1. A. True | False If A is a 5×5 matrix and the rank of A is 4 then $\det(A) = 0$.

   B. True | False There exists an isomorphism from P_{17} to $\mathbb{R}^{17}$.

   C. True | False $\mathbb{R}^4$ has a basis X such that each vector in $\mathbb{R}^4$ can be written in more than one way as a linear combination of the elements of X.

   D. True | False P_4 has a basis X such that each polynomial (vector) in P_4 can be written in more than one way as a linear combination of the elements of X.

   E. True | False The set of functions $\{c_2x^2 + c_3x^3 + c_4x^4 \mid c_2, c_3, c_4 \in \mathbb{R}\}$ is a subspace of P_4.

   F. True | False If X is a collection of vectors in a vector space W, then $\text{Span}(X)$ is a subspace of W.

   G. True | False $\text{Span}\left(\left\{\begin{bmatrix} 4 \\ 8 \\ -2 \end{bmatrix}\right\}\right)$ is a subspace of $\mathbb{R}^3$.

   H. True | False $\text{Span}(\{1, 1 - x^2\})$ is a subspace of P_2.

   I. True | False If the set S is linearly independent in P_4 then $S \cup \{1 + x\}$ is always linearly independent.

   J. True | False If S is a spanning set of P_4 then S always contains the vector (polynomial) x^3.

   K. True | False If S is a spanning set of P_4 then $\text{Span}(S)$ always contains the vector (polynomial) x^3.

   L. True | False The linear transformation $T{:}\mathbb{R}^2 \to \mathbb{R}^2$ with associated matrix $\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$ is an isomorphism.

2. Suppose A and B are both 5×5 matrices where $\det(A) = 5$ and $\det(B) = 3$. Evaluate $\det(B^{-1}AB^{\mathrm{T}}AB^2)$.

3. (a) Show that the set $B = (1 + 2x^2, 1 + x, 1 + x + x^2)$ is a basis of P_2.

   (b) There is a polynomial $p(x)$ in P_2 which has the coordinate vector $K_B(p(x)) = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$ with repect to the basis B from part (a). Find $p(x)$.

   (c) Find $K_B(7 + 3x - x^2)$ where B is the basis from part (a).

4. Show the set $Y = \{a_0 + a_1x + a_2x^2 \in P_2 \mid a_0 + a_1 + a_2 = 0\}$ is a subspace of P_2.

5. Let $S = \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$ be the standard basis of $\mathbb{R}^2$. Let $X = \left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ be a basis of $\mathbb{R}^2$ (you do not need to show this). Let $T{:}\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by the matrix (with respect to the basis S)

$$ {}_ST_S = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} $$

   (a) Find ${}_SI_X$.

   (b) Find ${}_XI_S$.

   (c) Using parts (a) and (b) find ${}_XT_X$.

   (d) Show that T is an isomorphism.