Math 304 Spring 2010
3/18/2010
Quiz #8

Name: Key

Show all work clearly and in order. Please box your answers. Use answer lines where provided. 10 minutes.

1. Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

(a) Find a basis X for the column space of A.

A is in RREF so $\boxed{X = \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)}$ OR $X = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$

(b) What is the dimension of the column space of A?

(b) 1

(c) Find a basis Y for the null space of A.

Solving the homogeneous system $A\vec{x} = \vec{0}$

$[A|\vec{0}] = \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ so $x_1 - x_2 = 0$, $x_2 =$ anything

so $x_1 = x_2$, $x_2 = x_2$ so $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ so $\boxed{Y = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)}$ OR $Y = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$

(d) What is the dimension of the null space of A?

(d) 1

(e) Find a basis Z for the row space of A.

A is in RREF so $\boxed{Z = \left(\begin{bmatrix} 1 & -1 \end{bmatrix}\right)}$ OR $Z = \left\{\begin{bmatrix} 1 & -1 \end{bmatrix}\right\}$

OR $Z = \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ OR $Z = \left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$

(f) What is the dimension of the row space of A?

(f) 1

2. $X = \left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$ is an ordered basis of $\mathbb{R}^2$. Let K be the coordinate transformation defined by the ordered basis X. The vector $\mathbf{w} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ in in $\mathbb{R}^2$. Calculate $K(\mathbf{w})$ (i.e., find the coordinate vector of $\mathbf{w}$ with respect to the ordered basis X).

$\left[\begin{array}{cc|c} -1 & 1 & -2 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{R1 \to -R1} \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{R1 \to R1+R2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$

so $\boxed{K(\vec{w}) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}}$

(since $\vec{w} = 3\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 1 \end{bmatrix}$)