



Laboratório Nacional de Computação Científica  
Programa de Pós-Graduação em Modelagem Computacional

# **Uncertainty Quantification Management System**

Noel Moreno Lemus

Petrópolis, RJ - Brasil

Abril de 2018

Noel Moreno Lemus

# **Uncertainty Quantification Management System**

Thesis submitted to the examining committee  
in partial fulfillment of the requirements for  
the degree of Doctor of Sciences in Computa-  
tional Modeling.

Laboratório Nacional de Computação Científica  
Programa de Pós-Graduação em Modelagem Computacional

Supervisor: Fábio André Machado Porto

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Petrópolis, RJ - Brasil  
Abril de 2018

**Dedication**

*To my little and special family.*

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O autor manifesta reconhecimentos às pessoas e instituições que colaboraram para a execução de seu trabalho.

*“Essentially, all models are wrong, but some are useful.”*  
*(George Edward Pelham)*

# Abstract

Segundo a ??, 3.1-3.2), o resumo deve ressaltar o objetivo, o método, os resultados e as conclusões do documento. A ordem e a extensão destes itens dependem do tipo de resumo (informativo ou indicativo) e do tratamento que cada item recebe no documento original. O resumo deve ser precedido da referência do documento, com exceção do resumo inserido no próprio documento. (...) As palavras-chave devem figurar logo abaixo do resumo, antecedidas da expressão Palavras-chave:, separadas entre si por ponto e finalizadas também por ponto.

**Keywords:** latex. abntex. editoração de texto.



# Abstract

This is the english abstract.

**Keywords:** latex. abntex. text editoration.

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# List of abbreviations and acronyms

UQ                      Uncertainty Quantification

FP                      Forward Problem

*QoI*                    Quantity of Interest

# List of symbols

$\Gamma$	Letra grega Gama
$\Lambda$	Lambda
$\zeta$	Letra grega minúscula zeta
$\in$	Pertence

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# 1 Introduction

## 1.1 Research Objectives

## 1.2 Highlights of the Dissertation

## 1.3 Organization of the Dissertation



## 2 Background

### 2.1 Overview

HPC and computational modeling play a dominant role in shaping the methodological developments and research in uncertainty qualification. Depending on the complexity of the uncertainty qualification investigation, anywhere from  $10^2$  to  $10^8$  runs of the computational model may be required. Thus, uncertainty qualification investigations may require extreme-computing environments (e.g., exascale) to obtain results in a useful time frame, even if a single run of the computational model does not require such resources.

Advances in computing over the past few decades—both in availability and power—have led to an explosion in computational models available for simulating a wide variety of complex physical (and social) systems. These complex models—which may involve millions of lines of code, and require extreme-computing resources—have led to numerous scientific discoveries and advances. This is because these models allow simulation of physical processes in environments and conditions that are difficult or even impossible to access experimentally. However, scientists’ abilities to quantify uncertainties in these model-based predictions lag well behind their abilities to produce these computational models. This is largely because such simulation-based scientific investigations present a set of challenges that is not present in traditional investigations.

Until recently, the original approach of describing model parameters using single values has been retained, and consequently the majority of mathematical models in use today provide point predictions, with no associated uncertainty. (JOHNSTONE et al., 2016)

An immediate challenge in the development of an appropriate treatment of uncertainty in an analysis of a complex system is the selection of a mathematical structure to be used in the representation of uncertainty. (HELTON et al., 2010a) Traditionally, probability theory has provided this structure [48-55]. However, in the last several decades, additional mathematical structures for the representation of uncertainty such as evidence theory [56-63], possibility theory [64- 70], fuzzy set theory [71-75], and interval analysis [76-81] have been introduced. This introduction has been accompanied by a lively discussion of the strengths and weaknesses of the various mathematical structures for the representation of uncertainty [82-90]. For perspective, several comparative discussions of these different approaches to the representation of uncertainty are available [72; 91-98]

a ‘typical’ UQ problem involves one or more mathematical models for a process of interest, subject to some uncertainty about the correct form of, or parameter values for,

those models.

