

Diffusion and Kinetics

Lecture: Binary phase diagrams and Gibbs free energy curves

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Textbook: Phase transformations in metals and alloys (Third Edition), By: Porter, Easterling, and Sherif (CRC Press, 2009).

Binary phase diagrams and Gibbs free energy curves

- Binary solutions with unlimited solubility
- Relative proportion of phases (tie lines and the lever principle)
- Development of microstructure in isomorphous alloys
- Binary eutectic systems (limited solid solubility)
- Solid state reactions (eutectoid, peritectoid reactions)
- Binary systems with intermediate phases/compounds
- The iron-carbon system (steel and cast iron)
- Temperature dependence of solubility
- Equilibrium vacancy concentration
- Three-component (ternary) phase diagrams

Reading: Chapter 1.5 of Porter, Easterling, Sherif

Binary phase diagram and Gibbs free energy

A binary phase diagram is a temperature - composition map which indicates the equilibrium phases present at a given temperature and composition.

The equilibrium state can be found from the **Gibbs free energy dependence on temperature and composition**.

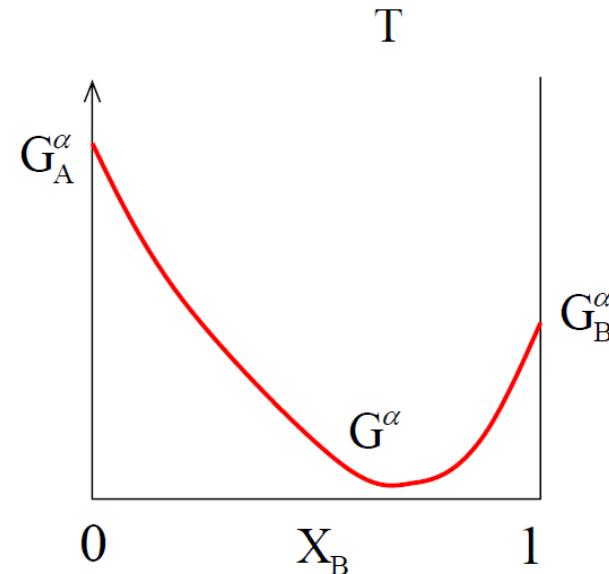
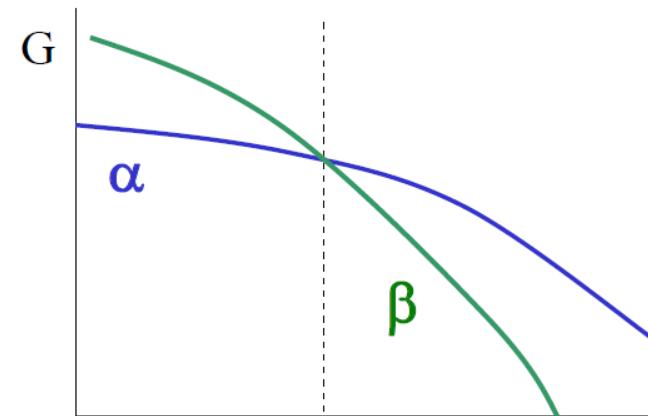
We have discussed the dependence of G of a one-component system on T:

$$\left(\frac{\partial G}{\partial T} \right)_P = -S$$

$$\left(\frac{\partial^2 G}{\partial T^2} \right)_P = -\left(\frac{\partial S}{\partial T} \right)_P = -\frac{c_p}{T}$$

We have also discussed the dependence of the Gibbs free energy from composition at a given T:

$$G = X_A G_A + X_B G_B + \Delta H_{mix} - T \Delta S_{mix}$$



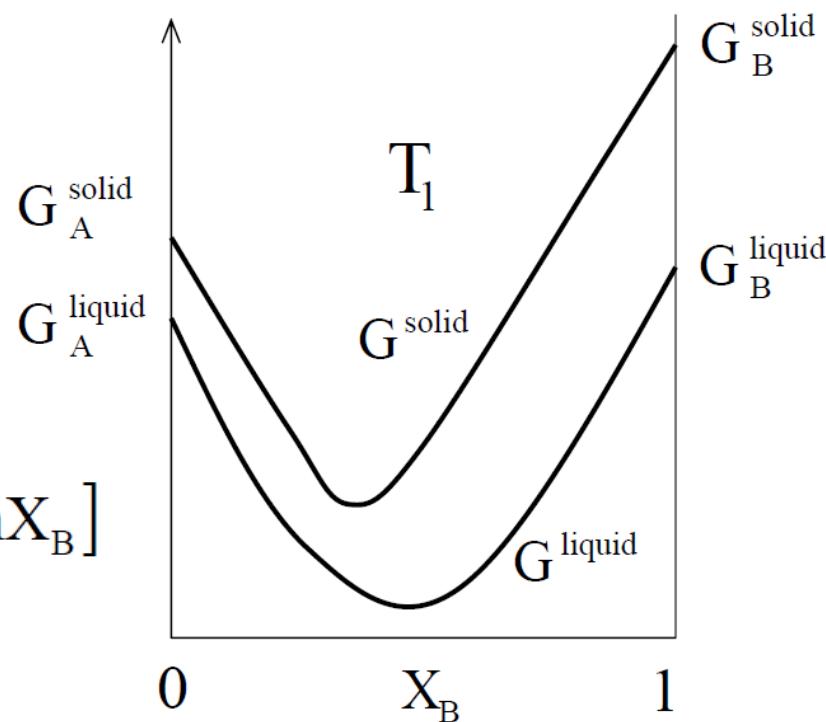
Binary solutions with unlimited solubility (I)

Let's construct a binary phase diagram for the simplest case: A and B components are mutually soluble in any amounts in both solid (**isomorphous system**) and liquid phases, and form ideal solutions ($\Delta H_{mix} = 0$).

We have 2 phases – liquid and solid. Let's consider Gibbs free energy curves for the two phases at different T.

- T_1 is above the equilibrium melting temperatures of both pure components: $T_1 > T_m(A) > T_m(B) \rightarrow$ the liquid phase will be the stable phase for any composition.

$$G^{id} = X_A G_A + X_B G_B + RT[X_A \ln X_A + X_B \ln X_B]$$



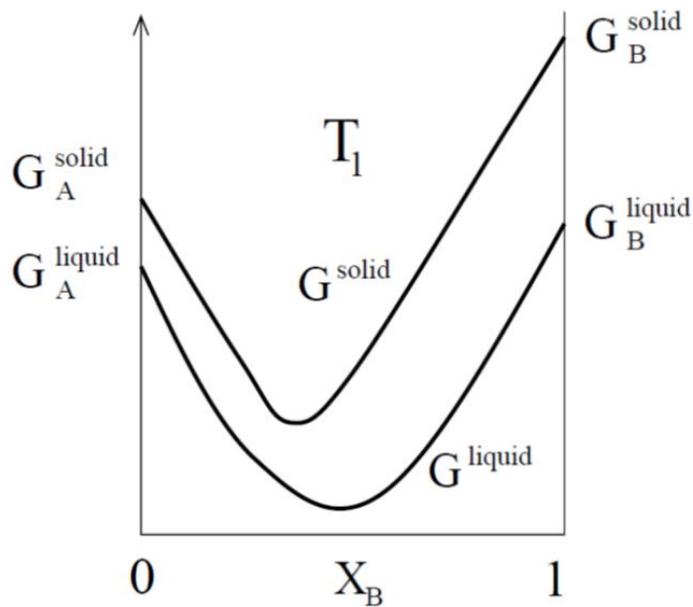
Binary solutions with unlimited solubility (II)

Decreasing the temperature below T_1 will have two effects:

❖ G_A^{liquid} and G_B^{liquid} will increase more rapidly than G_A^{solid} and G_B^{solid} Why?

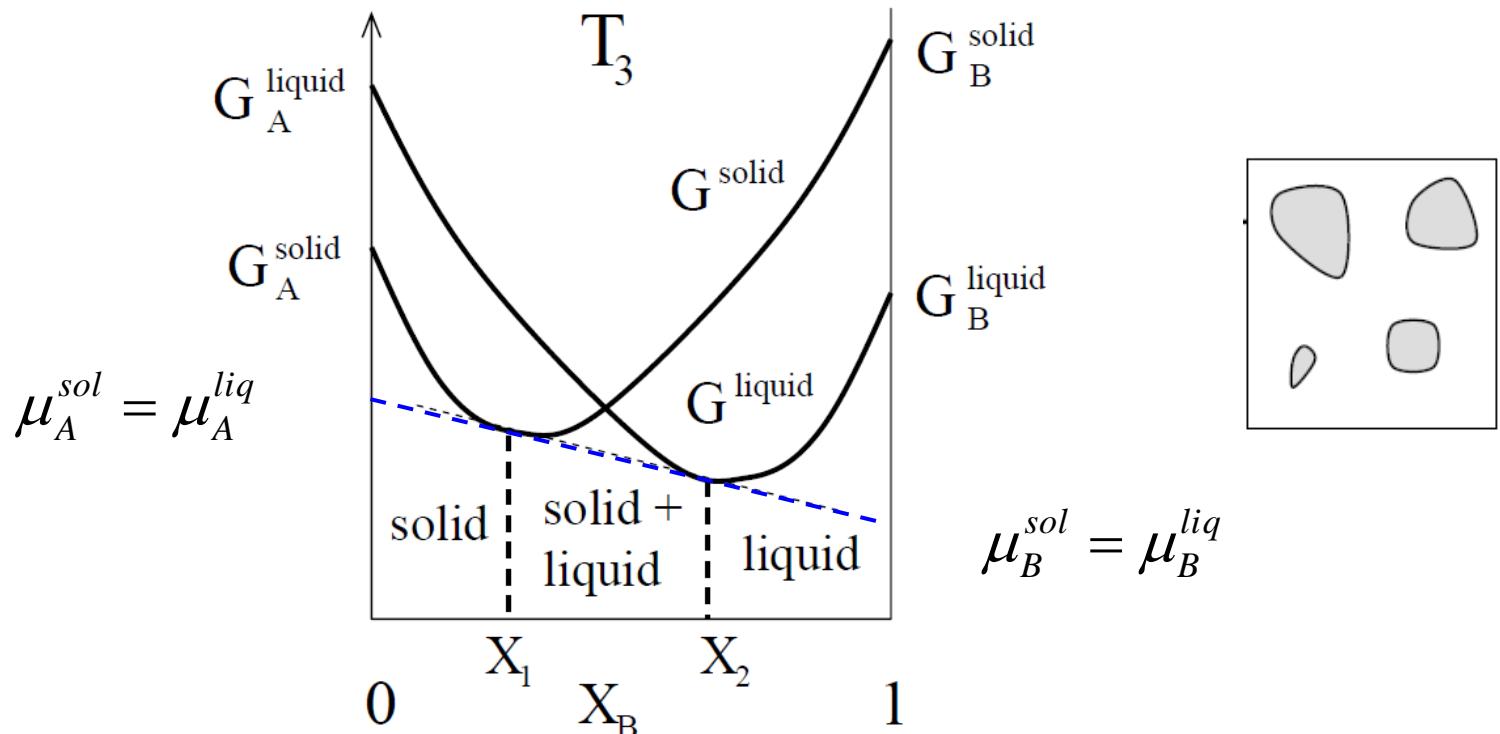
❖ The curvature of the $G(X_B)$ curves will decrease. Why?

➤ Eventually we will reach T_2 – melting point of pure component A, where $G_A^{\text{liquid}} = G_A^{\text{solid}}$



Binary solutions with unlimited solubility (III)

- For even lower temperature $T_3 < T_2 = T_m(A)$ the Gibbs free energy curves for the liquid and solid phases will cross.

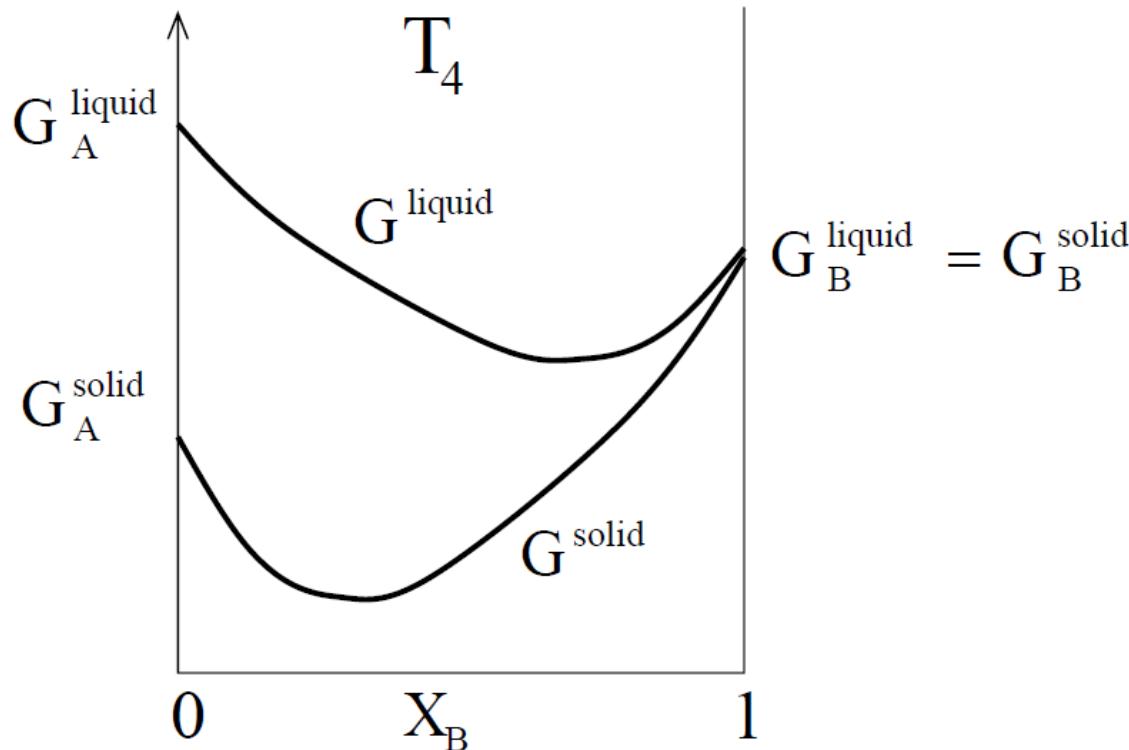


As we discussed before, the common tangent construction can be used to show that for compositions near cross-over of G^{solid} and G^{liquid} , the total Gibbs free energy can be minimized by *separation into two phases*.

Binary solutions with unlimited solubility (IV)

As temperature decreases below T_3 , G_A^{liquid} and G_B^{liquid} continue to increase more rapidly than G_A^{solid} and G_B^{solid}

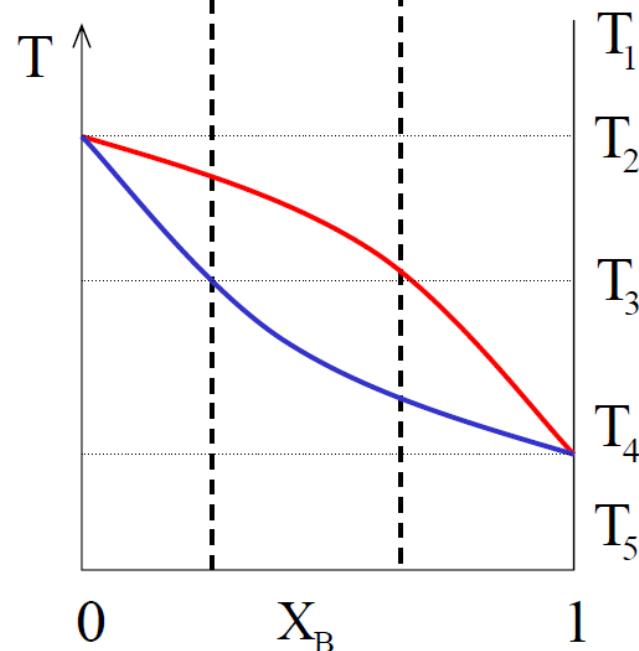
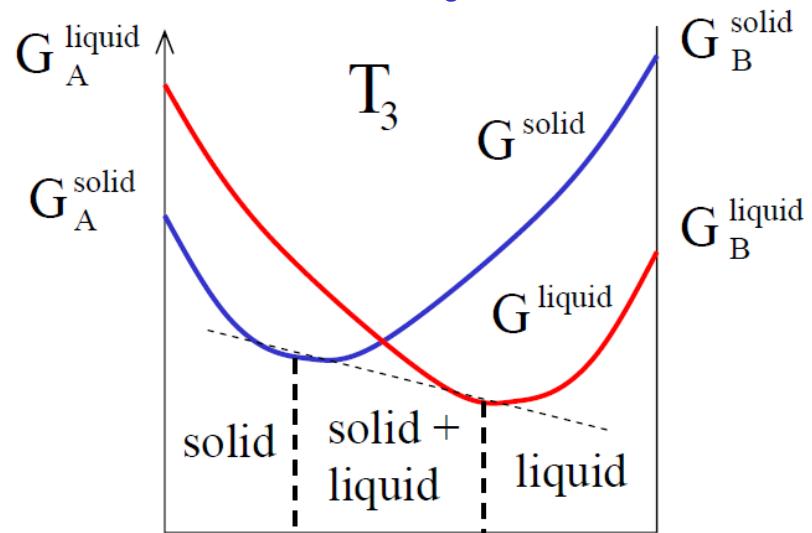
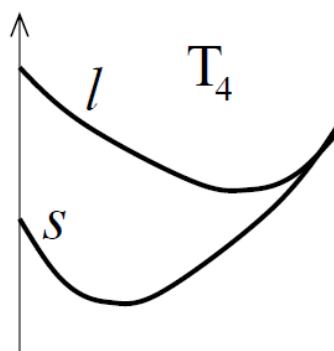
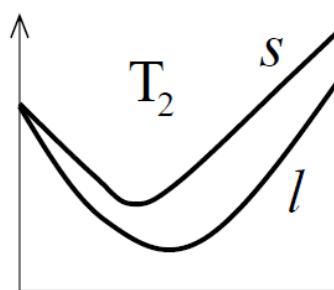
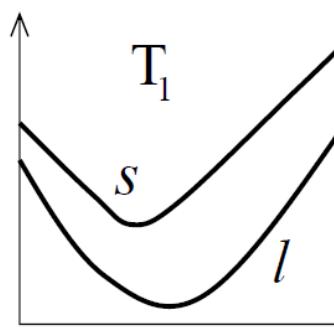
- Therefore, the intersection of the Gibbs free energy curves, as well as points X_1 and X_2 are shifting to the right, until, at $T_4 = T_m(B)$ the curves will intersect at $X_1 = X_2 = 1$



At T_4 and below this temperature the Gibbs free energy of the solid phase is lower than the G of the liquid phase in the whole range of compositions – **the solid phase is the only stable phase.**

Binary solutions with unlimited solubility (V)

Based on the Gibbs free energy curves we can now construct a phase diagram for a binary isomorphous systems (complete solubility)

