

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1.  $\int \sec(x) dx.$

A.  $\ln|\sec(x)| + C$

B.  $\ln|\sec(x) + \tan(x)| + C$

C.  $-\ln|\cos(x)| + C$

D.  $\tan(x) + C$

2.  $\int \sec^2(x) dx.$

A.  $\ln|\sec(x)| + C$

B.  $\ln|\sec(x) + \tan(x)| + C$

C.  $-\ln|\cos(x)| + C$

D.  $\tan(x) + C$

**SOLVE 2 OF THE FOLLOWING INTEGRALS:**

Please indicate which problems you do NOT want me to grade by putting a GIANT X through them, otherwise I will grade the first two worked on:

3. Evaluate  $\int \sin^2(3x) dx.$   $= \int \left(\frac{1}{2} - \frac{1}{2} \cos(6x)\right) dx$

$= \frac{1}{2}x - \frac{1}{12} \sin(6x) + C$

(to get this answer you can do a u-substitution of  $u = 6x$ )

4. Evaluate  $\int \sec^4(x) dx.$   $= \int \sec^2(x) \sec^2(x) dx = \int (\tan^2(x) + 1) \sec^2(x) dx$

$u = \tan(x) \Rightarrow \frac{du}{dx} = \sec^2(x) \Rightarrow dx = \frac{du}{\sec^2(x)}$

$= \int (u^2 + 1) \sec^2(x) \cdot \frac{du}{\sec^2(x)}$

$= \int (u^2 + 1) du = \frac{u^3}{3} + u + C$

$= \frac{\tan^3(x)}{3} + \tan(x) + C$

5. Evaluate  $\int \sin^3(3x) \cos^2(3x) dx.$

$= \int \sin(3x) \sin^2(3x) \cos^2(3x) dx$

$= \int \sin(3x) (1 - \cos^2(3x)) \cos^2(3x) dx$

$u = \cos(3x) \Rightarrow \frac{du}{dx} = -3 \sin(3x) \Rightarrow dx = \frac{du}{-3 \sin(3x)}$

$= \int \cancel{\sin(3x)} (1 - u^2) u^2 \cdot \frac{du}{-3 \sin(3x)} = -\frac{1}{3} \int (u^2 - u^4) du = -\frac{1}{3} \left[ \frac{u^3}{3} - \frac{u^5}{5} \right] + C$

$= -\frac{\cos^3(3x)}{9} + \frac{\cos^5(3x)}{15} + C$

6. Evaluate  $\int_0^{\pi/8} \sqrt{1 + \cos(4x)} dx.$

$\cos^2(2x) = \frac{1}{2} + \frac{1}{2} \cos(4x) \Rightarrow \cos(4x) = 2\cos^2(2x) - 1$

$\text{so } \int_0^{\pi/8} \sqrt{1 + \cos(4x)} dx = \int_0^{\pi/8} \sqrt{1 + (2\cos^2(2x) - 1)} dx = \int_0^{\pi/8} \sqrt{2\cos^2(2x)} dx = \sqrt{2} \int_0^{\pi/8} |\cos(2x)| dx$

on the interval  $[0, \pi/8]$ ,  $\cos(2x) \geq 0$  so we have  $\sqrt{2} \int_0^{\pi/8} \cos(2x) dx = \sqrt{2} \left[ \frac{\sin(2x)}{2} \right]_0^{\pi/8} = \boxed{\frac{1}{2}}$