Often, though not always, these uncertainties are treated probabilistically.

but how will you actually go about evaluating that expected value when it is an integral over a million-dimensional parameter space? Practical problems from engineering and the sciences can easily have models with millions or billions of inputs (degrees of freedom).

the language of probability theory is a powerful tool in describing uncertainty

UQ cannot tell you that your model is ‘right’ or ‘true’, but only that, if you accept the validity of the model (to some quantified degree), then you must logically accept the validity of certain conclusions (to some quantified degree). (SULLIVAN, 2015)

“UQ studies all sources of error and uncertainty, including the following: systematic and stochastic measurement error; ignorance; limitations of theoretical models; limitations of numerical representations of those models; limitations of the accuracy and reliability of computations, approximations, and algorithms; and human error. A more precise definition is UQ is the end-to-end study of the reliability of scientific inferences.”

UQ is not a mature field like linear algebra or single-variable complex analysis, with stately textbooks containing well-polished presentations of classical theorems bearing August names like Cauchy, Gauss and Hamilton. Both because of its youth as a field and its very close engagement with applications, UQ is much more about problems, methods and ‘good enough for the job’. There are some very elegant approaches within UQ, but as yet no single, general, over-arching theory of UQ.

In

Probability theorists usually denote the sample space of a probability space by  $\Omega$ ; PDE theorists often use the same letter to denote a domain in  $\mathbb{R}^n$  on which a partial differential equation is to be solved. In UQ, where the worlds of probability and PDE theory often collide, the possibility of confusion is clear. Therefore, this book will tend to use  $\Theta$  for a probability space and  $\mathbf{X}$  for a more general measurable space, which may happen to be the spatial domain for some PDE.

## 2.2 Types of Uncertainty

It is sometimes assumed that uncertainty can be classified into two categories, (KIUREGHIAN; DITLEVSEN, 2009) although the validity of this categorization is open to debate.

Aleatory uncertainty arises from an inherent randomness in the properties or behavior of the system under study. For example, the weather conditions at the time

of a reactor accident are inherently random with respect to our ability to predict the future. Other examples include the variability in the properties of a population of weapon components and the variability in the possible future environmental conditions that a weapon component could be exposed to. Alternative designations for aleatory uncertainty include variability, stochastic, irreducible and type A. (HELTON, 2009)

Epistemic uncertainty derives from a lack of knowledge about the appropriate value to use for a quantity that is assumed to have a fixed value in the context of a particular analysis. For example, the pressure at which a given reactor containment would fail for a specified set of pressurization conditions is fixed but not amenable to being unambiguously defined. Other examples include minimum voltage required for the operation of a system and the maximum temperature that a system can withstand before failing. Alternative designations for epistemic uncertainty include state of knowledge, subjective, reducible and type B. (HELTON, 2009)

### 2.2.1 Aleatory uncertainty

Aleatory uncertainty arises from an inherent randomness in the properties or behavior of the system under study. For example, the weather conditions at the time of a reactor accident are inherently random with respect to our ability to predict the future. Other examples include the variability in the properties of a population of weapon components and the variability in the possible future environmental conditions that a weapon component could be exposed to. Alternative designations for aleatory uncertainty include variability, stochastic, irreducible and type A. (HELTON, 2009)

### 2.2.2 Epistemic uncertainty

## 2.3 Uncertainty Representation

The question of how to represent and communicate uncertainties is a topic of research both from a practical and theoretical point of view. A fair bit of theoretical research is aimed at the mathematical calculus of uncertainty. This includes extensions and alternatives to standard probabilistic reasoning, such as Dempster-Schafer theory and imprecise probabilities. When uncertainties are needed for investigations requiring computational models, additional considerations arise. For example, if the simulation output is a daily surface-temperature field over the globe for the next 200 years, representing uncertainty and dependencies is complex. Should ensembles be used to represent plausible outcomes? How should these ensembles of simulation output be stored? How can high-consequence/low-probability outcomes be discovered in this massive output? Here some research investigations attempt to leverage theory that exploits high dimensionality to bound probabilities and system behavior. Finally, even when uncertainties are well captured,

