Vector Mechanics for Engineers: Dynamics

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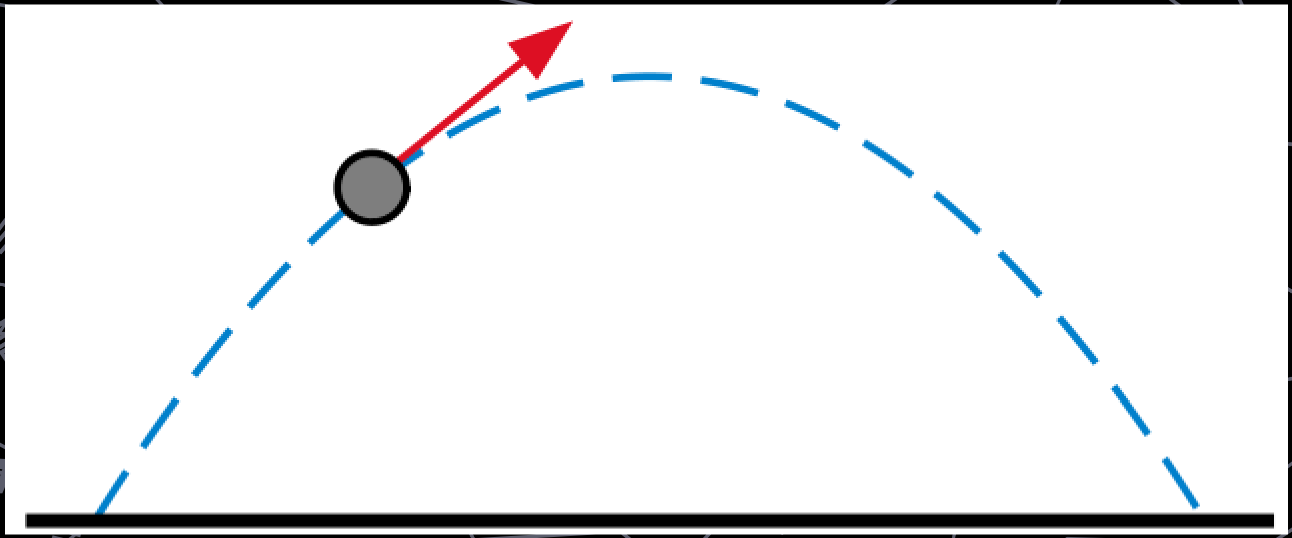
Textbook: *Vector Mechanics for Engineers: Dynamics,* Beer, Johnston, Mazurek and Cornwell, McGraw-Hill, 10th edition, 2012.

Kinematics: concerned with the geometric aspects of motion (do not take into account forces or moments)

Goal: relations between s(t), v(t), and a(t) for general motion



SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

- Differentiate position to get velocity and acceleration:

$v = ds/dt$ (graph) $a = dv/dt$ or $a = dv/ds \cdot ds/dt = v\,dv/ds$

- Integrate acceleration for velocity and position:

Velocity:

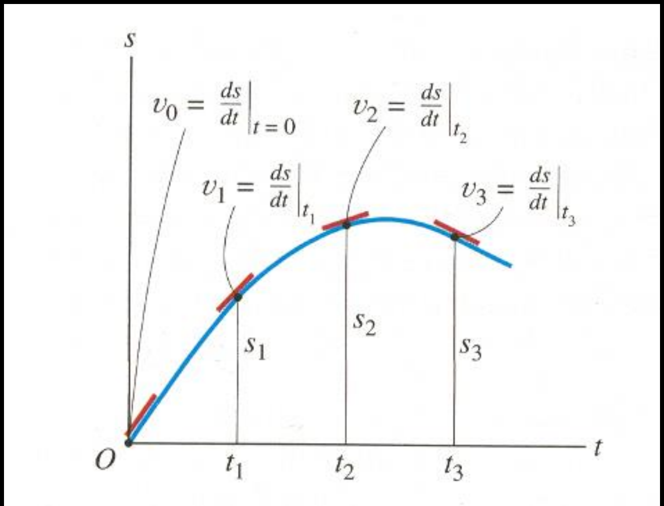
$$\int_{v_o}^{v} dv = \int_{o}^{t} a(t)\,dt \quad \text{or} \quad \int_{v_o}^{v} v\,dv = \int_{s_o}^{s} a(s)\,ds$$

Position:

