Vector Mechanics for Engineers: Dynamics

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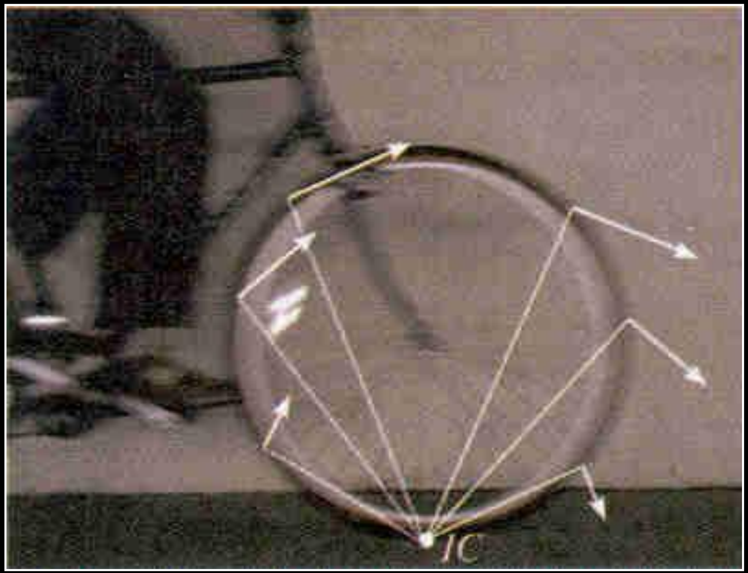
Textbook: *Vector Mechanics for Engineers: Dynamics,* Beer, Johnston, Mazurek and Cornwell, McGraw-Hill, 10th edition, 2012.

INSTANTANEOUS CENTER (IC) OF ZERO VELOCITY

Today's Objectives:

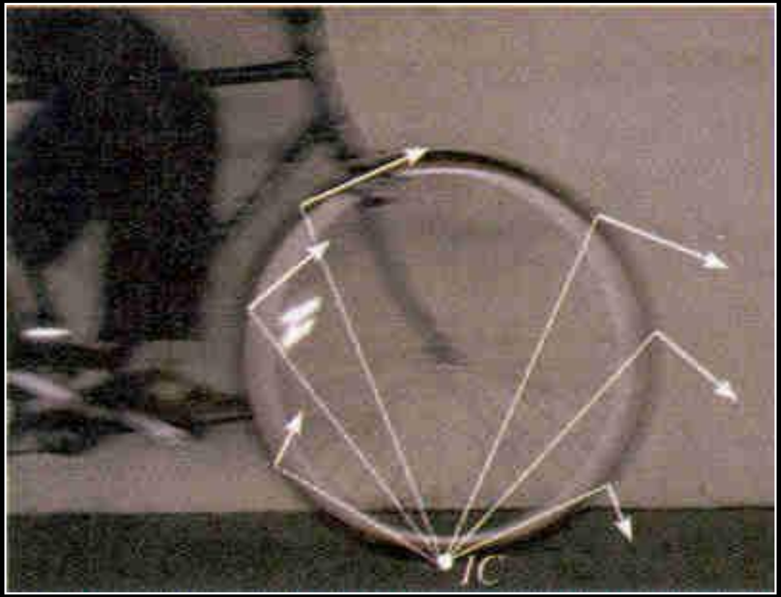
a) Locate the instantaneous center (IC) of zero velocity.
b) Use the IC to determine the velocity of any point on a rigid body in general plane motion.



In-Class Activities:

- Reading quiz
- Applications
- Location of the IC
- Velocity analysis
- Concept quiz
- Group problem solving
- Attention quiz

APPLICATIONS

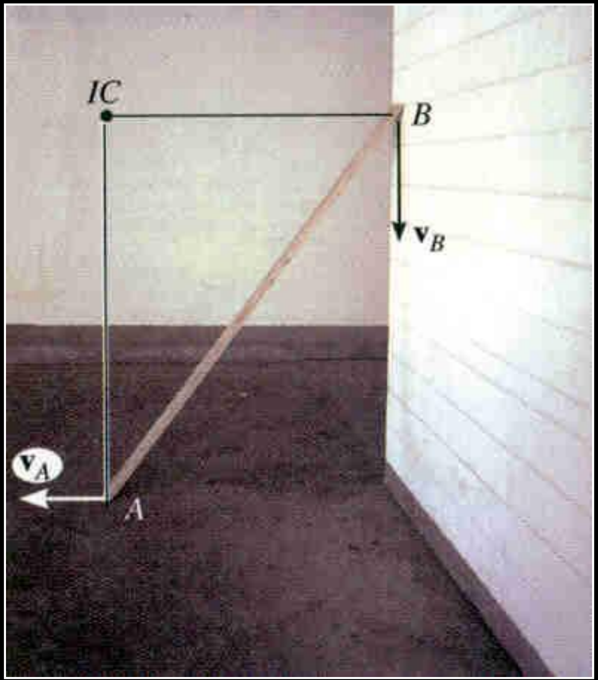


The instantaneous center of zero velocity for this bicycle wheel is at the point in contact with ground.

