Math 304 Spring 2010
4/9/2010
Quiz #10

Name: Key

Due on Monday 4/12/2010

1. Circle "True" at each statement that is always true, and circle "False" at each statement is not always true. In the following questions we will always denote P_n as the vector space of polynomials of degree at most n.

   A. True | False If V is a finite dimensional vector space then the *dimension* of V is the number of vectors in any finite basis of V.

   B. True | False The set $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ is a basis of $\mathbb{R}^4$.

   C. True | False The set $\{1, x, x^2, x^3, x^4\}$ is a basis of P_4.

   D. True | False The dimension of $\mathbb{R}^4$ is 4.

   E. True | False The dimension of P_4 is 5.

   F. True | False There exists an isomorphism from P_4 to $\mathbb{R}^4$.

   G. True | False $\mathbb{R}^4$ has a basis X such that each vector in $\mathbb{R}^4$ can be written in more than one way as a linear combination of the elements of X.

   H. True | False P_4 has a basis X such that each polynomial (vector) in P_4 can be written in more than one way as a linear combination of the elements of X.

   I. True | False The set of functions $\{c_2x^2 + c_3x^3 + c_4x^4 \mid c_2, c_3, c_4 \in \mathbb{R}\}$ is a subspace of P_4.

   J. True | False The set $\{1, 1-x, 1+x^2\}$ is a basis of P_2.

   K. True | False If X is a collection of vectors in a vector space W, then Span(X) is a subspace of W.

   L. True | False $\text{Span}\left(\left\{\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}\right\}\right)$ is a subspace of $\mathbb{R}^3$.

   M. True | False Span($\{1, 1-x\}$) is a subspace of P_2.

   N. True | False Span($\{5\}$) is a subspace of P_2.

   O. True | False If the set S is linearly independent in P_4 then $S \cup \{x\}$ is always linearly independent.

   P. True | False If S is a spanning set of P_4 then S always contains the vector (polynomial) 1.

   Q. True | False If S is a spanning set of P_4 then Span(S) always contains the vector (polynomial) 1.

   R. True | False A linear transformation is an isomorphism if and only if it is a one-to-one correspondence.

   S. True | False The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with associated matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ is an isomorphism.

   T. True | False Any isomorphism from $\mathbb{R}^4$ to $\mathbb{R}^4$ takes the standard basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ to itself.