Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Let $\{s_n\}$ be the sequence defined by

$$s_0 = 0, s_1 = 1, \text{ AND } \forall n \geq 2, s_n = 3s_{n-1} - 2s_{n-2}$$

Show: $\forall n \geq 0$, $s_n = 2^n - 1$.

proof: (By strong induction)

Base Cases: $(n=0)$: $2^0 - 1 = 1 - 1 = 0 = s_0$ ✓

$(n=1)$: $2^1 - 1 = 2 - 1 = 1 = s_1$ ✓

Induction Step: For $k \geq 1$, suppose

$s_i = 2^i - 1$ for all $0 \leq i \leq k$. } induction hypothesis.

(We want to show: $s_{k+1} = 2^{k+1} - 1$)

observe,

$$\begin{aligned} s_{k+1} &= 3s_k - 2s_{k-1} \\ &= 3(2^k - 1) - 2(2^{k-1} - 1) \quad \leftarrow \text{by the induction hypothesis.} \\ &= 3 \cdot 2^k - 3 - 2 \cdot 2^{k-1} + 2 \\ &= 3 \cdot 2^k - 1 - 2^k \\ &= 2 \cdot 2^k - 1 \\ &= 2^{k+1} - 1 \quad ✓ \end{aligned}$$

□