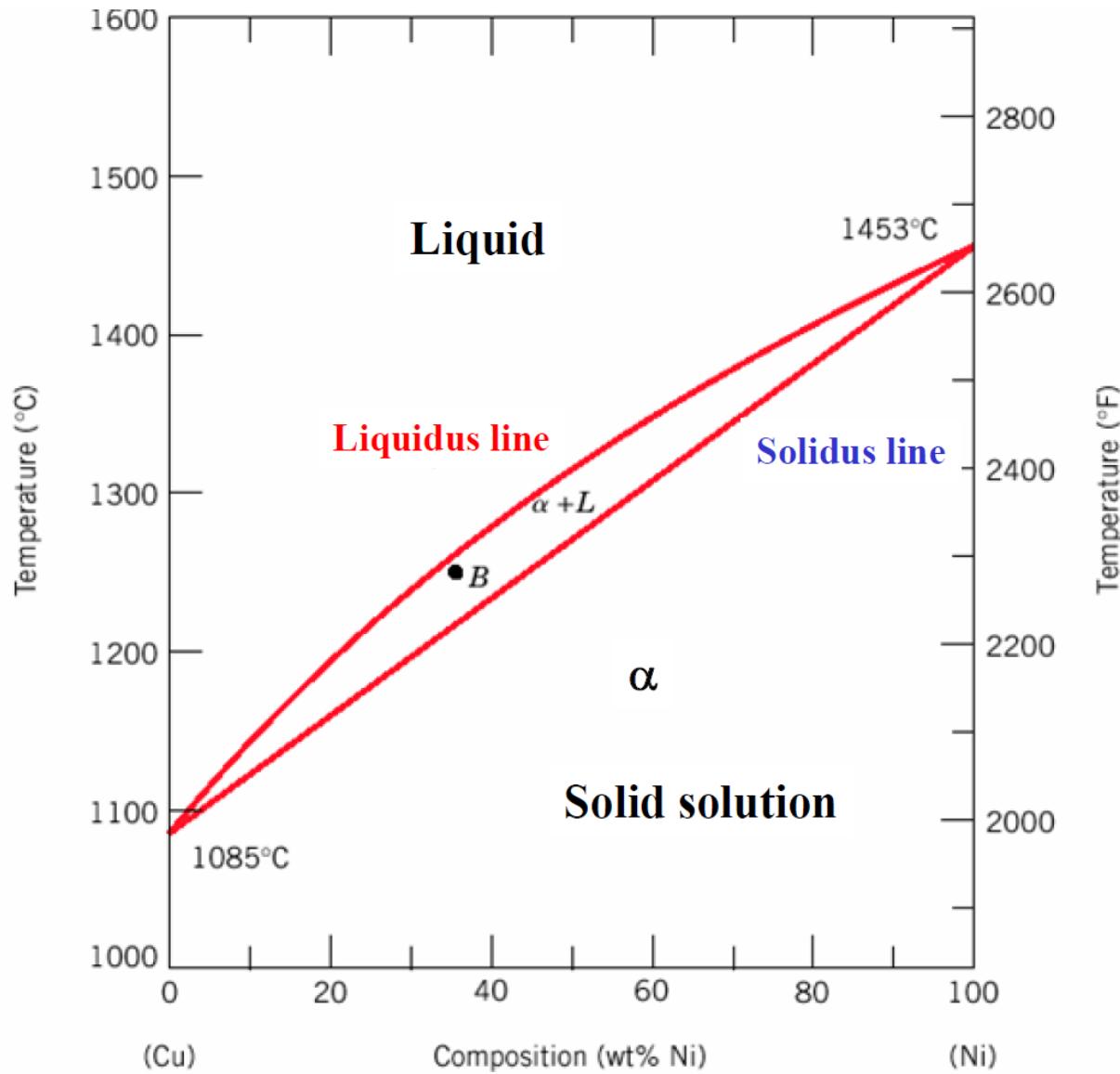


Binary solutions with unlimited solubility (VI)

Example of isomorphous system: Cu-Ni (the complete solubility occurs because both Cu and Ni have the same crystal structure, FCC, similar radii, electronegativity and valence).

Liquidus line separates liquid from liquid + solid

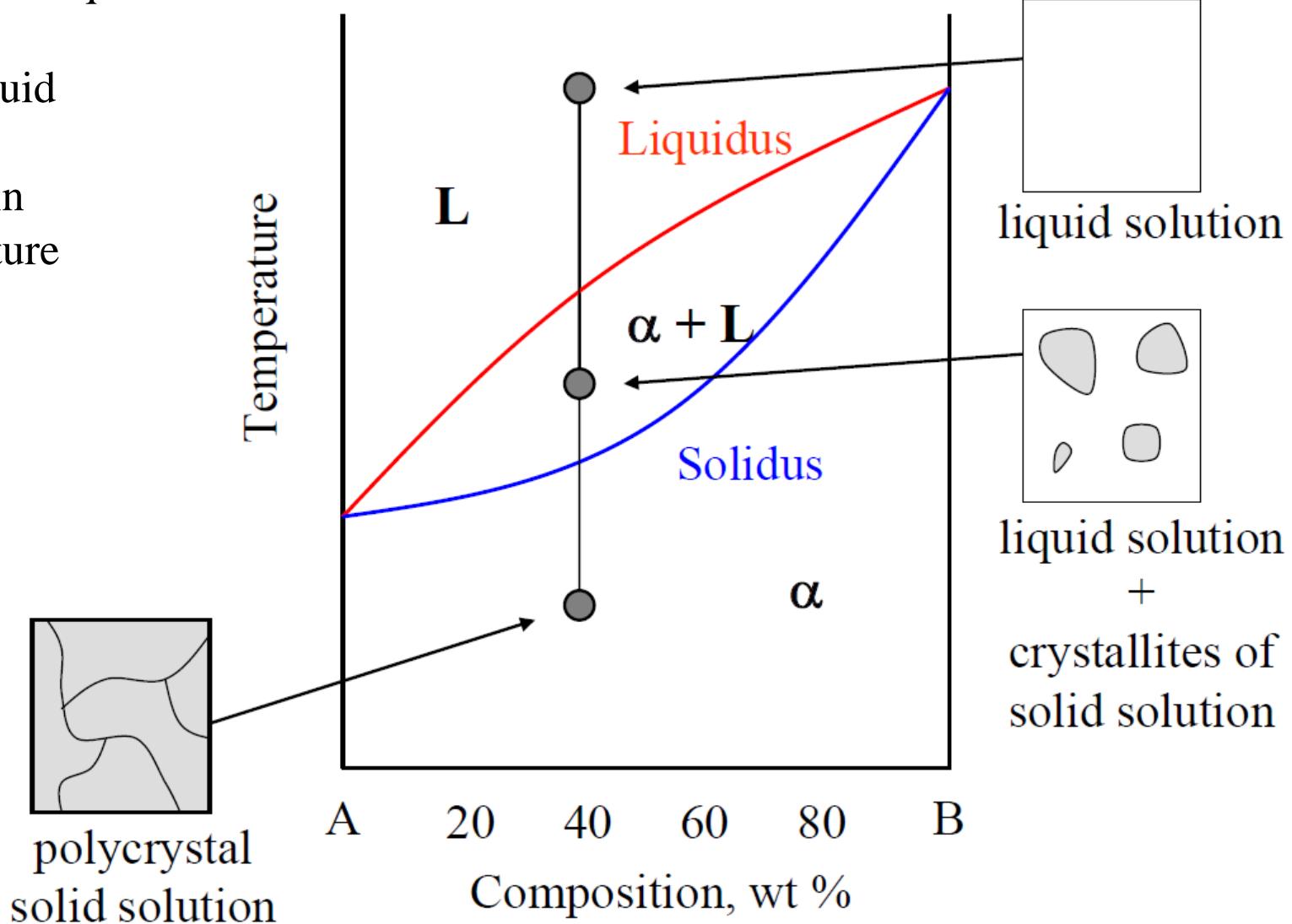
Solidus line separates solid from liquid + solid



Binary solutions with unlimited solubility (VII)

In one-component system melting occurs at a well-defined melting temperature. In multi-component systems melting occurs over the range of temperatures, between the solidus and liquidus lines.

Solid and liquid phases are at equilibrium in this temperature range.

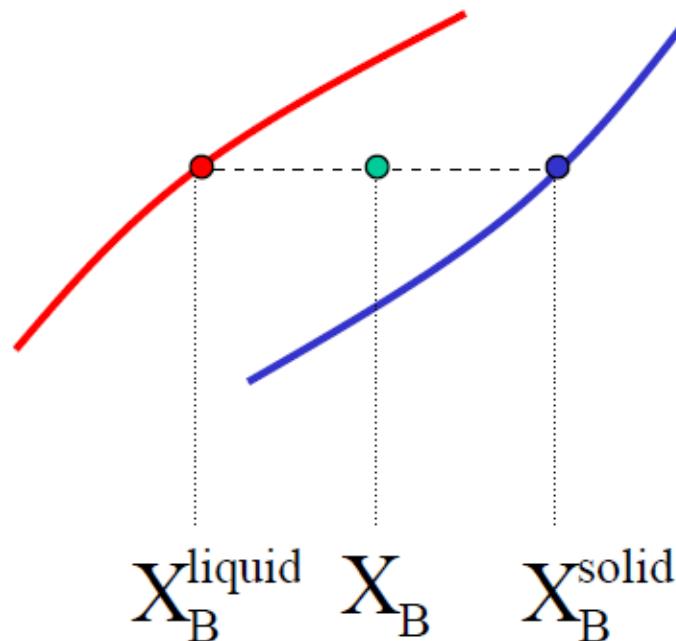


Interpretation of Phase Diagrams

For a given temperature and composition we can use phase diagram to determine: 1) The phases that are present, 2) Compositions of the phases, 3) The relative fractions of the phases

Finding the composition in a two phase region: (1) Locate composition and temperature in diagram, (2) In two phase region draw the **tie line** or isotherm, (3) Note intersection with phase boundaries. Read compositions at the intersections.

The liquid and solid phases have these compositions:

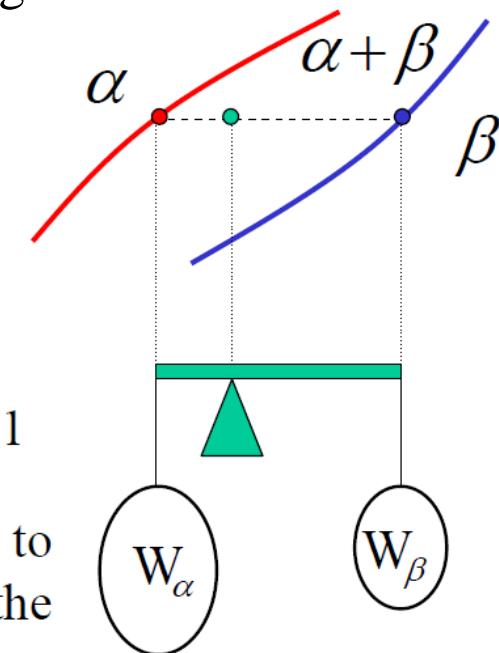


Interpretation of Phase Diagrams: the Lever Rule

Finding the amounts of phases in a two phase region:

- Locate composition and temperature in diagram
- In two phase region draw the tie line or isotherm
- Fraction of a phase is determined by taking the length of the tie line to the phase boundary for **the other phase**, and dividing by the total length of tie line

The lever rule is a mechanical analogy to the mass balance calculation. The tie line in the two-phase region is analogous to a lever balanced on a fulcrum.



Derivation of the lever rule:

- 1) All material must be in one phase or the other: $W_\alpha + W_\beta = 1$
- 2) Mass of a component that is present in both phases equal to the mass of the component in one phase + mass of the component in the second phase: $W_\alpha C_\alpha + W_\beta C_\beta = C_o$
- 3) Solution of these equations gives us the Lever rule.

$$W_\beta = (C_0 - C_\alpha) / (C_\beta - C_\alpha) \text{ and } W_\alpha = (C_\beta - C_0) / (C_\beta - C_\alpha)$$

W = mass fraction
 C = weight percent

Composition/Concentration: weight fraction vs. molar fraction

Composition can be expressed in

Molar fraction, X_B , or **atom percent** (at %) that is useful when trying to understand the material at the atomic level. Atom percent (at %) is a number of moles (atoms) of a particular element relative to the total number of moles (atoms) in alloy. For two-component system, concentration of element B in at. % is

$$C^{at} = \frac{n_m^B}{n_m^B + n_m^A} \times 100 \quad C^{at} = X_B \times 100$$

Where n_m^A and n_m^B are numbers of moles of elements A and B in the system.

Weight percent (C, wt %) that is useful when making the solution. Weight percent is the weight of a particular component relative to the total alloy weight. For two-component system, concentration of element B in wt. % is

$$C^{wt} = \frac{m_B}{m_B + m_A} \times 100$$

where m_A and m_B are the weights of the components in the system.

$$n_m^A = \frac{m_A}{A_A} \quad n_m^B = \frac{m_B}{A_B} \quad \text{where } A_A \text{ and } A_B \text{ are atomic weights of elements A and B.}$$

Composition Conversions

Weight % to Atomic %:

$$C_B^{\text{at}} = \frac{C_B^{\text{wt}} A_A}{C_B^{\text{wt}} A_A + C_A^{\text{wt}} A_B} \times 100$$

$$C_A^{\text{at}} = \frac{C_A^{\text{wt}} A_B}{C_B^{\text{wt}} A_A + C_A^{\text{wt}} A_B} \times 100$$

Atomic % to Weight %:

$$C_B^{\text{wt}} = \frac{C_B^{\text{at}} A_B}{C_B^{\text{at}} A_B + C_A^{\text{at}} A_A} \times 100$$

$$C_A^{\text{wt}} = \frac{C_A^{\text{at}} A_A}{C_B^{\text{at}} A_B + C_A^{\text{at}} A_A} \times 100$$

Composition Conversions

Of course **the lever rule** can be formulated for any specification of composition:

$$M^L = (X_B^\alpha - X_B^0) / (X_B^\alpha - X_B^L) = (C_{\alpha}^{at} - C_0^{at}) / (C_{\alpha}^{at} - C_L^{at})$$

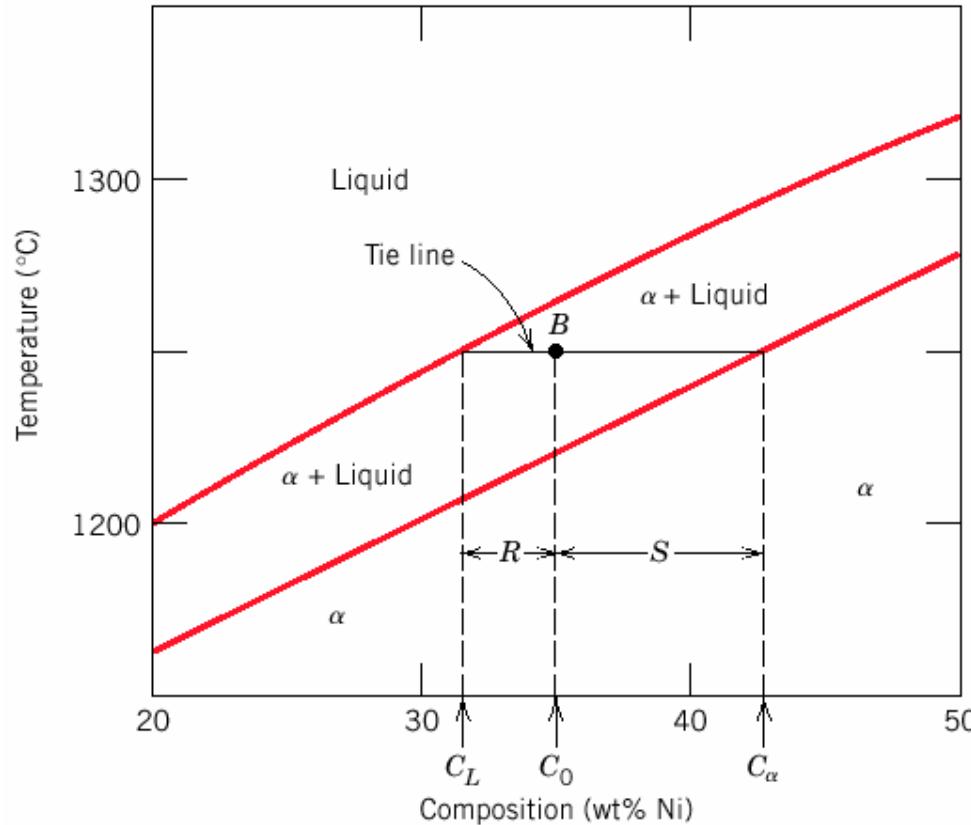
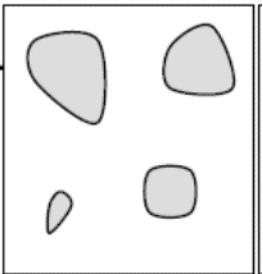
$$M^\alpha = (X_B^0 - X_B^L) / (X_B^\alpha - X_B^L) = (C_0^{at} - C_L^{at}) / (C_{\alpha}^{at} - C_L^{at})$$

$$W_L = (C_{\alpha}^{wt} - C_0^{wt}) / (C_{\alpha}^{wt} - C_L^{wt})$$

$$W_\alpha = (C_0^{wt} - C_L^{wt}) / (C_{\alpha}^{wt} - C_L^{wt})$$

Phase compositions and amounts: An example

Cu-Ni



$$C_o = 35 \text{ wt. \%}, \quad C_L = 31.5 \text{ wt. \%}, \quad C_\alpha = 42.5 \text{ wt. \%}$$