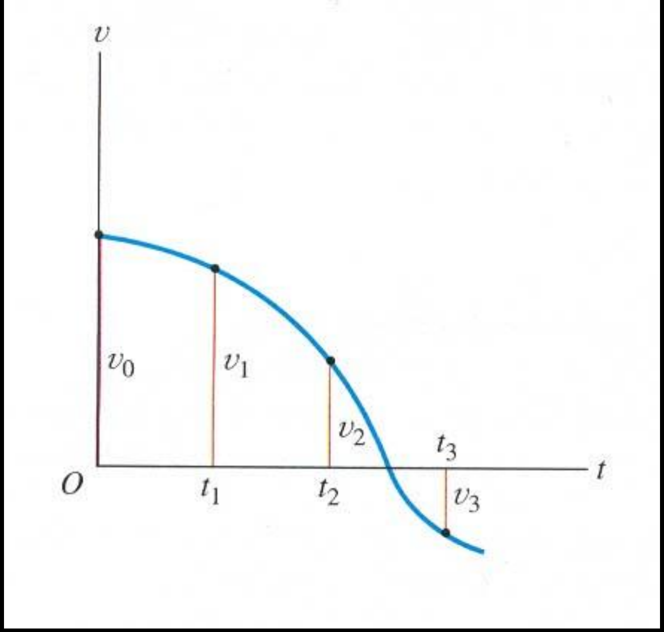
$$\int_{s_o}^{s} ds = \int_{o}^{t} v(t)\,dt$$

- Note that s_o and v_o represent the initial position and velocity of the particle at $t = 0$.

S-T GRAPH



Plots of position vs. time can be used to find velocity vs. time curves. Finding the slope of the line tangent to the motion curve at any point is the velocity at that point (or v = ds/dt).



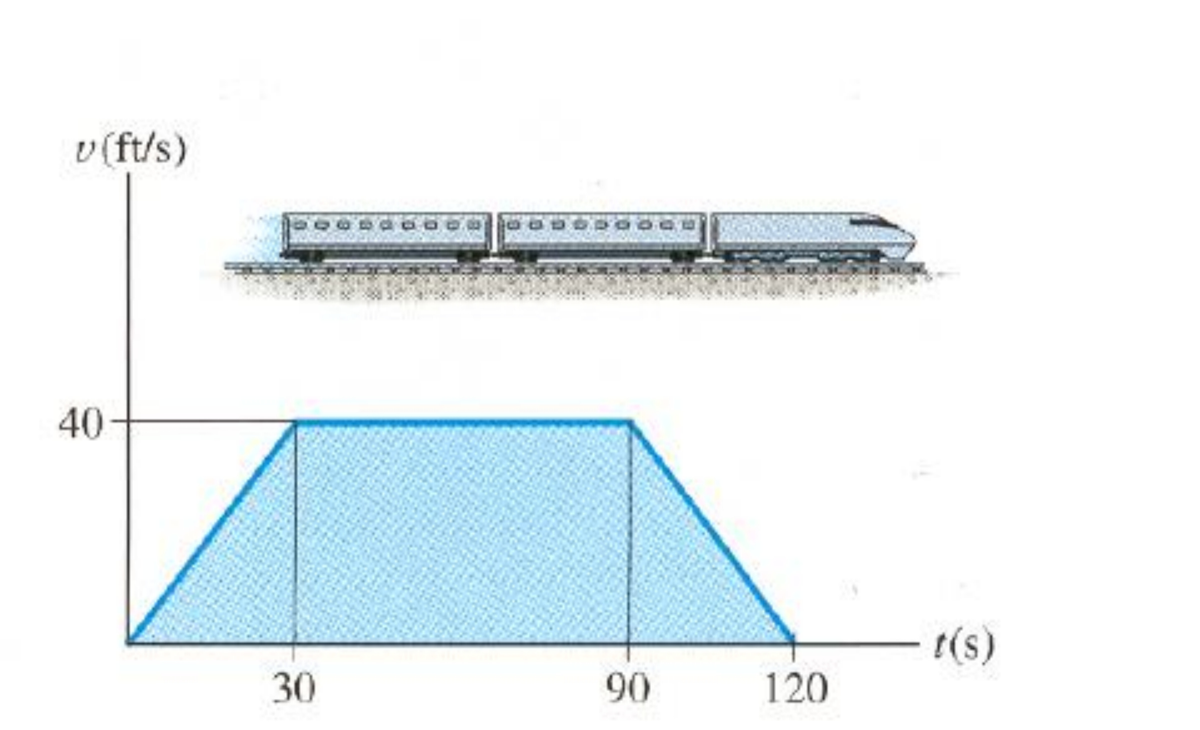
Therefore, the v-t graph can be constructed by finding the slope at various points along the s-t graph.

