A Comparative Study on Digital Mamography Enhancement Algorithms Based on Fuzzy Theory

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Abstract: Diagnosing cancer tissues using Digital mammograms are a time consuming task even for highly skilled radiologists because mammograms are low contrast, noisy images. Therefore, in digital mammogram there is a need for enhancing imaging before a reasonable segmentation can be achieved. In recent years, many researchers have applied the fuzzy logic to develop new image processing algorithms. Meanwhile, the fuzzy image processing is one of the important application areas of fuzzy logic. This paper gives a comparative study of fuzzy image enhancement techniques applied on digital mammogram images. Compared to other non-linear techniques, fuzzy filters are able to represent knowledge in a comprehensible way.

Keywords: Fuzzy set, Fuzzy image processing, fuzzy enhancement, Digital mammography.

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1. Introduction

Image enhancement in medical computing is the use of computers to make an image clearer [5]. This may be to aid interpretation by humans or computers. Types of image enhancement include, noise reduction, edge enhancement and contrast enhancement. Enhancement may be used to restore an image that has suffered some kind of deterioration or to enhance certain features of an image. Enhancement techniques can be useful in all areas of medicine. In an enhanced image it is often easier for a specialist to spot anomalies in an X-ray or CT scan, etc. For example, a medical specialist who is analyzing mammograms may have to get through each mammogram quickly to keep up with demand. In this case it is highly beneficial if the images are clear and any anomalies are easy to spot. However, because mammograms have limited contrast it may be hard to see an anomaly. In this case an enhanced image could both help the specialist see different structures in the image and check each mammogram faster [9, 12].

This paper presents a comparative study on digital mammography image enhancement based on fuzzy set theory in image processing. There are many reasons to do this. The most important of them are as follows:

1. Fuzzy techniques are powerful tools for knowledge representation and processing
2. Fuzzy techniques can manage the vagueness and ambiguity efficiently

In many image processing applications, we have to use expert knowledge to overcome the difficulties (e.g. object recognition, scene analysis). Fuzzy set theory and fuzzy logic offer us powerful tools to represent and process human knowledge in form of fuzzy if-then rules. On the other side, many difficulties in image processing arise because the data/tasks/results are uncertain. This uncertainty, however, is not always due to the randomness but to the ambiguity and vagueness. Beside randomness which can be managed by probability theory, there are three other kinds of uncertainty in the image processing, they are:

1. Grayness ambiguity
2. Geometrical fuzziness
3. Uncertain knowledge

These problems are fuzzy in the nature. The question whether a pixel should become darker or brighter than it already is, the question where is the boundary between two image segments, and the question what is a tree in a scene analysis problem, all of these and other similar questions are examples for situations where a fuzzy approach can be the more suitable way to manage the uncertainties.

Digital mammography refers to the application of digital system techniques on digital mammograms [8, 11]. Digital systems have the capacity to bring revolutionary advantages to breast cancer detection. Radiologists turn to digital mammography for an alternative diagnostic method due to the problems created by conventional screening programs. A digital mammogram is created when a conventional mammogram is digitized so a computer can use it. Digitization can be performed through the use of a specific mammogram digitizer or a camera [5,12]. 12 bits of detection resolution are usually needed to produce a high-resolution digital mammogram without the loss of information from the original mammogram. Generally, most digital mammograms have 4096 gray levels per pixel over the whole area of the mammogram. Currently, digital mammography is one of the most promising cancer control strategies since the cause of breast cancer is still unknown.

Enhancement algorithms are used to reduce image noise and increase the contrast of structures of interest in image. Where the distinction between normal and abnormal tissue is subtle, accurate interpretation may become difficult if noise levels are relatively high. In many cases enhancement improves the quality of the image and facilitates diagnosis enhancement techniques and generally provides a clearer image for a human observer but it can also form a preprocessing step for subsequent automated analysis.

Tumor detection in digital mammograms through image processing is a difficult task due to the following reasons:

1. Intensity levels vary greatly across different regions in a mammogram
2. Features for segmentation are hard to formulate
3. Subtle gray level variations across different parts of the image make the segmentation of tumor areas by gray level alone difficult
4. Tumors are not always obvious, especially where they are subtle or extremely subtle under the glandular tissues, which makes the task of interpretation difficult even for the radiologists themselves
5. Mammograms contain low signal to noise ratio (low contrast) and a complicated structured background.
6. Breast tissue contrast and density vary with age, thus mammography produces varying image qualities.
7. Mammography images are not bimodal. As a result, any segmentation method [6,7,8], which utilizes an a priori or single threshold value method, is highly likely to generate serious segmentation errors.

