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A new predictive thermal sensation index of human response

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Abstract

The purpose of this paper is to investigate the problem of determining a human thermal sensation index that can be used in feedback control of HVAC systems. We present a new approach based on fuzzy logic to estimate the thermal comfort level depending on the state of the following six variables: the air temperature, the mean radiant temperature, the relative humidity, the air velocity, the activity level of occupants and their clothing insulation. The new fuzzy thermal sensation index is calculated implicitly as the consequence of linguistic rules that describe human's comfort level as the result of the interaction of the environmental variables with the occupant's personal parameters. The fuzzy comfort model is deduced on the basis of learning Fanger's 'Predicted Mean Vote' (PMV) equation. Unlike Fanger's PMV, the new fuzzy PMV calculation does not require an iterative solution and can be easily adjusted depending on the specific thermal sensation of users. These characteristics make it an attractive index for feedback control of HVAC systems. The simulation results show that the new fuzzy PMV is as accurate as Fanger's PMV. © 1999 Elsevier Science S.A. All rights reserved.

Keywords: Thermal sensation index; Predicted mean vote; HVAC systems

1. Introduction

Over the years, a large number of thermal comfort indices have been set up for the analysis of indoor climates and the design of HVAC systems [6,7,9,14]. But only a few of them have been used to evaluate the ability of an existing room climate to create satisfactory thermal conditions for occupants. The most common and best understood one is Fanger's 'Predicted Mean Vote' (PMV). In 1970, Fanger [6] published a general comfort equation which makes it possible, for any activity level and any clothing insulation, to calculate all combinations of the environmental variables (air temperature, air humidity, mean radiant temperature and relative air velocity) which will create optimal thermal comfort. In order to derive the comfort equation, Fanger supposed that for long exposures to a constant thermal environment with a constant metabolic rate, a heat balance can be established for the human body and the bodily heat production is equal to its heat dissipation [6]. Based on these assumptions, Fanger developed a thermal sensation index to predict the mean thermal sensation vote on a standard scale for a large group of persons depending on the four thermal environmental variables, the activity level and the clo-value of clothing worn by the occupants. Because the thermal comfort equation is not an explicit function of the six variables that affect thermal comfort, Fanger utilised psycho-physical scales of subjective responses that describe an occupant's feeling of warmth or coolness in a given situation. To compute a value of PMV, it is necessary to know the values of the clothing insulation and the activity level of occupants and to measure the environmental variables at a suitable number of points in the occupied zone. On the basis of these measurements, the PMV index determines whether a given indoor climate is satisfactory or not using a seven-point thermal sensation scale.

Fanger's work is adopted by the ISO 7730 standard [12] for providing an index of comfort/discomfort levels taking all of the six above-mentioned factors into account. In order to ensure a comfortable indoor climate, ISO recommends to maintain the PMV at 0 with a tolerance of 0.5. Fig. 1 summarises the overall process of using the six variables associated with thermal comfort sensation to evaluate the PMV.

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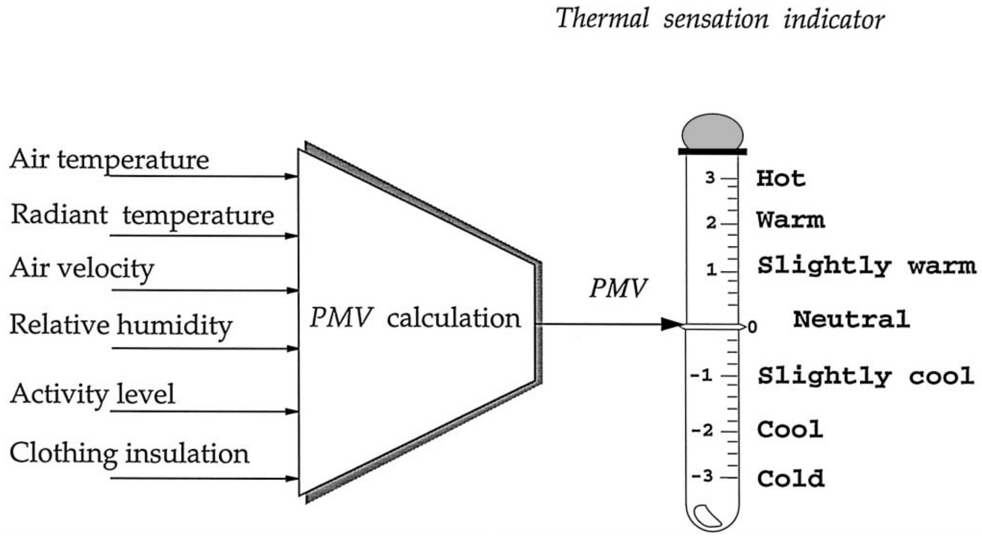

Fig. 1. PMV and thermal sensation.

The general comfort equation developed by Fanger [6] to describe the conditions under which a large group of people will feel in a thermal neutrality is too complex and cannot be used in real time applications. In fact, the predicted mean vote calculation, as ASHRAE uses it today, is non-linear and necessitates iterative solutions because the involved heat transfer processes are relatively complicated. Thus, the mathematical expression derived for the calculation of the PMV is cumbersome and not suitable for feedback control of HVAC systems [6,7,11,14]. To overcome these problems, Fanger and ISO proposed in Refs. [6,12] to use tables and diagrams to simplify the determination of the PMV in practical applications. Alternatively, many researchers proposed simplified models of PMV to avoid the iterative process [7,14,19]. These researchers also recognized, however, that the simplification of Fanger's PMV equation results in a significant error when the assumptions are not respected.

