Math 314 Summer 2010
6/23/2010 Quiz #16

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. A partial order relation on a set X is a relation on X that has what three properties?

reflexive
antisymmetric
transitive

2. An equivalence relation on a set X is a relation on X that has what three properties?

reflexive
symmetric
transitive

3. Define the relation R on $\mathbb{R}$ by

$$xRy \text{ if and only if } \lceil x \rceil = \lceil y \rceil.$$

(a) Show that R is an equivalence relation on $\mathbb{R}$.

proof:

R is reflexive: $\forall x \in \mathbb{R}$, $\lceil x \rceil = \lceil x \rceil$, Therefore xRx. ✓

R is symmetric: Let $x, y \in \mathbb{R}$ and suppose xRy
So $\lceil x \rceil = \lceil y \rceil$
So $\lceil y \rceil = \lceil x \rceil$
Therefore yRx. ✓

R is transitive: Let $x, y, z \in \mathbb{R}$ and suppose xRy and yRz
So $\lceil x \rceil = \lceil y \rceil$ and $\lceil y \rceil = \lceil z \rceil$
So $\lceil x \rceil = \lceil y \rceil = \lceil z \rceil$, so $\lceil x \rceil = \lceil z \rceil$
Therefore xRz. ✓ □

(b) Show that R is not an order relation on $\mathbb{R}$.

proof: R is not antisymmetric: (counterexample). Consider for example
$x = 6.3$ and $y = 6.4$, xRy since $\lceil 6.3 \rceil = 7 = \lceil 6.4 \rceil$
and yRx since $\lceil 6.4 \rceil = 7 = \lceil 6.3 \rceil$
but $x = 6.3 \neq 6.4 = y$ i.e. $x \neq y$ □

(c) Find a representative for the equivalence class [4.67] that is not equal to 4.67.

4.32189 (any number with ceiling 5 would be correct other than 4.67)

(d) Draw the real line $\mathbb{R}$, then draw an label the equivalence class [4.67] under the relation R.