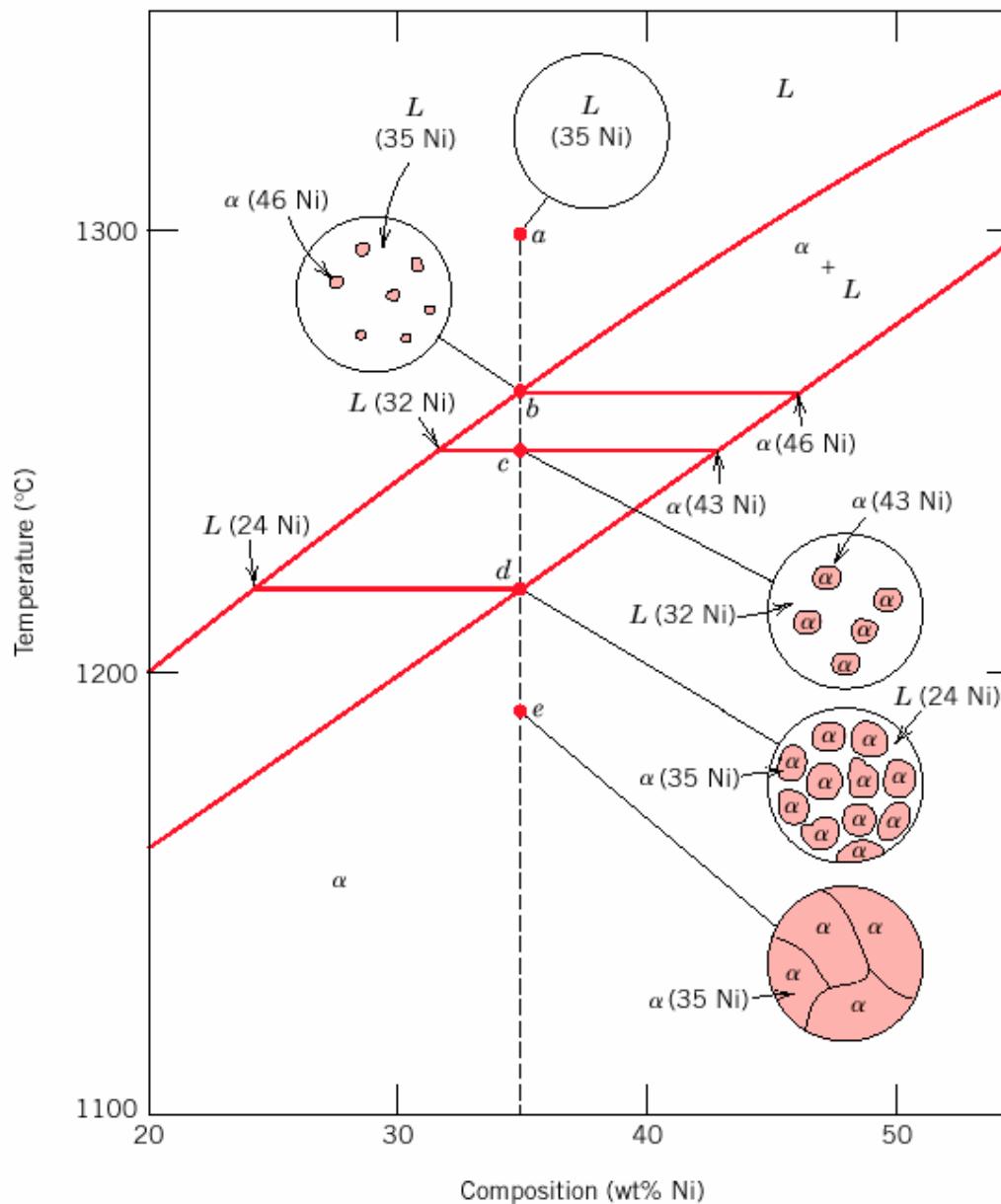
$$\text{Mass fractions: } W_L = S / (R+S) = (C_\alpha - C_o) / (C_\alpha - C_L) = 0.68$$

$$W_\alpha = R / (R+S) = (C_o - C_L) / (C_\alpha - C_L) = 0.32$$

Development of microstructure in isomorphous alloys

Equilibrium (very slow) cooling

Cu-Ni



Development of microstructure in isomorphous alloys

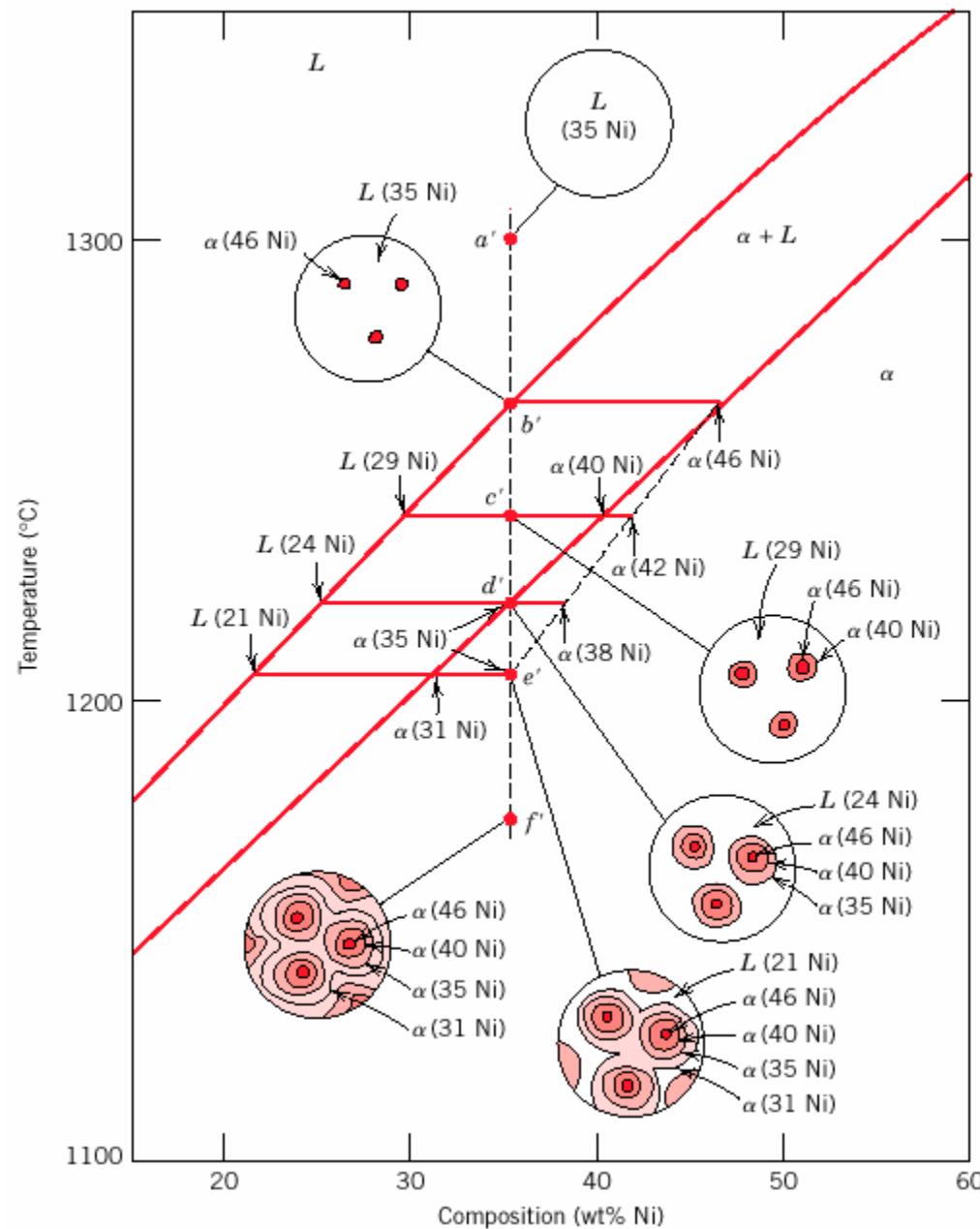
Equilibrium (very slow) cooling

- Solidification in the solid + liquid phase occurs gradually upon cooling from the liquidus line.
- The composition of the solid and the liquid change gradually during cooling (as can be determined by the tie-line method.)
- Nuclei of the solid phase form and they grow to consume all the liquid at the solidus line.

Development of microstructure in isomorphous alloys

Non-equilibrium cooling

Cu-Ni

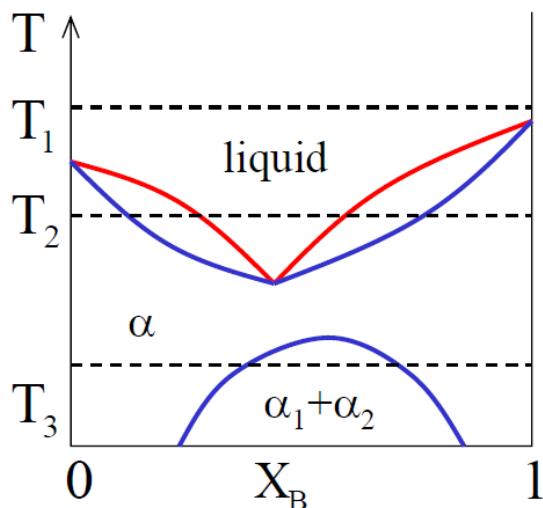
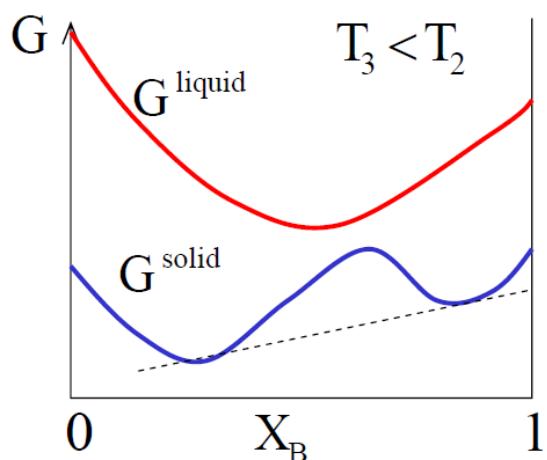
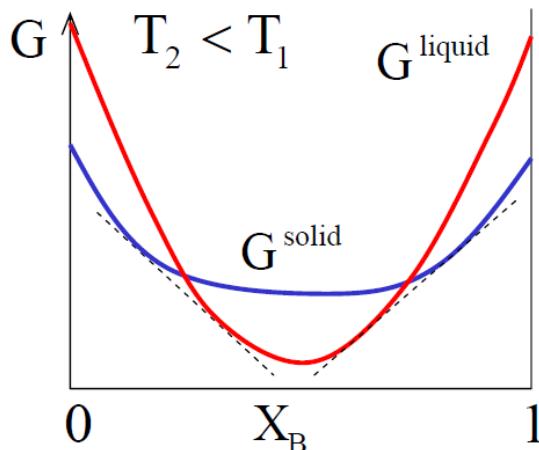
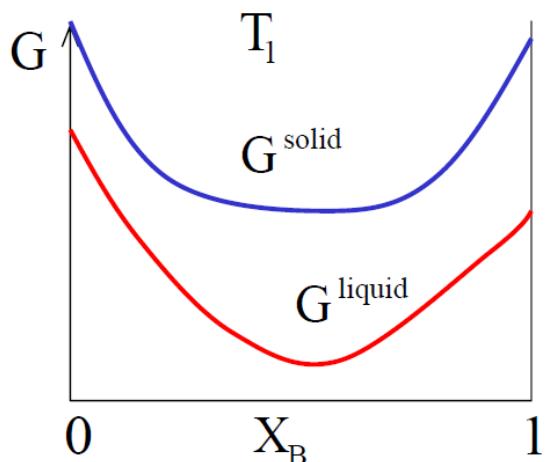


Development of microstructure in isomorphous alloys

- Compositional changes require diffusion in solid and liquid phases
- Diffusion in the solid state is very slow. \Rightarrow The new layers that solidify on top of the existing grains have the equilibrium composition at that temperature but once they are solid their composition does not change. \Rightarrow Formation of layered grains and the invalidity of the tie-line method to determine the composition of the solid phase.
- The tie-line method still works for the liquid phase, where diffusion is fast. Average Ni content of solid grains is higher. \Rightarrow Application of the lever rule gives us a greater proportion of liquid phase as compared to the one for equilibrium cooling at the same T. \Rightarrow Solidus line is shifted to the right (higher Ni contents), solidification is complete at lower T, the outer part of the grains are richer in the low-melting component (Cu).
- Upon heating grain boundaries will melt first. This can lead to premature mechanical failure.

Non-equilibrium cooling

Let's consider a system in which the liquid phase is approximately ideal, but for the solid phase we have $\Delta H_{\text{mix}} > 0$



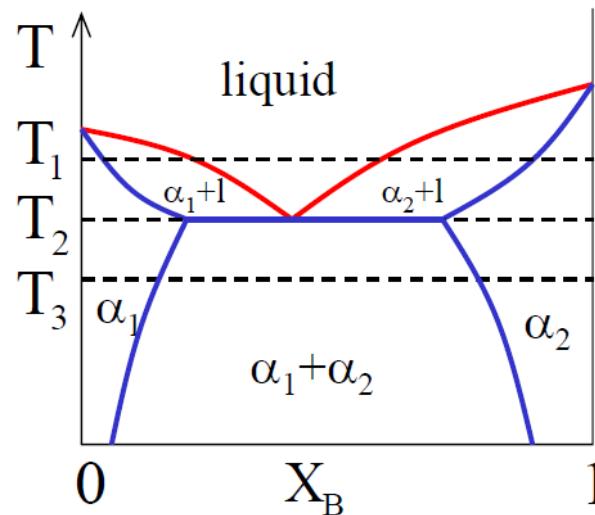
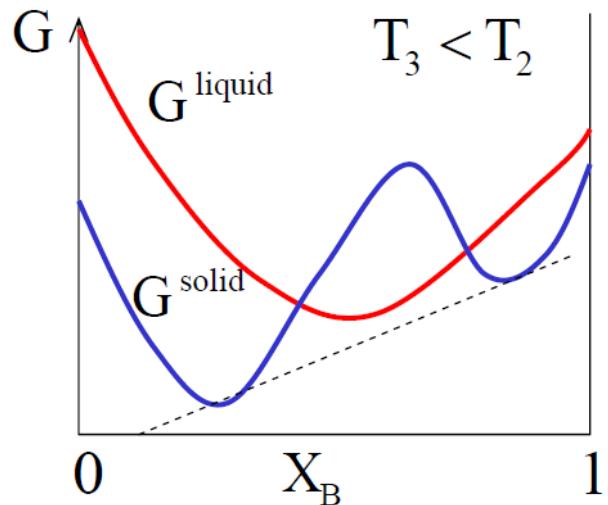
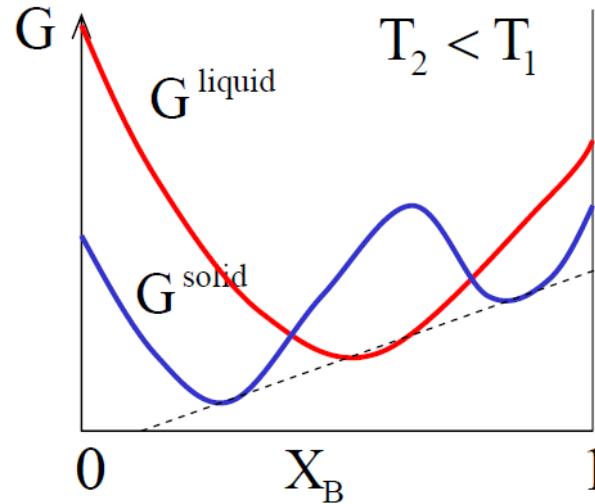
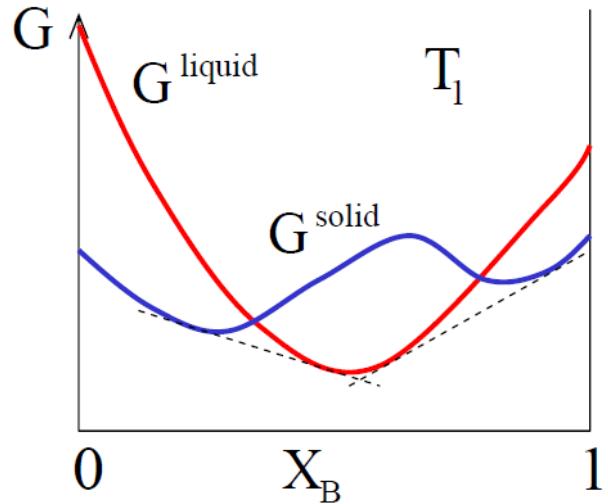
Binary solutions with a miscibility gap

A and B dislike each other in the solid phase

$T_m < T_A < T_B$ for all alloys

At low temperatures, there is a region where the solid solution is most stable as a mixture of two phases α_1 and α_2 with compositions X_1 and X_2 . This region is called a **miscibility gap**.

For an even larger ΔH_{mix} the miscibility gap can extend into the liquid phase region. In this case we have **eutectic phase diagram**.

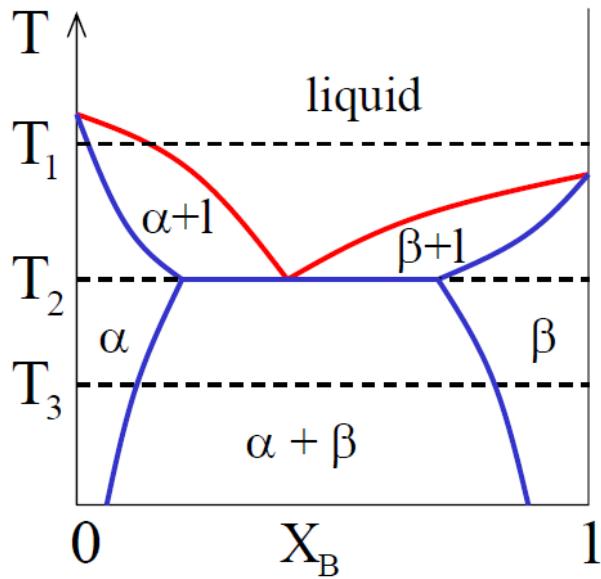
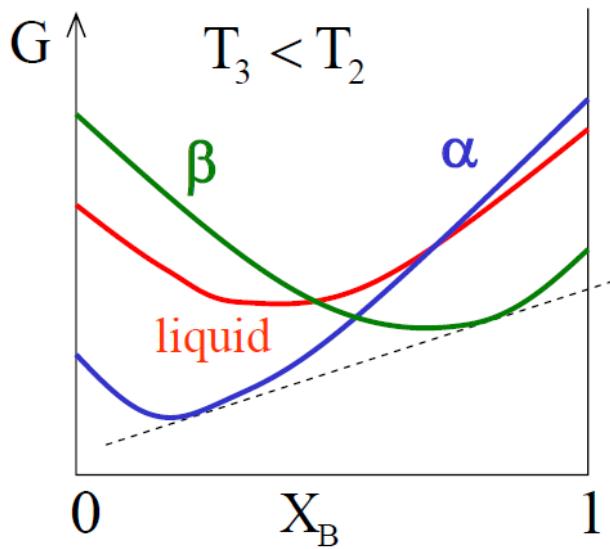
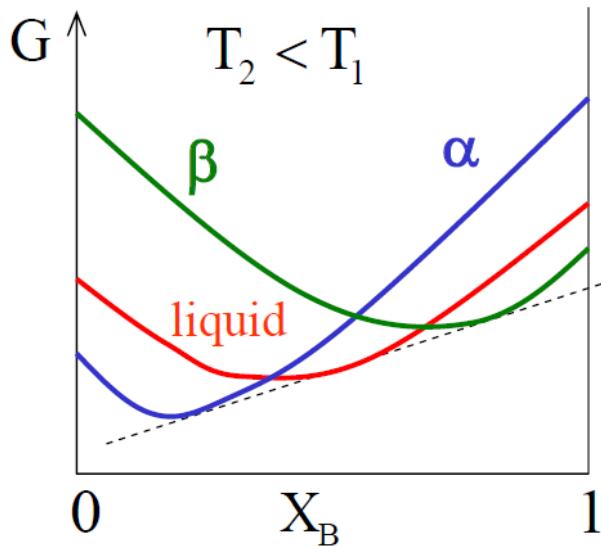
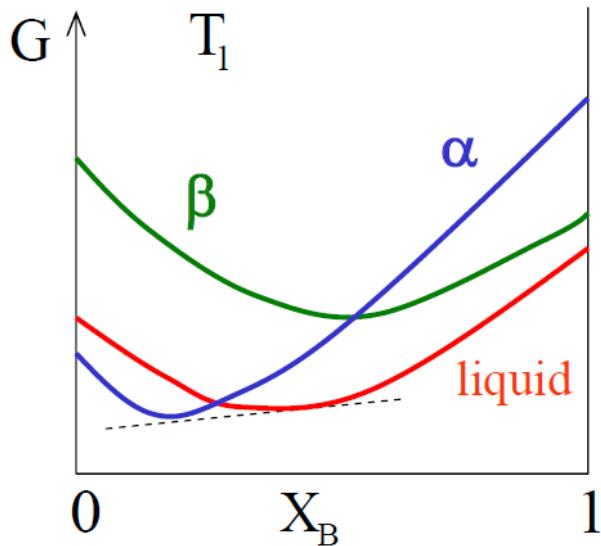


Eutectic phase diagram

(both solid phases have the same crystal structure)

The melting point of the eutectic alloy is lower than that of the components (*eutectic = easy to melt in Greek*).

