Math 314 Summer 2010

5/28/2010
Quiz #3

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Write the following statements efficiently using quantifiers and standard notation.
   (a) For every integer $m$, $2^{m+1}$ is positive.

   $\forall m \in \mathbb{Z}, 2^{m+1} > 0$

   (b) There exists an intger $n$ such that $n$ is not a natural number.

   $\exists n \in \mathbb{Z}$ such that $n \notin \mathbb{N}$

   (c) The product of any two real numbers is a real number.

   $\forall x, y \in \mathbb{R}, xy \in \mathbb{R}$

   (d) For every positive real number $x$, there is a real number $y$ such that $xy = 1$.

   $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}$ such that $xy = 1$

2. Which statements from question 1 are true?

   All of them: (a), (b), (c) and (d)

3. Negate the the following statements.
   (a) $\exists n \in \mathbb{Z}^-$ such that $5n + 2 > 1$.

   $\forall n \in \mathbb{Z}^-, 5n+2 \leq 1$

   (b) $\forall x \in \mathbb{R}^+$, if $x^2 > 4$ then $x > 2$. ← this is equivalent to: $\forall x \in \mathbb{R}^+, x^2 \leq 4$ or $x > 2$ (by the equivalence of logical statements $p \to q \equiv \neg p \vee q$)

   $\exists x \in \mathbb{R}^+, x^2 > 4$ and $x \leq 2$

   (c) $\forall p, q \in \mathbb{Z}, p + q \in \mathbb{Z}$.

   $\exists p, q \in \mathbb{Z}$ such that $p+q \notin \mathbb{Z}$

   (d) $\forall n \in \mathbb{R}, \exists x \in \mathbb{R}$ such that $e^{x^y} \in \mathbb{Z}$.

   $\exists n \in \mathbb{R}$ such that $\forall x \in \mathbb{R}, e^{x^y} \notin \mathbb{Z}$

4. ♠ Let $A$ be a set. Let $f : A \subseteq \mathbb{R} \to \mathbb{R}$. Negate the following statement.

$$\forall x \in A, \forall \epsilon > 0, \exists \delta > 0 \text{ such that } \forall y \in A, \text{ if } |x - y| < \delta \text{ then } |f(x) - f(y)| < \epsilon.$$

$\exists x \in A, \exists \varepsilon > 0, \forall \delta > 0, \exists y \in A$ such that $|x-y| < \delta$ and $|f(x) - f(y)| \geq \varepsilon$