What about acceleration? a=dv/dt

EXAMPLE

Given: v-t graph for a train moving between two stations

Find: a-t graph and s-t graph over this time interval



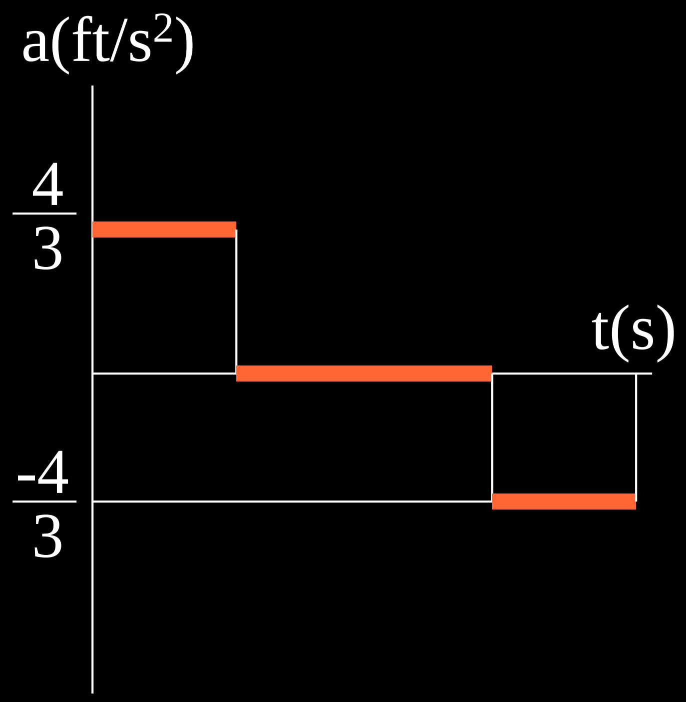
Think about your plan to solve the problem!

EXAMPLE (continued)

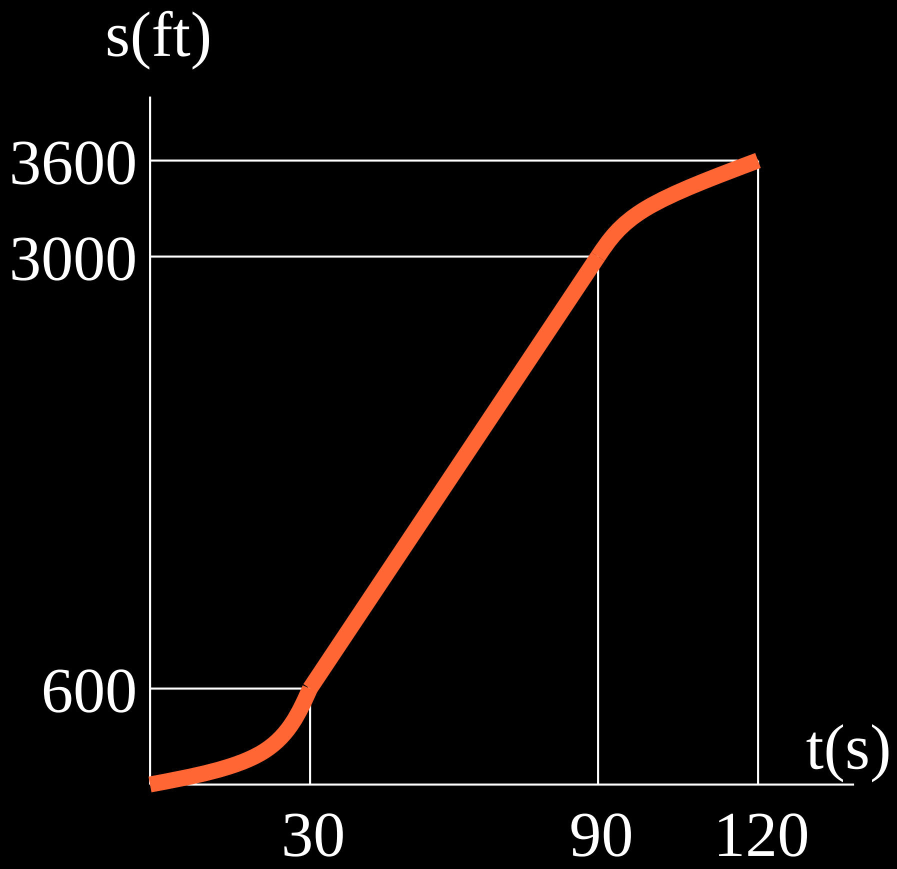
Solution: For the first 30 seconds the slope is constant and is equal to:

$$a_{0\text{-}30} = dv/dt = 40/30 = 4/3 \ ft/s^2$$

Similarly, $a_{30\text{-}90} = 0$ and $a_{90\text{-}120} = -4/3 \ ft/s^2$



EXAMPLE (continued)



The area under the v-t graph represents displacement.

