Math 222 Spring 2011
3/11/2011
Quiz #7 Name: Key

Please box your answers. Show all work clearly and in order.

[6] 1. Determine whether each sequence $\{a_n\}_{n=1}^{\infty}$ converges or diverges, and if it converges, then find its limit.

(a) $a_n = \dfrac{2n^3 - 15}{n^3 - n^2 + n}$

$$\lim_{n\to\infty} \frac{2n^3-15}{n^3-n^2+n} = \lim_{n\to\infty} \frac{(2n^3-15)\left(\frac{1}{n^3}\right)}{(n^3-n^2+n)\left(\frac{1}{n^3}\right)} = \lim_{n\to\infty} \frac{2-\frac{15}{n^3}}{1-\frac{1}{n}+\frac{1}{n^2}}$$

$$= \frac{2-0}{1-0+0} = \boxed{2}$$

so a_n $\boxed{\text{converges}}$

(b) $a_n = \cos(n)$

$\lim_{n\to\infty} \cos(n) = \text{D.N.E.}$ (as n increases $\cos(n)$ does not approach any one value, but oscillates between 1 and -1) or ($\cos(n)$ is periodic)

so a_n $\boxed{\text{diverges}}$

(c) $a_n = \dfrac{\ln(n)}{n}$

$\lim_{n\to\infty} \dfrac{\ln(n)}{n}$ consider the function $f(x) = \dfrac{\ln(x)}{x}$ where x is a real number (specifically let $x > 0$).

Notice $\lim_{x\to\infty} \underbrace{\dfrac{\ln(x)}{x}}_{\text{type } \frac{\infty}{\infty}} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{\left(\frac{1}{x}\right)}{1} = 0$. Therefore, since

$a_n = f(n)$ where n is a positive integer we have $\lim_{n\to\infty} \dfrac{\ln(n)}{n} = \boxed{0}$ so a_n $\boxed{\text{converges}}$

[4] 2. Pick ONE of the following. Cross out the problem you do not want graded. Otherwise I will grade the first problem worked on.

Determine if the following integral that you choose converges or diverges, and evaluate if it converges.

(a) $\displaystyle\int_0^2 \frac{3}{x-1}\,dx$

Notice that $f(x) = \dfrac{3}{x-1}$ is discontinuous at $x = 1$.

so write: $\displaystyle\int_0^2 \frac{3}{x-1}\,dx = \int_0^1 \frac{3}{x-1}\,dx + \int_1^2 \frac{3}{x-1}\,dx$

Each of these are improper integrals of Type II

so $\displaystyle\int_0^2 \frac{3}{x-1}\,dx = \underbrace{\lim_{t\to 1^-} \int_0^t \frac{3}{x-1}\,dx}_{} + \lim_{s\to 1^+} \int_s^2 \frac{3}{x-1}\,dx$

Solving this one first:

$\displaystyle\lim_{t\to 1^-} \int_0^t \frac{3}{x-1}\,dx = \lim_{t\to 1^-} \left(\Big[3\ln|x-1| \Big]_0^t \right)$

↓ see below

(b) $\displaystyle\int_1^{\infty} \frac{1}{x^2+1}\,dx$

This is an improper integral of type I.

so write: $\displaystyle\int_1^{\infty} \frac{1}{x^2+1}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^2+1}\,dx$

$\displaystyle= \lim_{t\to\infty} \left(\Big[\tan^{-1}(x)\Big]_1^t \right)$

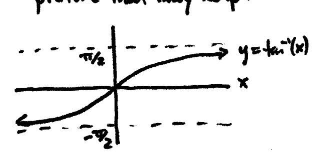
$\displaystyle= \lim_{t\to\infty} \left(\tan^{-1}(t) - \tan^{-1}(1) \right)$

$\displaystyle= \lim_{t\to\infty} \left(\tan^{-1}(t) - \frac{\pi}{4} \right)$

$\displaystyle= \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$

so $\boxed{\text{converges}}$

picture that may help:



$= \lim_{t \to 1^-} \left(3\ln|t-1| - 3\ln|-1| \right)$

$= \lim_{t \to 1^-} \left(3\ln|t-1| - \cancel{3\ln(1)}^{0} \right)$

$= \lim_{t \to 1^-} 3\ln|t-1| = -\infty$

as $t \to 1^-$ $|t-1| \longrightarrow 0^+$ so $\uparrow$

so this means $\int_0^1 \frac{3}{x-1}\,dx$ is divergent

so this means $\int_0^2 \frac{3}{x-1}\,dx$ is divergent

(we don't even need to find $\int_1^2 \frac{3}{x-1}\,dx$

we are done.)

so $\boxed{\text{diverges}}$.