Math 221 Fall 2009
9/15/2009
Quiz #3

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

[6] 1. Suppose $f(x) = \dfrac{x+1}{x-1}$

(a) $\lim_{x\to 4} f(x) = \lim_{x\to 4} \dfrac{x+1}{x-1} = \dfrac{4+1}{4-1} = \boxed{\dfrac{5}{3}}$ OR use the limit laws (pg. 77)

By the "Direct substitution property" (pg. 80) We can use this because f(x) is a rational function that is defined at x=4.

(b) Find the vertical asymptote(s) of $f(x)$.

$\boxed{x=1}$, why? since $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\left(\dfrac{x+1}{x-1}\right) = +\infty$ it follows from the definition of a vertical asymptote on (pg. 73) Remember you need to show one of those 6 possible limits on pg. 73. so another reason x=1 is a vertical asymptote is because $\lim_{x\to 1^-} f(x) = -\infty$

(c) Find the slope of the tangent line to the curve $y = f(x)$ at the point $(0, -1)$.

$$m_{\text{tangent line}} = f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\frac{h+1}{h-1} - (-1)}{h} = \lim_{h\to 0} \frac{\frac{(h+1)+1(h-1)}{h-1}}{h} = \lim_{h\to 0} \frac{2h}{h(h-1)}$$

$$= \lim_{h\to 0} \frac{2}{h-1} = \boxed{-2}$$

by AET.

[4] 2. Evaluate the following limits, if they exist.

(a) $\lim_{x\to 0} x^4 \sin\left(\dfrac{1}{x}\right) = \boxed{0}$ ← This was an example I did in class!

Notice: $-1 \le \sin\left(\frac{1}{x}\right) \le 1$, therefore $-x^4 \le x^4 \sin\left(\frac{1}{x}\right) \le x^4$

Now since $\lim_{x\to 0}(-x^4) = 0$ and $\lim_{x\to 0}(x^4) = 0$, by the squeeze theorem (pg. 83) $\lim_{x\to 0} x^4 \sin\left(\frac{1}{x}\right) = 0$

(b) $\lim_{h\to 0} \dfrac{\sqrt{1+h}-1}{h}$

$$= \lim_{h\to 0}\left(\frac{\sqrt{1+h}-1}{h}\right)\cdot 1 = \lim_{h\to 0} \frac{(\sqrt{1+h}-1)(\sqrt{1+h}+1)}{h\,(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{(\sqrt{1+h})^2 - \sqrt{1+h} + \sqrt{1+h} - 1}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{(1+h)-1}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{\lim_{h\to 0}\sqrt{1+h}+1}$$

$$= \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$