

Show all work clearly and in order. Please box your answers. 10 minutes.

- 5 1. Find the absolute maximum and absolute minimum values of $f(x) = (x^2 - 4)^2$ on the interval $[-3, 1]$

Use the closed interval method.

Find critical numbers:

$$f'(x) = 2(x^2 - 4)'(2x) = 4x(x^2 - 4) = 4x(x-2)(x+2)$$

$$f'(x) = 0 = 4x(x-2)(x+2)$$

$$x = 0 \quad \text{OR} \quad \underbrace{x = 2}_{\text{Not in } [-3, 1]} \quad \text{OR} \quad x = -2$$

$f'(x)$ is never undefined so no other critical numbers.

evaluate f at critical numbers and endpoints of the closed interval.

$$f(0) = (0 - 4)^2 = 16$$

$$f(-2) = (4 - 4)^2 = 0 \quad \leftarrow \text{absolute min value.}$$

$$f(-3) = (9 - 4)^2 = 25 \quad \leftarrow \text{absolute max value.}$$

$$f(1) = (1 - 4)^2 = 9$$

- 5 2. Find the critical numbers of the $F(x) = x^{4/5}(x-4)^2$.

$$F'(x) = x^{4/5} 2(x-4)'(1) + (x-4)^2 \left(\frac{4}{5}\right) x^{-1/5}$$

$$= x^{-1/5} (x-4) \left(2x^{5/5} + (x-4) \left(\frac{4}{5}\right) \right)$$

$$= x^{-1/5} (x-4) \left(2x + \frac{4}{5}x - \frac{16}{5} \right)$$

$$= x^{-1/5} (x-4) \left(\frac{10x}{5} + \frac{4}{5}x - \frac{16}{5} \right)$$

$$= \frac{(x-4) \left(\frac{14}{5}x - \frac{16}{5} \right)}{x^{1/5}}$$

$$F'(x) = 0 = \frac{(x-4) \left(\frac{14}{5}x - \frac{16}{5} \right)}{x^{1/5}}$$

$$x = 4 \quad \text{OR} \quad \frac{14}{5}x - \frac{16}{5} = 0$$

$$x = \left(\frac{16}{5}\right) \cdot \left(\frac{5}{14}\right)$$

$$x = \frac{8}{7}$$

or $F'(x)$ is undefined when

$$x^{1/5} = 0$$

$$x = 0$$

critical numbers: $x = 0$ OR $x = 4$ OR $x = \frac{8}{7}$