Math 221 Fall 2009
11/3/2009
Quiz #10

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

$\boxed{5}$ 1. Suppose $f(x) = \dfrac{x}{x^2-9}$.

Most of this question has been done for you. Fill in the missing information in parts (c) and (e)

(a) Natural Domain: $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$.

(b) Intercept:

$x$-intercept: (0,0)

$y$-intercept: (0,0)

(c) Asymptotes:

$$\lim_{x\to\infty} \frac{x}{x^2-9} = 0 \quad \text{and} \quad \lim_{x\to-\infty} \frac{x}{x^2-9} = 0$$

$$\lim_{x\to3^+} \frac{x}{x^2-9} = \infty \quad \text{and} \quad \lim_{x\to3^-} \frac{x}{x^2-9} = -\infty$$

$$\lim_{x\to-3^+} \frac{x}{x^2-9} = \infty \quad \text{and} \quad \lim_{x\to-3^-} \frac{x}{x^2-9} = -\infty$$

Vertical Asymptote(s):

→ tells us $\boxed{x=3 \text{ and } x=-3}$ are V.A.

Horizontal Asymptote(s):

→ tells us that only $\boxed{y=0}$ is a H.A.

(d) $f'(x) = \dfrac{-(x^2+9)}{(x^2-9)^2}$

$f(x)$ is increasing nowhere

$f(x)$ is decreasing on $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$

Local Max: none

Local Min: none

(e) $f''(x) = \dfrac{2x(x^2+27)}{(x^2-9)^3}$

find the critical numbers of $f''(x)$ and do a "sign analysis":

$f(x)$ is concave up on: $\boxed{(-3,0)\cup(3,\infty)}$

$f(x)$ is concave down on: $\boxed{(-\infty,-3)\cup(0,3)}$

Points of inflection: $\boxed{(0,0)}$

when is $f''(x)=0$

$2x(x^2+27)=0$

$2x=0$ OR $x^2+27=0$

$x=0$ no solutions

when is $f''(x)$ undefined

when $(x^2-9)^3=0$

$x^2-9=0$

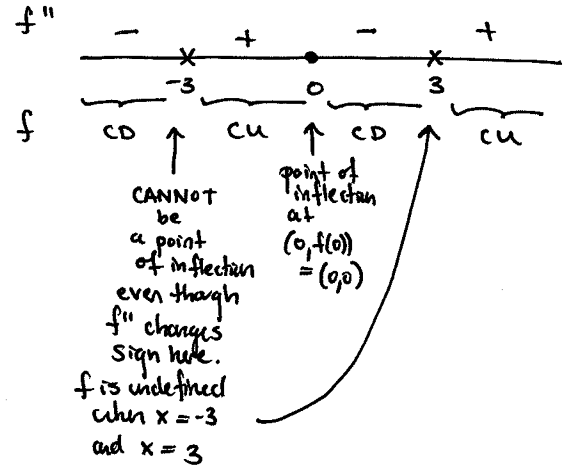
$(x-3)(x+3)=0$

$x=3$ OR $x=-3$

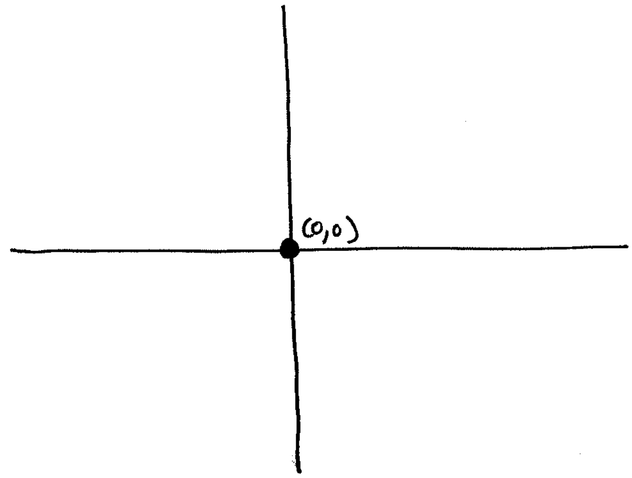
so the critical numbers are $x=0$, $x=3$ and $x=-3$

$\boxed{5}$ 2. Use all the information in Question 1 to sketch the graph of $f(x) = \dfrac{x}{x^2-9}$.

See the next two pages

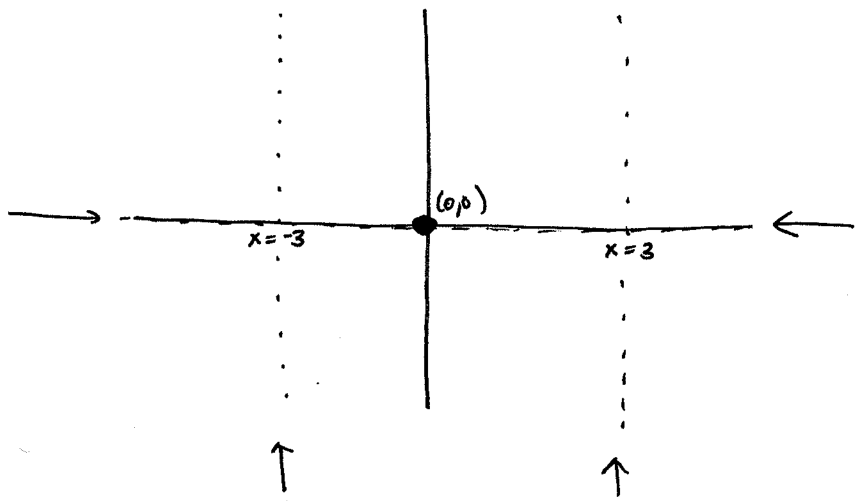


First plot the points you know f goes through and note the domain:




