Math 324 - Additional Problems HW#13

1. Show that similar matrices have the same characteristic polynomial, and hence, the same eigenvalues. (N.B. I gave you a hint for this in class).

2. Let $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$

   (a) Find the eigenvalues of A

   (b) For each eigenvalue of A find a basis for the corresponding eigenspace

   (c) For each eigenvalue of A compute the algebraic and geometric multiplicities.

   (d) Is A diagonalizable? Justify your answer (this should be short).

   (e) Is A invertible? Justify your answer (this should be short).

   (f) Compute $\mathrm{tr}(A)$ using the eigenvalues only

   (g) Compute $\det(A)$ using the eigenvalues only

3. Suppose A be an $n \times n$ matrix. Let P be a fixed $n \times n$ invertible matrix. Consider the mapping $T : M_{nn} \to M_{nn}$, defined by

$$T(A) = P^{-1}AP$$

   Is T a linear transforamtion? Provide a proof of your answer.

4. Let $F = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

   (a) Calculate several powers F^k by hand (go at least $k = 6$). The sequence of integers that appear in the $(1,1)-$entry form a famous sequence of numbers. Do you know what this is? See the On-Line Encyclopedia of Integer Sequences (OEIS): `https://oeis.org/A000045`

   (b) What are the eigenvalues of F?

   (c) How can we tell F is diagonalizable?

   (d) ♠ (Optional) Diagonalize F

   (e) ♠ (Optional) Use the diagonalization to write out F^k

   (f) ♠ (Optional) Can you use this form of F^k to get a closed formula for the famous sequence appearing in the $(1,1)$-entry.