A similar eutectic phase diagram can result if pure A and B have different crystal structures.

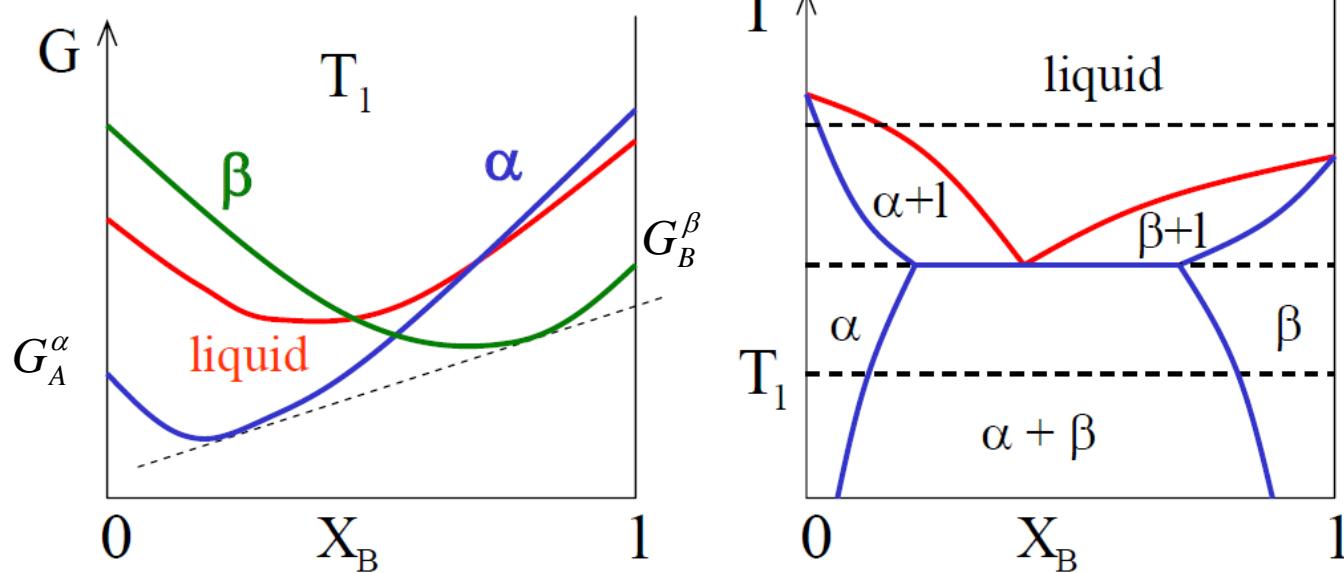


Eutectic phase diagram with different crystal structures of pure phases

A eutectic system describes a homogeneous solid mix of atomic and/or chemical species, to form a joint super-lattice, by striking a unique atomic percentage ratio between the components — as each pure component has its own distinct bulk lattice arrangement.

Temperature dependence of solubility

There is **limited solid solubility** of A in B and B in A in the alloy having eutectic phase diagram:



The closer is the minimum of the Gibbs free energy curve $G^\alpha(X_B)$ to the axes $X_B = 0$, the smaller is the maximum possible concentration of B in phase α . Therefore, to discuss the temperature dependence of solubility let's find the minimum of $G^\alpha(X_B)$.

$$G^{\text{reg}} = X_A G_A + X_B G_B + \Omega X_A X_B + RT[X_A \ln X_A + X_B \ln X_B]$$

$$\frac{dG}{dX_B} = 0 \quad - \text{Minimum of } G(X_B)$$

$$\Omega = zN_{\text{tot}} \left(E_{AB} - \frac{E_{AA} + E_{BB}}{2} \right)$$

Temperature dependence of solubility (II)

$$\begin{aligned} G^{\text{reg}} &= X_A G_A + X_B G_B + \Omega X_A X_B + RT[X_A \ln X_A + X_B \ln X_B] \\ &= (1 - X_B)G_A + X_B G_B + \Omega(1 - X_B)X_B + RT[(1 - X_B) \ln(1 - X_B) + X_B \ln X_B] \end{aligned}$$

$$\frac{dG}{dX_B} = -G_A + G_B + \Omega - 2\Omega X_B + RT \left[-\ln(1 - X_B) - \frac{(1 - X_B)}{(1 - X_B)} + \ln X_B + \frac{X_B}{X_B} \right] =$$

$$= -G_A + G_B + \Omega - 2\Omega X_B + RT[-\ln(1 - X_B) + \ln X_B] =$$

$$= G_B - G_A + \Omega(1 - 2X_B) + RT \ln \left(\frac{X_B}{1 - X_B} \right) \approx G_B - G_A + \Omega + RT \ln(X_B) = 0$$

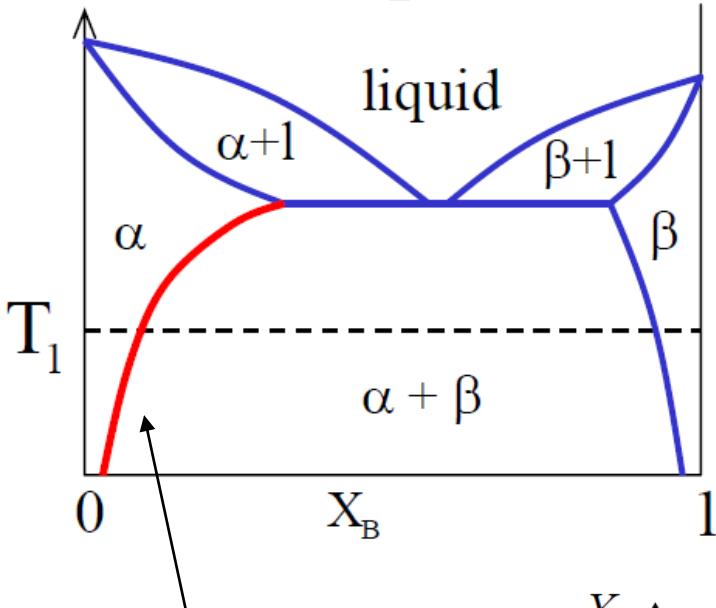
$$X_B \approx \exp \left(-\frac{G_B - G_A + \Omega}{RT} \right)$$

if X_B is small ($X_B \rightarrow 0$) Minimum of $G(X_B)$

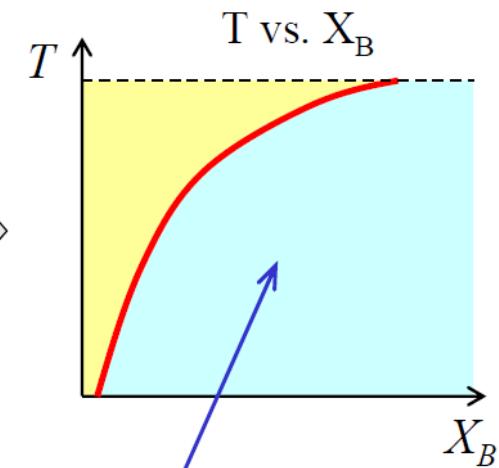
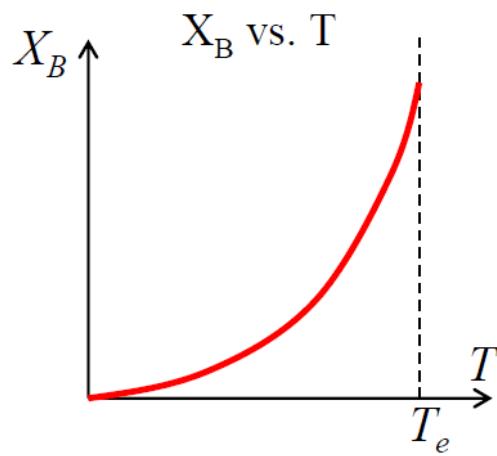
Solid solubility of B in α increases exponentially with temperature
(similar to vacancy concentration)

$$\Omega = zN_{\text{tot}} \left(E_{AB} - \frac{E_{AA} + E_{BB}}{2} \right)$$

Temperature dependence of solubility (III)



$$X_B \approx \exp\left(-\frac{G_B - G_A + \Omega}{RT}\right)$$

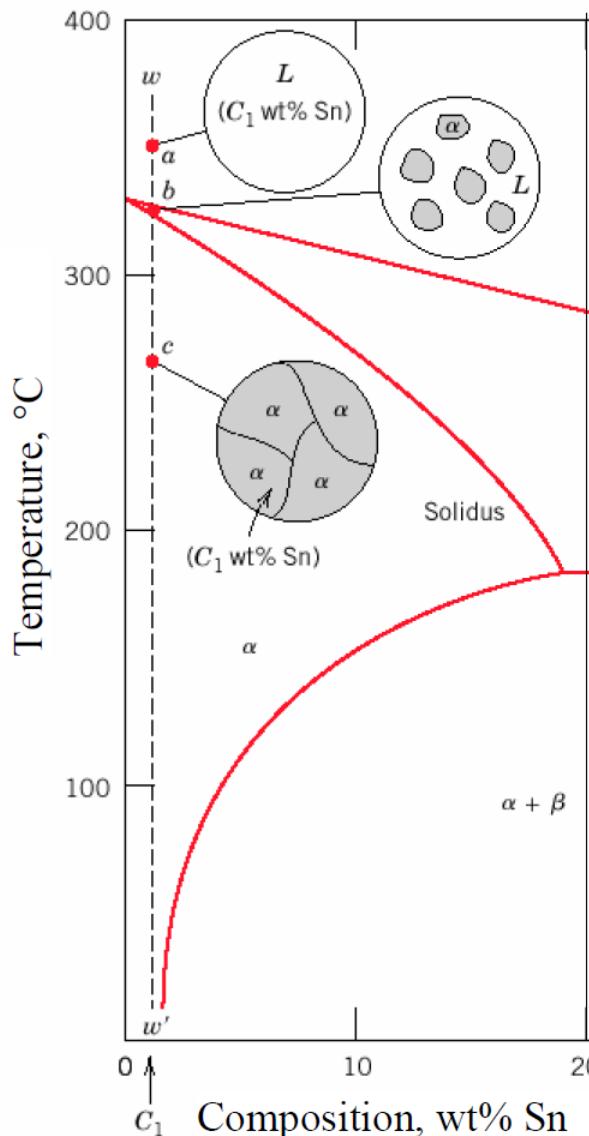


supersaturation with B

- phase separation α and β
- precipitation of B-rich intermediate phase or compound

Development of microstructure in eutectic alloys (I)

Several different types of microstructure can be formed in slow cooling an different compositions. Let's consider cooling of liquid lead – tin system as an example.



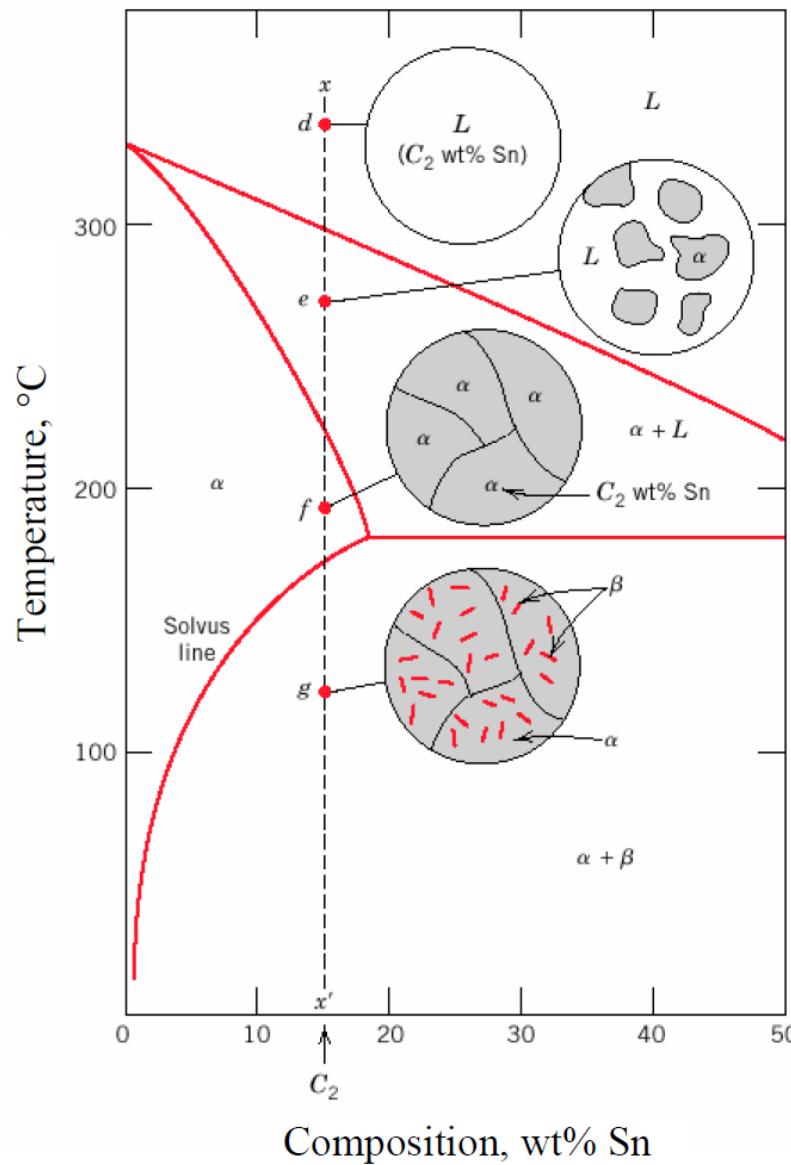
Pb-Sn

In the case of lead-rich alloy (0-2 wt. % of tin) solidification proceeds in the same manner as for isomorphous alloys (e.g. Cu-Ni) that we discussed earlier.

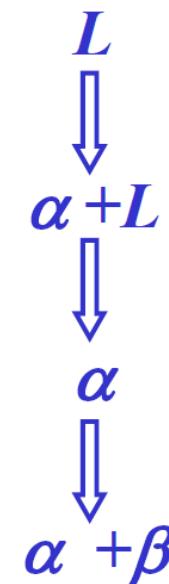
$L \rightarrow \alpha + L \rightarrow \alpha$

Development of microstructure in eutectic alloys (II)

At compositions between the room temperature solubility limit and the maximum solid solubility at the eutectic temperature, β phase nucleates as the α solid solubility is exceeded upon crossing the solvus line.



Pb-Sn



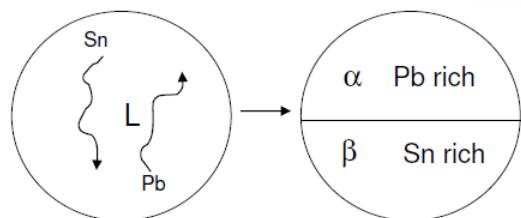
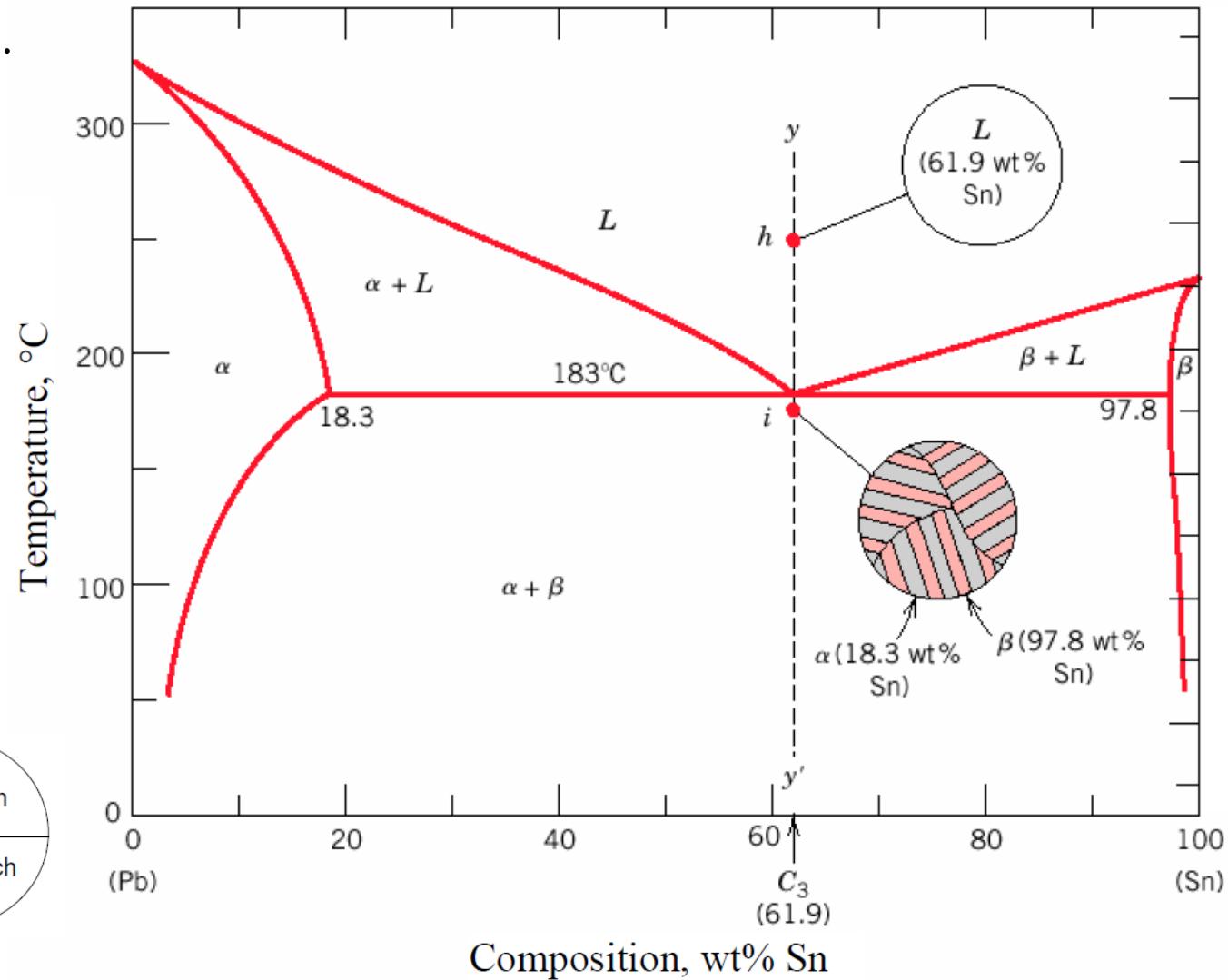
Development of microstructure in eutectic alloys (III)

Solidification at the eutectic composition

No changes above the eutectic temperature T_E . At T_E all the liquid transforms to α and β phases
(eutectic reaction).