$\Delta s_{0\text{-}30} = \frac{1}{2}(40)(30) = 600$ ft

$\Delta s_{30\text{-}90} = (60)(40) = 2400$ ft

$\Delta s_{90\text{-}120} = \frac{1}{2}(40)(30) = 600$ ft

CURVILINEAR MOTION: RECTANGULAR COMPONENTS

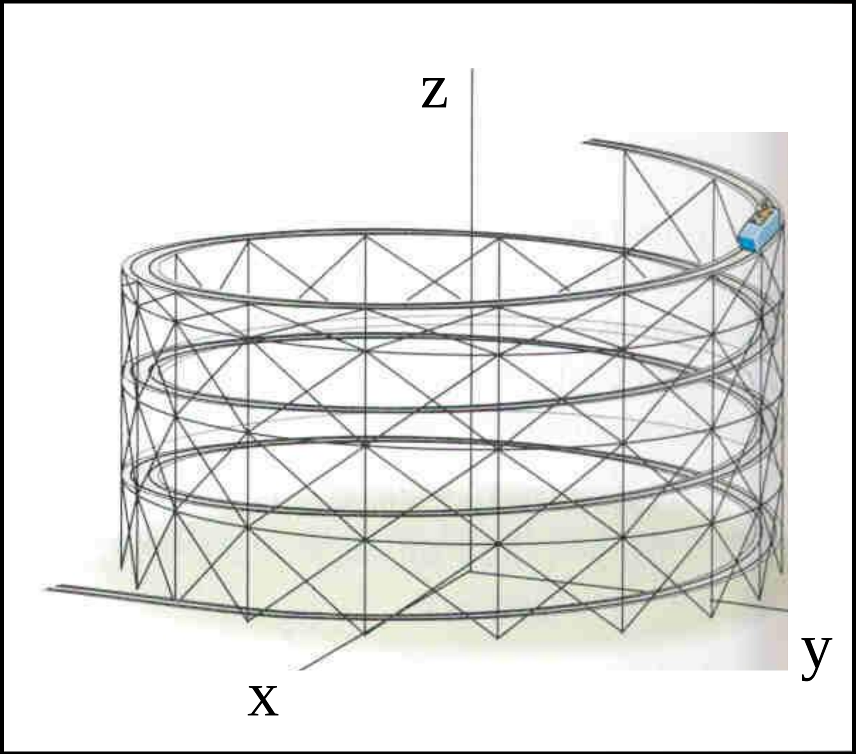
Today's Objectives:

a) Describe the motion of a particle traveling along a curved path.
b) Relate kinematic quantities in terms of the rectangular components of the vectors.

In-Class Activities:

- Reading quiz
- Applications
- General curvilinear motion
- Rectangular components of kinematic vectors
- Concept quiz
- Group problem solving
- Attention quiz

APPLICATIONS



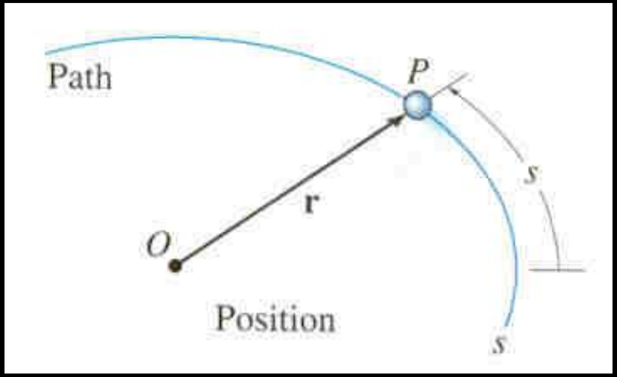
A roller coaster car travels down a fixed, helical path at a constant speed.

How can we determine its position or acceleration at any instant?

If you are designing the track, why is it important to be able to predict the acceleration of the car?

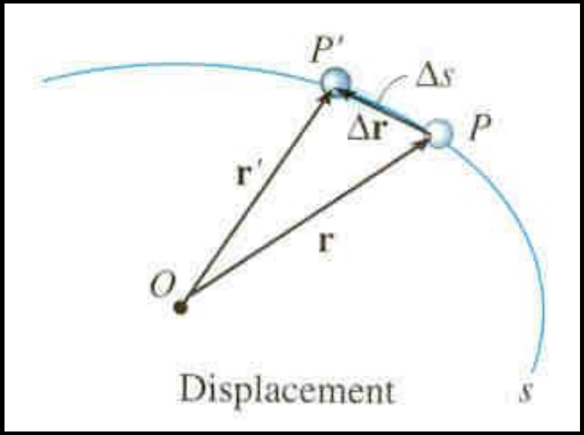
POSITION AND DISPLACEMENT

A particle moving along a curved path undergoes curvilinear motion. Since the motion is often three-dimensional, vectors are used to describe the motion.



