Supplementary homework problems for week 5.

1. Let $X = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ where

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

(a) Is the vector $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ in Span(X)? If so, write $\mathbf{u}$ as a linear combination of $\mathbf{x}_1$,$\mathbf{x}_2$, and $\mathbf{x}_3$.

(b) Is the vector $\mathbf{w} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ in Span(X)? If so, write $\mathbf{w}$ as a linear combination of $\mathbf{x}_1$,$\mathbf{x}_2$, and $\mathbf{x}_3$.

2. Show that the set

$$V = \left\{ \begin{bmatrix} 0 \\ x \\ 0 \end{bmatrix} \mid x \text{ is a real number} \right\}$$

is a subspace of $\mathbb{R}^3$. (Hint: use theorem 3.3.2 on p121)

3. Show that the set

$$W = \left\{ \begin{bmatrix} 1 \\ x \end{bmatrix} \mid x \text{ is a real number} \right\}$$

is NOT a subspace of $\mathbb{R}^2$. (Hint: use theorem 3.3.2 on p121)

Solutions :

1. (a) & (b) Using the "membership test" (p122) let's solve both (a) and (b) at the same time. (by creating an augmented matrix $[A|\vec{u}|\vec{w}]$)

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 1 & 2 & 1 \\ -1 & 1 & 0 & 1 & -2 \\ 0 & 2 & 2 & 1 & -2 \end{array}\right] \xrightarrow{R2 \to R2+R1} \left[\begin{array}{ccc|c|c} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 3 & -1 \\ 0 & 2 & 2 & 1 & -2 \end{array}\right] \xrightarrow{R3 \to R3-2R2}$$

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 3 & -1 \\ 0 & 0 & 0 & -5 & 0 \end{array}\right]$$

(a) $\vec{u}$ is not in Span(x) (since the equation $A\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \vec{u}$ has no solution. i.e.) it is inconsistent, i.e) $[A|\vec{u}]$ has a pivot column in the right most column)

(b) $\vec{w}$ is in span(x); The equation $A\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \vec{v}$ has solution :

$d_1 + d_3 = 1$
$d_2 + d_3 = -1$
d_3 = anything

So

$d_1 = 1 - t$
$d_2 = -1 - t$
$d_3 = t$

to write $\vec{w}$ as a lin. comb. of $\vec{x_1}, \vec{x_2}$ and $\vec{x_3}$ pick some t value, say $t = 0$:

$d_1 = 1$
$d_2 = -1$
$d_3 = 0$

Now $\vec{w} = (+1)\vec{x_1} + (-1)\vec{x_2} + 0\vec{x_3}$

So $\boxed{\vec{w} = \vec{x_1} - \vec{x_2}}$

(2) using thm 3.3.2 on p 121 we need to show:

(i) $\vec{0}$ is in V

(ii) if $\vec{x}$ and $\vec{y}$ are in V
<u>then</u> $\vec{x}+\vec{y}$ is in V

(iii) if c is a real number and $\vec{x}$ is in V
<u>then</u> $c\vec{x}$ is in V

<u>solution:</u>

(i)
$$\vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ ← a real number! so $\vec{0} \in V$ ✓

(ii) Let $\vec{x} \in V$ and $\vec{y} \in V$

so we can write: $\vec{x} = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix}$ for some $k \in \mathbb{R}$

<u>and</u> $\vec{y} = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix}$ for some $r \in \mathbb{R}$

then $\vec{x}+\vec{y} = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ k+r \\ 0 \end{bmatrix}$ ← a real number!

so $\vec{x}+\vec{y} \in V$ ✓

(iii) Let $\vec{x} \in V$ and $c \in \mathbb{R}$

so we can write: $\vec{x} = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix}$ for some $k \in \mathbb{R}$

then $c\vec{x} = c\begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ ck \\ 0 \end{bmatrix}$ ← a real number!

so $c\vec{x} \in V$ ✓

③

Notice

$$\vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ not equal to 1 !

hence (i) fails

i.e.) $\vec{0}$ is not in W

so W is not a subspace.

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