Math 221 Fall 2009
10/27/2009
Quiz #9

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

[5] 1. Find the horizontal asymptote(s) of $f(x) = \dfrac{3x^3 - x^5 - 15190898}{14x^5 - 14x^4 + 15324x^2 - 18012}$.

To find the horizontal asymptotes we find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{(3x^3 - x^5 - 15190898)\left(\frac{1}{x^5}\right)}{(14x^5 - 14x^4 + 15324x^2 - 18012)\left(\frac{1}{x^5}\right)} = \lim_{x\to\infty} \frac{\frac{3}{x^2} - 1 - \frac{15190898}{x^5}}{14 - \frac{14}{x} + \frac{15324}{x^3} - \frac{18012}{x^5}}$$

$$= \frac{0 - 1 - 0}{14 - 0 + 0 - 0} = -\frac{1}{14}$$

so $\boxed{y = -\frac{1}{14}}$ is a H.A.

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\frac{3}{x^2} - 1 - \frac{15190898}{x^5}}{14 - \frac{14}{x} + \frac{15324}{x^3} - \frac{18012}{x^5}} = \frac{0 - 1 - 0}{14 - 0 + 0 - 0} = -\frac{1}{14}$$

[5] 2. Suppose $f(x) = \dfrac{x^2}{x^2+1}$.

(a) Find the intervals of increase or decrease (if there are none then write "none").

$$f'(x) = \frac{(x^2+1)(2x) - x^2(2x)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$

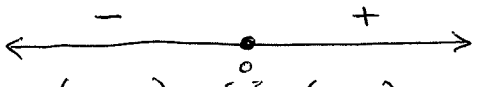
Critical numbers: $f'(x) = 0 = \dfrac{2x}{(x^2+1)^2}$

so $2x = 0$

$x = 0$

$f'(x)$ does not exist when $x^2 + 1 = 0$, no solution over $\mathbb{R}$

Sign analysis:



intervals: $(-\infty, 0)$ $\{0\}$ $(0, \infty)$

f: decreasing, increasing

f is increasing on $(0, \infty)$
f is decreasing on $(-\infty, 0)$

(b) Find the local maximum and minimum values (if there are none then write "none").

f has local minimum value at $f(0) = 0$
f has no local maximum value