Math 314 Summer 2010
6/18/2010 Quiz #13

Name: key

Solve two of the following problems. Place an X through the problem you do not want me to grade, otherwise I will grade the first two problems worked on. Show all work clearly and in order. Please box your answers. 10 minutes.

1. Show: $\forall n \geq 1, \sum_{j=1}^{n}(2j-1) = n^2$.

proof (by induction)

Base Case: $(n=1)$. Observe $\sum_{j=1}^{1}(2j-1) = 2(1)-1 = 1 = 1^2 = 1$ ✓

Inductive Step: Induction hypothesis: Let $k \geq 1$ and $\sum_{j=1}^{k}(2j-1) = k^2$

(Goal: $\sum_{j=1}^{k+1}(2j-1) = (k+1)^2$)

Observe: $\sum_{j=1}^{k+1}(2j-1) = \left[\sum_{j=1}^{k}(2j-1)\right] + (2(k+1)-1) \overset{(IH)}{=} k^2 + (2(k+1)-1)$
$= k^2 + 2k + 2 - 1$
$= k^2 + 2k + 1$
$= (k+1)^2$ $\square$

2. Show: $\forall n \geq 7, n! > 3^n$.

proof (by induction)

Base Case: $(n=7)$. Observe $7! = 5040 < 2187 = 3^7$ ✓

Inductive Step: Induction hypothesis: Let $k \geq 7$ and $k! > 3^k$

(Goal: $(k+1)! > 3^{k+1}$)

Observe: $(k+1)! = (k+1)\cdot k! \overset{(IH)}{>} (k+1)\cdot 3^k$. Notice that since $k \geq 7$
$> (3)\cdot 3^k$ $\quad$ so $k+1 \geq 8 > 3$
$= 3^{k+1}$

Hence, $(k+1)! > 3^{k+1}$ $\square$

3. Let $\{s_n\}$ be the sequence defined by

$$s_0 = 0, s_1 = 1, \text{ and } \forall n \geq 2, s_n = 3s_{n-1} - 2s_{n-2}$$

Show: $\forall n \geq 0, s_n = 2^n - 1$.

proof (by Strong Induction)

Base Cases: $(n=0, n=1)$

$S_0 = 2^0 - 1 = 1 - 1 = 0$ ✓
$S_1 = 2^1 - 1 = 2 - 1 = 1$ ✓

Inductive Step: Induction hypothesis: Let $k \geq 1$, and $S_i = 2^i - 1$ for all $0 \leq i \leq k$

(Goal: $S_{k+1} = 2^{k+1} - 1$)

Observe $S_{k+1} = 3S_k - 2S_{k-1}$
$\overset{(IH)}{=} 3(2^k - 1) - 2(2^{k-1} - 1)$
$= 3\cdot 2^k - 3 - 2\cdot 2^{k-1} + 2$
$= 3\cdot 2^k - 2^k - 1$
$= 2\cdot 2^k - 1$
$= 2^{k+1} - 1$

$\square$