Math 314 Summer 2010
6/11/2010 Quiz #10

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Use Eulclid's Algorithm (GCD Reduction) to compute the following:

   (a) gcd(110,44)

$$\gcd(110,44) = \gcd(44,22) = \gcd(22,0) = 22$$

$110 = 44 \cdot 2 + 22$ ← equation 1

$44 = 22 \cdot 2 + 0$

   (b) Use (a) to write gcd(110,44)=$110x + 44y$ from some $x, y \in \mathbb{Z}$

The first equation from (a) gives us

$$110 = 44 \cdot 2 + 22$$

so $22 = 110 - 44 \cdot 2$

so $22 = 110(1) + 44(-2)$

so $\boxed{x = 1, \; y = -2}$

   (c) gcd(50,35)

$$\gcd(50,35) = \gcd(35,15) = \gcd(15,5) = \gcd(5,0) = 5$$

$50 = 35 \cdot 1 + 15$ ← equation 1

$35 = 15 \cdot 2 + 5$ ← equation 2

$15 = 5 \cdot 3 + 0$

   (d) Use (c) to write gcd(50,35)=$50x + 35y$ from some $x, y \in \mathbb{Z}$

Write each equation in reverse order solving for the remainder:

equation 2: $35 = 15 \cdot 2 + 5$ gives us $5 = 35 - 15 \cdot 2$

equation 1: $50 = 35 \cdot 1 + 15$ gives us $15 = 50 - 35 \cdot 1$

Now substitute the expression for 15 from equation 2 into equation 1:

$$5 = 35 - (50 - 35 \cdot 1) \cdot 2$$
$$5 = 35 - 50 \cdot 2 + 35 \cdot 2$$
$$5 = 3 \cdot 35 - 50 \cdot 2$$
$$5 = 50(-2) + 35(3)$$

so $\boxed{x = -2, \; y = 3}$