

Part 1 - BM

$W_t := W(t, \omega)$ a brownian motion. Discretize this and approximate as a random walk, then $dW_j := W(t_{j+1}, \omega) - W(t_j, \omega)$, $dW_j \in \{-h, +h\}$, with $t_j = h \cdot j$, $P(dW_j = +h) = P(dW_j = -h) = \frac{1}{2}$. Then $t_0 = 0$, $W_{t_k} = \sum_{j=1}^k dW_j$

Implementation:

Let $y_j = \{1 \text{ if } dW_j = +h, 0 \text{ if } dW_j = -h\}$, then $dW_j = 2h y_j - h$, $W_{t_k} = \sum_{j=0}^k (2h y_j - h) = -k \cdot h + 2h \cdot \sum_{j=0}^k y_j$, so this looks like a binomial random variable. Then, seeing as $y_j \in \{1, 0\}$, with even probability, picking 32 y_j s (or whatever the word size if of the computer) is equivalent to picking a 32 bit integer with uniform distribution over the integer's full scale.

$\Omega_n = \{\omega_k : 0 \leq k < 2^n, \omega_k = (a_{k,0}, a_{k,1}, \dots, a_{k,n-1}), k = \sum_{j=0}^{n-1} a_{n,j} 2^{-j}, a_{k,j} \in \{0, 1\}\}$, $P_n(\omega_k) = 2^{-n}$.
 $X_n : \Omega_n \rightarrow \mathbb{R}$, $X_n(\omega_k) = \sum_{j=0}^{n-1} a_{n,j}$. $Y_n(\omega_k) := 2X_n(\omega_k) - n$. $b(x; n) = P_n(X_n^{-1}(\{x\})) = \binom{n}{x} 2^{-n}$.
 $g(x; n) := P_n(Y_n^{-1}(\{x\})) = P_n((2X_n - n)^{-1}(\{x\})) = b(\frac{x+n}{2}; n) = \binom{n}{\frac{x+n}{2}} 2^{-n}$.

Then, it is know that for $E(X_n) = \frac{1}{2}n$, $\sigma_{X_n}^2 = \frac{1}{4}n$, so $E(Y_n) = 0$, $\sigma_{Y_n}^2 = n$, so $\sqrt{h} \cdot Y_k$ has the same basic properties $W(hk)$.