Table 1 – The range of the GLD parameters and the minimum and maximum values corresponding to the labeling of the regions given in Figure

Region	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	Minimum	Maximum
1 and 5	all	$< 0$	$< -1$	$> 1$	$-\infty$	$\lambda_1 + \frac{1}{\lambda_2}$
2 and 6	all	$< 0$	$> 1$	$< -1$	$\lambda_1 - \frac{1}{\lambda_2}$	$\infty$
3	all	$> 0$	$> 0$	$> 0$	$\lambda_1 - \frac{1}{\lambda_2}$	$\lambda_1 + \frac{1}{\lambda_2}$
	all	$> 0$	$= 0$	$> 0$	$\lambda_1$	$\lambda_1 + \frac{1}{\lambda_2}$
	all	$> 0$	$> 0$	$= 0$	$\lambda_1 - \frac{1}{\lambda_2}$	$\lambda_1$
4	all	$< 0$	$< 0$	$< 0$	$-\infty$	$\infty$
	all	$< 0$	$= 0$	$< 0$	$\lambda_1$	$\infty$
	all	$< 0$	$< 0$	$= 0$	$-\infty$	$\lambda_1$

how best to communicate such uncertainties to the public or to decision-makers is also a topic of ongoing research.

([HELTON et al., 2010b](#))

### 2.3.1 Representation of Uncertainty with Probability

### 2.3.2 Dempster-Shafer theory

### 2.3.3 The Bayesian Methodology

## 2.4 Methods for Uncertainty Propagation

## 2.5 Probabilistic Background

### 2.5.1 The Generalized Lambda Distribution

The Generalized Lambda Distribution (GLD) was defined by Ramberg and Schmeiser in 1974 by the quantil function:

$$F^{-1}(p|\lambda) = F^{-1}(p|\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2} \quad (2.1)$$

where  $p$  are the probabilities,  $p \in [0, 1]$ ,  $\lambda_1$  and  $\lambda_2$  are the location and scale parameteres, and  $\lambda_3$  and  $\lambda_4$  determine the skewness and kurtosis of the  $GLD(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ .

Some restrictions in the values of  $\lambda_1, \lambda_2, \lambda_3$  and  $\lambda_4$  define if the  $GLD$  is valid. Those restrictions define 6 regions as is shown in table

The probability density function of the  $GLD$  at the point  $x = F^{-1}(p)$  is given by:

$$f(x) = f(F^{-1}(p)) = \frac{\lambda_2}{\lambda_3 p^{\lambda_3-1} + \lambda_4 (1-p)^{\lambda_4-1}} \quad (2.2)$$

Note that the valid parameters of  $\lambda$  guaranty that:

$$f(x) \geq 0 \tag{2.3}$$

$$\int f(x)dx = 1 \tag{2.4}$$

## 2.5.2 Fitting Mixture Distributions Using a Mixture of Generalized Lambda Distributions

Esto esta en ([TOBERGTE; CURTIS, 2013](#))

## 2.5.3 Sampling Methods

### 2.5.3.1 Monte Carlo

## 2.6 Summary

## 2.7 Concepts

high-dimensional parameter spaces computationally demanding forward models  
nonlinearity and/or complexity in the forward model

