**Simpler Example:**

Find the solution set to the following system of linear equations:

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ -1 & 2 & -1 & 1 \\ 2 & 1 & 3 & 0 \end{array}\right] \xrightarrow{R2\to R2+R1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{array}\right] \xrightarrow{R3\to R3-2R1}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 3 & 1 & -4 \end{array}\right] \xrightarrow{R3\to R3-3R2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -13 \end{array}\right] \xrightarrow{R1\to R1+R2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -13 \end{array}\right]$$

$$\xrightarrow{R1\to R1-R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 18 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -13 \end{array}\right]$$

So we have exactly one solution to the system: $x_1 = 18$, $x_2 = 3$ and $x_3 = 13$. Now just to make sure that we did not make a mistake we should check our answer...

**Good Practice with fractions:**

Add -2/3 times row 1 to row 2
Add 2/3 times row 1 to row 3
Add -8 times row 2 to row 3

Multiply row 1 by 1/3
Multiply row 2 by 3
Add -1/3 times row 2 to row 1
Multiply row 3 by -1/19
Add 3 times row 3 to row 1
Add -8 times row 2 to row 3

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 10 \\ 2 & 1 & 2 & 5 \\ -2 & 2 & 3 & 1 \end{array}\right] \xrightarrow{R2\to R2-\frac{2}{3}R1} \left[\begin{array}{ccc|c} 3 & 1 & -1 & 10 \\ 0 & 1/3 & 8/3 & -5/3 \\ -2 & 2 & 3 & 1 \end{array}\right] \xrightarrow{R3\to R3+\frac{2}{3}R1}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 10 \\ 0 & 1/3 & 8/3 & -5/3 \\ 0 & 8/3 & 7/3 & \frac{23}{3} \end{array}\right] \xrightarrow{R3\to R3-8R2} \left[\begin{array}{ccc|c} 3 & 1 & -1 & 10 \\ 0 & 1/3 & 8/3 & -5/3 \\ 0 & 0 & -19 & 21 \end{array}\right] \xrightarrow{R1\to \frac{1}{3}R1}$$

$$\left[\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 10/3 \\ 0 & 1/3 & 8/3 & -5/3 \\ 0 & 0 & -19 & 21 \end{array}\right] \xrightarrow{R2\to 3R2} \left[\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 10/3 \\ 0 & 1 & 8 & -5 \\ 0 & 0 & -19 & 21 \end{array}\right] \xrightarrow{R1\to R1-\frac{1}{3}R2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 5 \\ 0 & 1 & 8 & -5 \\ 0 & 0 & -19 & 21 \end{array}\right] \xrightarrow{R3\to \frac{-1}{19}R3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 5 \\ 0 & 1 & 8 & -5 \\ 0 & 0 & 1 & -\frac{21}{19} \end{array}\right] \xrightarrow{R1\to R1+3R3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{32}{19} \\ 0 & 1 & 8 & -5 \\ 0 & 0 & 1 & -\frac{21}{19} \end{array}\right] \xrightarrow{R2\to R2-8R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{32}{19} \\ 0 & 1 & 0 & \frac{73}{19} \\ 0 & 0 & 1 & -\frac{21}{19} \end{array}\right]$$