Notes on the derivation of Finite Differences kernels, on regularly spaced grids, using aritrary sample points

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$$u(s,0) = u_0(s) \tag{.1}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(s,t) = \frac{1}{s}\frac{\partial^1}{\partial s^1}u(s,t) + \frac{\partial^2}{\partial s^2}u(s,t) \tag{.2}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(s,t) = \frac{1}{s}\frac{\partial^1}{\partial s^1}u(s,t) + \frac{\partial^2}{\partial s^2}u(s,t) \tag{.3}$$

$$u(s,t) = T(s,t;u_0) = f(s,t)\,u_0(g(s,t)) \tag{.4}$$

$$\frac{\partial^0}{\partial s^0}u(s,t) = f(s,t)\,u_0(g(s,t)) \tag{.5}$$

$$\frac{\partial^1}{\partial s^1}u(s,t) = \left(\frac{\partial^1}{\partial s^1}f(s,t)\right)u_0(g(s,t)) + f(s,t)\;u_0'(g(s,t))\left(\frac{\partial^1}{\partial s^1}g(s,t)\right) \tag{.6}$$

$$\begin{aligned}
\frac{\partial^2}{\partial s^2}u(s,t) = &\left(\frac{\partial^2}{\partial s^2}f(s,t)\right)u_0(g(s,t)) + \\
&2\left(\frac{\partial^1}{\partial s^1}f(s,t)\right)u_0'(g(s,t))\left(\frac{\partial^1}{\partial s^1}g(s,t)\right) + \\
&f(s,t)\;u_0''(g(s,t))\left(\frac{\partial^1}{\partial s^1}g(s,t)\right)^2 + \\
&f(s,t)\;u_0'(g(s,t))\left(\frac{\partial^2}{\partial s^2}g(s,t)\right)
\end{aligned}$$

$$\frac{1}{s}\frac{\partial^1}{\partial s^1}u(s,t) = \frac{1}{s}\left(\frac{\partial^1}{\partial s^1}f(s,t)\right)u_0(g(s,t)) + \frac{1}{s}f(s,t)\;u_0'(g(s,t))\left(\frac{\partial^1}{\partial s^1}g(s,t)\right)$$

$$\begin{aligned}
\frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(s,t) = &\frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2}f(s,t)\right)u_0(g(s,t)) + \\
&2\frac{1}{c^2}\left(\frac{\partial^1}{\partial t^1}f(s,t)\right)u_0'(g(s,t))\left(\frac{\partial^1}{\partial t^1}g(s,t)\right) + \\
&\frac{1}{c^2}f(s,t)\;u_0''(g(s,t))\left(\frac{\partial^1}{\partial t^1}g(s,t)\right)^2 + \\
&\frac{1}{c^2}f(s,t)\;u_0'(g(s,t))\left(\frac{\partial^2}{\partial t^2}g(s,t)\right)
\end{aligned}$$

$$\frac{1}{s}\left(\frac{\partial^1}{\partial s^1}f(s,t)\right)u_0(g(s,t))+$$

$$\frac{1}{s}f(s,t)\left(\frac{\partial^1}{\partial s^1}g(s,t)\right)u_0'(g(s,t))+$$

$$\left(\frac{\partial^2}{\partial s^2}f(s,t)\right)u_0(g(s,t))+$$

$$2\left(\frac{\partial^1}{\partial s^1}f(s,t)\right)\left(\frac{\partial^1}{\partial s^1}g(s,t)\right)u_0'(g(s,t))+$$

$$f(s,t)\left(\frac{\partial^1}{\partial s^1}g(s,t)\right)^2 u_0''(g(s,t))+$$

$$f(s,t)\left(\frac{\partial^2}{\partial s^2}g(s,t)\right)u_0'(g(s,t))=$$

$$\frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2}f(s,t)\right)u_0(g(s,t))+$$

$$2\frac{1}{c^2}\left(\frac{\partial^1}{\partial t^1}f(s,t)\right)\left(\frac{\partial^1}{\partial t^1}g(s,t)\right)u_0'(g(s,t))+$$

$$\frac{1}{c^2}f(s,t)\left(\frac{\partial^1}{\partial t^1}g(s,t)\right)^2 u_0''(g(s,t))+$$

$$\frac{1}{c^2}f(s,t)\left(\frac{\partial^2}{\partial t^2}g(s,t)\right)u_0'(g(s,t))$$

$$\frac{1}{s}\left(\frac{\partial^1}{\partial s^1}f(s,t)\right)u_0(g(s,t))+$$

$$\left(\frac{\partial^2}{\partial s^2}f(s,t)\right)u_0(g(s,t))=$$

$$\frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2}f(s,t)\right)u_0(g(s,t))$$

$$\frac{1}{s}f(s,t)\left(\frac{\partial^1}{\partial s^1}g(s,t)\right)u_0'(g(s,t))+$$

$$2\left(\frac{\partial^1}{\partial s^1}f(s,t)\right)\left(\frac{\partial^1}{\partial s^1}g(s,t)\right)u_0'(g(s,t))+$$

$$f(s,t)\left(\frac{\partial^2}{\partial s^2}g(s,t)\right)u_0'(g(s,t))=$$

$$2\frac{1}{c^2}\left(\frac{\partial^1}{\partial t^1}f(s,t)\right)\left(\frac{\partial^1}{\partial t^1}g(s,t)\right)u_0'(g(s,t))+$$

$$\frac{1}{c^2}f(s,t)\left(\frac{\partial^2}{\partial t^2}g(s,t)\right)u_0'(g(s,t))$$

$$f(s,t)\left(\frac{\partial^1}{\partial s^1}g(s,t)\right)^2 u_0''(g(s,t))=$$

$$\frac{1}{c^2}f(s,t)\left(\frac{\partial^1}{\partial t^1}g(s,t)\right)^2 u_0''(g(s,t))$$

$$\pm c \left(\frac{\partial^1}{\partial s^1} g(s,t) \right) = \left(\frac{\partial^1}{\partial t^1} g(s,t) \right) \rightarrow g(s,t) = s \pm ct$$

$$\frac{1}{s} \frac{\partial^1}{\partial s^1} f(s,t) + \frac{\partial^2}{\partial s^2} f(s,t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} f(s,t)$$

$$\frac{1}{s} f(s,t) +$$

$$2 \left(\frac{\partial^1}{\partial s^1} f(s,t) \right) =$$

$$2 \frac{1}{c^2} \left(\frac{\partial^1}{\partial t^1} f(s,t) \right) (\pm c) +$$

$$\frac{1}{s} f(s,t) + 2 \frac{\partial^1}{\partial s^1} f(s,t) = \frac{\pm 2}{c} \frac{\partial^1}{\partial t^1} f(s,t)$$