Presently, the challenge is to derive a thermal sensation index based on the original work of Fanger that can be used in feedback HVAC control applications with an on-line calibration and without requiring any simplification. With this goal in mind, this paper presents a new strategy for the design of an accurate thermal sensation index that does not require any iteration solutions and that can be used as a high level performance variable in the control of HVAC systems. According to the state of the six parameters that affect human's thermal comfort, the proposed thermal sensation index can be calculated directly on the basis of knowledge extracted form the original work of Fanger. To reach this objective, fuzzy logic modelling, which is defined as a method of describing characteristics of systems using fuzzy reasoning [16], is used to approximate the human's comfort/discomfort level in a given indoor climate. By analysing the influence of the individual variables on the thermal sensation index, it becomes possible to evaluate linguistically how each variable influences the thermal sensation. It was shown [19] that it is impossible to consider the effect of the six variables on the human thermal sensation independently, as the effect of each of them depends on the level and the state of the other variables; the thermal comfort level is a complicated result of the interaction of the six variables [6]. On the basis of this analysis, a new fuzzy PMV is designed and a general fuzzy rule base is derived to describe the state of human's thermal sensation. For any activity level and any clothing, the fuzzy rule base is able to calculate all combinations of the four environmental variables which will create optimal thermal comfort. The new designed thermal sensation index can then be easily used in feedback control of HVAC systems.

This paper is organized as follows. Section 2 presents the problem formulation of the PMV model. Then, a brief overview of fuzzy set theory and fuzzy logic approximation principal are presented and applied to the estimation of the human thermal sensation. Section 4 outlines the design technique of the new fuzzy predicted mean vote. Section 5 presents the simulation results and outlines the performance of the designed fuzzy PMV by comparing it to Fanger's PMV. Finally, the Section 6 gives conclusions and a summary of the characteristics of the proposed thermal sensation index model.

2. Problem formulation

For many years, it has been desirable to determine directly human's thermal sensation in a given environment condition and for a specified activity level and clothing insulation. Until the 60's, thermal comfort calculation was limited by the lack of a well-defined unit to represent the degree of the thermal sensation. Such a unit appeared in 1970 when Fanger defined the PMV ‘Predicted Mean Vote’ as the index that gives the expected degree of thermal comfort in relation to all the above-mentioned six thermal parameters [6]. Besides that, Fanger presented a general comfort equation which describes the conditions under which the average sensation of a large group of people will feel thermal neutrality. He defined the thermal neutrality of a person as the condition of mind in which the subject would prefer neither warmer nor cooler surroundings [6]. Eq. (1) represents the comfort equation proposed by Fanger [6].

$$\begin{aligned} \mathrm{PMV} = {} & (0.028 + 0.3033 e^{-0.036M}) \cdot \{ (M - W) - 3.05[5.733 - 0.000699(M - W) - \mathrm{Pa}] \\ & -0.42[(M - W) - 58.15] - 0.0173M(5.867 - \mathrm{Pa}) - 0.0014M(34 - T_a) \\ & -3.96.10^{-8}\mathrm{fcl}\left[(T_{cl} + 273)^4 - (T_{mrt} + 273)^4\right] - \mathrm{fcl} \cdot h_c(T_{cl} - T_a) \} \end{aligned} \quad (1)$$

where

$$T_{cl} = 35.7 - 0.028(M - W) - 0.155 I_{cl}\left[3.9610^{-3}\mathrm{fcl}\left[(T_{cl} + 273)^4 - (T_{mrt} + 273)^4\right] - \mathrm{fcl} \cdot h_c(T_{cl} - T_a)\right] \quad (2)$$

$$h_c = \begin{cases} 2.38(T_{cl} - T_a)^{0.25} & \text{for } 2.38(T_{cl} + T_a)^{0.25} \geq 12.1\sqrt{V_{air}} \\ 12.1\sqrt{V_{air}} & \text{for } 2.38(T_{cl} - T_a)^{0.25} \leq 12.1\sqrt{V_{air}} \end{cases} \quad (3)$$

the parameters are defined as follows:

PMV: predicted mean vote.
M: metabolism (W/m^2).
W: external work, equal to zero for most activity (W/m^2).
I_{cl}: thermal resistance of clothing (Clo).
fcl: ratio of body’s surface area when fully clothed to body’s surface area when nude.
T_a: air temperature (°C).
T_{mrt}: mean radiant temperature (°C).
V_{air}: relative air velocity (m/s).
Pa: partial water vapour pressure (Pa).
h_c: convectional heat transfer coefficient (W/m^2 K).
T_{cl}: surface temperature of clothing (°C).

Since the involved heat transfer processes are relatively complicated, the mathematical expression derived for the calculation of the PMV is complicated and not suitable for feedback control systems [6,7,11,14]. To overcome these problems, Fanger and ISO proposed in Refs. [6,12] to use tables and diagrams to simplify the determination of PMV in practical applications. Other researchers proposed to use simplified models of PMV to avoid the iterative process. Such thermal sensation indexes have been proposed in Refs. [1,3,4,7,8,10,14] and they are deduced after significant modifications of Fanger’s comfort equation. For example, Sherman [14] proposed a simplified model of thermal comfort based on the original work of Fanger. In order to reach his objective, to calculate the value of PMV without any iteration, Sherman linearized the radiation exchange terms to remove the T^4 dependence on temperature. Then, he simplified the convection coefficient to eliminate the iterative solutions and finally, he used the dew point temperature instead of relative humidity to avoid its dependence on air temperature. Sherman [14] indicated that these simplifications should not affect the precision of the PMV calculation only when the occupants are near the comfort zone.

