Math 314 Summer 2010
6/8/2010 Quiz #8

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Show: For every even integer n, $(-1)^n = 1$.

Proof. Let $n \in \mathbb{Z}$ be even.

so $\exists k \in \mathbb{Z}$ such that $n = 2k$

so $(-1)^n = (-1)^{2k} = ((-1)^2)^k = (1)^k = 1$

□

2. Let $n \in \mathbb{Z}$. Show: If n is even, then $4 \mid n^2$.

Proof. Let $n \in \mathbb{Z}$ be even.

so $\exists k \in \mathbb{Z}$ such that $n = 2k$

So $n^2 = (2k)^2 = 4k^2$

Since $k^2 \in \mathbb{Z}$ we have $4 \mid n^2$

□

3. (a) Write all the divisors of 28: NOTE: I did not ask for the positive divisors.

$\boxed{-28, -14, -7, -4, -2, -1, 1, 2, 4, 7, 14, 28}$

(b) Which of the divisors found in part (a) are prime?

$\boxed{2, 7}$

(c) Which of the divisors found in part (a) are composite?

$\boxed{4, 14, 28}$

4. Find gcd(56, 42).

$56 = 28 \cdot 2 = 14 \cdot 2 \cdot 2 = 7 \cdot 2 \cdot 2 \cdot 2 = 2^3 \cdot 7$

$42 = 21 \cdot 2 = 7 \cdot 3 \cdot 2 = 2 \cdot 3 \cdot 7$

Hence $\gcd(56, 42) = 2^1 \cdot 3^0 \cdot 7^1 = 2 \cdot 1 \cdot 7 = \boxed{14}$