Math 221 Fall 2009
12/8/2009
Quiz #15

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

[5] 1. Set up but DO NOT EVALUATE the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the x-axis. Your answer should be a definite integral which you do not need to simplify.

using the washer method:

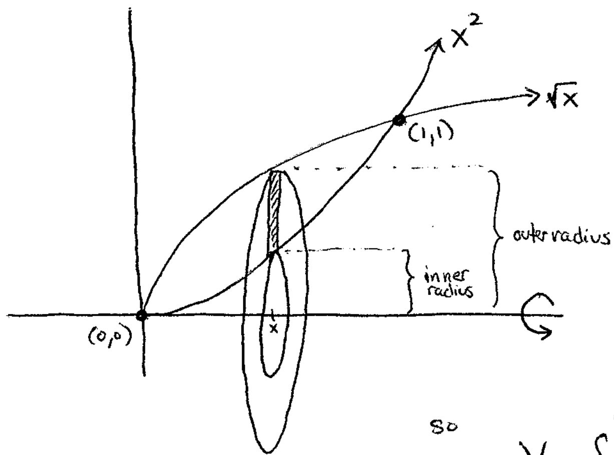
points of intersection: $\sqrt{x} = x^2$

$x = x^4$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0$ OR $x = 1$



inner radius: x^2

outer radius: $\sqrt{x}$

cross-sectional area: $A(x) = \pi(\sqrt{x})^2 - \pi(x^2)^2$

so

$$V = \int_a^b A(x)\,dx = \boxed{\int_0^1 \pi\left((\sqrt{x})^2 - (x^2)^2\right) dx}$$

other solution

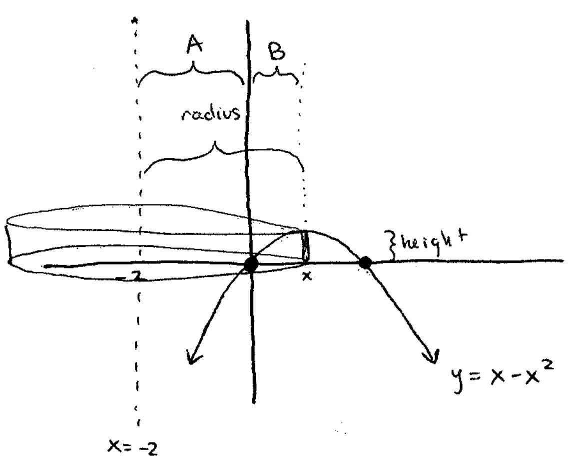
[5] 2. Set up but DO NOT EVALUATE the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = -2$. Your answer should be a definite integral which you do not need to simplify.

using the shell method

points of intersection: $x - x^2 = 0$

$x(1 - x) = 0$

$x = 0$ OR $x = 1$



radius: $x + 2$

(notice that from the picture radius $= A + B$. $A = 2$ and $B = x$ hence radius $= x + 2$)

height: $x - x^2$

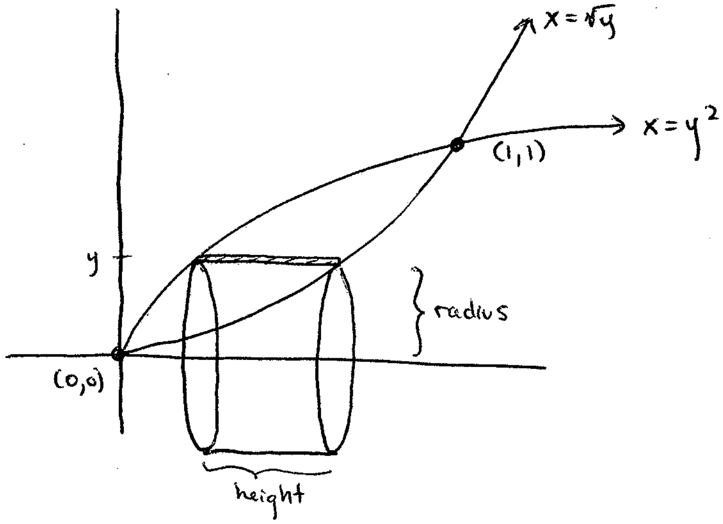
cross-sectional area: $A(x) = 2\pi(x+2)(x - x^2)$

so

$$V = \int_a^b A(x)\,dx = \boxed{\int_0^1 2\pi(x+2)(x - x^2)\,dx}$$

other solution

① using the shell method:



points of intersection:

$$\sqrt{y} = y^2$$
$$y = y^4$$
$$y^4 - y = 0$$
$$y(y^3 - 1) = 0$$
$$y = 0 \ \underline{\text{OR}} \ y = 1$$

radius: y

height: $\sqrt{y} - y^2$

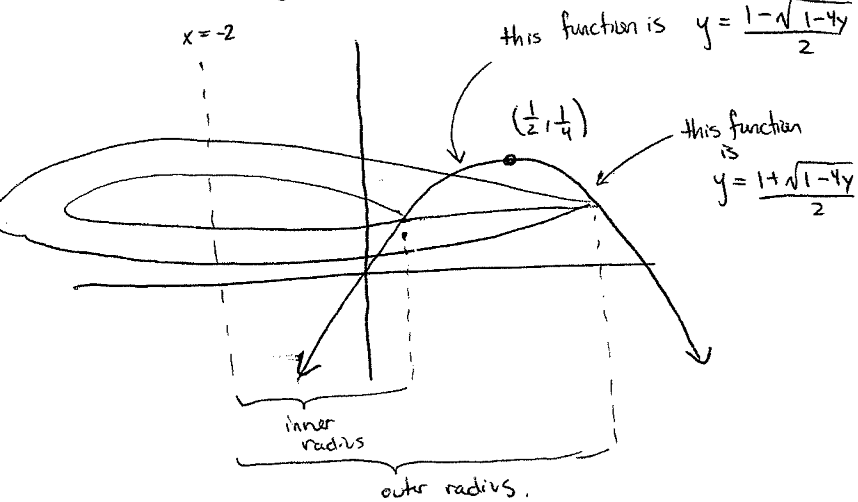
cross-sectional area:

$$A(y) = 2\pi y(\sqrt{y} - y^2)$$

so

$$V = \int_c^d A(y)\,dy = \boxed{\int_0^1 2\pi y(\sqrt{y} - y^2)\,dy}$$

② using the washer method:



we need to integrate with respect to y. Since $y = x - x^2$ we need to use some technique to find functions of x! use the quadratic formula on the equation $0 = x^2 - x + y$ to find

$$x = \frac{1 \pm \sqrt{1-4y}}{2}$$

use calculus or axis of symmetry to find limits of integration as 0 to $1/4$. (axis of sym: $x = \frac{-b}{2a} = \frac{1}{2}$)

inner radius: $\dfrac{1 - \sqrt{1-4y}}{2} + 2$

outer radius: $\dfrac{1 + \sqrt{1-4y}}{2} + 2$

so

$$V = \boxed{\int_0^{1/4} \pi\left(\left(\frac{1+\sqrt{1-4y}}{2} + 2\right)^2 - \left(\frac{1-\sqrt{1-4y}}{2} + 2\right)^2\right) dy}$$