## 3 Uncertainty Quantification Process

### 3.1 Measures of Information and Uncertainty

#### 3.1.1 Variance, Information and Entropy

**Variance.**

**Information and Entropy.**

#### 3.1.2 Information Gain, Distances and Divergences

### 3.2 Sensitivity Analysis

Sensitivity analysis is the systematic study of how model inputs—parameters, initial and boundary conditions—affect key model outputs. Depending on the application, one might use local derivatives or global descriptors such as Sobol’s functional decomposition or variance decomposition. Also, the needs of the application may range from simple ranking of the importance of inputs to a response surface model that predicts the output given the input settings. Such sensitivity studies are complicated by a number of factors, including the dimensionality of the input space, the complexity of the computational model, limited forward model runs due to the computational demands of the model, the availability of adjoint solvers or derivative information, stochastic simulation output, and high-dimensional output. Challenges in sensitivity analysis include dealing with these factors while addressing the needs of the application. (??)

$$E = mc^2 \tag{3.1}$$

## 4 UQMS Data Model

### 4.1 Introduction

To create a data model to our Uncertainty Quantification Management System (UQMS), we need to start with the definition of all the concepts that are related to the UQ process. As we talk before, from a very general point of view, we can simplify the UQ problem as follow:

Given a mathematical model,

$$\mathcal{Y} = \mathcal{M}(\mathcal{I}, \mathcal{P}) \quad (4.1)$$

where:

I: parameter inputs that are certain.

P: set of input parameters that are uncertain.

M: mathematical model, and

Y: output (quantities of interes)

### 4.2 SaViMe

### 4.3 UQMS Data Model

### 4.4 UQMS Operations

## 5 Applicability

In the present chapter we are going to test the UQMS in three different scenarios, spatial only domain, section 5.1, spatio-temporal domain, section 5.2, and finally a multidisciplinary system, section 5.3.

### 5.1 Case Study: Wave Propagation Problem

The first one is a geophysical tests for wave propagation problems

As a first case study we use the “HPC4E Seismic Test Suite”, a collection of four 3D models and sixteen associated tests that can be downloaded freely at the project’s website (<https://hpc4e.eu/downloads/datasets-and-software>). The models include simple cases that can be used in the development stage of any geophysical imaging practitioner (developer, tester ...) as well as extremely large cases that can only be solved in a reasonable time using ExaFLOPS supercomputers. The models are generated to the required size by means of a Matlab/Octave script and hence can be used by users of any OS or computing platform. The tests can be used to benchmark and compare the capabilities of different and innovative seismic modelling approaches, hence simplifying the task of assessing the algorithmic and computational advantages that they pose.

In our case, we are going to use the “HPC4E Seismic Test Suite” as a case study of the proposed UQMS. As we mention in the introduction of this chapter this model is a spatial only domain problem, because we are going to consider a multidimensional array as an Input and a multidimensional array as an output, but of them time independent.

#### 5.1.1 Mathematical Formulation

#### 5.1.2 Model and Dataset Description

The models have been designed as a set of 16 layers with constant physical properties. The top layer delineates the topography and the other 15 different layer interface surfaces or horizons. In the following, an interface horizon is associated with properties that apply to the layer that exists between itself and the immediately next layer horizon. The model covers an area of 10 x 10 x 5 km, with maximum topography at about 500 m and maximum depth at about 4500 m. The layer horizons have been sampled very finely with 1.6667 m spacing so that a highly accurate representation can be honored at high frequencies. For simulation schemes based on unstructured grids, the layer horizons can be used easily to constrain model blocks. For simulation schemes based upon Cartesian grids, a simple



Table 2 – Layer constant properties and their depth range. “Star” layers are only used in the flat case, in substitution of their non-star equivalents

Layer Id	Vp (m/s)	Vs (m/s)	Density (Kg/m3)	Max. depth (m)	Min. depth (m)
1	1618.92	500.00	1966.38	-135.55	-476.35
2	1684.08	765.49	1985.88	41.50	-394.90
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
2*					
3*					

script is provided that can generate 3D grids for any desired spatial sampling. Table 2 shows the properties of each of the layers included in the models.

