COMMENTS FOR LECTURE 6 - 2.3.2010

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Start homework 2 as soon as possible!

"The Function".

Given a system of linear equations with an $m \times n$ coefficient matrix C, the system looks like:

$$\begin{aligned} c_{11}x_1 + c_{12}x_2 + \ldots + c_{1n}x_n &= k_1 \\ c_{21}x_1 + c_{22}x_2 + \ldots + c_{2n}x_n &= k_2 \\ \vdots \qquad \vdots \qquad \vdots \qquad &= \vdots \\ c_{m1}x_1 + c_{m2}x_2 + \ldots + c_{mn}x_n &= k_m \end{aligned}$$

We talked about how we can actually think of this as a function. The input here would be the *n-tuple*: $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and the output would be the *m-tuple*: $\mathbf{k} = (k_1, k_2, \ldots, k_m)$. So now we think of C as a function having domain $\mathbb{R}^n$ and codomain $\mathbb{R}^m$ written:

$$C : \mathbb{R}^n \to \mathbb{R}^m$$

(NOTE: The dimentions of C as a matrix are $m \times n$)
and we write

$$C(\mathbf{x}) = \mathbf{k}$$

As we discussed in class **Theorem 1.6.2** was extremely nice because we were able to determine exactly when C is *onto* and when it is *one-to-one*. An amazing thing is that we can get this information for free when we compute *rank* of C!

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