Math 221 Fall 2009
12/1/2009
Quiz #14

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

[5] 1. Find the area of the region bounded by $y = x^2$ and $y = 2 - x^2$.

points of intersection: $x^2 = 2 - x^2$

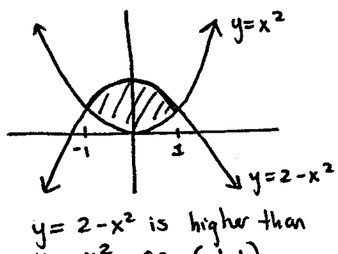
$2x^2 = 2$

$x^2 = 1$

$x = \pm\sqrt{1} = \pm 1$

which function is higher than the other on the interval $(-1,1)$: (2 ways)

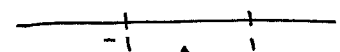
I) Graph:



$y = 2-x^2$ is higher than $y = x^2$ on $(-1,1)$

OR

II) sign diagram:



pick some value on the interval say $x=0$.

for $y=x^2$: $y(0) = 0^2 = 0$

for $y=2-x^2$: $y(0) = 2-0^2 = 2$

So $y = 2-x^2$ is higher than $y=x^2$ on $(-1,1)$.

Calculate area:

$$A = \int_{-1}^{1} ((2-x^2) - x^2)\,dx = \int_{-1}^{1} (2-2x^2)\,dx = \underbrace{2\left[2x - \frac{2x^3}{3}\right]_0^1}_{\text{This is because } 2-2x^2 \text{ is even. (You do not need to use this short cut)}} = 2\left[\left(2-\frac{2}{3}\right) - (0-0)\right] = \boxed{\frac{8}{3}}$$

[5] 2. Find the area of the region bounded by $x = y^2$, $x = 1$ and $0 \le y \le 2$.

SOLUTION 1

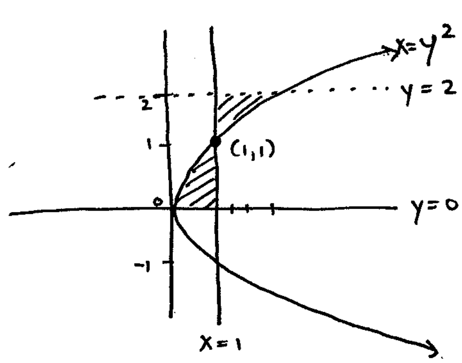
points of intersection: $y^2 = 1$

$y = \pm\sqrt{1} = \pm 1$

which function is higher than the other over the interval $(0,2)$: (notice where the points of intersection are)

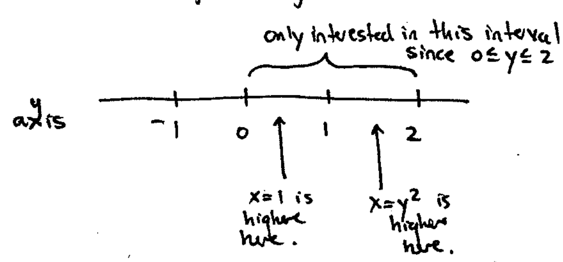
2 ways:

I) Graph:



OR

II) Sign diagram



from 0 to 1: $x=1$ is above $x=y^2$

from 1 to 2: $x=y^2$ is above $x=1$

Calculating area:

$$A = \int_0^2 |y^2-1|\,dy = \int_0^1 (1-y^2)\,dy + \int_1^2 (y^2-1)\,dy = \left[y - \frac{y^3}{3}\right]_0^1 + \left[\frac{y^3}{3} - y\right]_1^2 = \boxed{2}$$

second solution

$\boxed{\text{SOLUTION 2}}$: If we integrate with respect to x we need to change a few things:

<u>points of intersection</u>:

$x = y^2 \iff y = \pm\sqrt{x}$ ← two functions $y=\sqrt{x}$ and $y=-\sqrt{x}$

Since $0 \le y \le 2$ we only need to consider $y = \sqrt{x}$.

$x = 1$ ← not a function of y. (This will be used later)

$0 \le y \le 2 \iff y=0$ and $y=2$ are two bounding lines.

we need to look at the points $y^2 = 1 \iff y = \pm 1 \iff x = (+1)^2 = 1$ or $x = (-1)^2 = 1$

<u>and</u> when $\sqrt{x} = 2$
$x = 4$

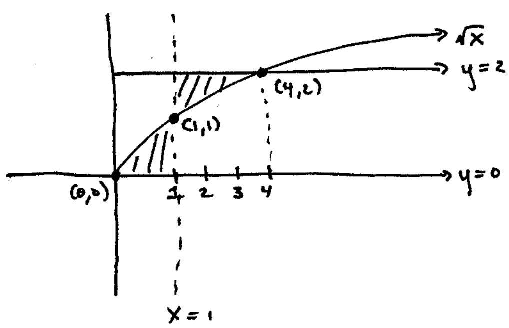
<u>and</u> when $\sqrt{x} = 0$
$x = 0$

<u>which function is above the other over the interval $(0,4)$</u>:

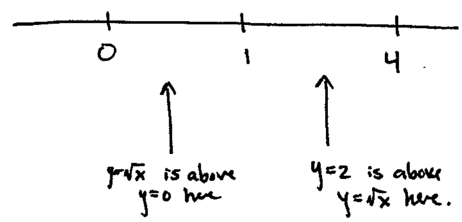
2 ways:

I) <u>graph</u>

II) sign diagram



$\boxed{\text{OR}}$



from 0 to 1 : $y=\sqrt{x}$ is above $y=0$
from 1 to 4 : $y=2$ is above $y=\sqrt{x}$

<u>Calculating Area</u>:

$$A = \int_0^1 \sqrt{x}\,dx + \int_1^4 (2-\sqrt{x})\,dx = \int_0^1 x^{1/2}\,dx + \int_1^4 (2 - x^{1/2})\,dx$$

$$= \left[\frac{x^{3/2}}{3/2}\right]_0^1 + \left[2x - \frac{x^{3/2}}{3/2}\right]_1^4$$

$$= \left[\left(\frac{1}{3/2}\right) - 0\right] + \left[\left(8 - \frac{4^{3/2}}{3/2}\right) - \left(2 - \frac{1}{3/2}\right)\right]$$

$$= \boxed{2}$$