Math 222:14 Spring 2011

Working with the infinite series: $\sum_{n=1}^{\infty} a_n$

This is a guide for how to evaluate if an infinite series is convergent or divergent.

1^{st} Check the n^{th} term of $\sum a_n$. If $\lim_{n\to\infty} a_n \neq 0$ then the series diverges, and you are done (Use T.F.D.). However if $\lim_{n\to\infty} a_n = 0$, then proceed to step 2.

2^{nd} Check to see if the series is harmonic, geometric, or a p-series.

- Geometric Series: $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=0}^{\infty} ar^n$. Now we check if $|r| < 1$.
- p-series/Harmonic: $\sum_{n=1}^{\infty} \frac{1}{n^p}$. Now we check to see if $p > 1$.

If it is none of these, go to step 3, step 6, or step 9.

If the series is all positive numbers (after some point):	
3^{rd}	Try comparing the series to one that you know. Usually the series you want to compare to are geometric series, p-series, and/or the series made by only using the dominating terms from your numerator and denominator. *Make sure the series you're comparing to is one you can evaluate.* Use either the Comparison Test or Limit Comparison Test.
4^{th}	If that didn't work, try either the integral test, ratio test, or root test. - Try the ratio test when there are terms like $n!$ or c^n. - Try the root test when there are terms like n^n, or even a function $f(n)^{5n}$. - Try the integral test when the a_n can be written as some easily integrable $f(n)$. (Integral test works well with logs and ln.)
5^{th}	If nothing has worked so far consider: - More creative comparison. - Can you split the series up along addition/subtraction into 2 *convergent* series? - Using partial sums (maybe they will telescope).

If the series alternates between positive and negative terms (after some point):	
6^{th}	If the b_n sequence is nice, try the Alternating Series Test
7^{th}	If there are terms like $n!$ or c^n, try the Ratio Test
8^{th}	Use $\sum |a_n|$ and then go to step 3. (You're hoping for absolute convergence here.)

If the series is not *eventually alternating and* not *eventually all positive:*	
9^{th}	Use Ratio/Root test if the series is a good candidate for one of them.
10^{th}	Other options: - Can you get Absolute Convergence using $\sum |a_n|$ (go to Step 3)? - Can you simplify the series? Split it into the sum of 2 *convergent* series? - Can you work with the partial sums?