A particle moves along a curve defined by the path function, s.

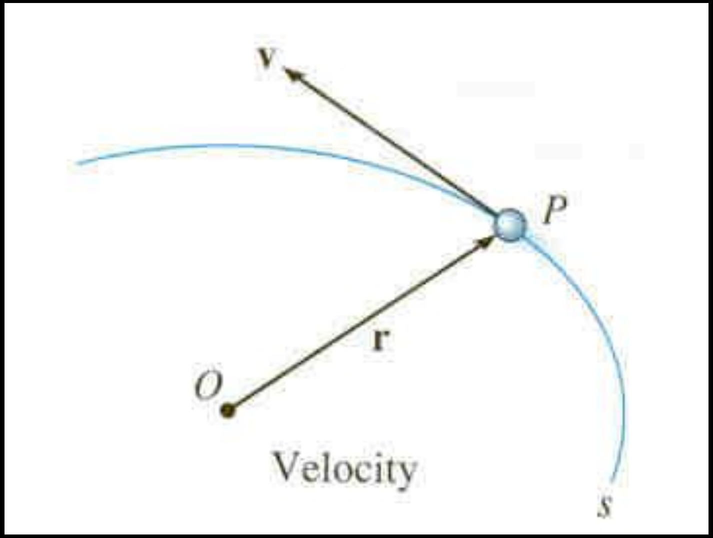
The position of the particle at any instant is designated by the vector $\boldsymbol{r} = \boldsymbol{r}(t)$. Both the magnitude and direction of $\boldsymbol{r}$ may vary with time.



If the particle moves a distance Δs along the curve during time interval Δt, the displacement is determined by vector subtraction: $\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$

VELOCITY

Velocity represents the rate of change in the position of a particle.



The average velocity of the particle during the time increment Δt is $\boldsymbol{v}_{avg} = \Delta\boldsymbol{r}/\Delta t$.

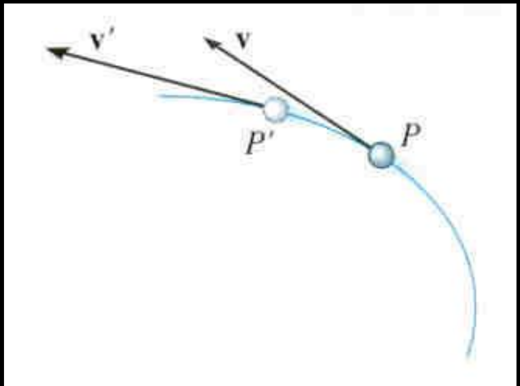
The instantaneous velocity is the time-derivative of position $\boldsymbol{v} = d\boldsymbol{r}/dt$.

The velocity vector, $\boldsymbol{v}$, is always tangent to the path of motion.

The magnitude of $\boldsymbol{v}$ is called the speed. Since the arc length Δs approaches the magnitude of $\Delta\boldsymbol{r}$ as $t \rightarrow 0$, the speed can be obtained by differentiating the path function ($v = ds/dt$). Note that this is not a vector!

ACCELERATION

Acceleration represents the rate of change in the velocity of a particle.

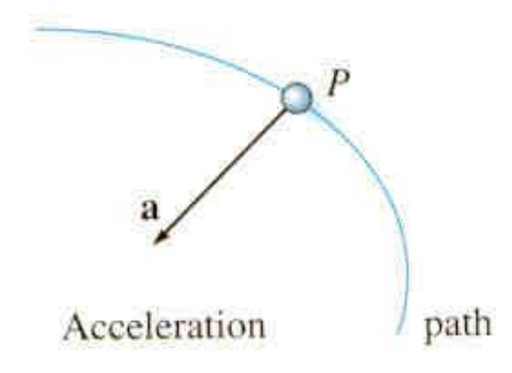


If a particle's velocity changes from $\boldsymbol{v}$ to $\boldsymbol{v'}$ over a time increment Δt, the average acceleration during that increment is:

$$\boldsymbol{a}_{avg} = \Delta \boldsymbol{v} / \Delta t = (\boldsymbol{v'} - \boldsymbol{v}) / \Delta t$$