Mammography image analysis is a challenging task due to poor illumination and high noise levels in the image that can vary up to 10-15% of the maximum pixel intensity. This is a problem because the image enhancement process may undesirably enhance noise component in the image [6,13,16]. Hence, mammograms are among the most difficult images to analyses and interpret. Moreover, the image always seems cluttered, and the background varies greatly between different breasts. Even the worst abnormalities appear quite subtle and irregular.

2. Fuzzy Image Enhancement

Fuzzy image enhancement is based on gray level mapping into a fuzzy plane, using a membership transformation function [10,12]. The aim is to generate an image of higher contrast than the original image by giving a larger weight to the gray levels that are closer to the mean gray level of the image than to those that are farther from the mean. In recent years, many researchers have applied the fuzzy set theory to develop new techniques for contrast improvement [10]. An image I of size M x N and L gray levels can be considered as an array of fuzzy singletons, each having a value of membership denoting its degree of brightness relative to some brightness levels. For an image I, we can write in the notation of fuzzy sets:

$$I = \bigcup_{mn} \mu_{mn} / g_{mn} \qquad m = 1,2,...,M \text{ and } n = 1,2,...,N \tag{1}$$

Where g_{mn} is the intensity of (m, n)th pixel and μ_{mn} its membership value. The membership function characterizes a suitable property of image (e.g. edginess, darkness, textural property) and can be defined globally for the whole image or locally for its segments. In recent years, some researchers have applied the concept of fuzziness to develop new algorithms for image enhancement. The principle of fuzzy enhancement scheme is illustrated in Figure (1).

Figure 1: The Main Principles of Fuzzy Image Enhancement

3. Enhancement Algorithms

Contrast enhancement is useful when an area of the image that is of particular importance has only subtle changes in pixel intensity. In these cases, it may be difficult for the human eye to make out the structures clearly, especially if the image is being displayed on a low quality screen. By exaggerating the changes in pixel intensity the image may become easier to interpret [1,2,3,4]. Applying the contrast enhancement filter will improve the readability of areas with subtle changes in contrast but will also destroy areas of the image where the intensity of the pixels is outside the range of intensities being enhanced. Therefore, in this section we will discuss and implement five fuzzy image enhancement algorithms and compare among them to improve the quality of the contrast digital mammogram images. The five algorithms are:

1. Possibility Distribution Algorithm
2. Contrast Improvement with Intensification Operator
3. Contrast Improvement with Fuzzy Histogram Hyperbolization
4. Contrast Improvement based on Fuzzy If-Then Rules
5. Locally Adaptive Contrast Enhancement

In this section each of these algorithms is described in details.

3.1 Possibility Distribution Algorithm- algorithm #1

The possibility distribution [18] of the gray levels in the original image can be characterized using five parameters:*(* α *, ß1,*γ*, ß2, max)* as shown in Figure (2).

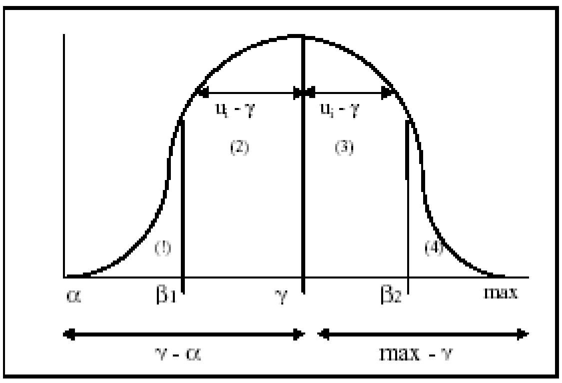


Figure 2: Possibility Distribution Function for Calculating Membership Values

Where the intensity value γ represents the mean value of the distribution, α is the minimum, and max is the maximum. The aim is to decrease the gray levels below *ß1*, and above *ß2*. Intensity levels between *ß1* and γ, and *ß2* and γ are stretched in opposite directions towards the mean γ.