### 5.1.3 Adding uncertainty into the model

The “HPC4E Seismic Test Suite” does not provide uncertainty sources, because all the input parameters of the model have fixed values. Then, to the purpose of our work we need to add some uncertainties into the inputs. Let’s suppose the variable  $V_p$  is uncertain. As this variable have 16 different values, one for each layer, we can consider it as a random vector, equation 5.1. We associate to each of the  $V_{p_i}$  a Normal distribution with  $\mu_i$  equal to the value reported in Table 2 and  $\sigma = 2$ .

$$V_p = \langle V_{p_i}, \mathcal{N}(\mu_i, \sigma_i) \rangle \quad (5.1)$$

## 5.2 Case Study: Austin, queso library

## 5.3 Case Study: Multidisciplinary System (NASA)

## 5.4 Case Study: Spatio-temporal Nicholson-Bailey model

Este esta en el software uqlab, en la carpeta Doc Manuals

## 6 Validation

## 7 Conclusions

## 8 Future Works

- Use of other distribution families in the regions where the GLD is not valide.
- Implement the same flow using other distribution families.
- Optimize all the process using Spark or other Big data System.
- Implement all the algorithms as operators inside SAVIME.

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# Appendix

# APPENDIX A – Título do apêndice A

Quisque facilisis auctor sapien. Pellentesque gravida hendrerit lectus. Mauris rutrum sodales sapien. Fusce hendrerit sem vel lorem. Integer pellentesque massa vel augue. Integer elit tortor, feugiat quis, sagittis et, ornare non, lacus. Vestibulum posuere pellentesque eros. Quisque venenatis ipsum dictum nulla. Aliquam quis quam non metus eleifend interdum. Nam eget sapien ac mauris malesuada adipiscing. Etiam eleifend neque sed quam. Nulla facilisi. Proin a ligula. Sed id dui eu nibh egestas tincidunt. Suspendisse arcu.

## A.1 Título da seção

Aqui temos uma seção dentro do Apêndice.



## APPENDIX B – Título do apêndice B

Nunc velit. Nullam elit sapien, eleifend eu, commodo nec, semper sit amet, elit. Nulla lectus risus, condimentum ut, laoreet eget, viverra nec, odio. Proin lobortis. Curabitur dictum arcu vel wisi. Cras id nulla venenatis tortor congue ultrices. Pellentesque eget pede. Sed eleifend sagittis elit. Nam sed tellus sit amet lectus ullamcorper tristique. Mauris enim sem, tristique eu, accumsan at, scelerisque vulputate, neque. Quisque lacus. Donec et ipsum sit amet elit nonummy aliquet. Sed viverra nisl at sem. Nam diam. Mauris ut dolor. Curabitur ornare tortor cursus velit.

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Aenean laoreet aliquam orci. Nunc interdum elementum urna. Quisque erat. Nullam tempor neque. Maecenas velit nibh, scelerisque a, consequat ut, viverra in, enim. Duis magna. Donec odio neque, tristique et, tincidunt eu, rhoncus ac, nunc. Mauris malesuada malesuada elit. Etiam lacus mauris, pretium vel, blandit in, ultricies id, libero. Phasellus bibendum erat ut diam. In congue imperdiet lectus.

## APPENDIX C – Título do apêndice C

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetur a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetur. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

## Annex

# ANNEX A – Título do anexo A

Donec et nisl id sapien blandit mattis. Aenean dictum odio sit amet risus. Morbi purus. Nulla a est sit amet purus venenatis iaculis. Vivamus viverra purus vel magna. Donec in justo sed odio malesuada dapibus. Nunc ultrices aliquam nunc. Vivamus facilisis pellentesque velit. Nulla nunc velit, vulputate dapibus, vulputate id, mattis ac, justo. Nam mattis elit dapibus purus. Quisque enim risus, congue non, elementum ut, mattis quis, sem. Quisque elit.

