Math 304 Spring 2010
2/18/2010
Quiz #4

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Solve $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ -2 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R2 \to R2+2R1} \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right] \quad \text{So} \quad \boxed{X = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}}$$

(Notice you actually found the inverse of $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, indeed $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓)

2. Let

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

(a) Compute A^{T}.

(b) Compute AA^{T}.

(c) Compute $(AB)^{\mathrm{T}}$.

(a) $A^T = \boxed{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}}$

(b) $AA^T = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}$

(c) <u>two solutions</u> : one way: $(AB)^T = \left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right)^T = \left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right)^T = \boxed{\begin{bmatrix} -1 & 0 \end{bmatrix}}$

Second way : $(AB)^T = B^T A^T = \begin{bmatrix} 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \end{bmatrix}$

3. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and suppose

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 6 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

(a) Write down the standard matrix of T (meaning write down the matrix A such that $A\mathbf{x} = T(\mathbf{x})$ for any $\mathbf{x}$ in $\mathbb{R}^2$).

(b) Compute $T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right)$.

$A = \begin{bmatrix} T(\vec{e}_1) & T(e_2) \end{bmatrix} = \begin{bmatrix} T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) & T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \end{bmatrix}$ so

(a) $\boxed{A = \begin{bmatrix} -1 & 3 \\ 6 & 0 \end{bmatrix}}$

(b) $T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = A\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3+3 \\ 3 \cdot 6 + 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ 18 \end{bmatrix}}$