$L \rightarrow \alpha + \beta$

Pb-Sn



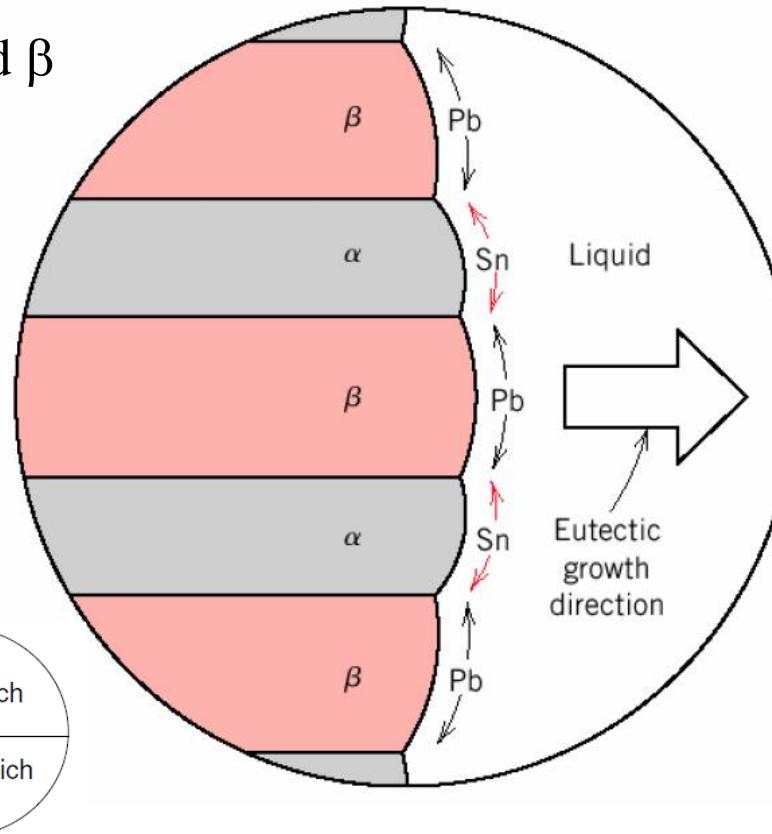
Development of microstructure in eutectic alloys (IV)

Pb-Sn

Solidification at the eutectic composition

Compositions of α and β phases are very different \rightarrow eutectic reaction involves redistribution of Pb and Sn atoms by atomic diffusion.

This simultaneous formation of α and β phases result in a layered (lamellar) microstructure that is called **eutectic structure**.



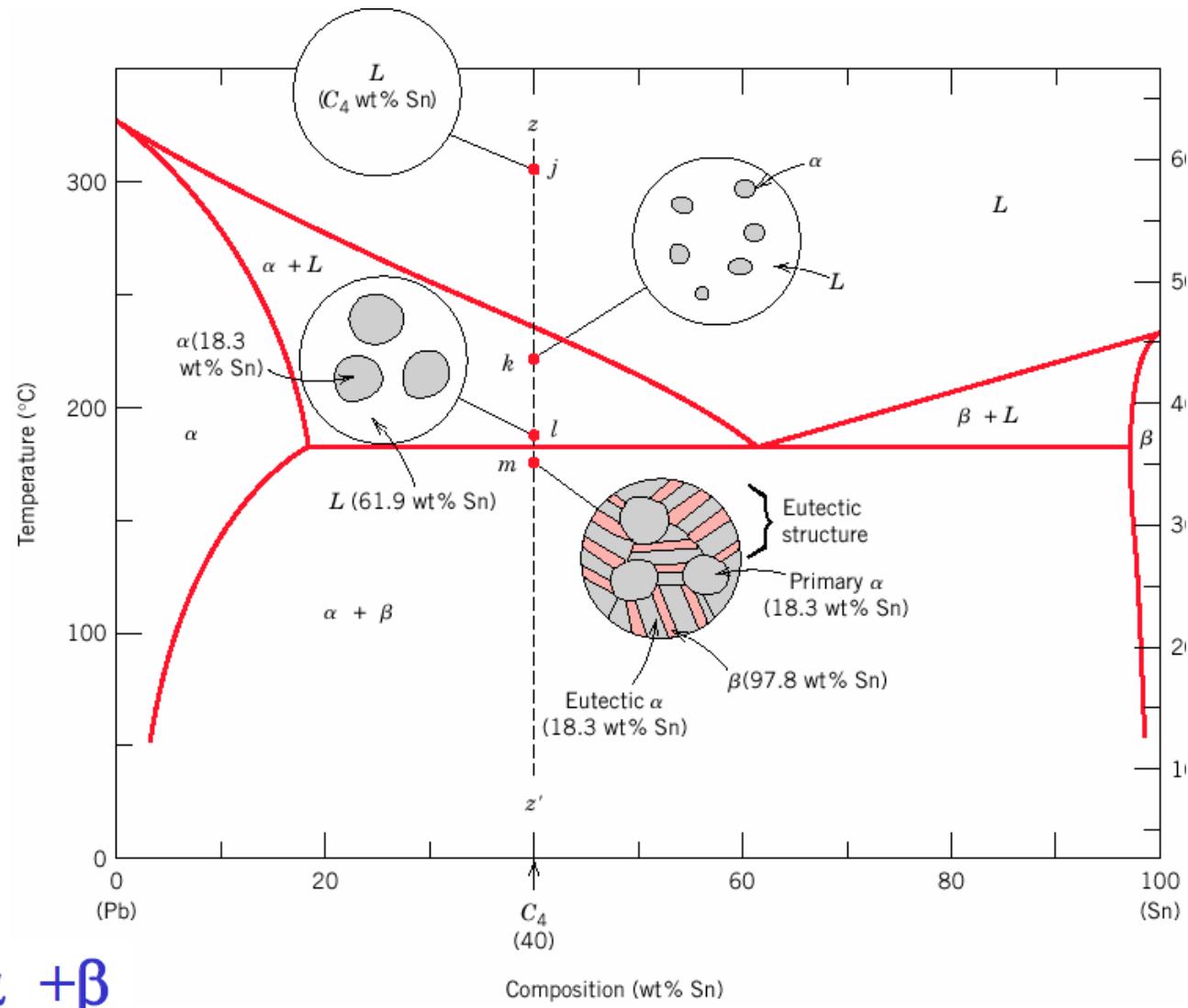
Formation of the eutectic structure in the lead-tin system. In the micrograph, the dark layers are lead-reach α phase, the light layers are the tin-reach β phase.

Development of microstructure in eutectic alloys (V)

Compositions other than eutectic but within the range of the eutectic isotherm

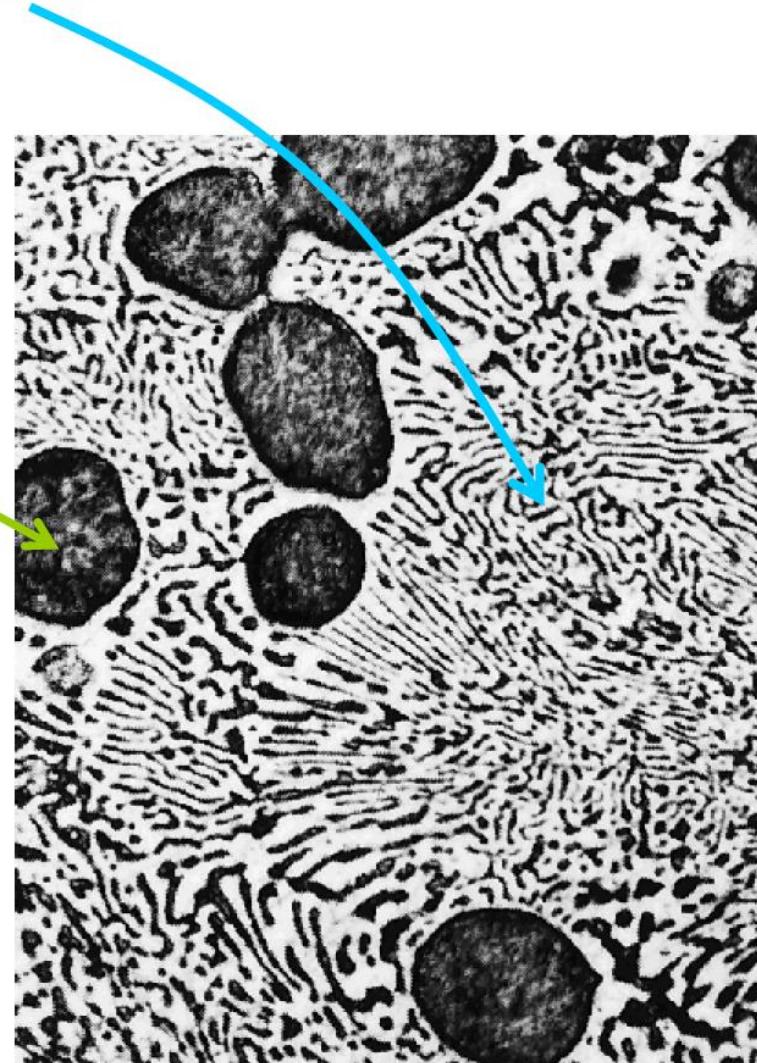
Primary α phase is formed in the $\alpha + L$ region, and the eutectic structure that includes layers of α and β phases (called **eutectic α** and **eutectic β** phases) is formed upon crossing the eutectic isotherm.

Pb-Sn



Development of microstructure in eutectic alloys (VI)

Microconstituent – element of the microstructure having a distinctive structure. In the case described in the previous page, microstructure consists of two microconstituents, **primary α phase** and the **eutectic structure**.

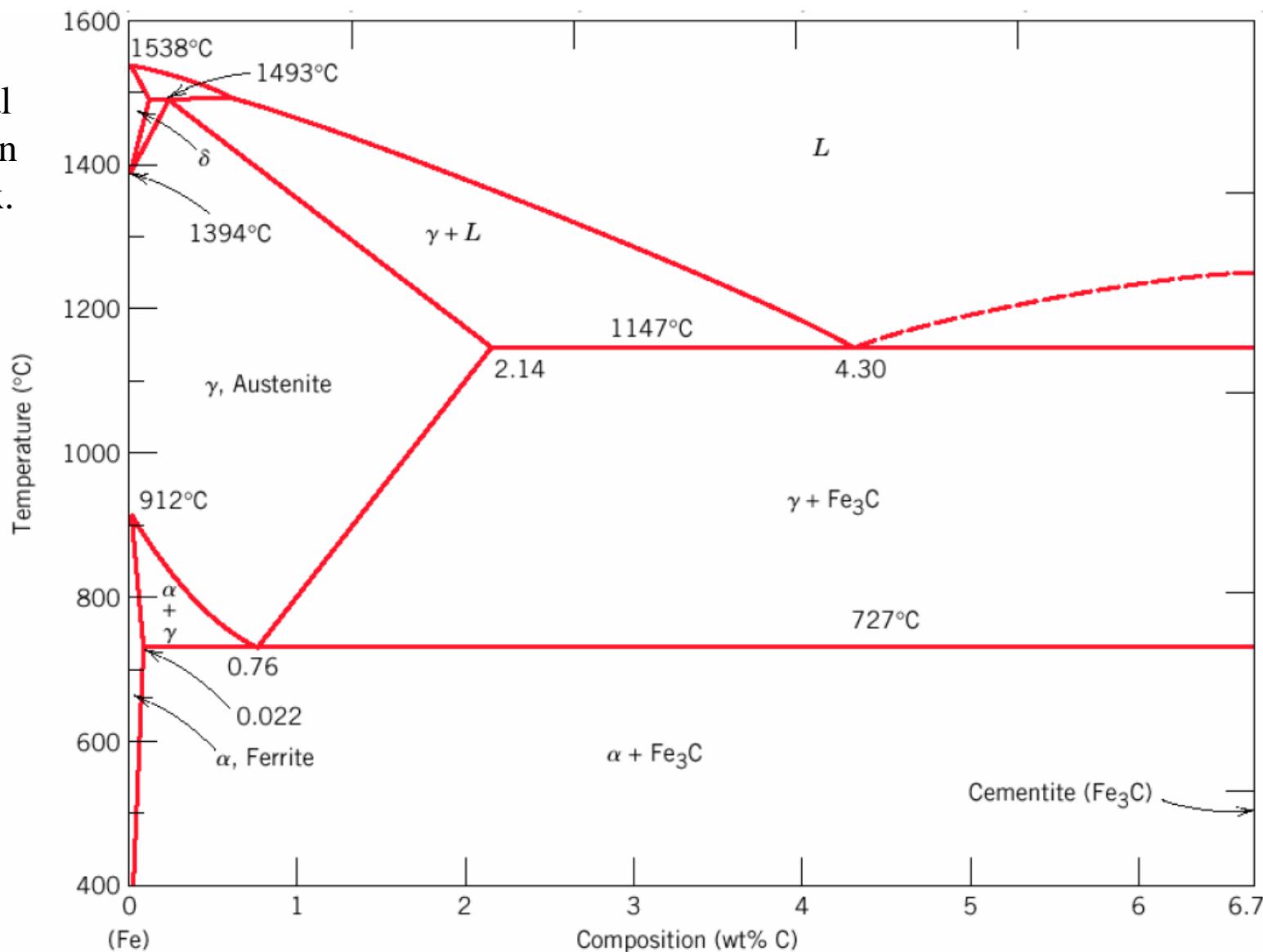


Although the eutectic structure consists of two phases, it is a microconstituent with distinct lamellar structure and fixed ratio of the two phases.

Example: The Iron–Iron Carbide (Fe–Fe₃C) Phase Diagram

In their simplest form, steels are alloys of Iron (Fe) and Carbon (C). The Fe-C phase diagram is a fairly complex one, but we will only consider the steel part of the diagram, up to around 7% Carbon.

C is an interstitial element in Fe matrix.



Phases in Iron-Iron Carbide (Fe–Fe₃C) phase diagram:

➤ α-ferrite - solid solution of C in BCC Fe

- Stable form of iron at room temperature.
- The maximum solubility of C is 0.022 wt%
- Transforms to FCC γ-austenite at 912 °C

➤ γ-austenite - solid solution of C in FCC Fe

- The maximum solubility of C is 2.14 wt %.
- Transforms to BCC δ-ferrite at 1394 °C
- Is not stable below the eutectic temperature (727 °C) unless cooled rapidly

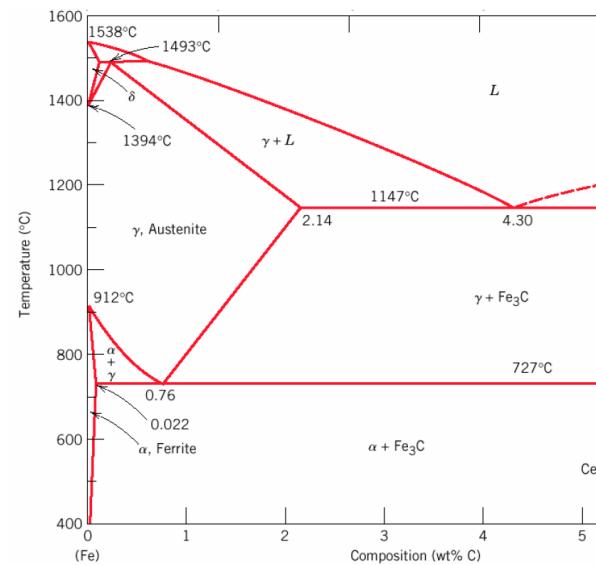
➤ δ-ferrite solid solution of C in BCC Fe

- The same structure as α-ferrite
- Stable only at high T, above 1394 °C
- Melts at 1538 °C

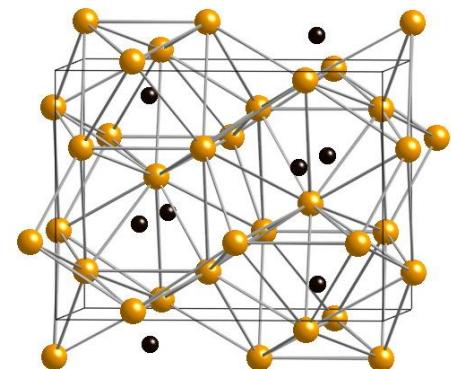
➤ Fe₃C (iron carbide or cementite) hard, brittle material

- This intermetallic compound is metastable, it remains as a compound indefinitely at room T, but decomposes (very slowly, within several years) into α-Fe and C (graphite) at 650 - 700 °C

➤ Fe-C liquid solution



(C black, Fe yellow)



By weight, it is 6.67% carbon and 93.3% iron.

C is an interstitial impurity in Fe. It forms a solid solution with α , γ , δ phases of iron

Maximum solubility in BCC α -ferrite is limited (max. 0.022 wt% at 727 °C) - BCC has relatively small interstitial positions

Maximum solubility in FCC austenite is 2.14 wt% at 1147 °C - FCC has larger interstitial positions

Mechanical properties: Cementite is very hard and brittle - can strengthen steels. Mechanical properties of steel depend on the microstructure, that is, how ferrite and cementite are mixed.

