Math 314 Summer 2011
6/17/2010 Quiz #11

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Compute $\sum_{i=1}^{100}(3i+1)$ $= \sum_{i=1}^{100} 3i + \sum_{i=1}^{100} 1$

$= 3\sum_{i=1}^{100} i + 100$

$= 3 \cdot \frac{100 \cdot 101}{2} + 100$

$= 15250$

2. Show: $\forall n \geq 1, \sum_{j=1}^{n}(2j-1) = n^2$.

proof. (by induction)

Base case: $(n=1)$: $\sum_{j=1}^{1}(2j-1) = 2(1)-1 = 1$

$1^2 = 1$ equal. ✓

Induction Step: Suppose for some $k \geq 1$, $\sum_{j=1}^{k}(2j-1) = k^2$ } induction hypothesis.

(Show: $\sum_{j=1}^{k+1}(2j-1) = (k+1)^2$)

observe, $\sum_{j=1}^{k+1}(2j-1) = \sum_{j=1}^{k}(2j-1) + (2(k+1)-1)$

$= k^2 + (2k+2-1)$ by the induction hypothesis.

$= k^2 + 2k + 1$

$= (k+1)^2$ □

3. Show: $\forall n \geq 4$, $n! > 2^n$.

proof. (by induction)

Base case: $(n=4)$: $4! = 4\cdot3\cdot2\cdot1 = 24 > 2^4 = 16$ ✓

Induction Step: suppose for some $k \geq 4$, $k! > 2^k$ } induction hypothesis.

(Show: $(k+1)! > 2^{k+1}$)

observe $(k+1)! = (k+1)k! > (k+1)\cdot 2^k$ by induction hypothesis.

Notice that since $k \geq 4$, $k+1 \geq 5 \geq 2$

Hence $(k+1)\cdot 2^k \geq 2\cdot 2^k = 2^{k+1}$

Therefore $(k+1)! > 2^{k+1}$

□