Math 314 Summer 2011
6/13/2011 Quiz #8

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

The following two proofs have been started for you. Please fill in the missing pieces to complete the proofs.

1. Show: $\mathbb{R}$ has no smallest element.

   Proof. (By Contradiction)
   Suppose not.

   So $\mathbb{R}$ has a smallest element $x \in \mathbb{R}$.
   Notice $x-1 \in \mathbb{R}$ and $x-1 < x$.
   This contradicts the fact that x was the smallest element of $\mathbb{R}$.
   Therefore $\mathbb{R}$ has no smallest element.

   □

2. Show: If $A \subseteq B$ and $C \subseteq D$, then $A \cup C \subseteq B \cup D$.

   Proof. Let $A \subseteq B$ and $C \subseteq D$.
   (We want to show $A \cup C \subseteq B \cup D$)
   Let $x \in A \cup C$
   so $x \in A$ or $x \in C$

   Case 1: $x \in A$
   since $A \subseteq B$ we have $x \in B$
   Hence $x \in B \cup D$

   Case 2: $x \in C$
   since $C \subseteq D$ we have $x \in D$
   Hence $x \in B \cup D$

   In either case $x \in B \cup D$.
   Therefore $A \cup C \subseteq B \cup D$

   □

3. ♠ Let $a, b, c \in \mathbb{Z}$. Show: If $a|b$ and $b|c$, then $a|c$.

   Proof.
   Let $a, b, c \in \mathbb{Z}$ and suppose $a|b$ and $b|c$
   so $\exists k \in \mathbb{Z}$ such that $b = ak$
   and $\exists j \in \mathbb{Z}$ such that $c = bj$

   Notice $c = bj = (ak)j = a(kj)$
   since $kj \in \mathbb{Z}$ (the product of two integers is an integer)
   we have $a|c$

   □