Magnetic properties: α -ferrite is magnetic below 768 °C, austenite is non-magnetic

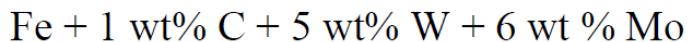
The Iron–Iron Carbide (Fe–Fe₃C) Phase Diagram

Phase	Crystal structure	Characteristics
Ferrite	BCC	Soft, ductile, magnetic
Austenite	FCC	Soft, moderate strength, non-magnetic
Cementite	Compound of Iron & Carbon Fe ₃ C	Hard & brittle

Classification. Three types of ferrous alloys:

- **Iron:** less than 0.008 wt % C in α -ferrite at room T
- **Steels:** 0.008 - 2.14 wt % C (usually < 1 wt %)
 α -ferrite + Fe₃C at room T

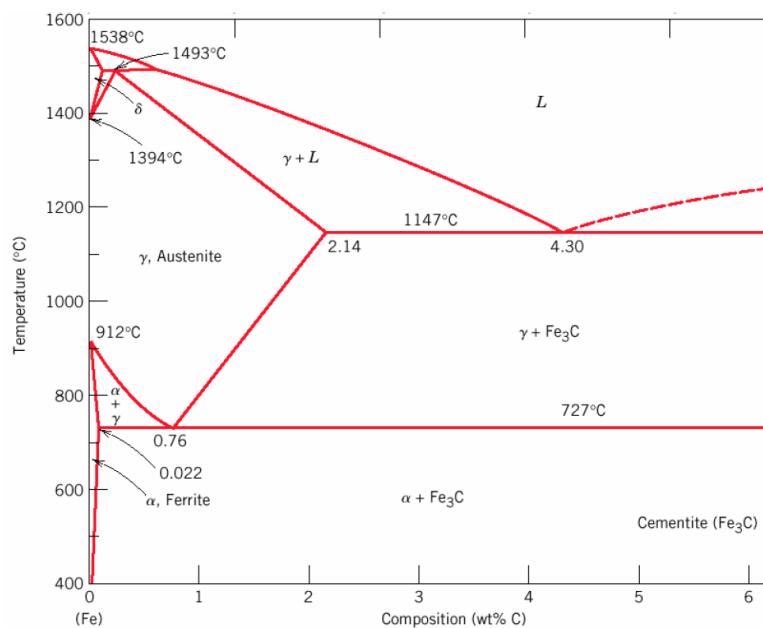
Examples of tool steel (machine tools for cutting other metals):



Stainless steel (food processing equipment, knives, petrochemical equipment, etc.): 12-20 wt% Cr, ~\$1500/ton

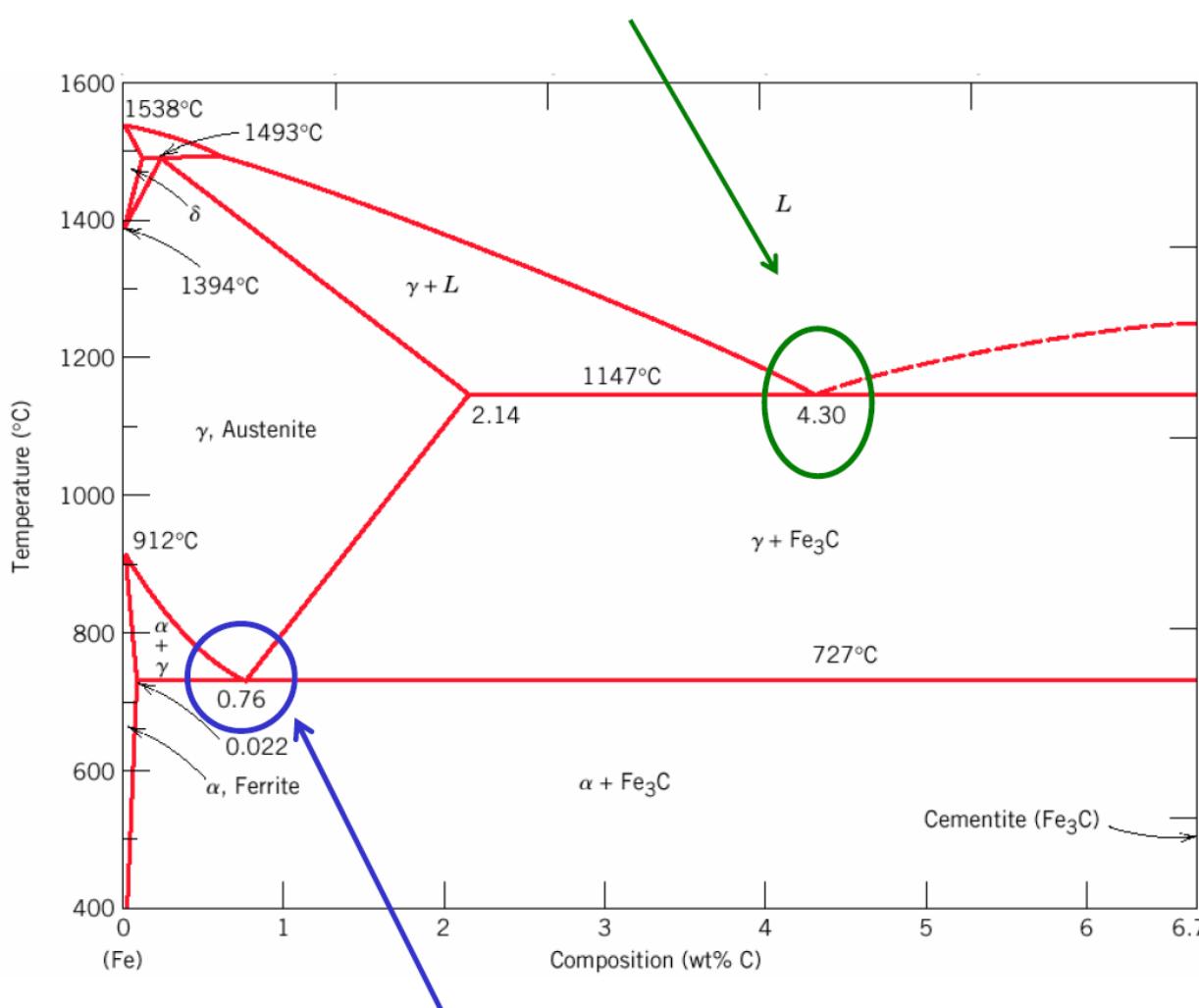
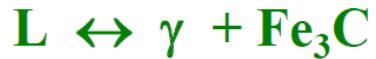
- **Cast iron:** 2.14 - 6.7 wt % (usually < 4.5 wt %)

heavy equipment casing



Eutectic and eutectoid reactions in Fe–Fe₃C phase diagram

Eutectic: 4.30 wt% C, 1147 °C



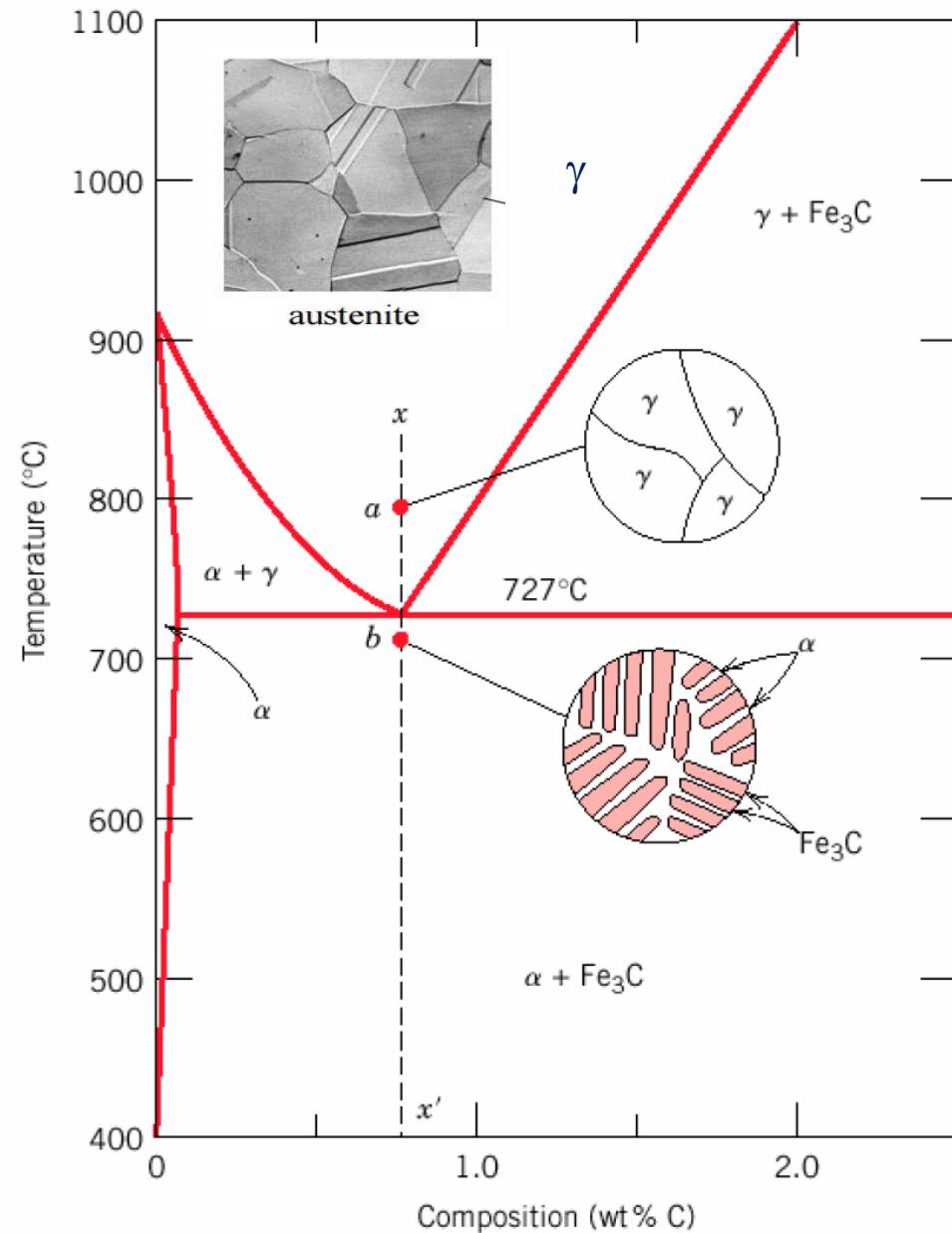
Eutectic (liquid to solid) and eutectoid (solid to solid) reactions are very important in heat treatment of steels

Eutectoid: 0.76 wt% C, 727 °C

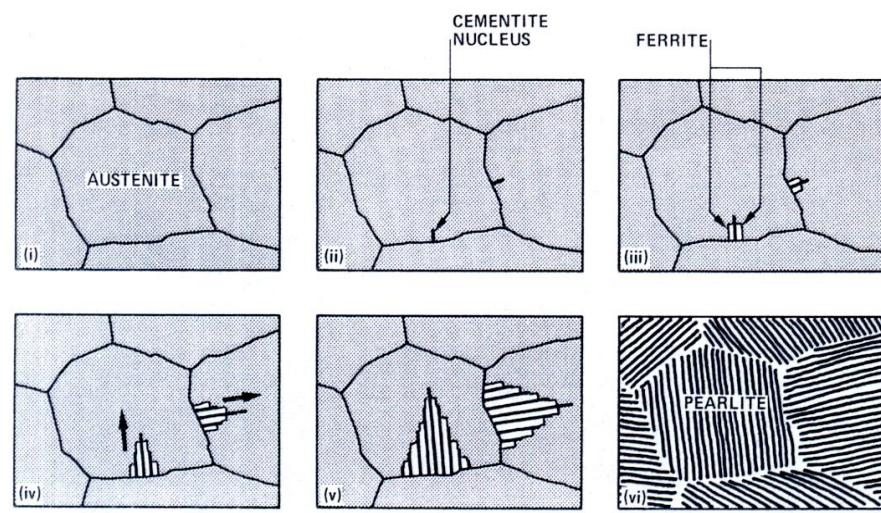


Development of Microstructure in Iron - Carbon alloys

Microstructure of eutectoid steel (I)



Microstructure depends on composition (carbon content) and heat treatment. In the discussion below we consider slow cooling in which equilibrium is maintained. (eutectoid = *eutectic-like* in Greek).

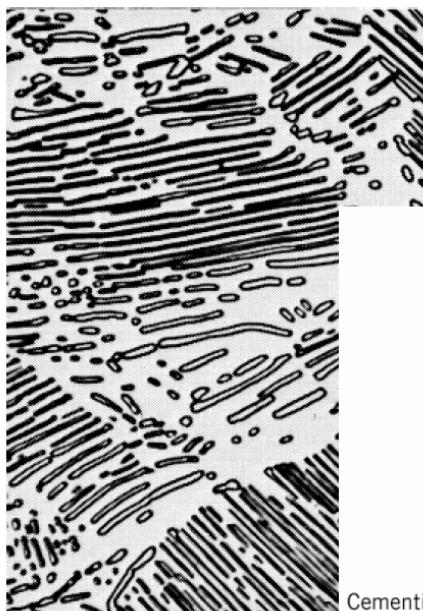


Microstructure of eutectoid steel (II)

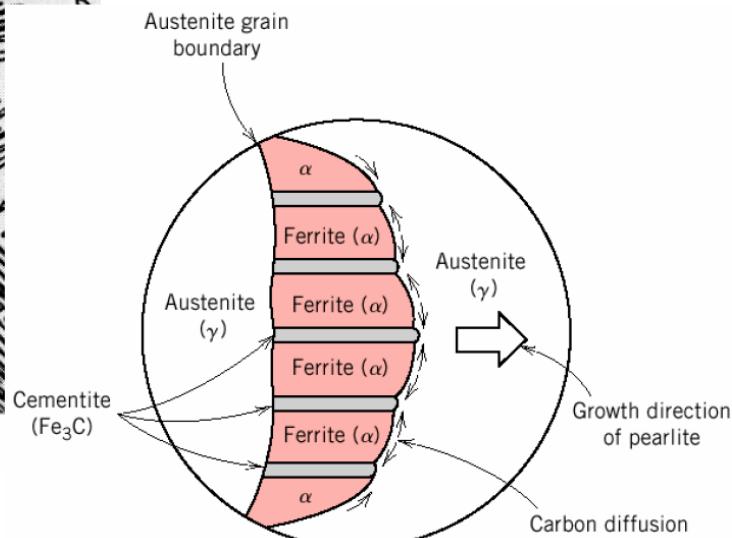
When alloy of eutectoid composition (0.76 wt % C) is cooled slowly it forms **perlite**, a lamellar or layered structure of two phases: α -ferrite and cementite (Fe_3C)

The layers of alternating phases in perlite are formed for the same reason as layered structure of eutectic structures: redistribution C atoms between ferrite (0.022 wt%) and cementite (6.7 wt%) by atomic diffusion.

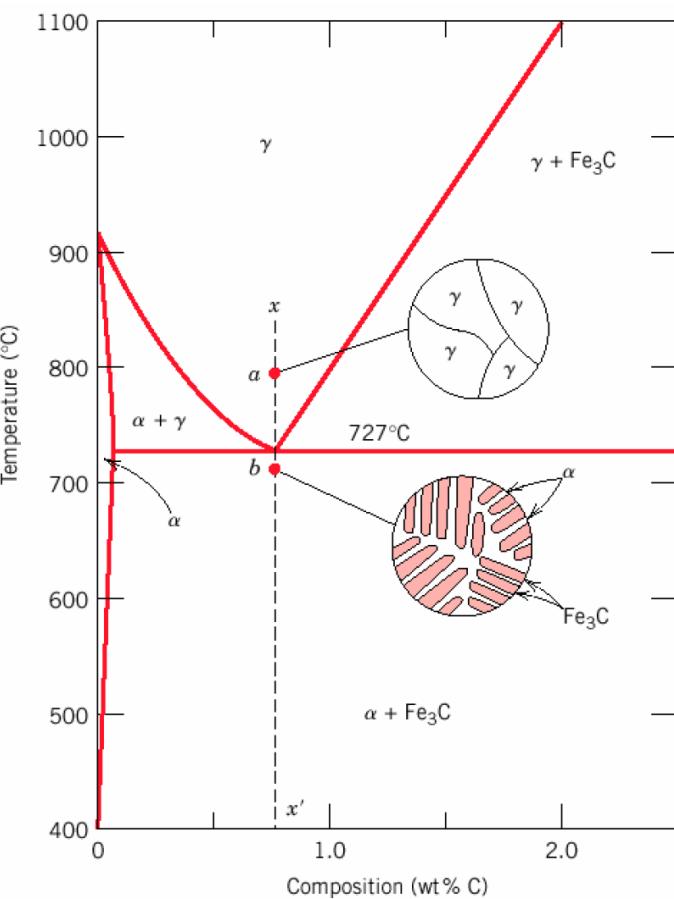
Mechanically, perlite has properties intermediate to soft, ductile ferrite and hard, brittle cementite.



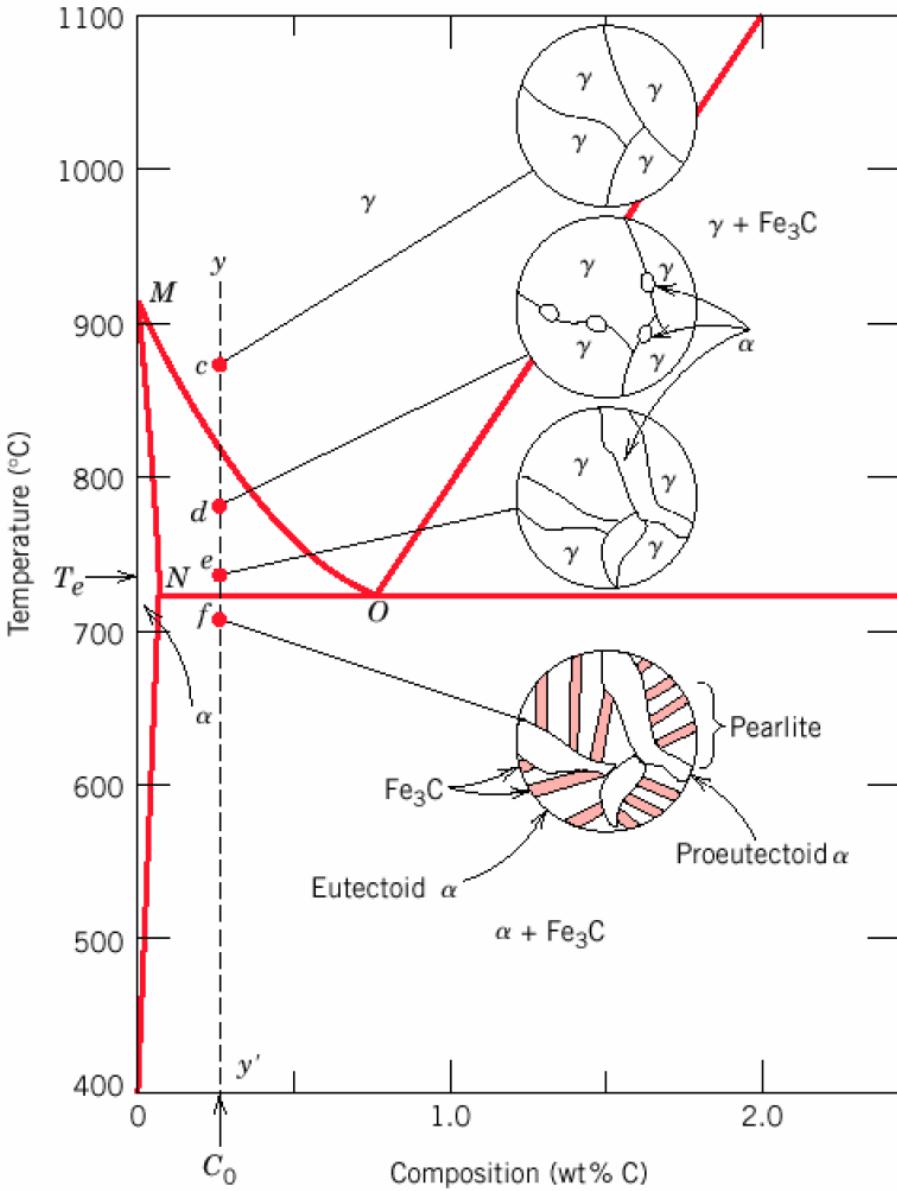
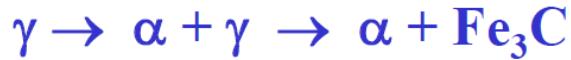
In the micrograph, the dark areas are Fe_3C layers, the light phase is α -ferrite



Microstructure of eutectoid steel (I)

