EXAM 2

Math 314 - Discrete Mathematics
6/15/2011

Name: ______________________

When you are finished please sign the following:

Signature: $\underbrace{\hspace{8cm}}$

By signing my name I pledge that I have not broken the Student Academic Honesty Code at any point during this examination.

Read all of the following information before starting the exam:

- You have 60 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please box/circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 11 problems and is worth 100 points. There is 1 bonus problem. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (a) Show: There is a set A such that $\{-1, 4, 2\} \backslash A = \{2\}$.

   (b) Show: There exist sets A and B such that $A \backslash B = B \backslash A$.

   (c) Show: $\exists n \in \mathbb{Z}$ such that $n^2 - 4 = 0$.

2. Show: $\forall x \in \mathbb{R}$, if $x \in (-1, 4]$, then $3x - 1 \in (-4, 11]$.

3. Let A, B and C be abritrary sets in some universe $\mathcal{U}$.

   Show: If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

4. This problem has been started for you. Fill in the missing pieces of the proof.

   Let A and B be arbitrary sets in some universe $\mathcal{U}$.
   Show: $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

   Proof. ($\subseteq$) Let $C \in \mathcal{P}(A \cap B)$

   ($\supseteq$) Let $C \in \mathcal{P}(A) \cap \mathcal{P}(B)$

   $\square$

5. Let A, B and C be arbitrary sets in some universe $\mathcal{U}$.
   Show: If $A \subseteq B \subseteq C$, then $A \times A \subseteq C \times C$.

6. Let A, and B be arbitrary sets in some universe $\mathcal{U}$.
   (a) Show: $A \backslash B \subseteq A$.

   (b) Show: $B \backslash A \subseteq B$.

   (c) Show: $A \Delta B \subseteq A \cup B$ (Hint: use parts (a) and (b) at some point).

7. Show: $\forall x \in \mathbb{R}^+$, $x^3 = x$ if and only if $x = 1$.

8. Show: The interval $(0, 1]$ has no smallest element.

9. Let $n \in \mathbb{Z}$. Show: If n^2 is odd, then n is odd. (Hint: Contrapositive)

10. Show: $\forall a, b \in \mathbb{Z}$, if $a \mid b$, then $a^2 \mid b^2$.

11. Prove that for all integers n, $n^2 - n + 3$ is odd.

12. (a) ♠ Let $n \in \mathbb{Z}$. Show: If n^2 is divisible by 3, then n is divisible by 3.

    (b) ♠♠ Prove that $\sqrt{3}$ is irrational.