The velocity direction at any point on the rim is perpendicular to the line connecting the point to the IC.

Which point on the wheel has the maximum velocity?

APPLICATIONS (continued)



As the board slides down the wall (to the left) it is subjected to general plane motion (both translation and rotation).

Since the directions of the velocities of ends A and B are known, the IC is located as shown.

What is the direction of the velocity of the center of gravity of the board?

INSTANTANEOUS CENTER OF ZERO VELOCITY

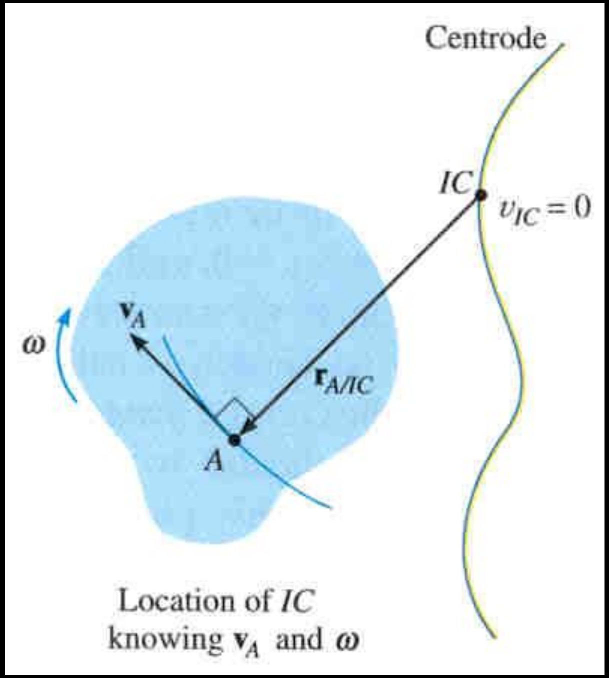
For any body undergoing planar motion, there always exists a point in the plane of motion at which the velocity is instantaneously zero (if it were rigidly connected to the body).

This point is called the instantaneous center of zero velocity, or IC. It may or may not lie on the body!

If the location of this point can be determined, the velocity analysis can be simplified because the body appears to rotate about this point at that instant.

LOCATION OF THE INSTANTANEOUS CENTER

To locate the IC, we can use the fact that the velocity of a point on a body is always perpendicular to the relative position vector from the IC to the point. Several possibilities exist:



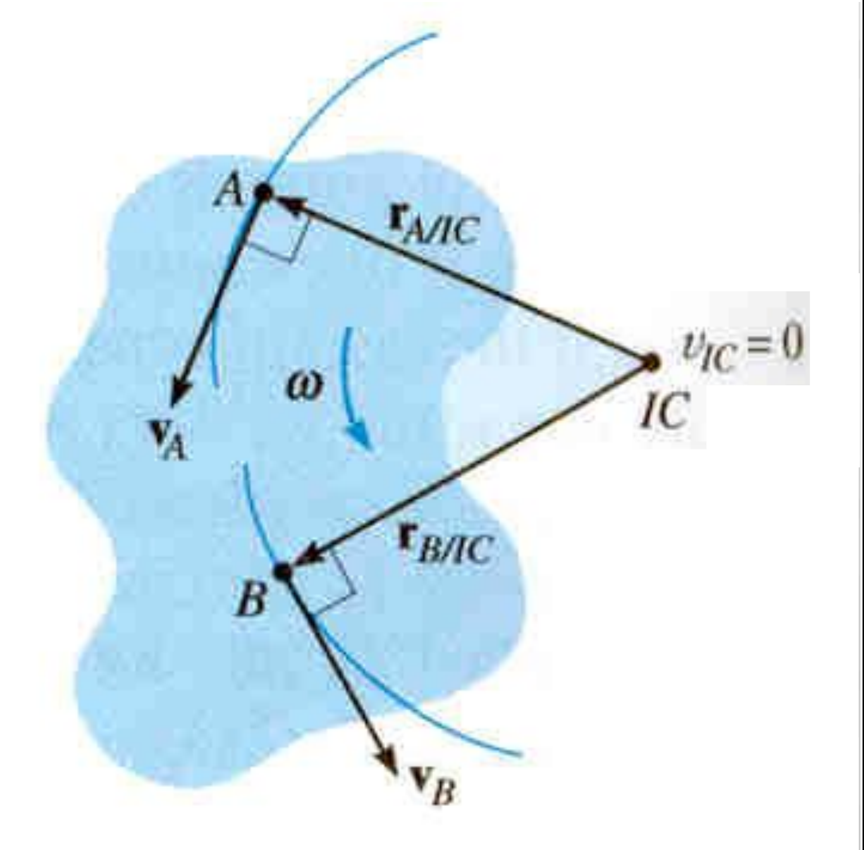
First, consider the case when velocity $\boldsymbol{v}_A$ of a point A on the body and the angular velocity $\boldsymbol{\omega}$ of the body are known: $\boldsymbol{v}_A = \boldsymbol{\omega} \times \boldsymbol{r}_{A/IC}$.

