Math 314 Summer 2011
6/28/2011 Quiz #16

Name: Key

Show all work clearly and in order. Please box your answers. Due 6/29/2011.

1. Let $X = \{0, 1, 2, 3, 4\}$. Let $\mathcal{A} = \{A_1, A_2, A_3\}$ where $A_1 = \{1, 2\}$, $A_2 = \{0, 3\}$, $A_3 = \{4\}$. Show that $\mathcal{A}$ forms a partition of X.

(1) $A_1 \neq \emptyset$
$A_2 \neq \emptyset$
$A_3 \neq \emptyset$

(2) $\bigcup_{A_i \in \mathcal{A}} A_i = A_1 \cup A_2 \cup A_3 = \{1,2\} \cup \{0,3\} \cup \{4\} = \{0,1,2,3,4\} = X$

(3) $A_1 \cap A_2 = \emptyset$
$A_1 \cap A_3 = \emptyset$
$A_2 \cap A_3 = \emptyset$

Hence, by (1), (2), (3) $\mathcal{A}$ forms a partition of X.

2. Define the relation R on $\mathbb{R}$ by

$$xRy \text{ if and only if } \lceil x \rceil = \lceil y \rceil.$$

(a) Show R is an equivalence relation.

R is reflexive: Let $x \in \mathbb{R} \Rightarrow \lceil x \rceil = \lceil x \rceil$, Hence, xRx ✓

R is symmetric: Let $x, y \in \mathbb{R}$. Suppose $xRy \Rightarrow \lceil x \rceil = \lceil y \rceil$
so $\lceil y \rceil = \lceil x \rceil$
Therefore, yRx ✓

R is transitive: Let $x, y, z \in \mathbb{R}$. Suppose $xRy \wedge yRz$
$\Rightarrow \lceil x \rceil = \lceil y \rceil$ and $\lceil y \rceil = \lceil z \rceil$
$\Rightarrow \lceil x \rceil = \lceil y \rceil = \lceil z \rceil \Rightarrow xRz$ ✓

(b) Find the partition on the set $\mathbb{R}$ that corresponds to the equivalence relation R.

the set of equivalence classes forms a partition on the set $\mathbb{R}$ that corresponds to the equivalence relation R.

i.e., $\mathcal{A} = \{ [x] \mid x \in \mathbb{R} \}$

Let's describe these equivalence classes though.

$[x] = \{ y \in \mathbb{R} \mid yRx \} = \{ y \in \mathbb{R} \mid \lceil y \rceil = \lceil x \rceil \}$

say $\lceil x \rceil = m$

then $[x] = \{ y \in \mathbb{R} \mid m-1 < y \leq m \}$

call this A_m

so now $\mathcal{A} = \{ A_m \mid m \in \mathbb{Z} \}$.