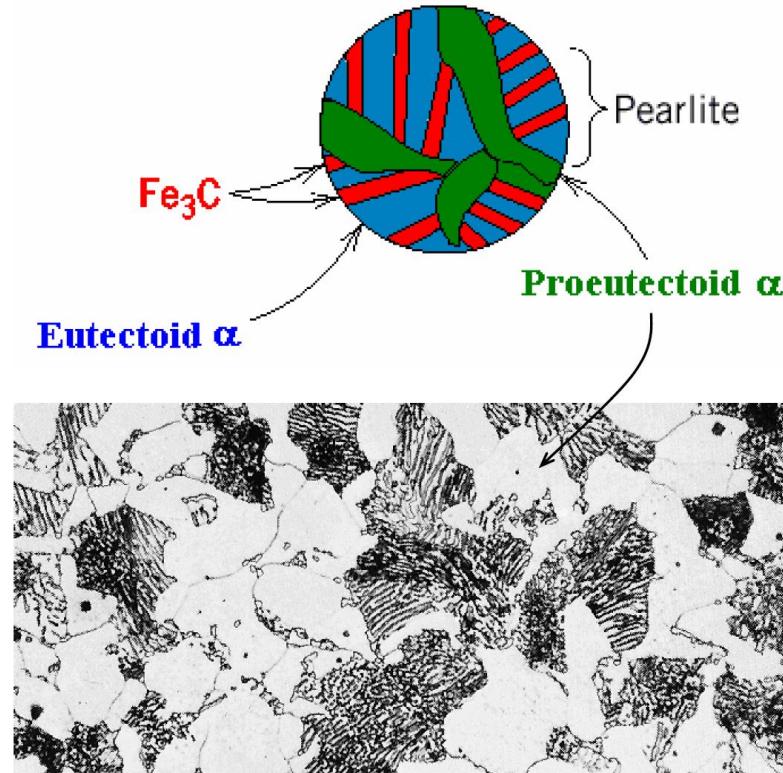


Microstructure of hypoeutectoid steel

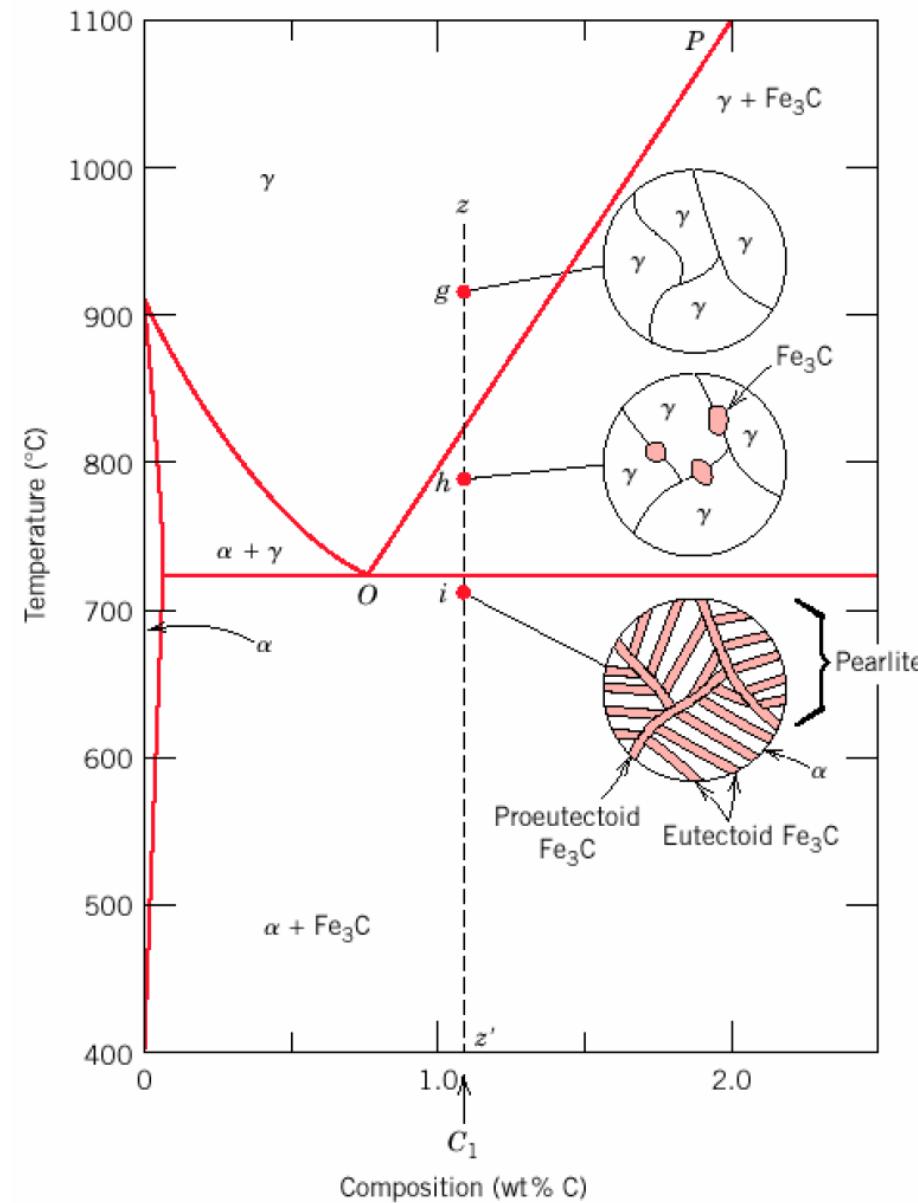


Compositions to the left of eutectoid (0.022 - 0.76 wt % C) **hypoeutectoid** (*less than eutectoid* -Greek) alloys.

Hypoeutectoid alloys contain proeutectoid ferrite (formed above the eutectoid temperature) plus the eutectoid perlite that contain eutectoid ferrite and cementite.

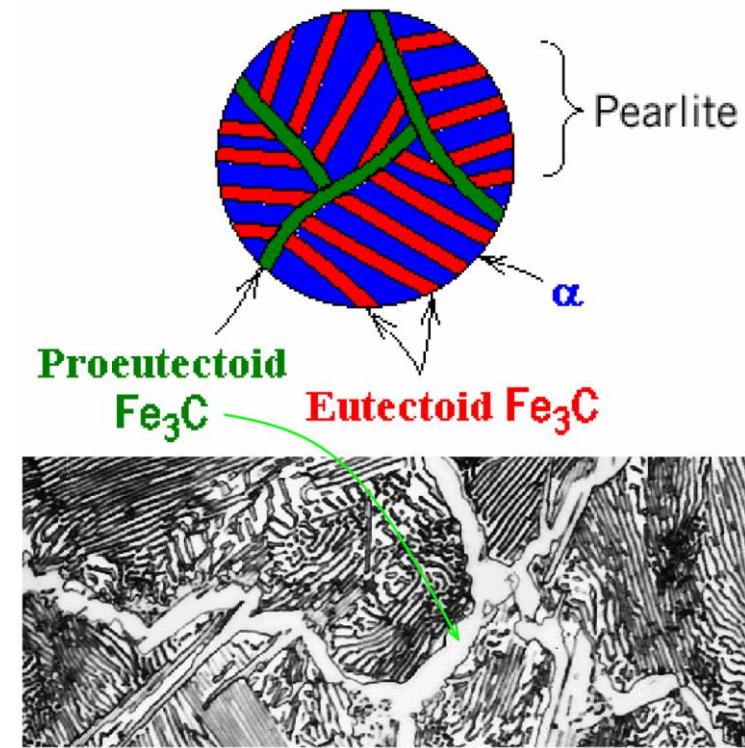


Microstructure of hypereutectoid steel



Compositions to the right of eutectoid (0.76 - 2.14 wt % C) **hypereutectoid** (*more than eutectoid* -Greek) alloys.

Hypereutectoid alloys contain proeutectoid cementite (formed above the eutectoid temperature) plus perlite that contain eutectoid ferrite and cementite.



Equilibrium Vacancy Concentration

Small : X_v^e (interactions v - v ignored)

$$\Delta H \approx \Delta H_v X_v$$

$$\Delta S \approx X_v \Delta S_v - R(X_v \ln X_v + (1-X_v) \ln(1-X_v))$$

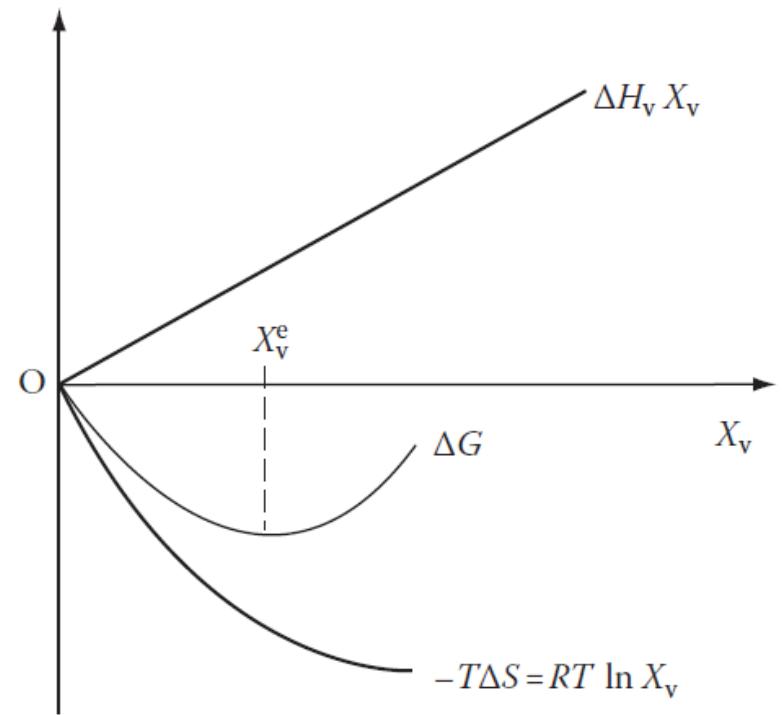
$$G \approx G_A + \Delta G = G_A + \Delta H - T\Delta S$$

$$\frac{dG}{dX_v} \Big|_{X_v^e} = 0$$

$$\Delta H_v - T\Delta S_v + RT \ln X_v^e = 0$$

$$X_v^e = \exp \frac{\Delta S_v}{R} \cdot \exp \frac{-\Delta H_v}{RT}$$

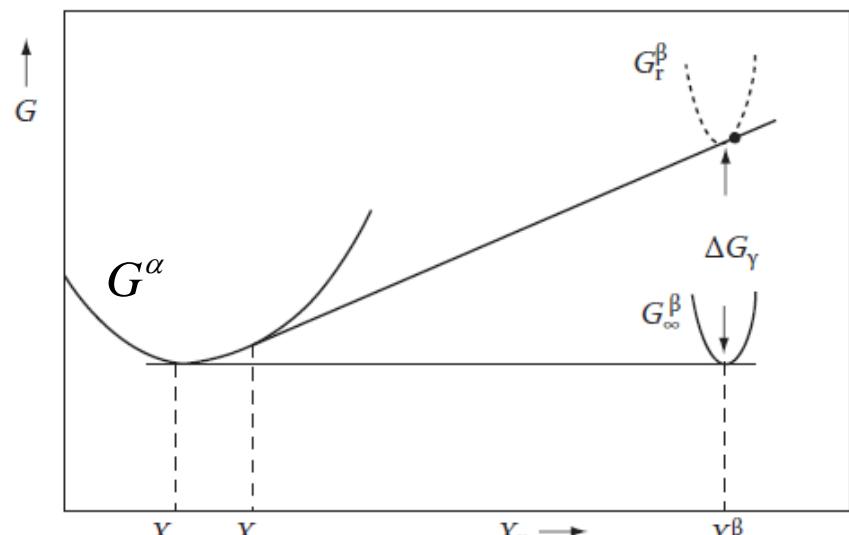
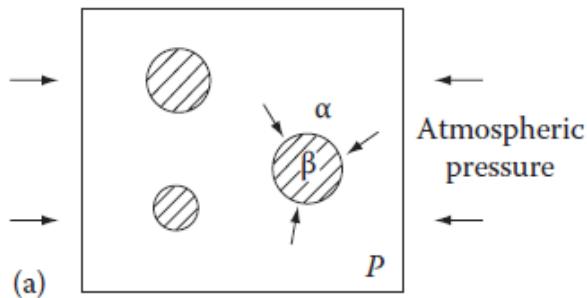
$$\sim 3 \quad \Delta H_v \approx 1 \text{ eV}$$



$$X_v^e \approx 10^{-3} \text{ or } 10^{-4}$$

for solids at the melting point

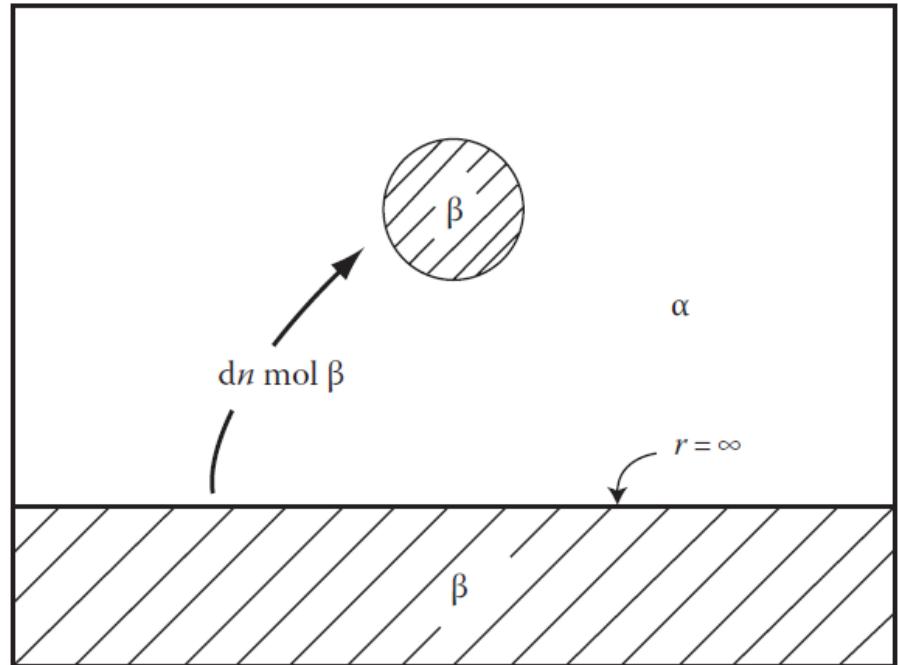
Influence of Interfaces on Equilibrium



(b)

The effect of interfacial energy on the solubility of small particles.

$$X_r = \exp\left(-\frac{\Delta G_B + \Omega - 2\gamma V_m / r}{RT}\right)$$



Transfer of dn mol of ↘ from large to a small particle

$$\Delta P = \frac{2\gamma}{r} \quad \Delta G = \Delta P \cdot V \quad \Delta G_\gamma = \frac{2\gamma V_m}{r}$$

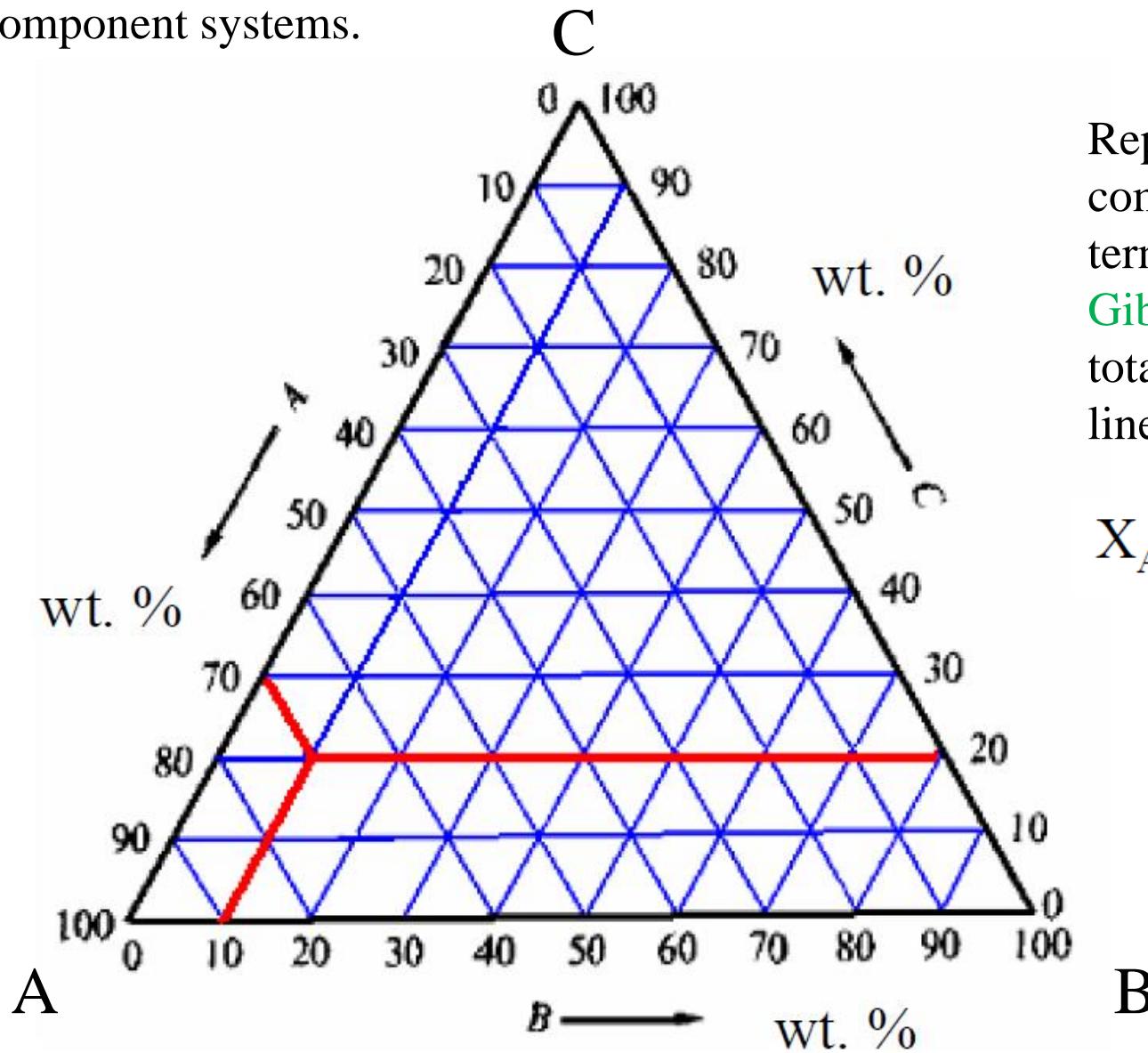
Gibbs-Thomson effect

$$X_r \approx X_\infty \left(1 + \frac{2\gamma V_m}{R T r}\right)$$

$$\frac{X_r}{X_\infty} \approx 1 + \frac{1}{r(\text{nm})}$$

Multicomponent systems (I)

The approach used for analysis of binary systems can be extended to multicomponent systems.



