

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Determine if the following system of linear equations is consistent:

$$\begin{array}{rrcrcl} x_1 & + & 3x_2 & + & 2x_3 & + & 9x_4 & = & 1 \\ x_1 & + & 2x_2 & - & x_3 & + & x_4 & = & 4 \\ -x_1 & - & 2x_2 & + & x_3 & - & x_4 & = & 2 \end{array}$$

$$A = \left[\begin{array}{cccc|c} 1 & 3 & 2 & 9 & 1 \\ 1 & 2 & -1 & 1 & 4 \\ -1 & -2 & 1 & -1 & 2 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 3 & 2 & 9 & 1 \\ 0 & -1 & -3 & -8 & 3 \\ -1 & -2 & 1 & -1 & 2 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + R_1} \left[\begin{array}{cccc|c} 1 & 3 & 2 & 9 & 1 \\ 0 & -1 & -3 & -8 & 3 \\ 0 & 1 & 3 & 8 & 3 \end{array} \right]$$

$$\xrightarrow{R_3 \rightarrow R_3 + R_2} \left[\begin{array}{cccc|c} 1 & 3 & 2 & 9 & 1 \\ 0 & -1 & -3 & -8 & 3 \\ 0 & 0 & 0 & 0 & 6 \end{array} \right]$$

The rightmost column here is a pivot column.
so the system is inconsistent
(so no solutions)

2. Consider the following system of linear equations:

$$\begin{array}{rrcrcl} x_1 & + & 2x_2 & + & 5x_3 & = & 3 \\ -x_1 & - & 2x_2 & - & 2x_3 & = & 0 \end{array}$$

- (a) Find the reduced row echelon form (RREF) of the augmented matrix A of the given system.
 (b) What are the pivot columns in A ?
 (c) What is the rank of A ?
 (d) If the system is consistent write the solution in parametric form.

$$(a) \left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ -1 & -2 & -2 & 0 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 0 & 3 & 3 \end{array} \right] \xrightarrow{R_2 \rightarrow \frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 \rightarrow R_1 - 5R_2} \left[\begin{array}{ccc|c} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

(b) The pivot columns in A are the first and third columns of A : $\left(\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 5 \\ -2 \end{pmatrix} \right)$

(c) The rank of $A = \boxed{2}$ (# of pivot columns)

(d) $x_1 + 2x_2 = -2$

so $x_1 = -2 - 2x_2$

x_2 is a free variable and $x_3 = 1$ so:

$$\begin{array}{l} x_1 = -2 - 2x_2 \\ x_2 = \text{anything} \\ x_3 = 1 \end{array}$$