Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

The following two proofs have been started for you. Please fill in the missing pieces to complete the proofs.

1. Suppose A and B are sets in some universe $\mathcal{U}$.
   Show: If $A \subseteq B$, then $A \cap B = A$.

   Proof. Let $A \subseteq B$.

   ($\subseteq$) Let $x \in A \cap B$.
   So $x \in A$ and $x \in B$.
   In particular $x \in A$.
   Therefore $A \cap B \subseteq A$.

   ($\supseteq$) Let $x \in A$.
   Since $A \subseteq B$, we have $x \in B$.
   So $x \in A$ and $x \in B$.
   Hence $x \in A \cap B$.
   Therefore $A \cap B \supseteq A$. □

2. Show: $\forall x \in \mathbb{R}, x \in (-2, 1]$ if and only if $-x \in [-1, 2)$.

   Proof. Let $x \in \mathbb{R}$ be arbitrary.

   ($\rightarrow$) Let $x \in (-2,1]$
   That is, $-2 < x \leq 1$
   Multiplication by -1 gives: $(-2)(-1) > (x)(-1) \geq (1)(-1)$
   That is, $2 > -x \geq -1$. Therefore $-x \in [-1,2)$

   ($\leftarrow$) Let $-x \in [-1,2)$
   That is, $-1 \leq -x < 2$
   Multiplication by -1 gives: $(-1)(-1) \geq (-x)(-1) > (2)(-1)$
   That is, $1 \geq x > -2$. Therefore $x \in (-2,1]$ □