1 problem statement

Problem: find a good estimate or exact expression for

$$\tilde{F}^{d,p}_{m,\sigma}(x) = \left(\int_x^\infty |\delta^{(d)}_{m,\sigma}(x')|^p dx' \right)^{1/p}$$

with

$$\delta^{(d)}_{m,\sigma}(x) = \frac{d^d}{d\,x^d}\,\delta_{m,\sigma}(x),$$
$$\delta_{m,\sigma}(x) = \frac{1}{\sqrt{\pi}}\,e^{\left(-\frac{x^2}{2\sigma^2}\right)} \sum_{n=0}^{m} L_n^{(-\frac{1}{2})}\left(\frac{x^2}{2\sigma^2}\right)$$

then we can use

$$F^{d,p}_{m,\sigma}(x) = \frac{\tilde{F}^{d,p}_{m,\sigma}(x)}{\tilde{F}^{d,p}_{m,\sigma}(0)}$$

so

$$F^{d,p}_{m,\sigma}(0) = 1,\ F^{d,p}_{m,\sigma}(\infty) = 0,$$

so $F^{d,p}_{m,\sigma}(x_0)$ is the error introduced by truncating the convolution integral as

$$f^{(d)}(x) = \int_{x-x_0}^{x+x_0} f(x')\,\delta^{(d)}(x-x')\,dx'.$$

2 relevant formulas

mupltiplication of polynomials:

$$p(x) = \sum_{k=0}^{n} p_k\,x^k\ , q(x) = \sum_{k=0}^{m} q_k\,x^k,\ pq(x) = \sum_{k=0}^{n+m} c_k\,x^k,\ c_k := \sum_{j=0}^{k} p_j\,q_{k-j} = \sum_{j=k-m}^{\min(k,n)} p_j\,q_{k-j}.$$

explicit formuas for hermite polynomials:

$$H_n = \sum_{k=0}^{n} h_{n,k} x^k$$

$$h_{n,k} = \frac{(-1)^n \, n! \, 2^k}{k! \, \lfloor \frac{n-k}{2} \rfloor !} \cos((n+k)\frac{\pi}{2})$$

$$h_{2n,2k} = \frac{(2n)!}{(2k)!(n-k)!} 4^k \, (-1)^{n+k}$$

$$h_{2n+1,2k+1} = \frac{(2n+!)!}{(2k+1)!(n-k)!} 2^{2k+1} \, (-1)^{n+k} = \frac{2(2n+1)}{2k+1} \, h_{2n,2k}$$

switch order of summation

$$\sum_{i=0}^{n} \sum_{j=0}^{i} = \sum_{j=0}^{n} \sum_{i=j}^{n}$$

explicit formula for σ-less HDAF

$$\delta_m(x) = e^{-x^2} \sum_{k=0}^{m} c_k^m \, x^{2k}$$

$$c_k^m = \frac{(-1)^k}{(2k)! \, \sqrt{\pi}} \sum_{n=0}^{m-k} \frac{(2n+2k)! \, 4^{-n}}{n! \, (n+k)!}$$

to do: simplify this.

explicit formula for $\delta_m^{(d)}(x)$

$$\delta_m(x) = e^{-x^2} \sum_{k=0}^{m} c_k^m \, x^{2k}$$

$$c_k^m = \frac{(-1)^k}{(2k)! \, \sqrt{\pi}} \sum_{n=0}^{m-k} \frac{(2n+2k)! \, 4^{-n}}{n! \, (n+k)!}$$

to do: simplify this.

3 case: p=2