

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Show: For every odd integer n , $(-1)^n = -1$.

Proof. Let $n \in \mathbb{Z}$ be odd.

so $\exists k \in \mathbb{Z}$ such that $n = 2k + 1$

Observe $(-1)^n = (-1)^{2k+1} = (-1)^{2k}(-1)^1 = ((-1)^2)^k(-1) = (+1)(-1) = -1$.

□

2. Let $n \in \mathbb{Z}$. Show: If n is even, then $4 \mid n^3$.

Proof. Let $n \in \mathbb{Z}$ be even.

so $\exists k \in \mathbb{Z}$ such that $n = 2k$

Notice that $n^3 = (2k)^3 = 2^3 k^3 = 8k^3 = 4(2k^3)$

since $2k^3 \in \mathbb{Z}$ we have $4 \mid n^3$

□

3. (a) Write all the divisors of 28:

$-28, -14, -7, -4, -2, -1, 1, 2, 4, 7, 14, 28$

- (b) Which of the divisors found in part (a) are prime?

$2, 7$

- (c) Which of the divisors found in part (a) are composite?

$4, 14, 28$