The fuzzy transformation function for computing the fuzzy plane value *P* is defined as follows:

α= min;
$\beta 1= (\alpha+ \gamma) /2$;
$\beta\ 2= (\max + \gamma) /2$;
γ = mean; max;

The following fuzzy rules are used for contrast enhancement based on Figure (2).

Rule-1: If $\alpha \leq u_i < \beta_1$ then $P = 2\ (\ (\ u_i - \alpha) / (\ \gamma - \alpha\ ))^2$ (2)

Rule-2: If $\beta_1 \leq u_i < \gamma$ then $P = 1- 2\ (\ (\ u_i - \gamma\ )/(\ \gamma - \alpha\ ))^2$ (3)

Rule-3: If $\gamma \leq u_i < \beta_2$ then $P= 1- 2(\ (\ u_i - \gamma\ ) / (\ \max - \gamma\ ))^2$ (4)

Rule-4: If $\beta_2 \leq u_i < \max$ then $P = 2\ ((\ u_i - \gamma\ )/(\ \max - \gamma\ ))^2$ (5)

Where $u_i = f(x,y)$ is the i^{th} pixel intensity.

The possibility distribution algorithm is described as follows:

Step-1: Parameter Initialization

- Set β_1 = (min + mean)/2
- Set β_2 = (max + mean)/2

Step-2: Fuzzification

- For all pixels (i,j) within the image Do
  - If ((data[i][j]>=min) && (data[i][j]< β_1))
    Compute NewGrayLevel =2*(pow(((data[i][j]-min)/(mean- min)),2))
  - If ((data[i][j]> = β_1) && (data[i][j] < mean))
    Compute NewGrayLevel=1-(2*(pow(((data[i][j]-mean)/(mean-min)),2)))
  - If ((data[i][j]>=mean)&&(data[i][j]< β_2))
    Compute NewGrayLevel=1-(2*(pow(((data[i][j]-mean)/(max-mean)),2)))
  - If ((data[i][j] >= β_2) && (data[i][j] <max))
    Compute NewGrayLevel=2*(pow(((data[i][j]-mean)/(max-mean)),2))

Step-3: Modification

- Compute FuzzyData[i][j]= pow(NewGrayLevel,2)

Step-4: Defuzzification

- For all pixels (i,j) within the image Do
  Compute EnhancedData[i][j]=FuzzyData[i][j]*data[i][j];

3.2 Contrast Improvement with Intensification Operator

This method uses the intensification operator [19] to reduce the fuzziness of the image which results in an increase of image contrast [20, 21] .

Histogram equalization is a widely used and well-established method of enhancing such image as X-rays and landscape photographs that are taken under poor illumination. This method involves increasing the dynamic range of pixels by stretching their gray level probability distribution. It works by define an N x N neighborhood and moves the center of this area from pixel to pixel. At each location, the histogram of the subimages is calculated to obtain the histogram equalization function. This function is finally used to map the level of the pixel centered in the neighborhood. While in contrast enhancement based fuzzy, we need some parameters in each neighborhood for adjacent of the membership function such as minimum and maximum gray levels in the image. Then, we can find the parameters of the membership function for some subimages and interpolate these values to obtain corresponding values for each pixel. In many cases, the global adaptive implementation is necessary to achieve better results. Fuzzy-based local contrast is very fast compared to global and classical image enhancement algorithms.

3.2.1 Contrast Improvement with Intensification Operator - algorithm #2

- Setting the parameters (Fe, Fd, $g_{\max}$) of membership function

$$F_e = 2 \quad \text{and} \quad F_d = \frac{g_{\max} - g_{mid}}{0.5^{-1/F_e} - 1} \tag{6}$$

- Define the membership function

$$\mu_{mn} = G(g_{mn}) = \left[1 + \frac{g_{\max} - g_{\min}}{F_d}\right]^{-F_e} \tag{7}$$

- Modify the membership values

$$\mu^{\backslash}_{mn} = \begin{cases} 2.[\mu_{mn}]^2 & 0 \le \mu_{\mathrm{mn}} \le 0.5 \\ 1 - 2.[1 - \mu_{mn}]^2 & 0.5 \le \mu_{\mathrm{mn}} \le 1 \end{cases} \tag{8}$$

- Generate new gray-levels

$$g^{\backslash}_{mn} = G^{-1}(\mu^{\backslash}_{mn}) = g_{\mathbf{max}} - F_d\left(\left(\mu^{/}_{mn}\right)^{\frac{-1}{F_e}} - 1\right) \tag{9}$$

