Solution to X8(d).

The answer is no. Let's call the solution set from part (c) W. To show W is a subspace of $\mathbb{R}^4$ we would need to show three things (see Theorem 3.3.2). I will show one of those things fails by coming up with a counter example.

Notice

$$\mathbf{a} = \begin{bmatrix} -1/2 \\ 3/4 \\ 0 \\ 0 \end{bmatrix} \text{ is a particular solution to } A\mathbf{x} = \mathbf{b} \text{ from part (c).}$$

So

$$2\mathbf{a} = \begin{bmatrix} -1 \\ 3/2 \\ 0 \\ 0 \end{bmatrix}$$

But we have $A(2\mathbf{a}) = A \begin{bmatrix} -1 \\ 3/2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix} \neq \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

Hence W is not a subspace of $\mathbb{R}^4$ (since the third property failed which said that for any vector $\mathbf{x} \in W$ and any $c \in \mathbb{R}$ we need to have $c\mathbf{x} \in W$ for W to be a subspace).