The instantaneous acceleration is the time-derivative of velocity:

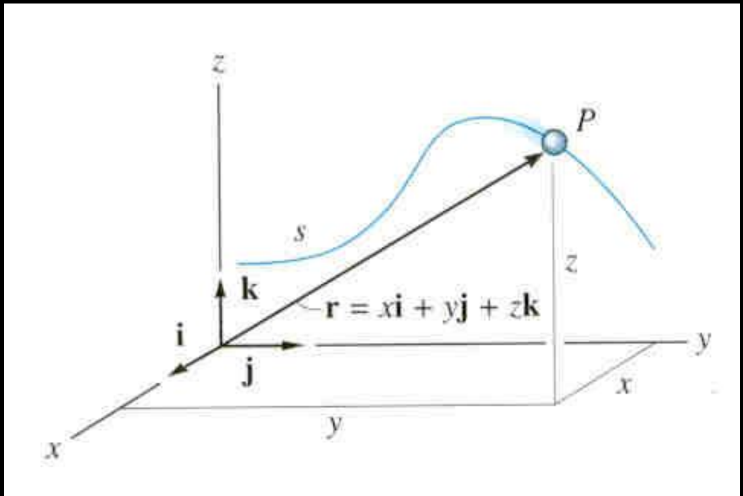
$$\boldsymbol{a} = d\boldsymbol{v}/dt = d^2\boldsymbol{r}/dt^2$$



<u>The acceleration vector is NOT, in general, tangent to the path function</u>!

RECTANGULAR COMPONENTS: POSITION

It is often convenient to describe the motion of a particle in terms of its x, y, z or rectangular components, relative to a fixed frame of reference.



Position

The position of the particle can be defined at any instant by the position vector

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$

The x, y, z components may all be functions of time, i.e.,

$x = x(t)$, $y = y(t)$, and $z = z(t)$

The magnitude of the position vector is: $r = (x^2 + y^2 + z^2)^{0.5}$

The direction of $\boldsymbol{r}$ is defined by the unit vector: $\boldsymbol{u_r} = (1/r)\,\boldsymbol{r}$

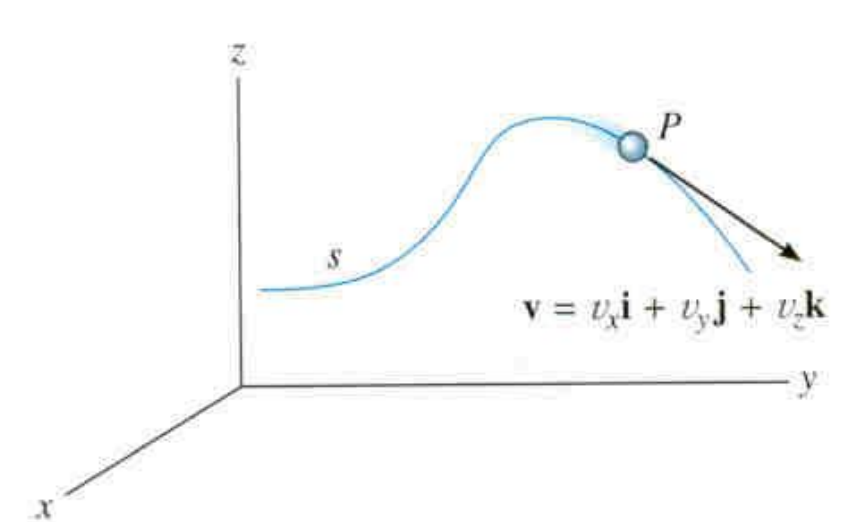
RECTANGULAR COMPONENTS: VELOCITY

The velocity vector is the time derivative of the position vector:

$$\boldsymbol{v} = d\boldsymbol{r}/dt = d(x\boldsymbol{i})/dt + d(y\boldsymbol{j})/dt + d(z\boldsymbol{k})/dt$$

Since the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are constant in magnitude and direction, this equation reduces to $\boldsymbol{v} = v_x\boldsymbol{i} + v_y\boldsymbol{j} + v_z\boldsymbol{k}$

where $v_x = \dot{x} = dx/dt$, $v_y = \dot{y} = dy/dt$, $v_z = \dot{z} = dz/dt$



Velocity

The magnitude of the velocity vector is

$$v = [(v_x)^2 + (v_y)^2 + (v_z)^2]^{0.5}$$

The direction of $\boldsymbol{v}$ is tangent to the path of motion!