The algorithm is described as follows:

Step-1: Parameter Initialization

- Set F_e=2;

Step-2: Fuzzification of the gray levels by the transformation G:

- For all pixels (i,j) within the image Do
  Compute FuzzyData[i][j]=pow((1+((maxgray-data[i][j])/Fd)),-Fe);

Step-3: Recursive modification of the memberships with no. of iteration (k=2)

- For all pixels (i,j) within the image Do
  - If ((FuzzyData[i][j]>=0)&&(FuzzyData[i][j]<=0.5))
    Compute FuzzyData[i][j]=2*(pow(FuzzyData[i][j],2));
  - Else If ((FuzzyData[i][j]>=0.5)&&(FuzzyData[i][j]<=1);
    Compute FuzzyData[i][j]=1-(2*(pow((1-FuzzyData[i][j]),2)));
    Else Return

Step-4: Generation of new gray levels g_{mn} by the inverse transformation G^{1-}:

- For all pixels (i,j) within the image Do
  Compute NewGrayLevel = maxgray-(F_d*((pow(FuzzyData[i][j]) 1/F_e)))-1));
  - If (NewGrayLevel<0)
    EnhancedData[i][j]=0;
  - Else if (NewGrayLevel>255)
    EnhancedData[i][j]=255;
  - Else EnhancedData[i][j]=NewGrayLevel

3.3 Contrast Improvement with Fuzzy Histogram Hyperbolization - algorithm #3

The idea of histogram hyperbolization, and fuzzy histogram hyperbolization is described in [22] and [23], respectively. Due to the nonlinear human brightness perception, this algorithm modifies the membership values of gray levels by a logarithmic function. The algorithm can be formulated as follows:

1. Setting the shape of membership function.
2. Setting the value of the fuzzifier β .
3. Calculation of membership values μ_{mn} .
4. Modification of the membership values by β .
5. Generation of new gray levels, as described below.

The choice of the membership function is very important, as the membership function characterize a certain property of the image (edginess, darkness, textual property).

In this algorithm the shape of membership function is set as a triangular to characterize the hedges, and the value of fuzzifier β as a linguistic hedge such that: β = - 0.75 + μ 1.5. Then by calculating the membership values μ_{mn} *and* modifying the membership values by β. Generate new gray levels values g'_{mn} by following equation:

$$g'_{mn} = \left(\frac{L-1}{e^{-1}-1} \right) \cdot \left[e^{-\mu_{mn}(g_{mn})^{\beta}} - 1 \right] \qquad (10)$$

The algorithm is described as follows:

Step-1: Parameter initialization

- Setting the shape of membership function (triangular).
- Setting the value of fuzzifier β such that β = - 0.75 + μ 1.5

Step-2: Fuzzy data //A linear Membership function

- for (i=0;i<height;i++)
- for(j=0;j<width;j++) {
  - if(data[i][j]<100)
    FuzzyData[i][j]=0
  - else if((data[i][j]>=100)&&(data[i][j]<=200))
    FuzzyData[i][j]=(0.01*data[i][j])-1;
  - else if((data[i][j]>200)&&(data[i][j]<=255))
    FuzzyData[i][j]=1; }

Step-3: Modify the membership values

- Set ModificationBeta=2;
- For(i=0;i<height;i++)
- For(j=0;j<width;j++) {
  power=pow(FuzzyData[i][j],ModificationBeta); }

Step-4: Generation of new gray levels

- Set m=maxgray/(0.367879-1);
- EnhancedData[i][j]=m*(exp((-1*power))-1);

3.4 Contrast Improvement based on Fuzzy If-Then Rules - algorithm #4

The fuzzy rule-based approach is a powerful and universal method for many tasks in the image processing. A very simple inference rule-based system was developed. The fuzzification function is depicted in Figure (3).

