hat{u} \longrightarrow \left( -\nabla_{xy}^2 - \frac{\omega^2}{c^2} \right) \hat{u} = \partial_z^2 \hat{u} \quad (1)$$

$$u_z = \partial_z u; \quad \hat{u}_z = \partial_z \hat{u}; \quad \mathbf{M} = \begin{pmatrix} 0 & 1 \\ -\nabla_{xy}^2 - \frac{\omega^2}{c^2} & 0 \end{pmatrix}$$

$$\partial_z \begin{pmatrix} \hat{u} \\ \hat{u}_z \end{pmatrix} (\omega, \vec{x}) = \mathbf{M} \begin{pmatrix} \hat{u} \\ \hat{u}_z \end{pmatrix} (\omega, \vec{x}) \quad (2)$$