Math 222 Spring 2011
2/11/2011
Quiz #3 Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1 1. Evaluate the following. No work is needed.

(a) $\frac{d}{dx}(\sin^{-1} x) = \boxed{\frac{1}{\sqrt{1-x^2}}}$

(b) $\frac{d}{dx}(\cos^{-1} x) = \boxed{\frac{-1}{\sqrt{1-x^2}}}$

(c) $\frac{d}{dx}(\tan^{-1} x) = \boxed{\frac{1}{1+x^2}}$

(d) $\int \frac{1}{\sqrt{1-x^2}} dx = \boxed{\sin^{-1}(x) + C}$

(e) $\int \frac{1}{1+x^2} dx = \boxed{\tan^{-1}(x) + C}$

(f) $\lim_{x\to\infty} \tan^{-1} x = \boxed{\frac{\pi}{2}}$

4 2. (a) $\tan(\sin^{-1}(\frac{2}{3})) =$

A. $\frac{2}{5}$

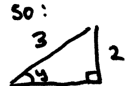
B. $-\frac{2}{\sqrt{5}}$

C. $\boxed{\frac{2}{\sqrt{5}}}$

D. $\frac{2}{3}$

Let $y = \sin^{-1}(\frac{2}{3})$

So:



$\sqrt{3^2-2^2} = \sqrt{5}$

So $\tan(y) = \frac{2}{\sqrt{5}}$

(b) $\lim_{x\to 0^+} x\ln(x) =$

$\boxed{\text{A. } 0}$

B. 1

C. $+\infty$

D. $-\infty$

$\lim_{x\to 0^+} \frac{\ln(x)}{(\frac{1}{x})} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}}$

$= \lim_{x\to 0^+} -x$

$= 0$

4 3. (a) Differentiate: $y = \cos^{-1}(e^x)$.

A. $\boxed{\frac{-e^x}{\sqrt{1-e^{2x}}}}$

B. $\frac{e^x}{\sqrt{1-e^{2x}}}$

C. $\frac{-1}{\sqrt{1-e^{2x}}}$

D. $\frac{e^x}{1+e^{2x}}$

$\frac{d}{dx} y = \frac{-1}{\sqrt{1-(e^x)^2}} \cdot \frac{d}{dx} e^x$

$= \frac{-1}{\sqrt{1-e^{2x}}} \cdot e^x$

(b) Evaluate: $\int \frac{1}{4+x^2} dx$.

A. $4\tan^{-1}(x) + C$

B. $\boxed{\frac{1}{2}\tan^{-1}(\frac{x}{2}) + C}$

C. $\frac{1}{4}\tan^{-1}(\frac{x}{4}) + C$

D. $\frac{1}{2}\tan^{-1}(2x) + C$

$= \int \frac{1}{4(1+\frac{x^2}{4})} dx$

$= \int \frac{1}{4(1+(\frac{x}{2})^2)} dx$

Let $u = \frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2}$

$\Rightarrow dx = 2du$

$= \int \frac{1}{4(1+u^2)} \cdot 2du = \frac{1}{2}\tan^{-1}(u) + C$

$= \frac{1}{2}\tan^{-1}(\frac{x}{2}) + C$

2 4. Pick ONE of the following. Cross out the problem you do not want graded. Otherwise I will grade the first problem worked on.

(a) How long will it take an investment to triple in value if the interest rate is 5% compounded continously.

$A(t) = A_0 e^{0.05t}$

Solve for t:

$3A_0 = A_0 e^{0.05t}$

$3 = e^{0.05t}$

$\ln(3) = \ln(e^{0.05t})$

$\ln(3) = 0.05t$

$\boxed{t = \frac{\ln(3)}{0.05}}$ (years)

(b) $\lim_{x\to\infty} \left(1 + \frac{4}{5x}\right)^x =$

SOL1: recall $\lim_{x\to\infty} (1+\frac{1}{x})^x = e$

$= \lim_{x\to\infty} \left(1 + \frac{1}{(\frac{5x}{4})}\right)^x = \lim_{x\to\infty} \left(1 + \frac{1}{(\frac{5x}{4})}\right)^{x\cdot\frac{5}{5}\cdot\frac{4}{4}}$

$= \lim_{x\to\infty} \left(1 + \frac{1}{(\frac{5x}{4})}\right)^{\frac{5x}{4}\cdot\frac{4}{5}}$

$= \left(\lim_{x\to\infty} \left(1 + \frac{1}{(\frac{5x}{4})}\right)^{\frac{5x}{4}}\right)^{4/5}$

$= \boxed{e^{4/5}}$

SOL2: This limit is of type 1^∞:

Let $y = (1+\frac{4}{5x})^x$

$\downarrow$

$$\ln(y) = \ln\left(1 + \frac{4}{5x}\right)^x$$
$$= x \ln\left(1 + \frac{4}{5x}\right)$$

$$\lim_{x\to\infty} \ln(y) = \lim_{x\to\infty} x \ln\left(1 + \frac{4}{5x}\right) \quad \leftarrow \text{Type } \infty\cdot 0$$

$$= \lim_{x\to\infty} \frac{\ln\left(1 + \frac{4}{5x}\right)}{\frac{1}{x}} \quad \leftarrow \text{Type } \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{\left(\frac{1}{1+\frac{4}{5x}}\right)\cdot \frac{d}{dx}\left(1+\frac{4}{5x}\right)}{\left(\frac{-1}{x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{1+\frac{4}{5x}}\right)\cdot \frac{-4}{5x^2}}{\left(\frac{-1}{x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{1}{1+\frac{4}{5x}} \cdot \frac{-4}{5\cancel{x^2}} \cdot -\cancel{x^2}$$

$$= \lim_{x\to\infty} \frac{\frac{4}{5}}{1+\frac{4}{5x}} = \frac{\frac{4}{5}}{1+0} = \frac{4}{5}$$

so $y \longrightarrow \boxed{e^{4/5}}$

(Formally, $\lim_{x\to\infty} \left(1 + \frac{4}{5x}\right)^x = \lim_{x\to\infty} y$

$$= \lim_{x\to\infty} e^{\ln(y)}$$
$$= e^{\lim_{x\to\infty} \ln(y)}$$
$$= \boxed{e^{4/5}}$$)