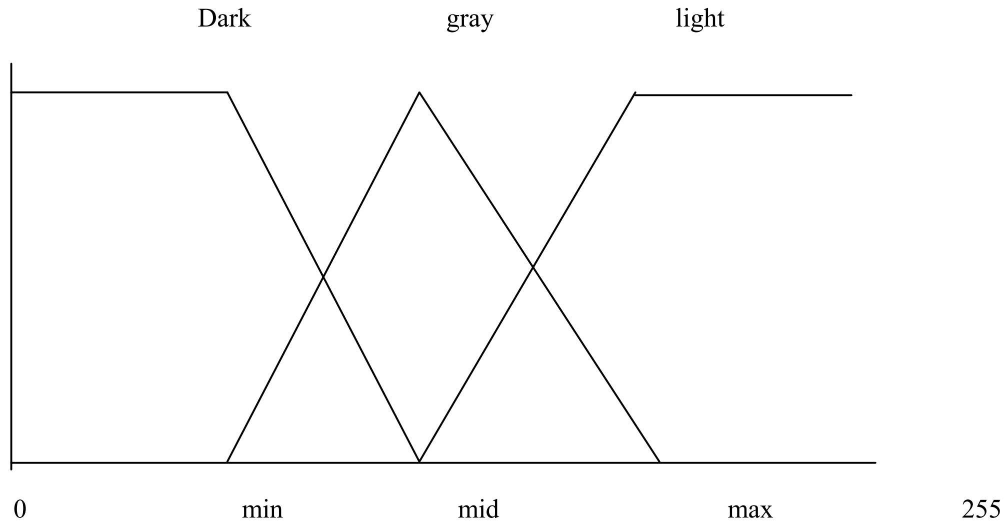


Figure 3: Membership function

The algorithm starts with the initialization of the image parameters; minimum and maximum gray level. Then by fuzzification of the gray levels (i.e., membership values to the dark, gray and bright) sets gray levels. The inference procedure evaluating appropriately the following rules:

- If dark then black
- If gray then gray
- If bright then white

Finally, defuzzification of the output using minimum (g_{min}), maximum (g_{max}) and medium (g_{mid}) of the gray levels such that the new enhanced gray level is computed by the following equation:

$$g = \frac{\mu_{dark} * g_{gray} + \mu_{gray} * g_{mid} + \mu_{bright} * g_{\max}}{\mu_{dark} + \mu_{gray} + \mu_{bright}} \tag{11}$$

Our implementation uses only three rules, however the use of an extended rule base will increase the performance of this method.

The algorithm is described as follows:

Step-1: Parameter initialization

- Finding the minimum and maximum gray level
- Calculating the mid gray level= (max+min)/2.

Step-2: Fuzzification

- For I=0; I<height; I++
- For J=0; J<width; J++
    - If 0<= data<=min then Fuzzydata_I=1;
        - Else if min<=data<=mid Fuzzydata_I=(1/mid-min)*min-(1/mid min)*data;
    - If mid<= data<=max then
        - Fuzzydata_I=(-1/max-mid)*mid+(1/max-mid)*data;
    - Else if max<=data<=255 then Fuzzydata_I=1;
    - If min<= data<=mid then
        - Fuzzydata_II=(-1/mid-min)*min+(1/mid-min)*data;
    - Else if mid<=data<=max then
        - Fuzzydata_II=(1/max-mid)*mid+1+(-1/max-mid)*data;

Step-3: Modification

- For I=0; I<height; I++
- For J=0; J<width; J++
    - If 0<= data<=min
        - If dark THEN darker and set Fuzzydata_I=1; **//dark.**
        - Else if min<=data<=mid *//Using contrast intensification.*
    - For x=0; x<3; x++
        - If 0<= Fuzzydata_I <=0 then
            - Fuzzydata_I=2*(Fuzzydata_I)^2;
        - else if 0.5<= Fuzzydata_I <=1 then
            - Fuzzydata_I=1-2*(1-Fuzzydata_I)^2;
    - If mid<= data<=max **// light.**
        - For x=0; x<3; x++
        - If 0<= Fuzzydata_I <=0.5 then
            - Fuzzydata_I=2*(Fuzzydata_I)^2;
        - else if 0.5<= Fuzzydata_I <=1 then
            - Fuzzydata_I=1-2*(1-Fuzzydata_I)^2;
        - Else if max<=data<=255
        - IF light THEN lighter and set Fuzzydata_I=1; **//gray.**
        - If min<= data<=mid then
            - Fuzzydata=min(Fuzzydata_I,Fuzzydata_II);
        - Else if mid<=data<=max then
            - Fuzzydata=MAX(Fuzzydata_I,Fuzzydata_II);