In this case, the IC is located along the line drawn perpendicular to $\boldsymbol{v}_A$ at A, a distance $r_{A/IC} = v_A/\omega$ from A.

Note that the IC lies up and to the right of A since $\boldsymbol{v}_A$ must cause a clockwise angular velocity $\boldsymbol{\omega}$ about the IC.

LOCATION OF THE INSTANTANEOUS CENTER

(continued)



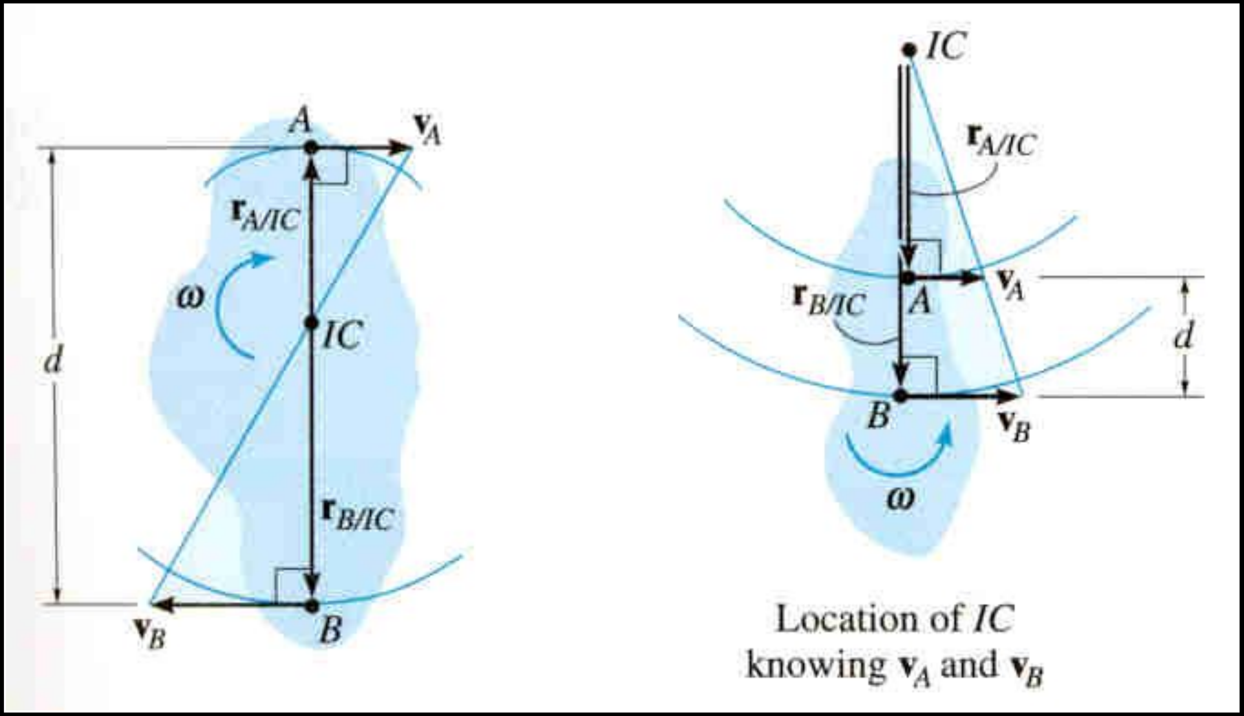
Location of *IC* knowing the lines of action of $\mathbf{v}_A$ and $\mathbf{v}_B$

A second case is when the lines of action of two non-parallel velocities, $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$, are known.

First, construct line segments from A and B perpendicular to $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$.

The point of intersection of these two line segments locates the IC of the body.

