Math 481 - Graded Problems #3 Name: _____
Fall 2013 - Nathan Reff

1. Let $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \;\middle|\; a, b, d \in \mathbb{R} \text{ and } ad \neq 0 \right\}$ under the operation of matrix multiplication.

   (a) Show that H is a subgroup of $GL(2, \mathbb{R})$. That is, show $H \leq GL(2, \mathbb{R})$.

   (b) Find $C\left(\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}\right)$ in H.

2. Let n be a positive integer. Let

$$n\mathbb{Z} := \{nm \mid m \in \mathbb{Z}\}.$$

   For example,

$$2\mathbb{Z} = \{2m \mid m \in \mathbb{Z}\} = \{\ldots, -4, -2, 0, 2, 4, \ldots\}.$$

   (a) Show that $(n\mathbb{Z}, +) \leq (\mathbb{Z}, +)$.

   (b) Show that $(\mathbb{Z}, +)$ is cyclic.

   (c) Show that $(n\mathbb{Z}, +)$ is cyclic.