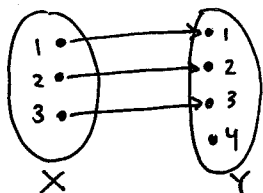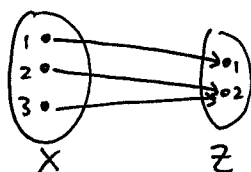Name: _____(Key)_____

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Let $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$, and $Z = \{1, 2\}$

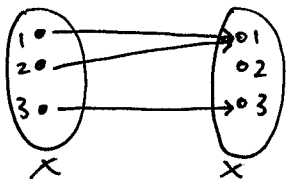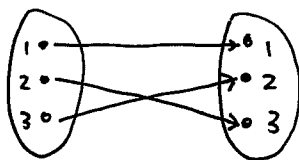   (a) Define a function $f : X \to Y$ that is one-to-one but not onto.

   (b) Define a function $g : X \to Z$ that is onto but not one-to-one.

   (c) Define a function $h : X \to X$ that is neither one-to-one nor onto.

   (d) Define a function $j : X \to X$ that is a one-to-one correspondence but is NOT the identity function on X.

2. Define $f : \mathbb{Z} \to \mathbb{Z}$ by $f(n) = 1 - 2n$, for all integers n.

   (a) Is f one-to-one? Prove or give a counterexample.

   Yes, proof: Let $n_1, n_2 \in \mathbb{Z}$. Suppose $f(n_1) = f(n_2)$

   So $1 - 2n_1 = 1 - 2n_2$

   $2n_1 = 2n_2$ ✓

   $n_1 = n_2$ ✓

   Hence, f is one-to-one. □

   (b) Is f onto? Prove or give a counterexample.

   ← the codomain.

   No consider $2 \in \mathbb{Z}$.

   If there were an $n \in \mathbb{Z}$ s.t. $f(n) = 2$

   then $1 - 2n = 2$

   So $-2n = 1$

   $n = -\frac{1}{2}$

   but $-\frac{1}{2} \notin \mathbb{Z}$ ← the domain. Therefore, f is NOT onto.