RECTANGULAR COMPONENTS: ACCELERATION

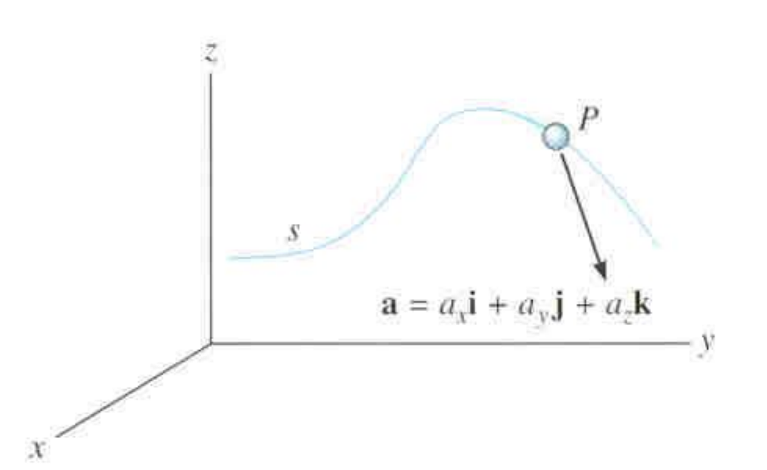
The acceleration vector is the time derivative of the velocity vector (second derivative of the position vector):

$$\boldsymbol{a} = d\boldsymbol{v}/dt = d^2\boldsymbol{r}/dt^2 = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$$

where $a_x = \dot{v}_x = \ddot{x} = dv_x/dt$, $a_y = \dot{v}_y = \ddot{y} = dv_y/dt$, $a_z = \dot{v}_z = \ddot{z} = dv_z/dt$

The magnitude of the acceleration vector is

$$a = [(a_x)^2 + (a_y)^2 + (a_z)^2]^{0.5}$$



Acceleration

The direction of $\boldsymbol{a}$ is usually NOT tangent to the path of the particle.

CONCEPT QUIZ

1. If the position of a particle is defined by

   r = [$(1.5t^2 + 1)$ ***i*** + $(4t - 1)$ ***j***] (m), its speed at t = 1 s is

   A) 2 m/s B) 3 m/s

   C) 5 m/s D) 7 m/s

EXAMPLE

Given: The motion of two particles (A and B) is described by the position vectors

$\boldsymbol{r_A} = [\ 3t\ \boldsymbol{i} + 9t(2 - t)\ \boldsymbol{j}\ ]$ m

$\boldsymbol{r_B} = [\ 3(t^2 - 2t + 2)\ \boldsymbol{i} + 3(t - 2)\ \boldsymbol{j}\ ]$ m

Find: The point at which the particles collide and their speeds just before the collision.

Plan:
1) The particles will collide when their position vectors are equal, or $\boldsymbol{r_A} = \boldsymbol{r_B}$.
2) Their speeds can be determined by differentiating the position vectors.

EXAMPLE (continued)

Solution:

1) The point of collision requires that $\boldsymbol{r_A} = \boldsymbol{r_B}$, so $x_A = x_B$ and $y_A = y_B$:

x-components: $3t = 3(t^2 - 2t + 2)$

Simplifying: $t^2 - 3t + 2 = 0$

Solving: $t = \{3 \pm [3^2 - 4(1)(2)]^{0.5}\}/2(1)$

=> $t = 2$ or 1 s

y-components: $9t(2 - t) = 3(t - 2)$

Simplifying: $3t^2 - 5t - 2 = 0$

Solving: $t = \{5 \pm [5^2 - 4(3)(-2)]^{0.5}\}/2(3)$

=> $t = 2$ or $-1/3$ s

So, the particles collide when t = 2 s. Substituting this value into $\boldsymbol{r_A}$ or $\boldsymbol{r_B}$ yields

$$x_A = x_B = 6 \text{ m} \quad \text{and} \quad y_A = y_B = 0$$

EXAMPLE (continued)

$\boldsymbol{r_A} = [\ 3t\ \boldsymbol{i} + 9t(2 - t)\ \boldsymbol{j}\ ]$ m
$\boldsymbol{r_B} = [\ 3(t^2 - 2t + 2)\ \boldsymbol{i} + 3(t - 2)\ \boldsymbol{j}\ ]$ m

2) Differentiate $\boldsymbol{r_A}$ and $\boldsymbol{r_B}$ to get the velocity vectors.