Based on the above mentioned assumptions, Sherman concluded that the resulting simplified index could be computed explicitly in a compact form. However, Sherman’s thermal sensation index was not linearly parameterized [7], and therefore, not suitable for on-line calibration and could not be used in a control algorithm of HVAC systems. Federspiel proposed an other thermal sensation index (V) that is a modification of Fanger’s PMV index [7]. To simplify the derivation, he supposed that the radiative exchange and the heat transfer coefficient are linear. In addition, it was assumed that the bodily heat production and the clothing insulation are constant. In addition, Federspiel supposed that the occupants are in a thermal neutrality condition. All of these assumptions were applied to the derivation of a thermal sensation index that is an explicit and linearly parameterized function of the four environmental variables. However, it was outlined that (V) becomes non-linear as soon as the activity level or the clothing insulation are changed. This problem limits the use of the thermal sensation index (V) in zones where the two above-mentioned factors are changing in time.

Other researchers recognized that the above-cited assumptions are difficult to reach and the simplification of the original PMV model results in a significant error when they are not respected [2,7,19]. Presently, the challenge is to derive a thermal sensation index based on the original work of Fanger without any simplification and which can be used in feedback control applications with an on-line calibration. With this goal in mind, the derivation in this paper results in an index that does not require any iteration solutions because it is implicitly dependent on the state of the air temperature, the mean radiant temperature, the air velocity, the relative humidity, the activity level occupants and the thermal resistance of their clothing. Since we propose to use Fanger's thermal comfort model itself, it was not necessary—at the expense of the accuracy—to simplify the comfort model that makes calculations easier. It is difficult to justify control schemes based on a simplified model of thermal comfort where it is necessary to verify the believability of the model simplifications. However, our proposed thermal comfort model can be used as a new indoor climate high-level performance control variable of the HVAC systems without any simplification of the original work of Fanger, and does not require iterative solutions. To our knowledge, such a thermal sensation index has not yet been developed.

3. Review of fuzzy logic

Since its introduction in 1965 [20], fuzzy logic has been successfully applied to a variety of fields. In most of these applications, the main design objective is to construct a fuzzy system that approximates a desired function up to a given level of accuracy. Fuzzy logic is found to be very appropriate in the estimation of complex non-linear plants where a precise mathematical model is difficult or impossible to obtain and to use. Recently, it was proven that fuzzy systems are capable of approximating any real function on a compact fuzzy subset [18]. Such approximators have often been found superior to conventional modelling, especially when information being processed is inexact and uncertain. The main advantages of the application of fuzzy logic in approximation theory, as compared to conventional approaches, is that no mathematical model is needed, and it is possible to use all the available information about the process in the design of the fuzzy approximator scheme. A fuzzy logic approximator converts a set of linguistic rules, based on expert knowledge, into an automatic approximation strategy. It has been outlined that employing fuzzy if–then rules can model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analysis [16–20]. Generally, fuzzy systems process the available information in a symbolic format instead of proceeding with a numerical calculus.

As shown in Fig. 2, a fuzzy logic approximator consists of three stages; in the fuzzification stage, the input variables are converted into fuzzy variables using the pre-defined membership functions. Once the input variables are fuzzified, the fuzzy system evaluates a set of approximation rules which are given by a set of conditions as the following classic example: If input 1 is *big* and input 2 is *small* then output 1 have to be *more or less than medium*.

Finally, in the defuzzification stage, the generated fuzzy outputs are converted into a discrete values ready to be acquired by the processor for driving certain control mechanism.

4. Fuzzy thermal sensation index

Ensuring thermal comfort of occupants and reducing energy consumption are the control challenges in the development of modern industrial HVAC systems. For thermal comfort and energy efficiency, it is desirable to design a HVAC control system that can guarantee high performance and good robustness with regard to the variation of the environmental variables as well as the activity level of occupants and their clothing insulation. Research has shown that it is possible to reach these objectives if HVAC control strategies are based on the thermal sensation index instead of air temperature alone [1,6–8,13,14,19]. Presently, the non-linear behaviour of human's thermal sensation and the unavailability of a direct quantitative PMV regarding the inputs–output relations make it very difficult (or impossible) to design a direct control strategy of HVAC systems that regulate thermal comfort levels.

To overcome this problem, the thermal sensation index should be calculated as an implicit result of the six previously mentioned variables influencing human's thermal sensation. To this end, we propose to use fuzzy logic theory to make

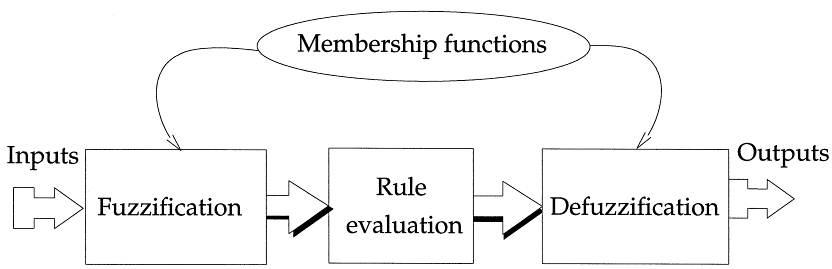

Fig. 2. Internal structure of fuzzy logic system.