LOCATION OF THE INSTANTANEOUS CENTER



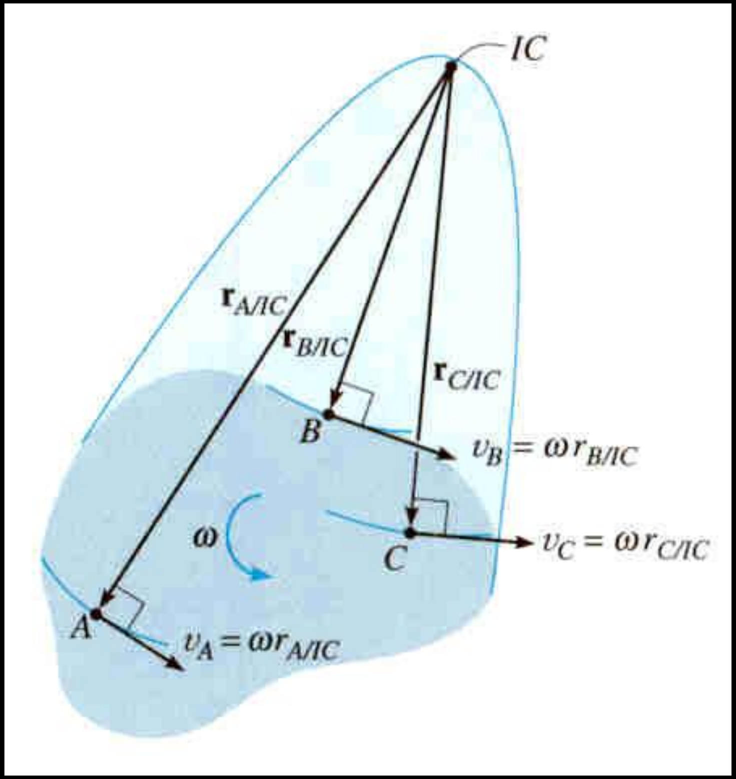
Location of *IC* knowing $\mathbf{v}_A$ and $\mathbf{v}_B$

A third case is when the magnitude and direction of two parallel velocities at A and B are known.

Here the location of the IC is determined by proportional triangles. As a special case, note that if the body is translating only ($\boldsymbol{v}_A = \boldsymbol{v}_B$), then the IC would be located at infinity. Then ω equals zero, as expected.

VELOCITY ANALYSIS

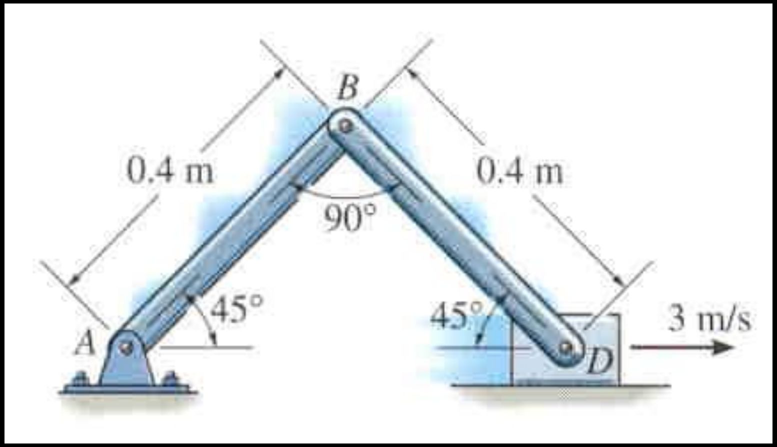
The velocity of any point on a body undergoing general plane motion can be determined easily once the instantaneous center (IC) of zero velocity of the body is located.



Since the body seems to rotate about the IC at any instant, as shown in this kinematic diagram, the magnitude of velocity of any arbitrary point is $v = \omega r$, where r is the radial distance from the IC to the point. The velocity's line of action is perpendicular to its associated radial line. Note the velocity has a sense of direction which tends to move the point in a manner consistent with the angular rotation direction.

This is much easier than relate velocities at A and C!