Maecenas non massa. Vestibulum pharetra nulla at lorem. Duis quis quam id lacus dapibus interdum. Nulla lorem. Donec ut ante quis dolor bibendum condimentum. Etiam egestas tortor vitae lacus. Praesent cursus. Mauris bibendum pede at elit. Morbi et felis a lectus interdum facilisis. Sed suscipit gravida turpis. Nulla at lectus. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Praesent nonummy luctus nibh. Proin turpis nunc, congue eu, egestas ut, fringilla at, tellus. In hac habitasse platea dictumst.

## A.1 Título da seção

Aqui temos uma seção dentro do Anexo.

Vivamus eu tellus sed tellus consequat suscipit. Nam orci orci, malesuada id, gravida nec, ultricies vitae, erat. Donec risus turpis, luctus sit amet, interdum quis, porta sed, ipsum. Suspendisse condimentum, tortor at egestas posuere, neque metus tempor orci, et tincidunt urna nunc a purus. Sed facilisis blandit tellus. Nunc risus sem, suscipit nec, eleifend quis, cursus quis, libero. Curabitur et dolor. Sed vitae sem. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Maecenas ante. Duis ullamcorper enim. Donec tristique enim eu leo. Nullam molestie elit eu dolor. Nullam bibendum, turpis vitae tristique gravida, quam sapien tempor lectus, quis pretium tellus purus ac quam. Nulla facilisi.

## ANNEX B – Título do anexo B

Curabitur nunc magna, posuere eget, venenatis eu, vehicula ac, velit. Aenean ornare, massa a accumsan pulvinar, quam lorem laoreet purus, eu sodales magna risus molestie lorem. Nunc erat velit, hendrerit quis, malesuada ut, aliquam vitae, wisi. Sed posuere. Suspendisse ipsum arcu, scelerisque nec, aliquam eu, molestie tincidunt, justo. Phasellus iaculis. Sed posuere lorem non ipsum. Pellentesque dapibus. Suspendisse quam libero, laoreet a, tincidunt eget, consequat at, est. Nullam ut lectus non enim consequat facilisis. Mauris leo. Quisque pede ligula, auctor vel, pellentesque vel, posuere id, turpis. Cras ipsum sem, cursus et, facilisis ut, tempus euismod, quam. Suspendisse tristique dolor eu orci. Mauris mattis. Aenean semper. Vivamus tortor magna, facilisis id, varius mattis, hendrerit in, justo. Integer purus.

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## ANNEX C – Título do anexo C

Nunc velit. Nullam elit sapien, eleifend eu, commodo nec, semper sit amet, elit. Nulla lectus risus, condimentum ut, laoreet eget, viverra nec, odio. Proin lobortis. Curabitur dictum arcu vel wisi. Cras id nulla venenatis tortor congue ultrices. Pellentesque eget pede. Sed eleifend sagittis elit. Nam sed tellus sit amet lectus ullamcorper tristique. Mauris enim sem, tristique eu, accumsan at, scelerisque vulputate, neque. Quisque lacus. Donec et ipsum sit amet elit nonummy aliquet. Sed viverra nisl at sem. Nam diam. Mauris ut dolor. Curabitur ornare tortor cursus velit.

Morbi tincidunt posuere arcu. Cras venenatis est vitae dolor. Vivamus scelerisque semper mi. Donec ipsum arcu, consequat scelerisque, viverra id, dictum at, metus. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut pede sem, tempus ut, porttitor bibendum, molestie eu, elit. Suspendisse potenti. Sed id lectus sit amet purus faucibus vehicula. Praesent sed sem non dui pharetra interdum. Nam viverra ultrices magna.

Aenean laoreet aliquam orci. Nunc interdum elementum urna. Quisque erat. Nullam tempor neque. Maecenas velit nibh, scelerisque a, consequat ut, viverra in, enim. Duis magna. Donec odio neque, tristique et, tincidunt eu, rhoncus ac, nunc. Mauris malesuada malesuada elit. Etiam lacus mauris, pretium vel, blandit in, ultricies id, libero. Phasellus bibendum erat ut diam. In congue imperdiet lectus.