Math 221 Fall 2009
9/22/2009
Quiz #4

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

[4] 1. Suppose $f(x) = \begin{cases} 1+x^2 & \text{if } x \le 0 \\ 2-x & \text{if } 0 < x \le 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$ ← This is #37 on pg. 106

(a) Find the x-value(s) for which f is discontinuous.

$\boxed{x=0}$

Question: could you explain why f is continuous at all other values of x? In other words could you show f is continuous on $(-\infty,0)\cup(0,\infty)$

(b) Show why f is discontinuous at the x-value(s) from part (a).

Check to see if $f(x)$ is continuous at $x=0$:

(i) $f(0) = 1+0^2 = 1$ so $f(0)$ exists ✓

(ii) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(2-x) = 2$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(1+x^2) = 1$

$2 \neq 1$ hence $\lim_{x\to 0} f(x) = \text{D.N.E.}$

so f is NOT cts. at $x=0$, hence: $\Rightarrow$ $\boxed{f \text{ is discontinuous at } x=0}$

[4] 2. Differentiate and please simplify.

(a) $y = \dfrac{u^6 - 2u^3 + 5}{u^2} = \dfrac{u^6}{u^2} - \dfrac{2u^3}{u^2} + \dfrac{5}{u^2} = u^4 - 2u + 5u^{-2}$

so $y' = 4u^3 - 2 + 5(-2)u^{-2-1}$ (OR use quotient rule)

$\boxed{y' = 4u^3 - 2 - 10u^{-3}}$ This is #40 on pg. 145

(b) $g(t) = 100\pi t^5 + \sqrt{t^3} + 216 = 100\pi t^5 + t^{3/2} + 216$

$g'(t) = 100\pi(5)t^4 + \frac{3}{2}t^{1/2} + 0$

$\boxed{g'(t) = 500\pi t^4 + \frac{3}{2}t^{1/2}}$

[2] 3. Find an equation of the tangent line to the curve $f(x) = x^2$ at $x = 3$ **using the definition of the derivative (you will receive NO credit otherwise).**

$f'(3) = \lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \dfrac{(3+h)^2 - 3^2}{h} = \lim_{h\to 0} \dfrac{9+6h+h^2-9}{h}$

$= \lim_{h\to 0} \dfrac{h(6+h)}{h}$

$= \lim_{h\to 0} 6+h = 6$

so an equation of the tangent line: $y - f(3) = 6(x-3)$

$\boxed{y - 9 = 6(x-3)}$ OR $\boxed{y = 6x - 9}$