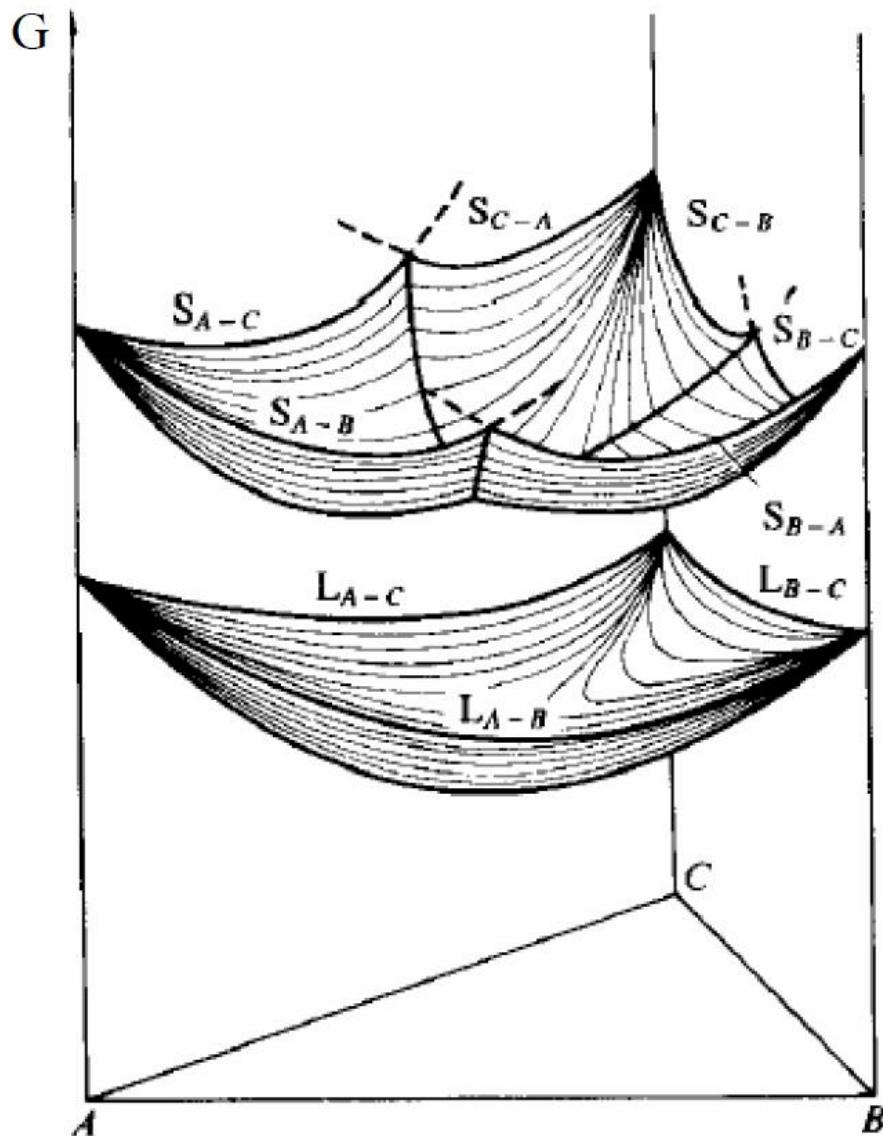
Representation of the composition in a ternary system (**the Gibbs triangle**). The total length of the red lines is 100% :

$$X_A + X_B + X_C = 1$$

Multicomponent systems (II)

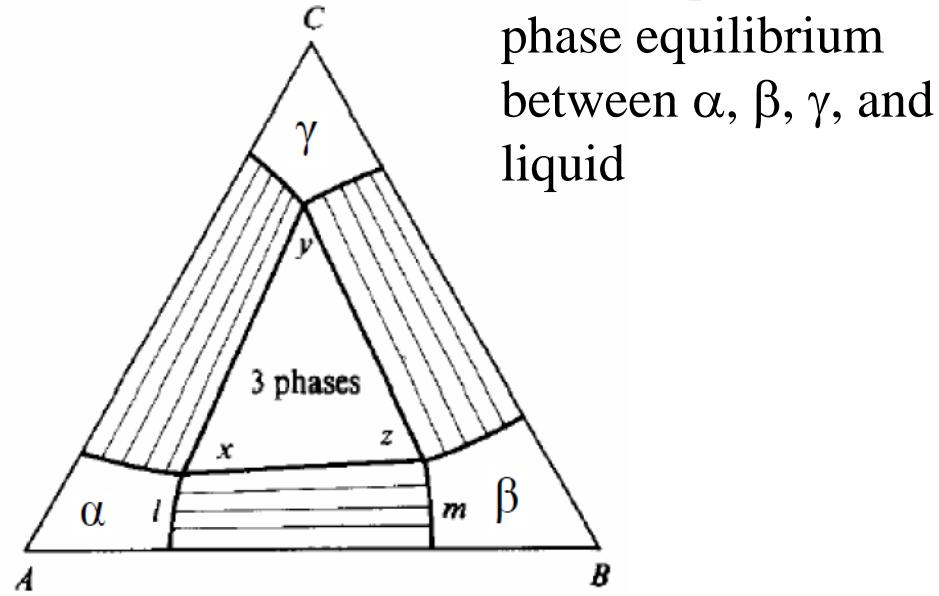
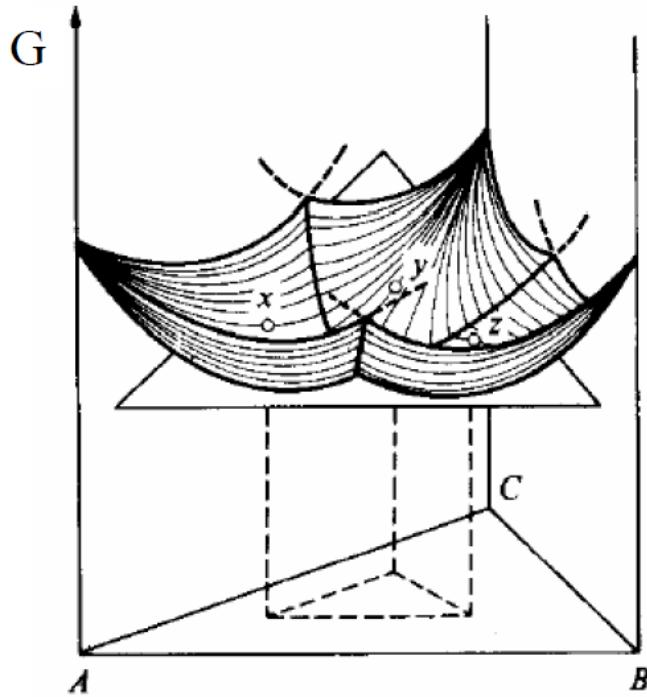
The *Gibbs free energy surfaces* (instead of curves for a binary system) can be plotted for all the possible phases and for different temperatures.

The chemical potentials of A, B, and C of any phase in this system are given by the points where the *tangential plane* to the free energy surfaces intersects the A, B, and C axis.



Multicomponent systems (III)

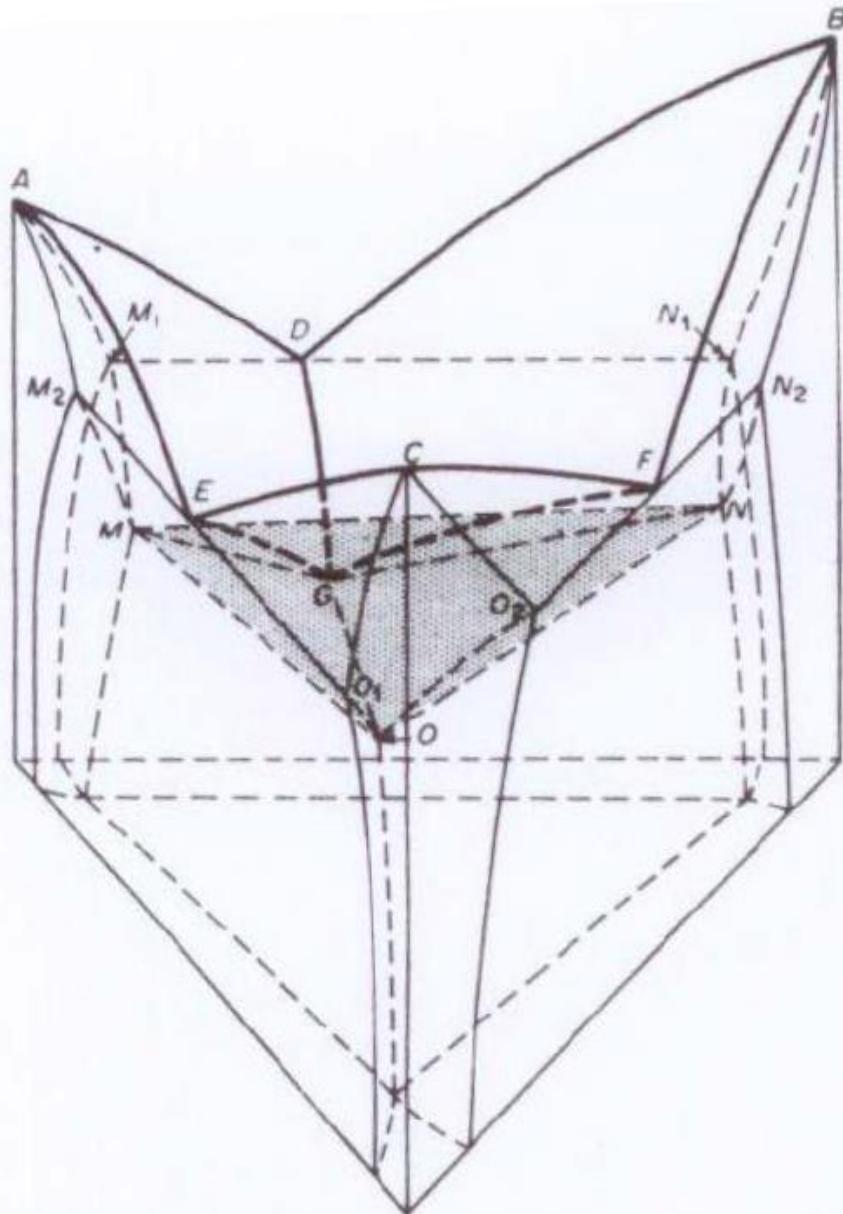
A three-phase equilibrium in the ternary system for a given temperature can be derived by means of the tangential plane construction.



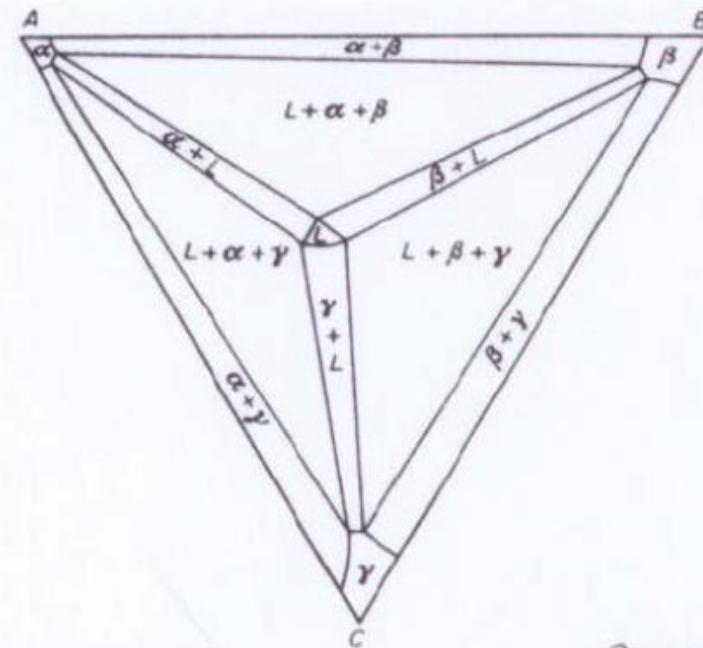
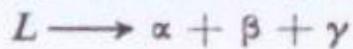
Eutectic point: four-phase equilibrium between α , β , γ , and liquid

For two phases to be in equilibrium, the chemical potentials should be equal, that is the compositions of the two phases in equilibrium must be given by points connected by a common tangential plane (e.g. l and m).

The relative amounts of phases are given by the lever rule (e.g. using tie-line l-m). A three phase triangle can result from a common tangential plane simultaneously touching the Gibbs free energies of three phases (e.g. points x, y, and z).



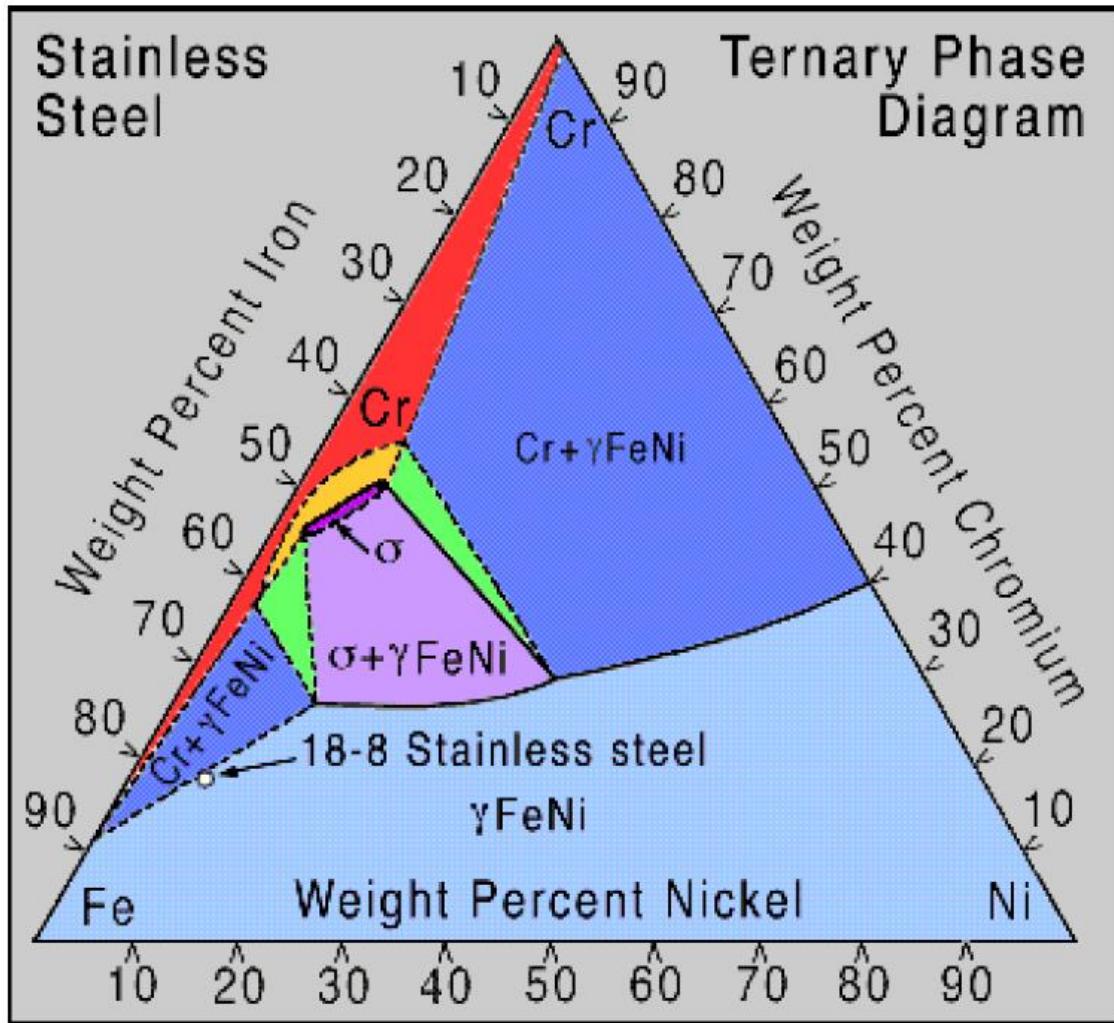
Space model of system showing a ternary eutectic reaction



(b)

Space diagram and isothermal sections

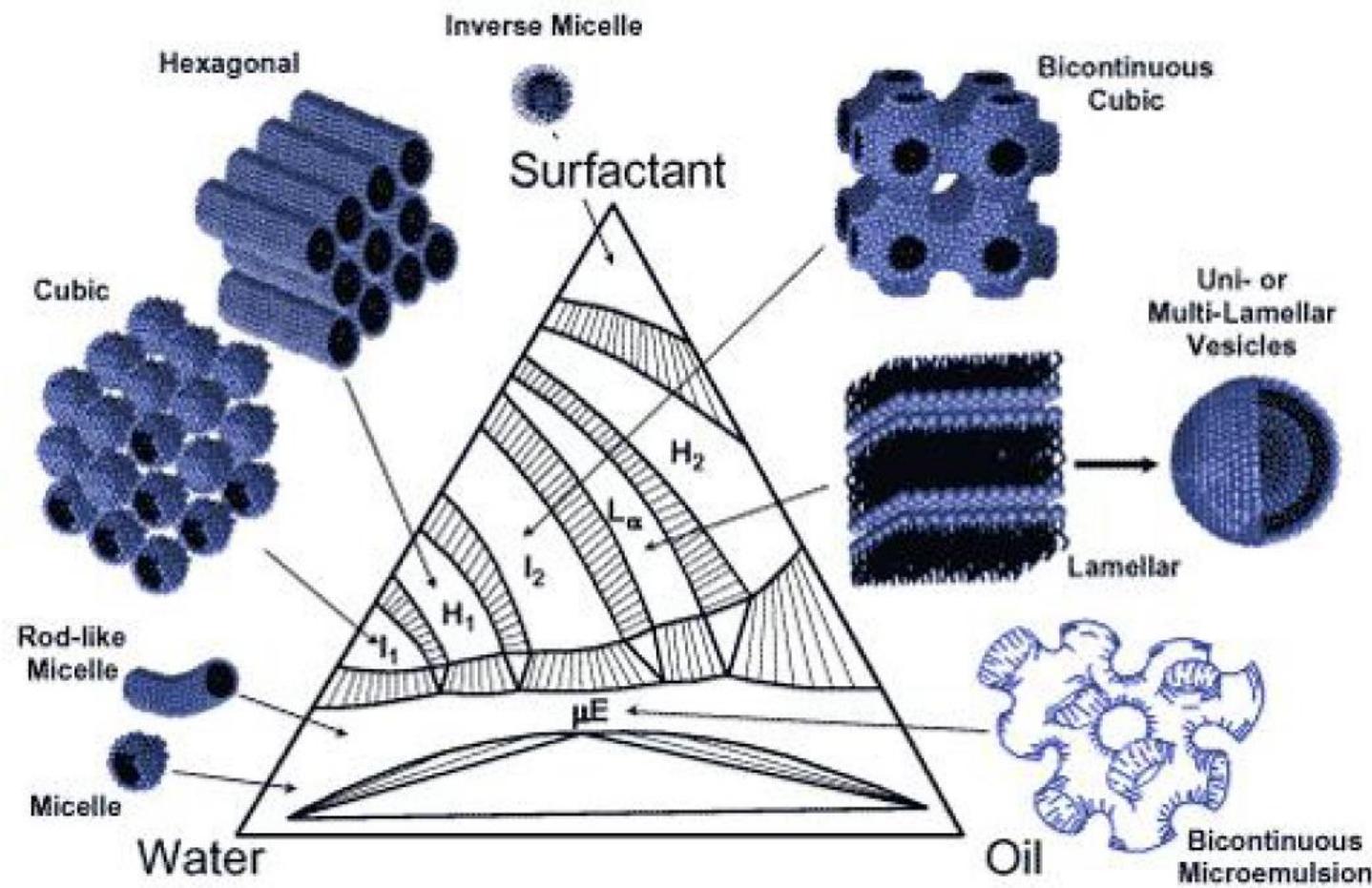
An example of ternary system



The ternary diagram of Ni-Cr-Fe. It includes Stainless Steel (wt.% of Cr > 11.5 %, wt.% of Fe > 50 %) and Inconel™ (Nickel based super alloys).

Inconel have very good corrosion resistance, but are more expensive and therefore used in corrosive environments where Stainless Steels are not sufficient (Piping on Nuclear Reactors or Steam Generators).

Another example of ternary phase diagram: oil – water – surfactant system



Drawing by Carlos Co, University of Cincinnati

Surfactants are surface-active molecules that can form interfaces between immiscible fluids (such as oil and water). A large number of structurally different phases can be formed, such as droplet, rod-like, and bicontinuous microemulsions, along with hexagonal, lamellar, and cubic liquid crystalline phases.