EXAMPLE 1

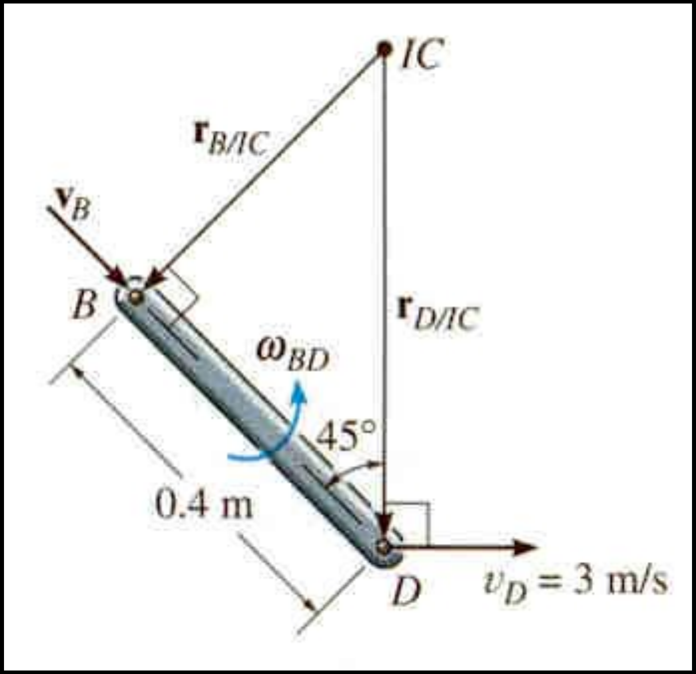


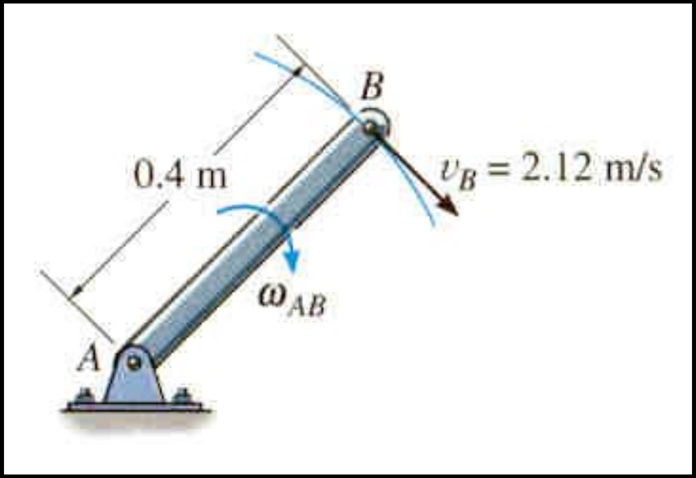
Given: A linkage undergoing motion as shown. The velocity of the block, $v_D = 3$ m/s.

Find: The angular velocities of links AB and BD.

Plan: Locate the instantaneous center of zero velocity of link BD.

Solution: Since D moves to the right, it causes link AB to rotate clockwise about point A. The instantaneous center of velocity for BD is located at the intersection of the line segments drawn perpendicular to $\boldsymbol{v}_B$ and $\boldsymbol{v}_D$. Note that $\boldsymbol{v}_B$ is perpendicular to link AB. Therefore we can see that the IC is located along the extension of link AB.





EXAMPLE 1 (continued)

Using these facts,

$r_{B/IC} = 0.4 \tan 45° = 0.4$ m
$r_{D/IC} = 0.4/\cos 45° = 0.566$ m

Since the magnitude of v_D is known, the angular velocity of <u>link BD</u> can be found from $v_D = \omega_{BD}\, r_{D/IC}$.

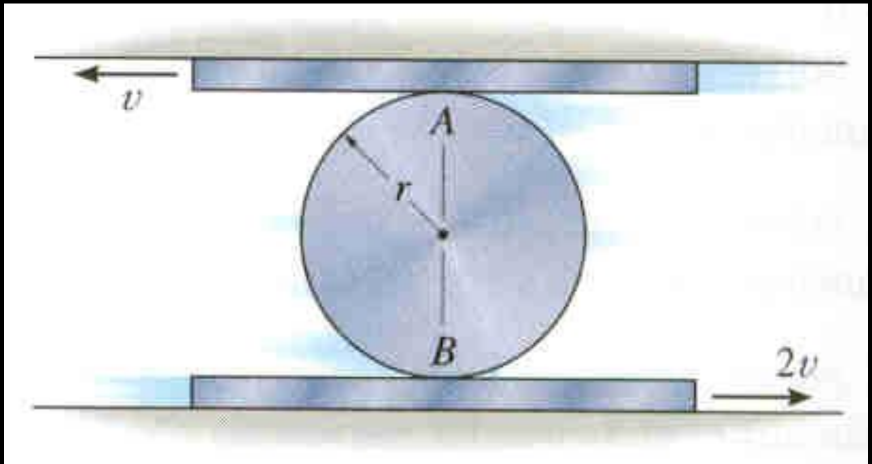
$$\omega_{BD} = v_D/r_{D/IC} = 3/0.566 = 5.3 \text{ rad/s}$$

$$v_B = \omega_{BD}\, r_{B/IC}$$

<u>Link AB</u> is subjected to rotation about A.

$$\omega_{AB} = v_B/r_{B/A} = (r_{B/IC})\omega_{BD}/r_{B/A} = 0.4(5.3)/0.4 = 5.3 \text{ rad/s}$$

EXAMPLE 2



Given: The disk rolls without slipping between two moving plates.