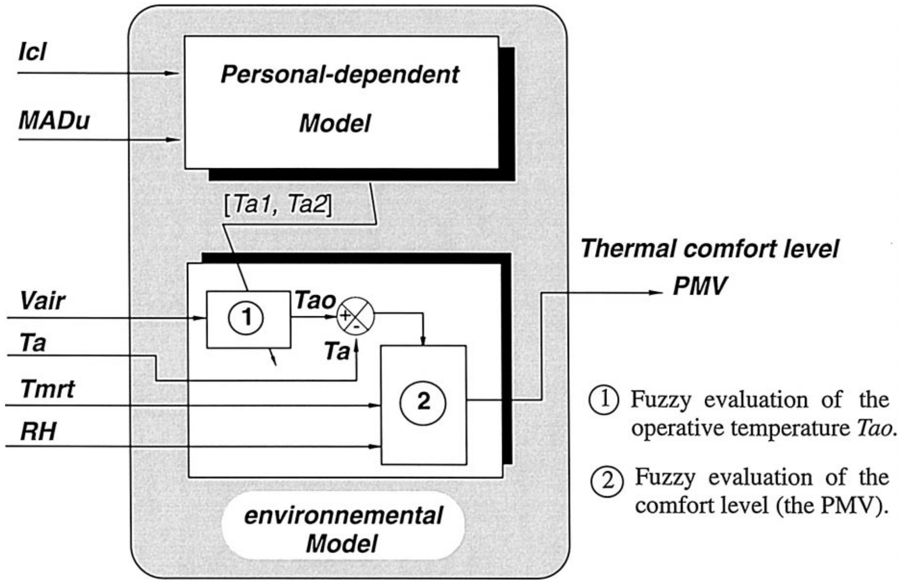

Fig. 3. Architecture of the fuzzy thermal sensation index.

quick and direct calculation of the thermal comfort level in a given indoor climate. The new fuzzy thermal sensation index (fuzzy PMV) can be designed by extracting knowledge from Fanger's comfort equation and by transforming it into rules and membership functions. The basic design idea is to transform all possible combinations of the variables that affect thermal comfort into linguistic fuzzy implications to describe the thermal sensation index. So that, the input–output relationships are transformed into a set of fuzzy rules and the human thermal sensation is evaluated as a result of a fuzzy evaluation of the state of the six input variables that affect thermal comfort. Instead of using Eq. (1) to calculate a PMV value, it becomes possible to calculate it directly by using some linguistic rules such as:

IF the air temperature (T_a) is *High*,
AND the relative humidity (RH) is *Around* 50%,
AND relative air velocity (V_{air}) is *Very small*,
AND radiant mean temperature (T_{mrt}) is *Close to* air temperature,
AND the activity level (MADu) is *Low*,
AND the clothing (I_{cl}) is *Very light*,
THEN PMV is *Near* zero (the indoor climate is comfortable).

where *High, Small, Low, Close to, Very light and Near zero* are fuzzy subsets.

While the six input variables are described by a set of fuzzy terms, the above-presented design strategy requires a high number of fuzzy rules and thus, a large amount of time calculation. This number of rules can be reduced significantly by considering that the fuzzy PMV model is composed of two subsystems: the personal-dependent model and the environmental model. Their interconnection is shown in Fig. 3. On one side, the personal-dependent model evaluates the air temperature range(ΔT_a), in which the predicted mean vote is found to be close to zero. ΔT_a is evaluated depending on the state of the occupants activity level and their clothing insulation. On the other side, the environmental model calculates the PMV value according to the state of T_{ao} and the four environmental variables.

4.1. The fuzzy PMV calculation

The design strategy of the fuzzy thermal sensation index is achieved in four steps process. First, the input and output variables are chosen. As shown in Fig. 3, the personal-dependent subsystem input variables are the activity level of occupants and their clothing insulation. Its output variable is the ambient temperature range in which the predicted mean vote is close to zero. However, the environmental-dependent subsystem input variables are air temperature (T_a), air velocity (V_{air}), mean radiant temperature (T_{mrt}) and relative air humidity (RH). The output variable is the value of the predicted mean vote (fuzzy PMV). The second design step is to derive the fuzzy rule base that should be used to evaluate the PMV depending on the state of the input variables. The general method developed by Wang and Mendel [17] is used to generate an accurate fuzzy rule base by extracting knowledge from Fanger's thermal sensation vote equation. To generate all fuzzy rules that represent all possible combinations of the six variables, each of the input and the output spaces are divided into symmetric triangular membership functions as shown in Fig. 4.

