Math 221 Fall 2009
11/17/2009
Quiz #12

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

(PRACTICE PROBLEM) Find the most general antiderivative of the function $f(\theta) = 5 + \frac{1}{\theta^2}$.

The domain of f is $(-\infty, 0) \cup (0, \infty)$ so the most general antiderivative of f is

$$F(\theta) = \begin{cases} 5\theta - \frac{1}{\theta} + C_1 & \text{if } \theta < 0 \\ 5\theta - \frac{1}{\theta} + C_2 & \text{if } \theta > 0 \end{cases}$$

(notice C_1 and C_2 are independent of each other)

3 1. Write $\int_0^3 \sin(\sqrt{x})dx$ as a limit of Riemann sums taking the sample points to be the right endpoints on the subintervals. **DO NOT EVALUATE THE LIMIT**

$a = 0$
$b = 3$ so $\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$ AND

(right endpoints of subintervals): $x_i^* = a + i\Delta x = 0 + i\left(\frac{3}{n}\right) = \frac{3i}{n}$

so $$\int_0^3 \sin(\sqrt{x})\, dx = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\sqrt{\frac{3i}{n}}\right)\left(\frac{3}{n}\right)}$$

4 2. Evaluate $\int_0^2 3x dx$ as a limit of Riemann sums taking the sample points to be the right endpoints on the subintervals.

$a = 0$
$b = 2$ So $\Delta x = \frac{2-0}{n} = \frac{2}{n}$ AND

(right endpoints of subintervals) $x_i^* = a + i\Delta x = 0 + i\left(\frac{2}{n}\right) = \frac{2i}{n}$

$$\int_0^2 3x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} 3\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{12i}{n^2} = \lim_{n\to\infty} \frac{12}{n^2} \sum_{i=1}^{n} i$$

I can pull this in front of the sum because it does NOT depend on "i"

$$= \lim_{n\to\infty} \frac{12}{n^2}\left(\frac{n(n+1)}{2}\right) = \lim_{n\to\infty} \frac{(12n^2 + 12n)\left(\frac{1}{n^2}\right)}{2n^2 \left(\frac{1}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{12 + \frac{12}{n}}{2} = \boxed{6}$$