$v_B = 2v \longrightarrow$

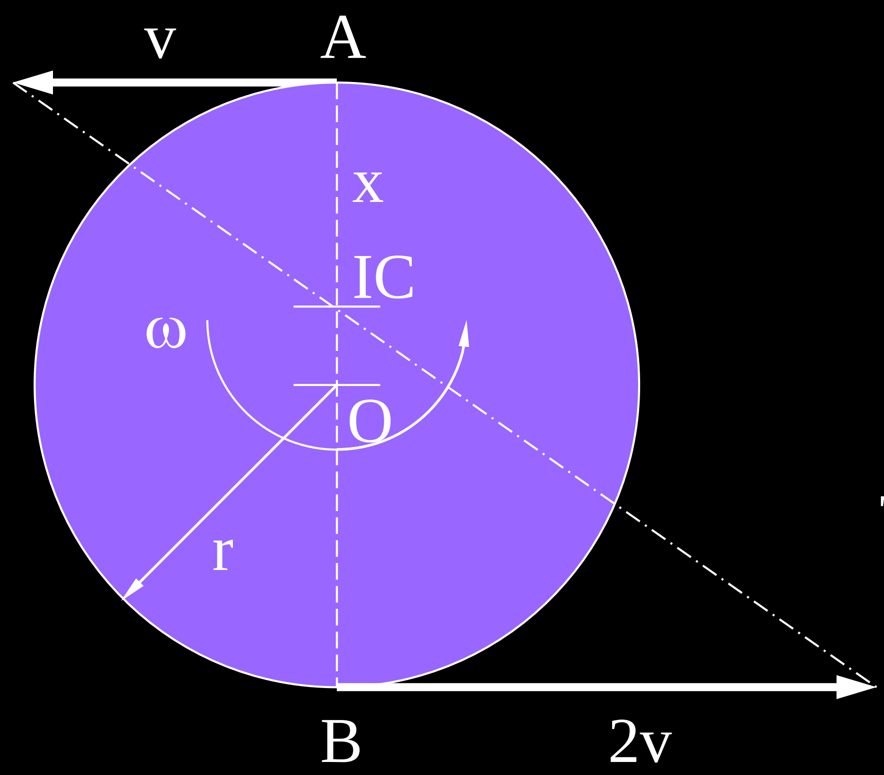
$v_A = v \longleftarrow$

Find: The angular velocity of the disk.

Plan: This is an example of the third case discussed in the lecture notes. Locate the IC of the disk using geometry and trigonometry. Then calculate the angular velocity.

EXAMPLE 2 (continued)

Solution:



Using similar triangles:

$$x/v = (2r-x)/(2v)$$

$$\text{or } x = (2/3)r$$

$$\text{Therefore } \omega = v/x = 1.5(v/r)$$

CONCEPT QUIZ

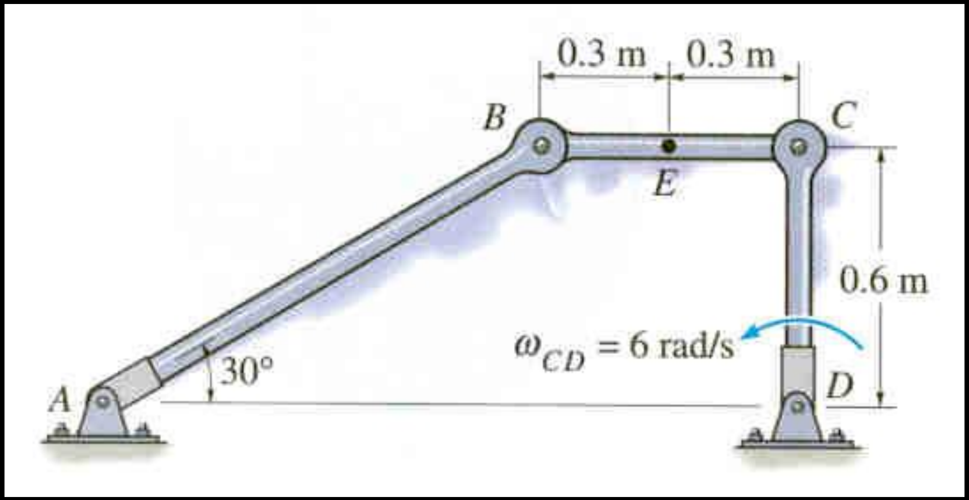
1. When the velocities of two points on a body are equal in magnitude and parallel but in opposite directions, the IC is located at

   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.

2. When the direction of velocities of two points on a body are perpendicular to each other, the IC is located at

   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.