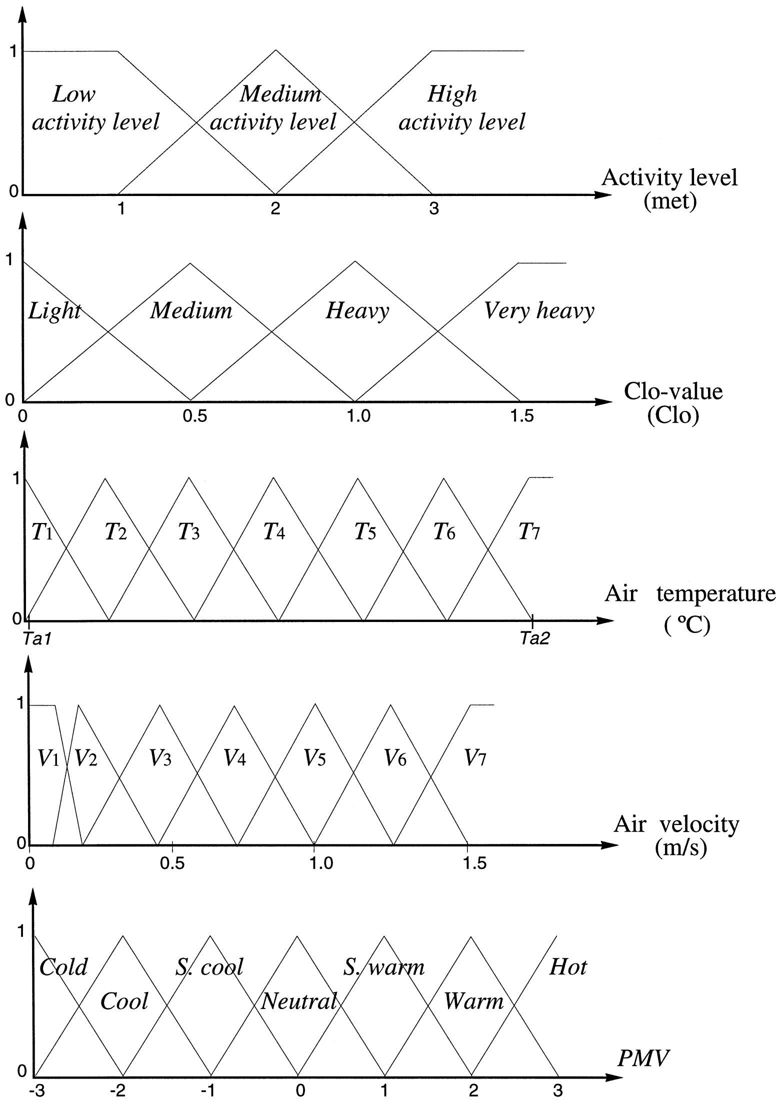

Fig. 4. Initial membership functions.

For accuracy purposes, the activity level and the clothing insulation are described by three and four triangular membership functions, respectively. On the other side, the air velocity, the air temperature and the predicted mean vote are transferred into fuzzy subsets by using seven triangular membership functions to describe each of them. For instance, the relative humidity is supposed to be 50% and the mean radiant temperature is supposed to be close to the ambient air temperature. Since thermal comfort can be obtained by many different combinations of the six above-mentioned variable, a conflict among the generated rules appears. This is due to the interdependence between thermal comfort influencing factors as the effect of each of them depends on the level and the conditions of the other factors. In order to resolve this conflict, we assigned a degree to each of the generated rules to keep only the rule from a conflict group that has maximum degree. By this way, not only the conflict problem is resolved, but also the number of rules is greatly reduced [17].

Once the fuzzy rules are generated on the basis of the membership functions in Fig. 4, the fuzzy PMV model is transformed into a neuro-fuzzy system to equip it with learning capability [18]. This step was necessary to ameliorate the accuracy of the final fuzzy PMV approximation by automatic adjustments of the membership functions of each of the input and output variables. Therefore, the fuzzy system becomes able to update and to fine-tune itself depending on Fanger's thermal comfort model and ISO tables [21]. This is achieved by modifying the connection weights of the network through learning with the back-propagation algorithm. Therefore, not only the error between the new fuzzy PMV and Fanger's PMV is reduced, but also the fuzzy rule base is optimised. Fig. 5 shows the final membership functions of the air temperature, the air velocity and the clothing insulation in the case where RH = 50% and $T_{mrt} = T_a$. The predicted mean vote triangular membership functions are replaced by seven singletons in order to simplify the defuzzification process. However, the activity level membership functions are kept as shown in Fig. 4.

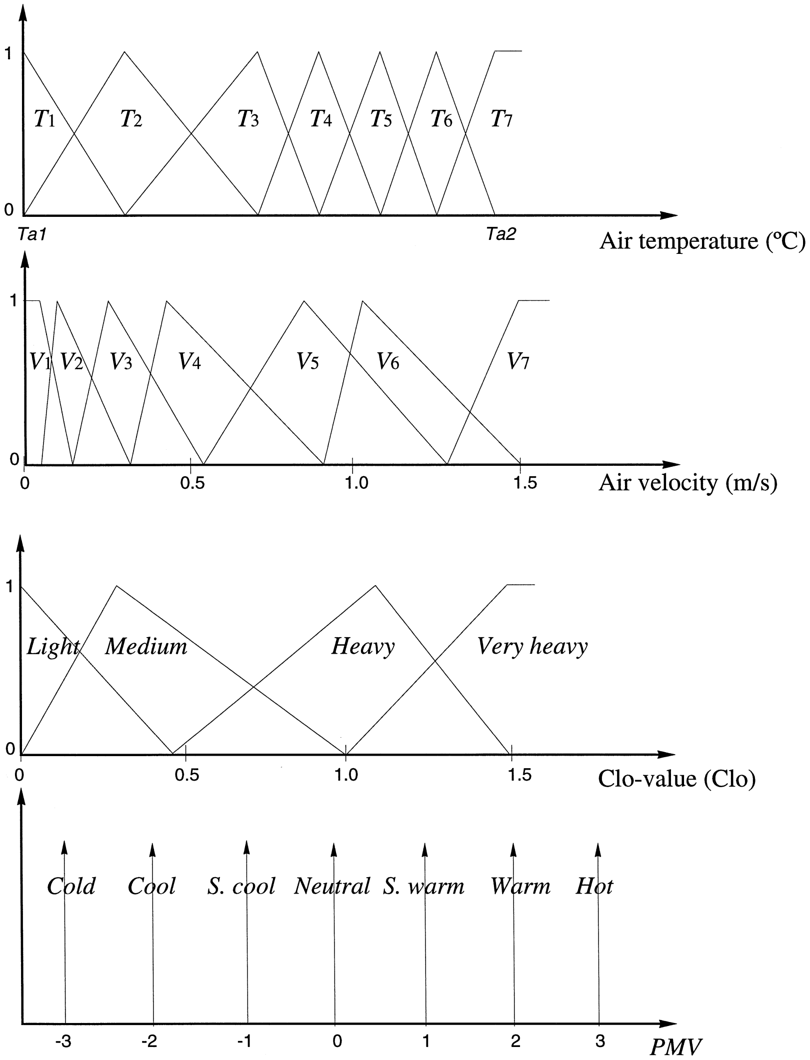

Fig. 5. Final membership functions.

In order to take into account the effect of the relative humidity and the mean radiant temperature on human's thermal sensation, a few additional rules are added to correct the PMV calculation when the relative humidity is different than 50% and/or the mean radiant temperature is different than the ambient air temperature.

