

Normal Subgroups / Factor Groups.

(Q1) in S_3 is $\langle (12) \rangle$ a normal subgroup?

SOL:

$$\langle (12) \rangle = \{ (1), (12) \}$$

Now consider using the result:

$$\{ H \triangleleft G \iff \forall x \in G, x H x^{-1} \subseteq H \}$$

In this particular case, if $H = \langle (12) \rangle$

$$\langle (12) \rangle = H \triangleleft S_3 \iff \forall x \in S_3, x H x^{-1} \subseteq H.$$

Let $x = (13)$ and let's compute:

$$\begin{aligned} x (12) x^{-1} &= (13) \underbrace{(12) (13)}_{\substack{\uparrow \\ \text{in } H}} \\ &= (13) (132) \\ &= (1)(23) \\ &= (23) \notin H \end{aligned}$$

↑ so we found an example where $x H x^{-1} \subseteq H$ FAILS.

Hence $\langle (12) \rangle$ is NOT a normal subgroup of S_3 .