Math 314 Summer 2010
6/24/2010 Quiz #17

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Let $X = \{1,2,3,4\}$. Let $\mathcal{A} = \{A_1, A_2, A_3\}$ where $A_1 = \{1,2\}$, $A_2 = \{3\}$, $A_3 = \{4\}$. Show that $\mathcal{A}$ forms a partition of X.

(i) $A_1, A_2, A_3 \neq \emptyset$ and $A_1, A_2, A_3 \subseteq X$ ✓ (so each A_i is nonempty and a subset of X)

(ii) $A_1 \cap A_2 = \{1,2\} \cap \{3\} = \emptyset$
$A_1 \cap A_3 = \{1,2\} \cap \{4\} = \emptyset$
$A_2 \cap A_3 = \{3\} \cap \{4\} = \emptyset$
(so the collection $\mathcal{A}$ consists of disjoint sets)

(iii) $\bigcup_{A \in \mathcal{A}} A = \underbrace{\{1,2\}}_{A_1} \cup \underbrace{\{3\}}_{A_2} \cup \underbrace{\{4\}}_{A_3} = \{1,2,3,4\} = X$ (the union of the collection $\mathcal{A}$ is X)

2. Define the relation R on $\mathbb{R}$ by

$$xRy \text{ if and only if } \lceil x \rceil = \lceil y \rceil.$$

We showed yesterday that R is an equivalence relation (so you do not need to show this here). Find the partition on the set $\mathbb{R}$ that corresponds to the equivalence relation R.

Lemma 5.4 on p242 gives us: the collection of equivalence classes $\mathcal{A} = \{[x] : x \in X\}$ is a partition of X.

So our partition on the set $\mathbb{R}$ corresponding to this relation R is:
$\mathcal{A} = \{[x] : x \in \mathbb{R}\}$
but $[x] = \{y : y \in \mathbb{R} \text{ and } yRx\}$
$= \{y : y \in \mathbb{R} \text{ and } \lceil y \rceil = \lceil x \rceil\}$ ← x and y have the same ceiling if for some $m \in \mathbb{Z}$ $m-1 < x \leq m$ and $m-1 < y \leq m$ so we can describe our collection $\mathcal{A}$ as the set of all $A_m = \{z : m-1 < z \leq m\}$ where $m \in \mathbb{Z}$.

3. $X = \mathbb{R} \times \mathbb{R}$, and, for each $a \in \mathbb{R}$, let A_a be the set of points on the vertical line through $(a, 0)$. The collection of subsets A_a forms a partition of X (you do not need to show this). Find the equivalence relation R on X that corresponds to this partition.

Let $\mathcal{A}$ be the collection of all A_a where $a \in \mathbb{R}$ (we know this is a partition.)
Lemma 5.5 on p244 gives us the relation we are looking for.

$$(x_1, y_1) \, R \, (x_2, y_2) \quad \text{iff} \quad \exists A_a \in \mathcal{A} \text{ such that } x, y \in A_a.$$

Notice: if $(x_1, y_1) \in A_a$ then $x_1 = a$
if $(x_2, y_2) \in A_a$ then $x_2 = a$
so both $(x_1, y_1), (x_2, y_2) \in A_a$ if $x_1 = x_2 = a$
so in general (x_1, y_1) and (x_2, y_2) are in the same set A in the collection $\mathcal{A}$ is $x_1 = x_2$.

Therefore our relation is:

$$\boxed{(x_1, y_1) \, R \, (x_2, y_2) \quad \text{iff} \quad x_1 = x_2}$$