now look at the limit information to determine some structure of f:

there will be a HA at $y=0$ as $x \to -\infty$

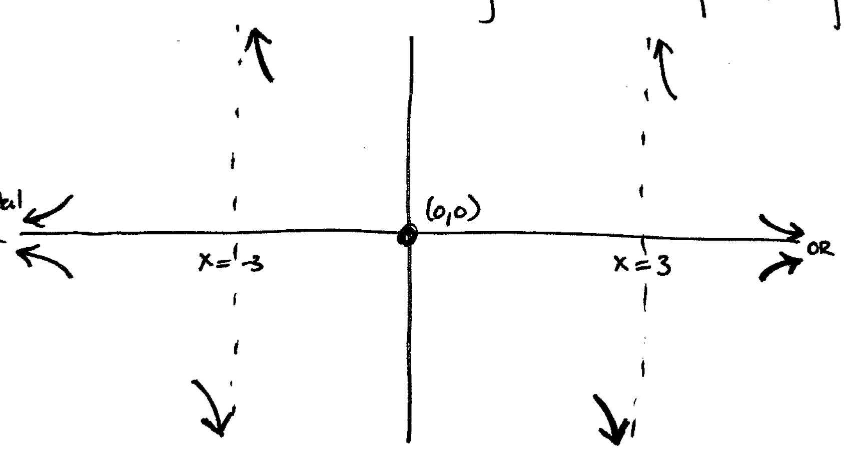



there will be a HA at $y=0$ as $x \to \infty$

there will be vertical asymptotes at $x = -3$ and $x = 3$

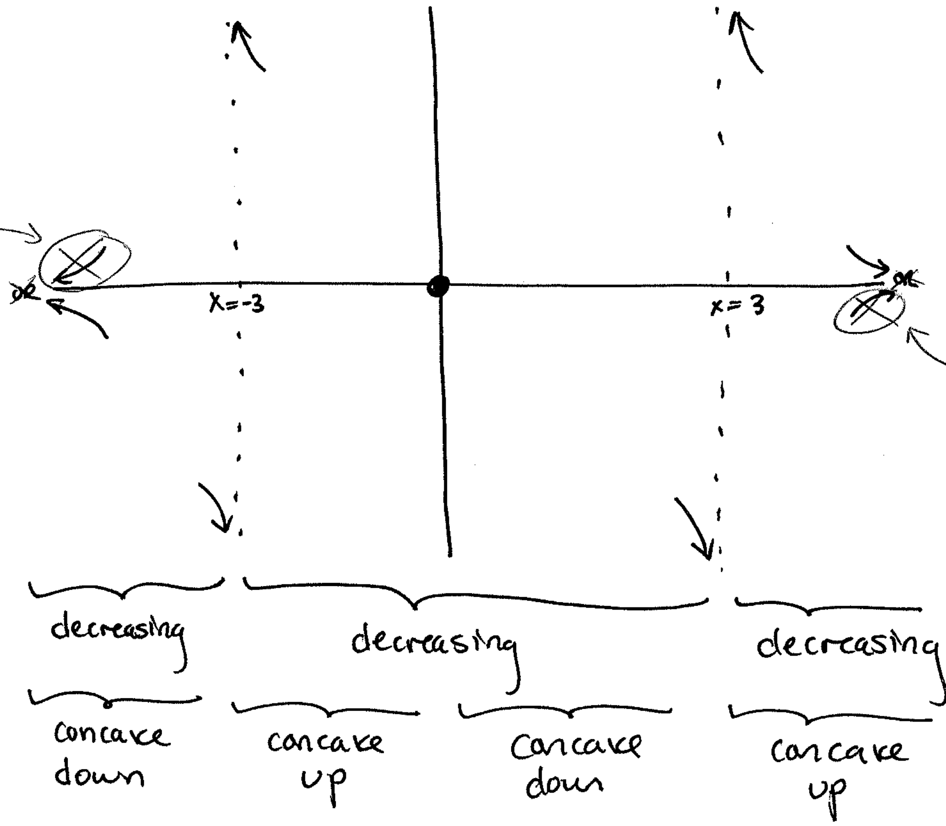
specifically the limits dealing with the vertical asymptotes tell us how f looks near them and the information about the HA is not exactly clear just yet

Horizontal asymptote shows f will look like one of these ends




Horizontal asymptote shows f will look like one of these ends now mark under the graph where the function is increasing or decreasing. Also mark under the graph where the function is concave up or concave down.

now we know f cannot look like this on the far left end because it would be increasing.




now we know f cannot look like this on the far right end because it would be increasing

Now fill in the missing pieces and make sure all of the information is correct. You should end up with: