Part 1 - Preliminaries

Pushforward measure:

Given a measure space, $(X, \mathcal{A}, \mu)$, a measurable space, $(Y, \mathcal{B})$, and an $(\mathcal{A}, \mathcal{B})-$measurable function, $f : X \to Y$, we may construct a measure on $(Y, \mathcal{B})$, $f_*(\mu) := \mu \circ f^{-1}$.
pf: (1) $\phi \in \mathcal{B}$, $f_*(\mu)(\phi) = \mu(F^{-1}(\phi)) = \mu(\phi) = 0$.
(2) $\{B_k\}_{k\in\mathbb{N}} \in \mathcal{B}$, disjoint, $B := \cup_{k\in\mathbb{N}} B_k$. $A := f^{-1}(B) = \cup_{k\in\mathbb{N}} A_k$, $A_k := f^{-1}(B_k)$. Then $A \in \mathcal{A}$ by f being $(\mathcal{A}, \mathcal{B})-$measurable, and $\{A_k\}_{k\in\mathbb{N}}$ is disjoint because, when $E_1 \cap E_2 = \phi$, $E_1, E_2 \in \mathcal{B}$, $\phi = f^{-1}(E_1 \cap E_2) = f^{-1}(E_1) \cap f^{-1}(E_2)$. Then, $f_*(\mu)(\cup_{k\in\mathbb{N}} B_k) = \mu(f^{-1}(\cup_{k\in\mathbb{N}} B_k)) = \mu(\cup_{k\in\mathbb{N}} f^{-1}(B_k)) = \mu(\cup_{k\in\mathbb{N}} A_k) = \sum_{k\in\mathbb{N}} \mu(A_k) = \sum_{k\in\mathbb{N}} (f_*(\mu))(B_k)$, by countable additivity of μ. So $(Y, \mathcal{B}, f_*(\mu))$ is a well defined measure space obtained by pushing forward μ via f.

Probability law of a Stochastic process:

Given $(\Omega, \mathcal{F}, P)$ a probability space, and T an index set, $(S, \mathcal{A})$ a measurable space. Then $X : T \times \Omega \to S$ is a stochastic process when the $t-$section of X, $X_t(\omega) := X(t, \omega)$ is $(\mathcal{F}, \mathcal{A})-$measurable for all $t \in T$. Let $S^T = \{g : T \to S\}$.
Each stochastic process, X induces a function, $\Phi_X : \Omega \to S^T$ by $\Phi_X(\omega) := t \mapsto X(t, \omega)$, so $\Phi_X(\omega)$ is the $\omega-$section of X. We're interested in defining a measure on a suitable sigma algebra on S^T, by pushing forward P via Φ_X.