Math 314 Summer 2011

6/6/2011
Quiz #4

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Write the following statements efficiently using quantifiers and standard notation.
   (a) For every integer m, 3^{m+1} is positive.

   $\forall m \in \mathbb{Z},\ 3^{m+1} > 0$

   (b) There exists an integer n such that n is not a rational number.

   $\exists n \in \mathbb{Z}$ such that $n \notin \mathbb{Q}$

   (c) The product of any two rational numbers is a rational number.

   $\forall m, n \in \mathbb{Q},\ mn \in \mathbb{Q}$

   (d) For every positive real number x, there is a real number y such that $xy = 2$.

   $\forall x \in \mathbb{R}^+,\ \exists y \in \mathbb{R}$ such that $xy = 2$

2. Which statements from question 1 are true? (No work is needed here).

   (a), (c), (d)

3. Negate the the following statements.
   (a) $\exists n \in \mathbb{Z}^-$ such that $2n + 3 < 1$.

   $\forall n \in \mathbb{Z}^-,\ 2n + 3 \geq 1$

   (b) $\forall x \in \mathbb{R}^+$, if $x^2 > 9$ then $x > 3$. (this is logically equivalent to: $\forall x \in \mathbb{R}^+,\ x^2 \leq 9$ or $x > 3$) so the negation is:

   $\exists x \in \mathbb{R}^+$ such that $x^2 > 9$ and $x \leq 3$

   (c) $\forall p, q \in \mathbb{Z},\ p + q \in \mathbb{Z}$.

   $\exists p \in \mathbb{Z}$ such that $\exists q \in \mathbb{Z}$ such that $p + q \notin \mathbb{Z}$

   (d) $\forall n \in \mathbb{R},\ \exists x \in \mathbb{R}$ such that $e^{xy} \in \mathbb{Z}$.

   $\exists n \in \mathbb{R}$ such that $\forall x \in \mathbb{R},\ e^{xy} \notin \mathbb{Z}$

4. ♠ Let A be a set. Let $f: A \subseteq \mathbb{R} \to \mathbb{R}$. Negate the following statement.

$$\forall x \in A,\ \forall \epsilon > 0,\ \exists \delta > 0 \text{ such that } \forall y \in A, \text{ if } |x - y| < \delta \text{ then } |f(x) - f(y)| < \epsilon.$$

$\exists x \in A$ such that $\exists \epsilon > 0$ such that $\forall \delta > 0,\ \exists y \in A$ such that $|x - y| < \delta$ and $|f(x) - f(y)| \geq \epsilon$