Math 314 Summer 2010

5/27/2010
Quiz #2

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Determine the following cardinalities:
   (a) $|\{1,2,3\}| =$ 3
   (b) $|\{\emptyset,\{\emptyset\}\}| =$ 2
   (c) $|\emptyset| =$ 0
   (d) $|\{5,5,6\}| =$ 2
   (e) $|\{\{\{5\}\},\emptyset\}| =$ 2

2. Let $A = \{1,2,5\}$
   (a) Give an example of an element of A.
   Acceptable answers: 1, 2 or 5
   (b) Give an example of a subset of A.
   Acceptable answers: $\emptyset$, $\{1\}$, $\{2\}$, $\{5\}$, $\{1,2\}$, $\{1,5\}$, $\{2,5\}$ or $A = \{1,2,5\}$

3. Suppose $B = \{x : x \in \mathbb{Z} \text{ and } x^2 - 1 = 0\}$. Rewrite B in list notation.
   $B = \{1, -1\}$
   notice the only x values where $x^2 - 1 = 0$ is $x = 1$ or $x = -1$
   these are both integers i.e., $1 \in \mathbb{Z}$ and $-1 \in \mathbb{Z}$ So both are in B.

4. Suppose $C = \{x : x \in \mathbb{Z} \text{ and } (x - \frac{1}{3})(x - 2) = 0\}$. Rewrite C in list notation.
   $C = \{2\}$
   only $x = 1/3$ and $x = 2$ are solutions to the equation $(x - 1/3)(x-2) = 0$
   but $1/3 \notin \mathbb{Z}$ and $2 \in \mathbb{Z}$ so only 2 will be in C.

5. Suppose $D = \{x : x \in \mathbb{Q} \text{ and } (x - \frac{1}{3})(x - 2) = 0\}$. Rewrite D in list notation.
   $D = \{1/3, 2\}$
   again $x = 1/3$ and $x = 2$ are the only solutions to the equation $(x - 1/3)(x-2) = 0$
   and both $1/3 \in \mathbb{Q}$ and $2 \in \mathbb{Q}$
   so both $1/3$ and 2 will be in D

Recall the definition of $\mathbb{Q}$:

A real number $x \in \mathbb{R}$ is said to be <u>rational</u> if we can write $x = \frac{m}{n}$ for some $m, n \in \mathbb{Z}$ and $n \neq 0$

the set of rational numbers is denoted $\mathbb{Q}$

Another way of writing $\mathbb{Q}$ is by set builder notation:

$\mathbb{Q} = \{x : x \in \mathbb{R}, \exists m, n \in \mathbb{Z} \text{ such that } x = \frac{m}{n} \text{ and } n \neq 0\}$