Math 304 Spring 2010
3/4/2010
Quiz #6

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Let $X = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k\}$ be a collection of vectors in $\mathbb{R}^n$. Give the definition of Span(X):

$$\boxed{\text{Span}(X) = \left\{ \alpha_1 \vec{x}_1 + \alpha_2 \vec{x}_2 + \cdots + \alpha_k \vec{x}_k \;\middle|\; \alpha_1, \ldots, \alpha_k \in \mathbb{R},\ \vec{x}_1, \ldots, \vec{x}_k \in X \right\}}$$

OR

Span(X) is the set of all linear combinations of vectors in X

2. Let $X = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ where

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

Is the vector $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ in Span(X)? If so, write $\mathbf{u}$ as a linear combination of $\mathbf{x}_1$,$\mathbf{x}_2$, and $\mathbf{x}_3$.

using the "membership test" discussed in class (and on p 122 of the book)

$$\vec{u} \in \text{Span}(X) \iff \text{there is a linear combination } \vec{u} = c_1\vec{x}_1 + c_2\vec{x}_2 + c_3\vec{x}_3 = \underbrace{\begin{bmatrix} \vec{x}_1 & \vec{x}_2 & \vec{x}_3 \end{bmatrix}}_{A} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

(so $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$ is a solution of the equation $A\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \vec{u}$)

so for this problem:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & -2 \\ 0 & 2 & 2 & -2 \end{array}\right] \xrightarrow{R2 \to R2+R1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 2 & -2 \end{array}\right] \xrightarrow{R3 \to R3-2R2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

so $c_1 + c_3 = 1$
$c_2 + c_3 = -1$
$c_3 =$ anything

so in parametric form $c_1 = 1 - t$, $c_2 = -1 - t$, $c_3 = t$ } so pick some t value to get a linear combination (or keep general form)

for example if $t=0$ then $c_1 = 1, c_2 = -1, c_3 = 0$ so

$\boxed{\vec{u} = \vec{x}_1 - \vec{x}_2}$ (NOTE: there are infinitely many solutions!)