Quiz 4 - Additional Problems

(1)(a) shows this is not exact (1)(b) we find the integrating factor $T.F. = e^{\times}$

SOL: multiply by I.F., e :

$$\underbrace{e^{\times}y(x+y+1)}_{M(x,y)}dx + \underbrace{e^{\times}(x+2y)}_{N(x,y)}dy = 0$$

$$f(x_1y) = \int H(x_1y) dx + g(y)$$

$$= \int e^{x} y(x_1y_1) dx + g(y)$$

$$= \int (e^{x}y_1x_1 + e^{x}y_2^2 + e^{x}y_1) dx + g(y)$$

$$= \int e^{x}(x_1y_1) dx + e^{x}y_2^2 + e^{x}y_1 dx + g(y)$$

$$= \int e^{x}(x_1y_1) dx + g(y_1) dx + g(y_1)$$

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$$= \int e^{x}y(x_1y_1) dx + g(y$$

$$\frac{\partial f}{\partial y} = e^{x}(x-1) + 2e^{x}y + e^{x} + g'(y) = e^{x}(x+2y)$$

$$xe^{x} - e^{x} + 2e^{x}y + e^{x} + 9'(y) = xe^{x} + 2ye^{x}$$

$$g'(y) = 0$$

$$g(y) = C$$

$$f(x,y) = ye^{x}(x-1) + e^{x}y^{2} + e^{x}y + C$$

= $yxe^{x} + y^{2}e^{x} + C$
 $solution : f(x,y) = D$

$$y \times e^{x} + y^{2}e^{x} + C = D$$

$$x y e^{x} + y^{2}e^{x} = E$$

(2)(a) shows this is homogeneous (2)(b) gives a possible substitution.

SOL:
Let's use the substitution
$$u = \frac{y}{x}$$
 (you can also use)

So
$$y = u \times dx + x du$$

substituting into
 $(y^2 + y \times) dx + x^2 dy = 0$
we get
 $((ux)^2 + (ux) \times) dx + x^2 (udx + x du) = 0$
 $(u^2 x^2 + ux^2) dx + x^2 u dx + x^3 du = 0$
 $u^2 x^2 dx + ux^2 dx + x^2 u dx + x^3 du = 0$
 $u^2 x^2 dx + 2u x^2 dx + x^3 du = 0$
 $(u^2 x^2 + 2u x^2) dx = -x^3 du$
 $x^2 (u^2 + 2u) dx = -x^3 du$
 $\frac{x^2}{x^3} dx = \frac{1}{u^2 + 2u} du$

$$\frac{1}{x} dx = -\frac{1}{u(u+2)} du = \left[\frac{A}{u} + \frac{B}{u+2} \right] du$$

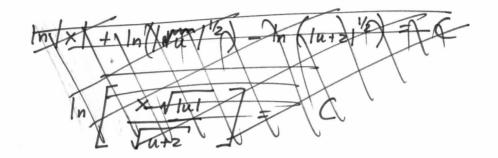
$$A(u+2) + Bu = -1$$

 $Au + 2A + Bu = -1$
 $(A+B)u + 2A = -1$
 $A = -\frac{1}{2}$
 $A+B = 0 \implies B = \frac{1}{2}$

$$\int \frac{1}{x} dx = \int \left[\frac{-1}{2} \cdot \frac{1}{u} + \frac{1}{2} \cdot \frac{1}{(u+2)} \right] du$$

$$\ln|x| + C_1 = -\frac{1}{2} \ln|u| + \frac{1}{2} \ln|u+2| + C_2$$

$$\ln|x| + \frac{1}{3} \ln|u| - \frac{1}{3} \ln|u+2| = C$$



$$2\ln|x| + \ln|u| - \ln|u+2| = D$$

$$\ln(x^2) + \ln|u| - \ln|u+2| = D$$

$$\ln\left(\frac{x^2|u|}{\ln + 2}\right) = D$$

$$\frac{x^2/u/}{|u+z/|} = e^D = E$$

$$\frac{x^2u}{u+2} = F$$

$$\frac{x^2(\frac{4}{2})}{\frac{4}{2}+2} = F$$

$$X^{2}\left(\frac{y}{x}\right) = F\left(\frac{y}{x} + 2\right)$$

$$x^2y = F(y + 2x)$$