

### System of Linear Equations

$$\begin{aligned}x_1 + x_2 + 3x_3 &= 2 \\ -x_1 + 3x_2 + x_3 &= 0 \\ 2x_1 - x_2 + x_3 &= 1\end{aligned}$$

Add equations 1 to equation 2:

$$\begin{aligned}x_1 + x_2 + 3x_3 &= 2 \\ 0x_1 + 4x_2 + 4x_3 &= 2 \\ 2x_1 - x_2 + x_3 &= 1\end{aligned}$$

Add -2 times equation 1 to equation 3:

$$\begin{aligned}x_1 + x_2 + 3x_3 &= 2 \\ 0x_1 + 4x_2 + 4x_3 &= 2 \\ 0x_1 - 3x_2 - 5x_3 &= -3\end{aligned}$$

Add 3/4 times equation 2 to equation 3:

$$\begin{aligned}x_1 + x_2 + 3x_3 &= 2 \\ 0x_1 + 4x_2 + 4x_3 &= 2 \\ 0x_1 + 0x_2 - 2x_3 &= -3/2\end{aligned}$$

Multiply equation 2 by 1/4:

$$\begin{aligned}x_1 + x_2 + 3x_3 &= 2 \\ 0x_1 + x_2 + x_3 &= 1/2 \\ 0x_1 + 0x_2 - 2x_3 &= -3/2\end{aligned}$$

Multiply equation 3 by -1/2:

$$\begin{aligned}x_1 + x_2 + 3x_3 &= 2 \\ 0x_1 + x_2 + x_3 &= 1/2 \\ 0x_1 + 0x_2 + x_3 &= 3/4\end{aligned}$$

Add -1 times equation 3 to equation 2:

$$\begin{aligned}x_1 + x_2 + 3x_3 &= 2 \\ 0x_1 + x_2 + 0x_3 &= -1/4 \\ 0x_1 + 0x_2 + x_3 &= 3/4\end{aligned}$$

### Augmented Matrix

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ -1 & 3 & 1 & 0 \\ 2 & -1 & 1 & 1 \end{array} \right]$$

Add row 1 to row 2:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 2 & -1 & 1 & 1 \end{array} \right]$$

Add -2 times row 1 to row 3:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 0 & -3 & -5 & -3 \end{array} \right]$$

Add 3/4 times row 2 to row 3:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 0 & 0 & -2 & -3/2 \end{array} \right]$$

Multiply row 2 by 1/4:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & -2 & -3/2 \end{array} \right]$$

Multiply row 3 by -1/2:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & 3/4 \end{array} \right]$$

Add -1 times row 3 to row 2:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array} \right]$$

### System of Linear Equations

Add -3 times equation 3 to equation 1:

$$\begin{aligned}x_1 + x_2 + 0x_3 &= -1/4 \\0x_1 + x_2 + 0x_3 &= -1/4 \\0x_1 + 0x_2 + x_3 &= 3/4\end{aligned}$$

Add -1 times equation 2 to equation 1:

$$\begin{aligned}x_1 + 0x_2 + 0x_3 &= 0 \\0x_1 + x_2 + 0x_3 &= -1/4 \\0x_1 + 0x_2 + x_3 &= 3/4\end{aligned}$$

So  $x_1 = 0$ ,  $x_2 = -1/4$  and  $x_3 = 3/4$ .

### Augmented Matrix

Add -3 times row 3 to row 1:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 0 & -1/4 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array} \right]$$

Add -1 times row 2 to row 1:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array} \right]$$

So  $x_1 = 0$ ,  $x_2 = -1/4$  and  $x_3 = 3/4$ .

### Elementary row operations (Definition 8, p10)

1. Switch two equations (rows).
2. Multiply one equation (row) by a non-zero constant.
3. Add a multiple of one equation (row) to a different equation (row). Specifically, replace a row by the sum of that row and a multiple of some different row.

**IMPORTANT!** On tests you will be required to identify the row operation that you are using in each step. The way to describe this is by annotating each step in the following style:

1.  $R2 \leftrightarrow R5$  (switch rows 2 and 5).
2.  $R4 \rightarrow \frac{1}{13}R4$  (divide row 4 by 13).
3.  $R1 \rightarrow R1 - 3R2$  (replace row 1 with row 1 minus 3 times row 2).

**WARNING!** The operation of type 3 is *ALWAYS* ‘replace a row by itself plus or minus a multiple of a different row’. So  $R2 \rightarrow R1 - 3R2$  is NOT a legal row operation.

**The previous example using the above convention:**

$$\begin{aligned} & \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ -1 & 3 & 1 & 0 \\ 2 & -1 & 1 & 1 \end{array} \right] \xrightarrow{R2 \rightarrow R2 + R1} \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 2 & -1 & 1 & 1 \end{array} \right] \xrightarrow{R3 \rightarrow R3 - 2R1} \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 0 & -3 & -5 & -3 \end{array} \right] \\ & \xrightarrow{R3 \rightarrow R3 + \frac{3}{4}R2} \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 0 & 0 & -2 & -3/2 \end{array} \right] \xrightarrow{R2 \rightarrow \frac{1}{4}R2} \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & -2 & -3/2 \end{array} \right] \xrightarrow{R3 \rightarrow \frac{-1}{2}R3} \\ & \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & 3/4 \end{array} \right] \xrightarrow{R2 \rightarrow R2 - R3} \left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array} \right] \xrightarrow{R1 \rightarrow R1 - 3R3} \left[ \begin{array}{ccc|c} 1 & 1 & 0 & -1/4 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array} \right] \\ & \xrightarrow{R1 \rightarrow R1 - R2} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array} \right] \end{aligned}$$

So  $x_1 = 0$ ,  $x_2 = -1/4$  and  $x_3 = 3/4$ .