Math 314 Summer 2010
6/7/2010 Quiz #7

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

The following two proofs have been started for you. Please fill in the missing pieces to complete the proofs.

1. Show: $\mathbb{N}$ has no largest element.

   Proof. (By Contradiction)
   Suppose not.
   So $\mathbb{N}$ has a largest element say $\ell \in \mathbb{N}$.
   Notice $\ell+1 \in \mathbb{N}$ and $\ell+1 > \ell$.
   This contradicts the fact that ℓ was the largest element of $\mathbb{N}$.
   Therefore $\mathbb{N}$ has no largest element. □

2. Show: If $A \subseteq B$ and $C \subseteq D$, then $A \cup C \subseteq B \cup D$.

   Proof. Let $A \subseteq B$ and $C \subseteq D$.
   (We want to show $A \cup C \subseteq B \cup D$)
   Let $x \in A \cup C$
   So $x \in A$ or $x \in C$
   Case 1: $x \in A$
   Since $A \subseteq B$ we have $x \in B$
   Hence $x \in B \cup D$
   Case 2: $x \in C$
   Since $C \subseteq D$ we have $x \in D$
   Hence $x \in B \cup D$
   In either case $x \in B \cup D$.
   Therefore $A \cup C \subseteq B \cup D$ □

3. ♠ Let $a, b, c \in \mathbb{Z}$. Show: If $a|b$ and $b|c$, then $a|c$.

   proof:
   Let $a, b, c \in \mathbb{Z}$ and $a|b$ and $b|c$
   So $\exists k \in \mathbb{Z}$ such that $b = ak$
   and $\exists j \in \mathbb{Z}$ such that $c = bj$
   Notice $c = bj = (ak)j = a(kj)$
   Since $kj \in \mathbb{Z}$ we have $a|c$ □