$\boldsymbol{v_A} = d\boldsymbol{r_A}/dt = \dot{x}_A\ \boldsymbol{i} + \dot{y}_A\ \boldsymbol{j} = [\ 3\boldsymbol{i} + (18 - 18t)\boldsymbol{j}\ ]$ m/s

At t = 2 s: $\boldsymbol{v_A} = [\ 3\boldsymbol{i} - 18\boldsymbol{j}\ ]$ m/s

$\boldsymbol{v_B} = d\boldsymbol{r_B}/dt = \dot{x}_B\boldsymbol{i} + \dot{y}_B\boldsymbol{j} = [\ (6t - 6)\boldsymbol{i} + 3\boldsymbol{j}\ ]$ m/s

At t = 2 s: $\boldsymbol{v_B} = [\ 6\boldsymbol{i} + 3\boldsymbol{j}\ ]$ m/s

Speed is the magnitude of the velocity vector.

$v_A = (3^2 + 18^2)^{0.5} = 18.2$ m/s
$v_B = (6^2 + 3^2)^{0.5} = 6.71$ m/s

CONCEPT QUIZ

1. The path of a particle is defined by $y = 0.5x^2$. If the component of its velocity along the x-axis at $x = 2$ m is $v_x = 1$ m/s, its velocity component along the y-axis at this position is

   A) 0.25 m/s   B) 0.5 m/s

   C) 1 m/s   D) 2 m/s

PROBLEM

Given: A particle travels along a path described by the parabola $y = 0.5x^2$. The x-component of velocity is given by $v_x = (5t)$ ft/s. When $t = 0$, $x = y = 0$.

Find: The particle's distance from the origin and the magnitude of its acceleration when $t = 1$ s.

Plan: Note that v_x is given as a function of time.

1) Determine the x-component of position and acceleration by integrating and differentiating v_x, respectively.
2) Determine the y-component of position from the parabolic equation and differentiate to get a_y.
3) Determine the magnitudes of the position and acceleration vectors.

PROBLEM (continued)

Solution:

1) x-components:

Velocity: $v_x = \dot{x} = dx/dt = (5t)$ ft/s

Position: $\int_0^x dx = \int_0^t 5t\, dt \Rightarrow x = (5/2)t^2 = (2.5t^2)$ ft

Integration constant? x(0)=0, C=0

Acceleration: $a_x = \ddot{x} = \dot{v}_x = d/dt\,(5t) = 5$ ft/s^2

2) y-components:

Position: $y = 0.5x^2 = 0.5(2.5t^2)^2 = (3.125t^4)$ ft

Velocity: $v_y = dy/dt = d\,(3.125t^4) / dt = (12.5t^3)$ ft/s

Acceleration: $a_y = v_y = d\,(12.5t^3) / dt = (37.5t^2)$ ft/s^2

PROBLEM (continued)

3) The distance from the origin is the magnitude of the position vector:

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} = [\,2.5t^2\,\boldsymbol{i} + 3.125t^4\,\boldsymbol{j}\,]\text{ ft}$$

At t = 1 s, $\boldsymbol{r} = [\,2.5\,\boldsymbol{i} + 3.125\,\boldsymbol{j}\,]$ ft

Distance: $d = r = (2.5^2 + 3.125^2)^{0.5} = 4.0$ ft

The magnitude of the acceleration vector is calculated as:

Acceleration vector: $\boldsymbol{a} = [\,5\,\boldsymbol{i} + 37.5t^2\,\boldsymbol{j}\,]\ \text{ft/s}^2$

Magnitude: $a = (5^2 + 37.5^2)^{0.5} = 37.8\ \text{ft/s}^2$

ATTENTION QUIZ

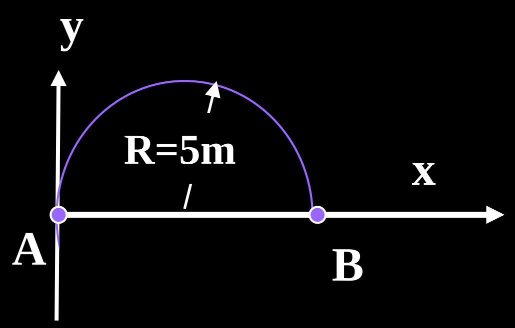
1. If a particle has moved from A to B along the circular path in 4s, what is the average velocity of the particle ?

   A) 2.5 ***i*** m/s

   B) 2.5 ***i*** +1.25***j*** m/s

   C) 1.25 π ***i*** m/s

   D) 1.25 π ***j*** m/s



2. The position of a particle is given as ***r*** = (4t^2 ***i*** – 2t ***j***) m. Determine the particle's acceleration.

   A) (4 ***i*** +8 ***j***) m/s^2

   B) (8 ***i*** -16 ***j***) m/s^2

   C) (8 ***i***) m/s^2

   D) (8 ***j***) m/s^2