Step-4: Deffuzzification

- For I=0; I<height; I++
- For J=0; J<width; J++
    - If 0<= data<=min then
        - Enhanceddata=data; //Dark
    - Else if max<=data<=255 then
        - Enhanceddata=data; //light.
    - If min<= data<=mid //gray.
    - If Fuzzydata==Fuzzydata_II then
        - Enhanceddata=(mid-min)*Fuzzydata+min;
        - else Enhanceddata=-(mid min)*Fuzzydata+min+(mid- min);
        - Else if mid<=data<=max
    - If Fuzzydata==Fuzzydata_II then
        - Enhanceddata=-(max-mid)*Fuzzydata+mid+(max-mid);
    - Else Enhanceddata=(max-mid)*Fuzzydata+mid;

3.5 Locally Adaptive Contrast Improvement - algorithm #5

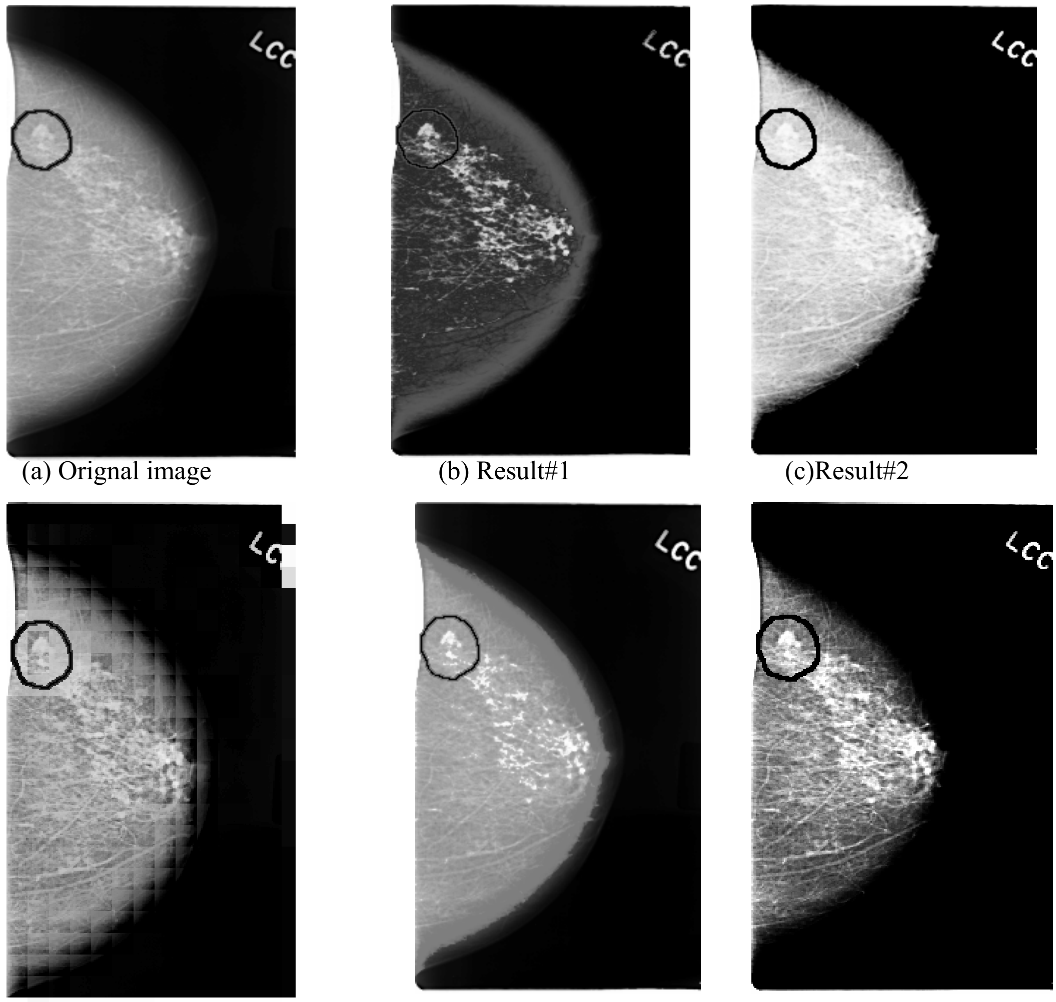
It is based on applying a locally adaptive image enhancement, by defining an n x m neighborhood and move the center of this area from pixel to pixel [24], at each location each algorithm parameter is calculated.