4.2. The personal-dependent model rules

Studies of the influence of the input variables on the thermal sensation index [15,19] demonstrated the powerful dependence between the personal-dependent variables and the environmental variables. Since the variation of I_{cl} and MADu affects the bodily heat production and consequently, the mean temperature of the outer surface of the clothed body (T_{cl}), the relative influence of the activity level and the clothing insulation on the thermal comfort is assembled in the personal-dependent model to evaluate the air temperature range that should ensure thermal comfort. This is realized by a fuzzy reasoning that uses three and five membership functions to describe respectively the activity level and the clothing insulation states. Table 1 shows the 15 fuzzy rules used to evaluate the temperature range in which the predicted mean vote is close to zero.

This rule base can be expressed linguistically as:

IF the occupant has *Light* clothing AND he or she is sedentary
THEN the ambient temperature should be *Very high* (in [28.2°C–31.5°C] range).
IF the occupant has *Medium* clothing AND his or her activity level is *Medium*
THEN the ambient temperature should be *Normal* (in [19.5°C–23.5°C] range).
IF the occupant has *Very heavy* clothing AND his or her activity level is *Medium*
THEN the ambient temperature should be *Low* (in [10.8°C–14°C] range).

Table 1
Fuzzy evaluation of the temperature range in which the thermal sensation is neutral

The clothing insulation	The activity level		
	Low	Medium	High
Light	[28.2°C–31.5°C]	[24°C–28.5°C]	[19.5°C–25.5°C]
Medium	[25.9°C–28.0°C]	[19.5°C–23.5°C]	[13°C–18.4°C]
Heavy	[23°C–26.0°C]	[15°C–24°C]	[7.0°C–12°C]
Very Heavy	[19.5°C–23.5°C]	[10.8°C–14°C]	[0.0°C–0.6°C]

The last fuzzy rule can be interpreted as an active person attired in very heavy clothing with a clo-value of 1.5 Clo prefers an ambient temperature that is lower than 14°C.

4.3. The environmental model rules

From a practical point of view, the personal-dependent model is used to adjust the environmental model which starts with the evaluation of the air velocity to determine the operative temperature that ensure a predicted mean vote equal to zero (PMV = 0). To this end, seven membership functions are used to describe the state of both the air velocity (V_{air}) and the operative temperature (T_{ao}). These variables generate a maximum of seven fuzzy rules such as the following examples:

IF the air velocity is V_1 THEN the operative temperature is T_1.
IF the air velocity is V_3 THEN the operative temperature is T_3.
IF the air velocity is V_7 THEN the operative temperature T_7.
etc.

where ($V_1, \ldots, V_7$) and ($T_1, \ldots, T_7$) are fuzzy terms that describe respectively the air velocity and the operative temperature that correspond to an optimal sensation of thermal comfort (Fig. 5).

Once the desired ambient air temperature is calculated, it is compared to the measured air temperature (T_a) to determine the state of the predicted mean vote (PMV) as a fuzzy result of the following three linguistic rules:

If (RH = 50%) and ($T_{mrt} = T_a$) then:
IF T_a is *close* to T_{ao} THEN the PMV is *zero*
IF T_a is *higher than* T_{ao} THEN the PMV is *positive*
IF T_a is *lower than* T_{ao} THEN the PMV is *negative*.

Where *Close to, higher than, lower than, positive* and *negative* are fuzzy subsets.

4.4. The correction rules

The above-presented 25 fuzzy rules are evaluated under the assumption that the relative humidity is equal to 50% and the mean radiant temperature is equal to the air temperature. In order to take into account their effect on human's thermal sensation, the influence of the relative humidity and the mean radiant temperature on the PMV is evaluated by using additional correction rules. As mentioned in Refs. [5,6,13,19], the relative humidity has only a moderate influence on the thermal sensation near comfort conditions. Thus, for other relative humidities which are different than 50%, eventual corrections can be made from the following fuzzy rules:

IF RH is higher than 50% THEN
the PMV should be a little bit higher than normal
IF RH is lower than 50% THEN
the PMV should be a little bit lower than normal

where higher and lower are fuzzy terms and a normal PMV is the PMV value corresponding to a relative humidity of 50%.

Table 2
Fuzzy evaluation of the effect of changes in radiant temperature on the thermal sensation

	The activity level		
The clothing insulation	Low activity level	Medium activity level	High activity level
Light clothing	0.15	0.12	0.07
Medium clothing	0.12	0.07	0.05
Heavy clothing	0.10	0.05	0.03
Very heavy	0.05	0.03	0.03

A detailed study [19] of the sensibility of PMV over the mean radiant temperature demonstrated that changes in radiant environment greatly affect thermal comfort in a given indoor climate. Therefore, the effect of the mean radiant temperature deviations on human's thermal sensation is incorporated within some correction rules. The linguistic correction rules are activated only when T_{mrt} is different than T_a and the relative influence of T_{mrt} on PMV is evaluated by the following rules:

IF T_{mrt} is higher than T_a
Then the PMV is higher than normal $(PMV|_{T_{mrt}=T_a})$
IF T_{mrt} is lower than T_a
Then the PMV is lower than normal $(PMV|_{T_{mrt}=T_a})$
The higher the relative velocity
The lower the effect of T_{mrt} on the PMV.

