

SE(3) WNOA Interp Jacobians

$$\begin{aligned}\hat{\varpi} &= \mathcal{J}(\ln(\hat{\mathbf{T}}(\tau)\hat{\mathbf{T}}_k^{-1})^\vee)(\mathbf{\Lambda}_2(\tau)\hat{\gamma}_k(t_k) + \mathbf{\Psi}_2(\tau)\hat{\gamma}_k(t_{k+1})) \\ &= \mathcal{J}(\mathbf{\Lambda}_1(\tau)\hat{\gamma}_k(t_k) + \mathbf{\Psi}_1(\tau)\hat{\gamma}_k(t_{k+1})) \underbrace{\left[\mathbf{\Lambda}_2(\tau)\hat{\gamma}_k(t_k) + \mathbf{\Psi}_2(\tau)\hat{\gamma}_k(t_{k+1}) \right]}_{\varpi_\tau}\end{aligned}$$

$$\hat{\gamma}_k(t_k) = \begin{bmatrix} \mathbf{0} \\ \hat{\varpi}_k \end{bmatrix}, \quad \hat{\gamma}_k(t_{k+1}) = \begin{bmatrix} \ln(\hat{\mathbf{T}}_{k+1,k})^\vee \\ \mathcal{J}(\ln(\hat{\mathbf{T}}_{k+1,k})^\vee)^{-1} \hat{\varpi}_{k+1} \end{bmatrix}$$

$$\hat{\gamma}_k(t_{k+1}) \approx \begin{bmatrix} \ln(\hat{\mathbf{T}}_{\text{op},k+1,k})^\vee + \mathcal{J}_{\text{op},k+1,k}^{-1}(\epsilon_{k+1} - \mathcal{T}_{\text{op},k+1,k}\epsilon_k) \\ \left(\mathcal{J}_{\text{op},k+1,k}^{-1} - \frac{1}{2} \left(\mathcal{J}_{\text{op},k+1,k}^{-1}(\epsilon_{k+1} - \mathcal{T}_{\text{op},k+1,k}\epsilon_k) \right)^\wedge \right) (\varpi_{\text{op},k+1} + \eta_{k+1}) \end{bmatrix}$$

$$\begin{aligned}\frac{\partial \hat{\varpi}(\tau)}{\partial \mathbf{x}} &= \mathcal{J}_{\text{op},\tau,k} \frac{\partial}{\partial \mathbf{x}} \left[\mathbf{\Lambda}_2(\tau)\hat{\gamma}_k(t_k) + \mathbf{\Psi}_2(\tau)\hat{\gamma}_k(t_{k+1}) \right] \\ &\quad - \frac{1}{2} \varpi_{\text{op},\tau}^\wedge \frac{\partial}{\partial \mathbf{x}} \left[\mathbf{\Lambda}_1(\tau)\hat{\gamma}_k(t_k) + \mathbf{\Psi}_1(\tau)\hat{\gamma}_k(t_{k+1}) \right]\end{aligned}$$

$$\frac{\partial \varpi_\tau}{\partial \epsilon_{k+1}} = \Psi_{21} \mathcal{J}_{\text{op},k+1,k}^{-1} + \frac{1}{2} \Psi_{22} \varpi_{\text{op},k+1}^\wedge \mathcal{J}_{\text{op},k+1,k}^{-1}$$

$$\frac{\partial \varpi_\tau}{\partial \epsilon_k} = -\Psi_{21} \mathcal{J}_{\text{op},k+1,k}^{-1} \mathcal{T}_{\text{op},k+1,k} - \frac{1}{2} \Psi_{22} \varpi_{\text{op},k+1}^\wedge \mathcal{J}_{\text{op},k+1,k}^{-1} \mathcal{T}_{\text{op},k+1,k}$$

$$\frac{\partial \varpi_\tau}{\partial \eta_k} = \Lambda_{22}$$

$$\frac{\partial \varpi_\tau}{\partial \eta_{k+1}} = \Psi_{22} \mathcal{J}_{\text{op},k+1,k}^{-1}$$

Note: the Jacobians for $\left[\mathbf{\Lambda}_1(\tau)\hat{\gamma}_k(t_k) + \mathbf{\Psi}_1(\tau)\hat{\gamma}_k(t_{k+1}) \right]$ look the same, you just need to use the appropriate interpolation constants: $\mathbf{\Lambda}_1, \mathbf{\Psi}_1$.