TEST 3

Math 104

Score: _____ out of 100

4/4/2013

Name: _____ Key _____

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.

- Show all work (if necessary), clearly and in order, if you want to recieve full credit. Please make sure you read the directions for each problem. If the problem requires work I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- Please box/circle or otherwise indicate your final answers.

- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.

- This test has 8 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!

- All answers must be exact, no rounding. Please keep things in fraction form whenever possible.

- Good luck!

1. Let $g(x) = 3 + (x+1)^2$. Evaluate:

(a) $g(-1) =$ $3 + (-1+1)^2 = 3 + 0^2 = \boxed{3}$

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(b) $g(a+1) =$ $3 + ((a+1)+1)^2 = 3 + (a+2)^2$

$$= 3 + (a+2)(a+2)$$
$$= 3 + a^2 + 4a + 4$$
$$= \boxed{a^2 + 4a + 7}$$

(c) $\dfrac{g(a+1) - g(a)}{3} = \dfrac{(a^2 + 4a + 7) - [3 + (a+1)^2]}{3}$

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$$= \dfrac{a^2 + 4a + 7 - (3 + a^2 + 2a + 1)}{3}$$

$$= \dfrac{a^2 + 4a + 7 - a^2 - 2a - 4}{3} = \boxed{\dfrac{2a+3}{3}}$$

2. Let $f(x) = 2x - 1$ and $g(x) = \sqrt{3x+3}$. Evaluate:

(a) $f(g(x)) = f\left(\sqrt{3x+3}\right)$

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$$= \boxed{2\sqrt{3x+3} - 1}$$

(b) $g(f(x)) = g(2x-1)$

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$$= \boxed{\sqrt{3(2x-1)+3}} \quad \overset{or}{\longrightarrow} = \sqrt{6x-3+3} \quad = \boxed{\sqrt{6x}}$$

(c) $f(f(x)) = f(2x-1)$

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$$= \boxed{2(2x-1) -1} \quad \overset{or}{\longrightarrow} = 4x - 2 - 1 = \boxed{4x - 3}$$

(d) $f(g(6)) =$ $2\sqrt{18+3} - 1$

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$$= \boxed{2\sqrt{21} - 1}$$

3. Solve $4x^2 - 12x + 5 = 0$ by factoring:

SOL 1 (by grouping)

$$ac = 4 \cdot 5 = 20$$

find two factors that sum to -12:

$$-10 \text{ and } -2$$

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$$0 = 4x^2 - 12x + 5 = 4x^2 - 10x - 2x + 5$$
$$= 2x(2x - 5) - (2x - 5)$$
$$= (2x - 5)(2x - 1).$$

$2x - 5 = 0$	$2x - 1 = 0$
$2x = 5$	$2x = 1$
$\boxed{x = \dfrac{5}{2}}$ and	$\boxed{x = \dfrac{1}{2}}$

SOL 2: by guessing

$$4x^2 - 12x + 5 = 0$$
$$(2x - 5)(2x - 1) = 0$$
$$\boxed{x = 5/2} \text{ and } \boxed{x = 1/2}$$

4. Solve $x^2 + 8x - 209 = 0$ using the quadratic formula:

$$a = 1 \qquad b = 8 \qquad c = -209$$

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$$x = \frac{-8 \pm \sqrt{64 - 4(1)(-209)}}{2(1)} = \frac{-8 \pm \sqrt{64 + 836}}{2}$$

$$= \frac{-8 \pm \sqrt{900}}{2}$$

$$= \frac{-8 \pm 30}{2}$$

$$x = \frac{-8 + 30}{2} \qquad \text{and} \qquad x = \frac{-8 - 30}{2}$$

$$x = \frac{22}{2} \qquad \text{and} \qquad x = \frac{-38}{2}$$

$$\boxed{x = 11} \qquad \text{and} \qquad \boxed{x = -19}$$

$\boxed{10}$

5. For the quadratic function
$$y = 2x^2 - 8$$

(a) Find the **axis of symmetry** $a = 2, \quad b = 0, \quad c = -8$

$$x = \frac{-b}{2a} = \frac{-0}{2(2)} = 0$$

$$\boxed{x = 0}$$

(b) Find the coordinates of the **vertex**

$$x = 0 \quad \xrightarrow{\text{plug into}} \quad y = 2x^2 - 8$$

$$y(0) = 0 - 8 = -8$$

$$\boxed{(0, -8)}$$

(c) Find any x-intercept(s)