PROBLEM

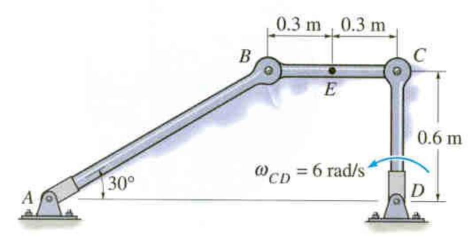


Given: The four bar linkage is moving with ω_{CD} equal to 6 rad/s CCW.

Find: The velocity of point E on link BC and angular velocity of link AB.

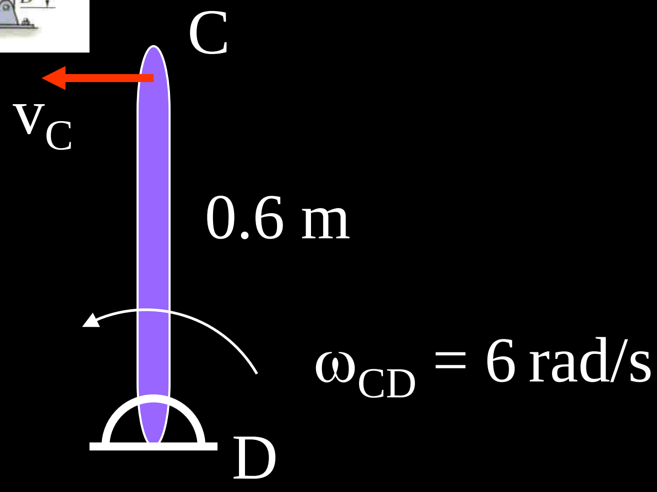
Plan: This is an example of the second case in the lecture notes.

Since the direction of Point B's velocity must be perpendicular to AB and Point C's velocity must be perpendicular to CD, the location of the instantaneous center, I, for link BC can be found.



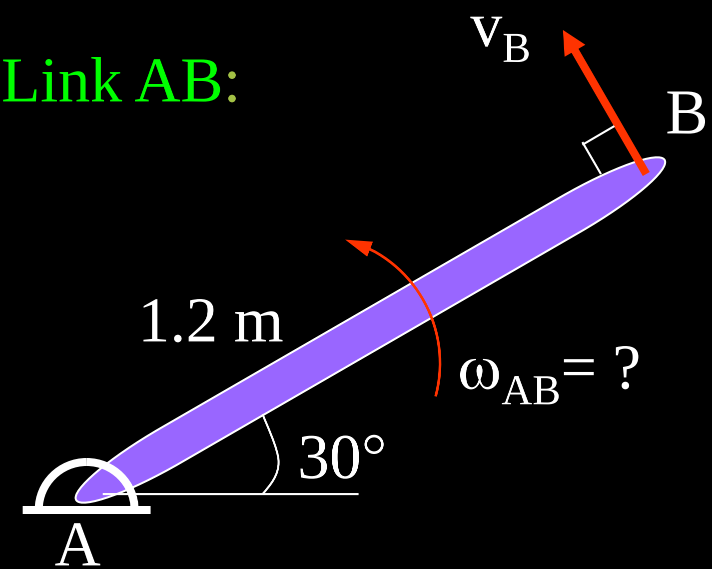
PROBLEM (continued)

Link CD:

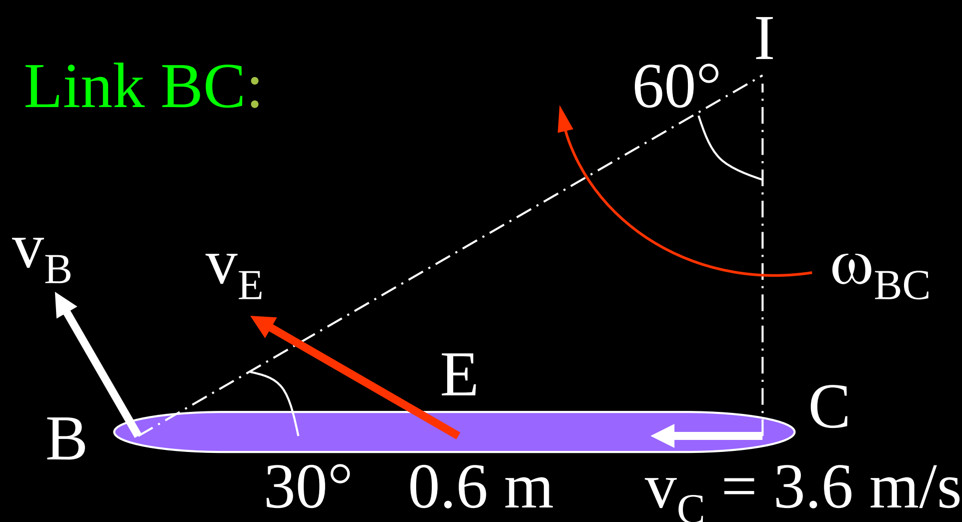


$v_C = 0.6(6) = 3.6$ m/s

Link AB:



Link BC:



From triangle CBI:

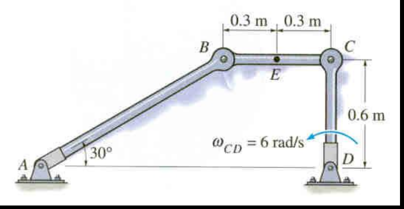
$IC = 0.6 \tan 60° = 0.346$ m

$IB = 0.6/\sin 60° = 0.693$ m

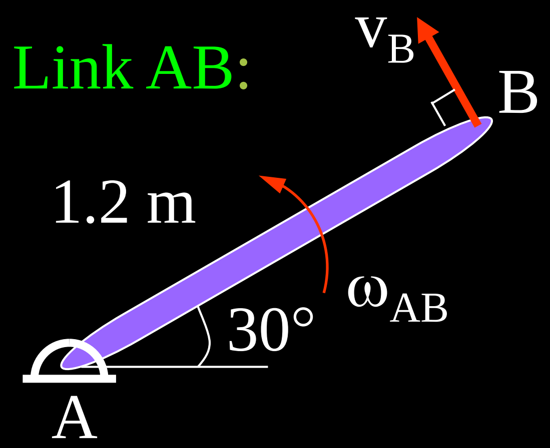
$v_C = (IC)\,\omega_{BC}$

$\omega_{BC} = v_C/IC = 3.6/0.346$

$\omega_{BC} = 10.39$ rad/s



PROBLEM (continued)



v_B = (IB) ω_{BC} = 0.693 (10.39) = 7.2 m/s

From link AB, v_B is also equal to 1.2 ω_{AB}.

Therefore, 7.2 = 1.2 ω_{AB} => ω_{AB} = 6 rad/s

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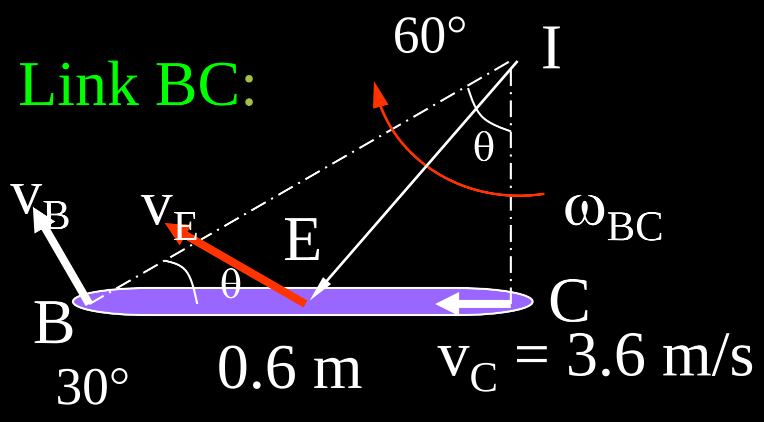
v_E = (IE) ω_{BC} where distance IE = $\sqrt{0.3^2 + 0.346^2}$ = 0.458 m

v_E = 0.458 (10.39) = 4.76 m/s θ

where θ = $\tan^{-1}$(0.3/0.346) = 40.9°

from triangle IEC

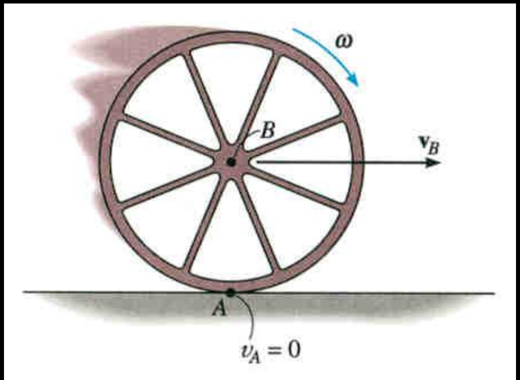


ATTENTION QUIZ

1. The wheel shown has a radius of 15 in and rotates clockwise at a rate of ω = 3 rad/s. What is v_B?

   A) 5 in/s　　B) 15 in/s

   C) 0 in/s　　D) 45 in/s



2. Point A on the rod has a velocity of 8 m/s to the right. Where is the IC for the rod?

   A) Point A.

   B) Point B.

   C) Point C.

   D) Point D.