Table 3
Comparison between the fuzzy PMV and Fanger's PMV when ($T_{mrt} = T_a$ and RH = 50%)

MADu (met)	I_{cl} (Clo)	T_a (°C)	V_{air} (m/s)				
			< 0.1	0.2	0.5	1.0	1.5
1	0.0	26	0.01	−0.05			
		28	−0.01	0.00			
		30	−0.04	0.01			
		32	−0.02	0.02			
	0.5	23	−0.03	0.00	−0.02	−0.01	0.01
		25	0.00	−0.01	0.00	0.01	0.04
		27	0.01	0.00	−0.01	0.02	0.01
		29	0.03	0.02	−0.02	0.01	0.03
	1.0	20	0.01	0.00	0.02	0.01	0.03
		22	−0.03	0.02	−0.04	−0.01	−0.01
		24	0.00	0.00	−0.02	−0.00	−0.01
		26	0.02	0.00	−0.02	−0.01	−0.02
	1.5	14	0.02	−0.01	−0.02	0.01	−0.01
		18	0.04	0.02	−0.02	0.01	−0.01
		22	−0.03	0.00	−0.00	−0.01	0.02
		26	0.02	0.00	−0.02	0.01	−0.01
2	0.0	18	0.03	−0.02	−0.02	−0.01	0.00
		22	0.01	−0.03	0.01	0.03	0.05
		26	0.04	−0.01	−0.01	−0.02	−0.01
		30	0.05	0.02	0.00	0.00	−0.01
	0.5	14	−0.02	0.01	0.02	−0.01	−0.02
		18	−0.01	0.00	−0.01	0.00	−0.01
		22	0.00	−0.01	−0.02	0.01	−0.01
		26	0.02	−0.03	−0.04	0.03	−0.02
	1.0	8	0.04	0.03	0.02	0.01	−0.01
		12	0.02	−0.01	0.04	0.03	−0.03
		16	0.00	−0.01	−0.02	0.01	−0.01
		20	−0.02	0.00	−0.03	0.00	0.00
	1.5	−4	0.00	−0.03	−0.02	0.01	0.02
		4	0.02	−0.01	−0.00	−0.01	−0.01
		18	0.03	0.02	0.03	−0.00	−0.04
		20	0.05	0.04	0.03	−0.02	−0.05
3	0.5	14	0.02	0.01	0.00	−0.01	−0.03
		18	0.01	−0.01	0.02	−0.00	0.00
		22	0.00	0.04	0.03	−0.00	0.01
		26	−0.02	0.02	0.03	0.01	0.02
	1.0	8	0.01	−0.02	0.03	−0.04	−0.04
		12	0.00	−0.01	−0.02	−0.03	0.01
		16	0.03	0.02	0.02	−0.00	0.04
		20	−0.00	−0.01	−0.02	0.01	−0.01
	1.5	−10	0.04	−0.02	−0.01	0.02	0.01
		−2	0.03	0.02	0.03	0.00	−0.04
		6	0.00	−0.01	−0.02	0.01	−0.01
		12	0.02	−0.03	−0.04	0.03	0.01

In addition, the correction rules take into account the influence of MADu and I_{cl} on the effect of the mean radiant temperature changes on the PMV.

IF I_{cl} is *Zero* AND MADu is *Low*
THEN the effect of T_{mrt} on PMV is important
IF I_{cl} is *Zero* AND MADu is *High*
THEN the effect of T_{mrt} on PMV is *medium*
IF I_{cl} is *High* AND MADu is *High*
THEN the effect of T_{mrt} on PMV is *Close to zero*.

Table 2 summarizes the influence of the mean radiant temperature on the predicted mean vote. It shows how the predicted mean vote should be corrected if the mean radiant temperature deviates from the air temperature.

The designed thermal sensation index is a general linguistic description of the input–output relationships between the predicted mean vote and the six input variables that affect thermal comfort. Overall, the interaction between the six input variables is transformed into a compact fuzzy rule base (35 linguistic rules) that can be evaluated in parallel to characterise the thermal sensation index. Unlike other thermal sensation indexes, the new model is realized by extracting knowledge from Fanger's comfort equation without any simplification and any iterative solutions.

5. Simulation results

The proposed fuzzy thermal comfort model estimates human's thermal sensation index, which gives the predicted mean vote (PMV) for a large group of persons, as a fuzzy function of activity level (W/m^2), clothing (Clo), air temperature (°C),

Table 4
Error between the fuzzy PMV and Fanger's PMV when $T_{mrt} \neq T_a$ (RH = 50%)

MADu (met)	I_{cl} (Clo)	T_a (°C)	T_{mrt} (°C)	V_{air} (m/s)				
				< 0.1	0.2	0.5	1.0	1.5
1	0.0	26	24	0.02	−0.01			
		28	26	−0.01	0.00			
		30	32	0.00	0.02			
	0.5	23	25	−0.02	0.00	0.02	−0.01	−0.01
		25	22	0.00	−0.01	0.00	0.01	0.04
		27	26	0.01	0.01	0.01	−0.03	−0.01
	1.0	20	18	0.02	0.00	0.02	0.01	0.03
		22	20	−0.01	−0.02	−0.04	−0.01	−0.01
		24	25	0.02	0.00	−0.02	−0.01	−0.01
	1.5	14	16	0.02	−0.01	−0.02	0.01	−0.01
		18	21	−0.02	0.02	−0.02	0.01	−0.01
		22	21	0.03	0.01	0.00	−0.01	0.02
2	0.0	18	15	0.03	−0.01	−0.05	−0.06	0.03
		22	24	0.02	−0.03	−0.04	0.03	−0.02
		26	29	0.00	−0.01	0.00	0.01	0.04
	0.5	14	17	0.01	−0.01	−0.02	−0.03	−0.04
		18	21	0.00	−0.01	0.00	0.01	0.05
		22	27	−0.02	−0.03	−0.02	−0.05	−0.06
	1.0	8	12	0.02	−0.03	−0.04	0.03	−0.02
		12	15	0.03	−0.03	−0.04	0.03	−0.02
		16	20	0.00	−0.01	0.00	0.01	0.04
	1.5	−4	0	0.00	−0.01	0.00	0.01	0.05
		4	10	−0.01	−0.03	−0.04	0.03	0.05
		12	16	0.01	−0.01	−0.02	−0.00	−0.04
3	0.5	14	15	0.02	0.01	0.03	−0.02	−0.04
		18	19	0.00	−0.01	0.00	0.01	0.04
		22	20	−0.02	−0.01	0.00	0.03	0.04
	1.0	8	9	0.00	0.02	−0.01	−0.02	0.01
		12	15	0.02	0.01	0.00	0.01	−0.04
		16	20	0.00	0.02	−0.01	−0.02	0.01
	1.5	−10	10	0.05	−0.02	−0.03	−0.04	0.06
		−2	10	0.02	0.01	−0.01	0.03	−0.02
		6	15	0.00	0.02	−0.01	−0.02	0.01