SOL 1:

$$0 = 2x^2 - 8$$
$$8 = 2x^2$$
$$4 = x^2$$
$$\pm\sqrt{4} = x$$
$$\pm 2 = x$$
$$\boxed{x = 2 \quad \text{and} \quad x = -2}$$

SOL 2:

$$0 = 2x^2 - 8$$
$$0 = 2(x^2 - 4)$$
$$0 = 2(x - 2)(x + 2)$$

$$x - 2 = 0 \quad | \quad x + 2 = 0$$
$$\boxed{x = 2} \quad \text{and} \quad \boxed{x = -2}$$

(d) Find the y-intercept

plug $x = 0$ into the function. (already did this.)

$$y(0) = 0 - 8 = -8$$

$$\boxed{b = -8}$$

OR

$$\boxed{(0, -8)}$$

[20]

6. Suppose a company has $R(x) = -5x^2 + 750x$ and cost $C(x) = 100x + 20000$.

   (a) Find the **profit function** $\pi(x)$

$$\pi(x) = R(x) - C(x)$$
$$\pi(x) = -5x^2 + 750x - (100x + 20000)$$
$$\boxed{\pi(x) = -5x^2 + 650x - 20{,}000}$$

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   (b) Find the **maximum level of profit**.

$$x = \frac{-b}{2a} = \frac{-650}{2(-5)} = \frac{-650}{-10} = 65$$

$$\pi(65) = \boxed{1125}$$

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   (c) Find the **break-even** levels of output.

$$\pi(x) = 0 \qquad \left(OR \qquad R(x) = C(x) \right)$$
$$-5x^2 + 650x - 20000 = 0$$

S $$x = \frac{-650 \pm \sqrt{650^2 - 4(-20000)(-5)}}{2(-5)} = \frac{-650 \pm \sqrt{22500}}{-10} = \frac{-650 \pm 150}{-10}$$

$$x = \frac{-650 + 150}{-10} = \boxed{50} \quad and \quad x = \frac{-650 - 150}{-10}$$
$$x = \boxed{80}$$

   (d) Find the **x-intercept(s)** for the function $R(x)$.

$$0 = R(x)$$
$$0 = -5x^2 + 750x$$
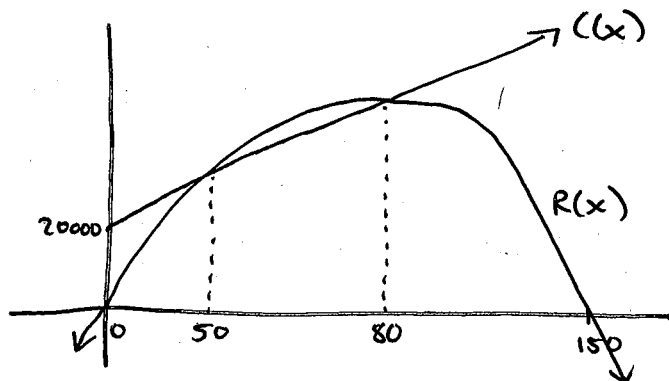$$0 = x(-5x + 750)$$
$$\boxed{x = 0} \quad and \quad -5x + 750 = 0$$
$$-5x = -750$$
$$\boxed{x = 150}$$

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   (e) Using the above information sketch a graph of both $R(x)$ and $C(x)$ on the same graph, and label the **break-even** points.

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7. Solve the following system using **elimination**

$$2\left(2x - 3y = 4\ \right)$$
$$4x + y = 5$$

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$$4x - 6y = 8$$
$$-(4x + y = 5)$$
_____
$$0 - 7y = 3$$

$$y = \frac{-3}{7} \longrightarrow 4x + \left(\frac{-3}{7}\right) = 5$$

$$4x = \cancel{} \ 5 + \frac{3}{7} = \frac{38}{7}$$

$$x = \frac{38}{28} = \cancel{}\ \frac{19}{14}$$

$$\boxed{x = \cancel{}\frac{19}{14}\ \text{and}\ y = -3/7}$$

8. Solve the following system using **substitution**

$$\left(\begin{array}{c} x - 5y = 10 \\ 3x + 2y = 6 \end{array}\right.$$

↓

$$x = 10 + 5y$$

So

$$3x + 2y = 6$$
$$3(10 + 5y) + 2y = 6$$
$$30 + 15y + 2y = 6$$
$$30 + 17y = 6$$
$$17y = -24$$
$$y = \frac{-24}{17} \longrightarrow x = 10 + 5\left(\frac{-24}{17}\right) = \frac{50}{17}$$

$$\boxed{x = \frac{50}{17} \quad \text{and} \quad y = \frac{-24}{17}}$$

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