For algorithm (3.1), we need the minimum, maximum, and mean value of each nxm block, α, γ, max respectively , to calculate the membership values. For calculation of membership values in algorithms (3.2.1), (3.3), and (3.4) we need only minimum, and maximum gray levels.

In many cases, the global fuzzy techniques fail to deliver satisfactory results [14,15,17]. Therefore, a locally adaptive implementation is necessary to achieve better results. The disadvantage of using adaptive techniques is that we need to calculate minimum and maximum gray levels, which may lead to noise affecting the membership value falsely. To avoid this we can either select sufficiently great sub images, or by eliminating noisy data in the histogram of each sub image.

4. Results and Discussion

Figure (4) shows the experimental results of the fuzzy-based enhancement techniques introduced in this paper. Figure (4-a) shows the original mammogram image. Figure (4-b) shows the result of the possibility distribution algorithm. Figure (4-c) shows the results of the global improvement with intensification operator. Figure (4-d) shows the result of the adaptive improvement with intensification operator. Figure (4-e) shows the result of the rule based algorithm. Figure (4-f) shows the histogram hyperbolization result.



(a) Orignal image (b) Result#1 (c)Result#2

(d) Result#3 (e) Result#4 (f) Result#5

Figure 4: Visual Contrast enhancement results

To measure the quality of the original and enhanced images we use the linear index of fuzziness γ and the fuzzy entropy *H*. Where

$$\gamma = \frac{2}{MN}\sum_M \sum_N \min(\mu_{mn}, 1-\mu_{mn}), \tag{12}$$

$$H = \frac{1}{MN \ln 2}\sum_M \sum_N -\mu_{mn}\ln(\mu_{mn}) - (1-\mu_{mn})\ln(1-\mu_{mn}) \tag{13}$$

The index of fuzziness was defined by Kaufmann [25], and fuzzy entropy by De Luca and Termini [26]. The index of fuzziness, for instance, reflects the ambiguity in an image by measuring the distance between its fuzzy property plane and the nearest ordinary plane. Both index of fuzziness and fuzzy entropy are measures for global grayness ambiguity (fuzziness) of an image. They can be regarded as a degree of difficulty in deciding whether a pixel should be treated as black (dark) or white (bright) [27]. It should be noted that the decrease in the index of fuzziness and fuzzy entropy does not ensure proper enhancement of the images.

Table (1) demonstrates the grayness ambiguity of the introduced algorithms. Quantitative justification of image quality is carried out by the use of measures of fuzziness.

Table 1:Grayness Ambiguity Between the Original Image and the Five Algorithms.

Image	**Grayness ambiguity**	
	Index of fuzziness	**Fuzzy entropy**
Original image	0.280374	0.392456
Algorithm #1	0.265769	0.361702
Algorithm #2	0.280268	0.521419
Algorithm #3	0.00738927	0.0141429
Algorithm #4	0.198138	0.268460
Algorithm #5	0.245001	0.402536

5. Conclusion

Fuzzy image processing is a powerful tool for formulation of expert knowledge edge and the combination of imprecise information from different sources. In this paper, we have studied different fuzzy image enhancement techniques to increase the contrast of the digital mammogram images for further processing. A comparison among the introduced fuzzy techniques based on the grayness ambiguity measure is demonstrated**.**

Table (2) demonstrates the effect of the decrease of grayness ambiguity of the introduced algorithms.

Table 2: The effect of the Decrease of Grayness Ambiguity.

Algorithm	**Conclusion**
Algorithm #1	Decreases both the index of fuzziness and the entropy, and the resulting image is appropriate for visual perception and future tracking.
Algorithm #2	Increased both grayness ambiguity, and therefore the resulting image is not appropriate for visual perception.
Algorithm #3	Compared to other algorithms, it gives the lowest grayness ambiguity, and its results are appropriate for visual perception.
Algorithm #4	Gives lower grayness ambiguity than the first algorithm, although the later are more appropriate for visual perception.
Algorithm #5	Does not decrease the grayness ambiguity much, and the resulting image is not appropriate for visual perception nor future tracking.

Basically, we can only say that a *good* enhancement algorithm should reduce the grayness ambiguity. However, a low amount of ambiguity does not automatically lead to the desired enhancement effect.

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