Table 5
Error between the fuzzy PMV and Fanger's PMV when RH ≠ 50% (I_{cl} = 0.5 Clo and MADu = 2 met)

RH (%)	$T_a = T_{mrt}$ (°C)	V_{air} (m/s)				
		< 0.1	0.2	0.5	1.0	1.5
00%	18	0.03	0.00	−0.01	−0.02	−0.04
	22	0.04	0.02	0.00	0.01	−0.01
	26	−0.02	−0.01	0.03	0.05	0.05
50%	18	−0.01	0.00	−0.01	0.00	−0.01
	22	0.00	−0.01	−0.02	0.01	−0.01
	26	0.02	−0.03	−0.04	0.03	−0.02
100%	18	−0.04	0.03	−0.01	0.03	0.03
	22	−0.02	−0.03	−0.02	0.03	−0.01
	26	0.02	0.01	−0.01	−0.03	0.04

mean radiant temperature (°C), relative air velocity (m/s) and relative air humidity (%). Unlike Fanger's PMV calculation, the new fuzzy thermal sensation index provides a quick and direct calculation of the predicted mean vote in any indoor climate depending on the six above mentioned factors.

For comparison purposes, both the proposed fuzzy model and Fanger's predicted mean vote have been evaluated on a digital simulation using MATLAB, and setup for a range of possible combinations of the six above-mentioned variables. The results of the simulation are tabulated in Tables 3–5. The fuzzy PMV value is calculated for three activity levels, four clo-values and five air velocities. As shown in Table 3, for ($T_{mrt} = T_a$ and RH = 50%), the difference between Fanger's PMV and the fuzzy PMV values is found to be lower than 0.05 for any state of the six input variables. These results show the high precision of the proposed fuzzy PMV model. In addition, the authors have calculated and given in tabular form the thermal comfort level value for six activity levels, seven clo-values, nine air velocities, five relative humidities and eight ambient and mean radiant temperatures [19]. These simulation results showed a good agreement between the fuzzy model and Fanger's PMV expression for all possible combination of the six input variables [19].

Table 4 shows the accuracy of the proposed fuzzy PMV model when the mean radiant temperature deviates from the air temperature. The predicted mean vote is calculated for four different clothing values, each at three activity levels and five air velocities (RH = 50%). The maximum error is found to be 0.08 when the mean radiant temperature is 5°C higher than the ambient air temperature [19]. The good accuracy of the fuzzy PMV is tested when the relative humidity is different than 50%. Table 5 shows that even if the relative humidity deviates from 50% to 100% (or from 50% to 0%), the error between the fuzzy PMV and Fanger's PMV is less than 0.05 PMV.

6. Conclusion

In this paper, a new approach to calculate human's thermal sensation index is presented. The proposed model evaluates the predicted mean vote as an implicit result of the interaction of the air temperature, the air velocity, the mean radiant temperature, the relative humidity, the activity level of occupants and their clothing insulation. The new model of the thermal sensation index is designed on the basis of fuzzy logic theory by learning Fanger's 'Predicted Mean Vote' equation. Unlike Fanger's thermal comfort model, the new fuzzy thermal sensation model provides a quick and direct calculation of the thermal sensation index which makes it an attractive index for feedback control of HVAC systems. The fuzzy rule base is composed of 35 linguistic rules that evaluates the state of each of the six variables that affect thermal comfort to calculate directly the PMV value. While the proposed fuzzy model treats thermal comfort as a fuzzy concept rather than a crisp concept, it is possible to take into account the quite imprecision knowledge of occupants activity level and their clothing insulation in the calculation of the predicted mean vote. Besides that, the environmental variables are evaluated as uncertain quantities rather than precise values. This statement is often encountered in practice because of the imprecision of the measurement of the environmental variables that affect thermal comfort.

Both of the architecture of the proposed fuzzy thermal comfort model and the simplicity of the center of area method which is used in the defuzzifying process lead to a fast PMV calculation. In addition, the flexibility of the proposed PMV model permits easily its adjustment depending on the specific thermal sensation of users. This characteristics make the proposed PMV model very suitable to take into account the influence of certain special, such as the national-geographic location, the age and the sex of the user, factors on the human's thermal sensation index.

The accuracy of the proposed fuzzy PMV model is examined on a reasonable number of combinations of the six factors that affect thermal comfort. Simulation results show a good agreement between the Fanger's PMV and the new fuzzy PMV values. The relative influence of each of the six input variables on thermal comfort can be analysed and used to construct a new HVAC control strategy on the basis of the proposed new fuzzy thermal sensation index. In fact, since it is simple to calculate, fast and precise, the new fuzzy thermal sensation index can be easily used as a new indoor high level performance variable in a general control strategy of HVAC systems. It is common knowledge that a HVAC control strategy based on thermal comfort levels rather than just air temperature provides both better comfort and lower cost than that provided by thermostatic control [5,7].

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