Math 304 Spring 2010
4/15/2010
Quiz #11

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. There is a polynomial $p(x)$ in P_2 which has the coordinate vector $K_B(p(x)) = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$ with respect to the basis $B = (1, 1-x, x+x^2)$. Find $p(x)$.

$p(x) = (-1)(1) + (1)(1-x) + (5)(x+x^2)$

$p(x) = -1 + 1 - x + 5x + 5x^2$

$\boxed{p(x) = 4x + 5x^2}$

(see lecture 41 comments)

2. Show that the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ with associated matrix $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is an isomorphism.

Thm 4.5.8 tells us T is an isomorphism if $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is invertible.

A is already in REF so $\text{rank}(A) = 2$ (notice there are 2 pivot columns $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$)

Hence A is invertible $\Rightarrow$ T is an isomorphism.

3. Show that the set $X = \{1, 1-x, 1+x+x^2\}$ is linearly independent in P_2.

Consider the basis $S = (1, x, x^2)$ of P_2. By Lem. 4.5.10 on p179 we have

X is linearly independent $\overset{\text{if and only if}}{\Longleftrightarrow}$ $K_S(X)$ is linearly independent

since $K_S: P_2 \to \mathbb{R}^3$ is an isomorphism. (see ch 4.6)

well $K_S(X) = \{K_S(1), K_S(1-x), K_S(1+x+x^2)\} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

to show $K_S(X)$ is linearly independent consider the matrix

$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, well A is already in REF and $\text{rank}(A) = 3$

Hence since $3 = \#$ columns of A $\Rightarrow$ $K_S(X)$ is linearly independent in $\mathbb{R}^3$

$\Rightarrow$ X is linearly independent in P_2