Math 222 Spring 2011
5/6/2011
Quiz #12

Name: Key

Please box your answers. Show all work clearly and in order.

1. The following are parametric equations for a curve:

$$x = 1 + e^t,$$
$$y = t^2, \qquad -3 \le t \le 3.$$

(a) Set up the integral for the length of that curve, but DO NOT EVALUATE OR SIMPLIFY THAT INTEGRAL.

$$\int_{-3}^{3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \boxed{\int_{-3}^{3} \sqrt{(e^t)^2 + (2t)^2}\, dt}$$

(b) Set up the integral for the surface area obtained by rotating that curve around the x-axis, but DO NOT EVALUATE OR SIMPLIFY THAT INTEGRAL.

$$\int_{-3}^{3} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \boxed{\int_{-3}^{3} 2\pi (t^2) \sqrt{(e^t)^2 + (2t)^2}\, dt}$$

(c) Find an equation for the tangent line to that curve at $t = 1$.

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2t}{e^t}$$

So $\left.\frac{dy}{dx}\right|_{t=1} = \frac{2}{e}$

also when $t=1$: $x(1) = 1 + e^1 = 1 + e$
$y(1) = 1^2 = 1$

so when $t = 1$ the corresponding point on the curve has coordinates $(1+e, 1)$
Therefore, an equation for the tangent line to the curve at $t = 1$ is:

$$\boxed{y - 1 = \frac{2}{e}(x - (1+e))}$$