Math 314 Summer 2010
6/21/2010 Quiz #14

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Use the binomial theorem to expand and simplify $(2x-y)^3$.

$$(2x-y)^3 = \sum_{k=0}^{3} \binom{3}{k}(2x)^{3-k}(-y)^k$$
$$= \binom{3}{0}(2x)^{3-0}(-y)^0 + \binom{3}{1}(2x)^{3-1}(-y)^1 + \binom{3}{2}(2x)^{3-2}(-y)^2 + \binom{3}{3}(2x)^{3-3}(-y)^3$$
$$= \binom{3}{0}(2x)^3\cdot 1 + \binom{3}{1}(2x)^2(-y)^1 + \binom{3}{2}(2x)^1(-y)^2 + \binom{3}{3}(2x)^0(-y)^3$$
$$= 1\cdot 2^3\cdot x^3 + 3\cdot 2^2 x^2\cdot(-y) + 3\cdot 2\cdot x\, y^2 + 1\cdot 1\cdot(-y)^3$$
$$= \boxed{8x^3 - 12x^2y + 6xy^2 - y^3}$$

2. Find the coefficient of $x^{60}y^{40}$ in $(3x+2y)^{100}$.

The Binomial Thm. gives us $(3x+2y)^{100} = \sum_{k=0}^{100}\binom{100}{k}(3x)^{100-k}(2y)^k$

Consider when $k=40$. Then we have the term:

$$\binom{100}{40}(3x)^{100-40}(2y)^{40} = \binom{100}{40}(3x)^{60}(2y)^{40} = \binom{100}{40}3^{60}\cdot x^{60}\cdot 2^{40}\cdot y^{40}$$
$$= \binom{100}{40}3^{60}\cdot 2^{40}\cdot x^{60}\cdot y^{40}$$

so the coefficient of $x^{60}y^{40}$ in $(3x+2y)^{100}$ is

$$\boxed{\binom{100}{40}3^{60}\cdot 2^{40}}$$

3. Verify the identity: $\sum_{i=0}^{n}\binom{n}{i}5^{n-i}3^i = 2^{3n}$.

proof.:

$$2^{3n} = (2^3)^n = 8^n = (5+3)^n \underset{\text{by binomial thm.}}{=} \sum_{i=0}^{n}\binom{n}{i}5^{n-i}3^i$$

$\square$