Comments for Lecture 5
2.1.2010

Rank.

Please read p27-28 again. The rank will be used several times throughout this course.

Homogeneous Linear Systems.

Recall that a linear equation is said to be *homogeneous* if it is of the the form:

$$c_1x_1 + c_2x_2 + \ldots + c_nx_n = 0$$

(One could think of this geometrically as a hyperplane through the origin).
We say that a system of linear equations is *homogeneous* if each equation in the system is homogeneous. So the system would be of the form:

$$\begin{aligned} c_{11}x_1 + c_{12}x_2 + \ldots + c_{1n}x_n &= 0 \\ c_{21}x_1 + c_{22}x_2 + \ldots + c_{2n}x_n &= 0 \\ \vdots \qquad \vdots \qquad \vdots \qquad &= \vdots \\ c_{m1}x_1 + c_{m2}x_2 + \ldots + c_{mn}x_n &= 0 \end{aligned}$$

Observe that every homogeneous systems must be consistent, since

$$x_1 = 0, x_2 = 0, \ldots, x_n = 0$$

is always a solution to the system (this is called the *trivial solution*).
Also observe that if a homogeneous system of linear equations has a *nontrivial solution*

$$x_1 = s_1, x_2 = s_2, \ldots, x_n = s_n$$

then it must have infinitely many solutions, since

$$x_1 = zs_1, x_2 = zs_2, \ldots, x_n = zs_n$$

is also a solution for any real number z (Check this yourself!). From this we get the following theorem:

Theorem. *A homogeneous system of linear equations has only the trivial solution or it has infinitely many solutions; there are no other possibilities.*

Please read 1.5 for more results we had on homogeneous systems of linear equations.