Math 304 Spring 2010
2/25/2010
Quiz #5

Name: ____________________

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$

   (a) Compute A^{-1}.

   (b) Write A^{-1} as a product of elementary matrices.

   (c) Write A as a product of elementary matrices.

   (d) Use the A^{-1} computed in part (a) to solve the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

(a)
$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R2 \to R2+2R1]{①} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R3 \to R3+3R2]{②}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 6 & 3 & 1 \end{array}\right] \quad \text{so} \quad \boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}}$$

(b) operation ① corresponds to $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

operation ② corresponds to $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ so $\boxed{A^{-1} = E_2E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$

(c) $E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$ so $\boxed{A = E_1^{-1}E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}}$

(d)
$$A^{-1}Ax = x = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}}$$

2. Give an example of a 2×2 matrix which is not the zero matrix and is NOT invertible.

$$\boxed{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}$$