

1. Find
- $f(x)$
- if
- $f''(x) = e^x + \cos(x)$
- ,
- $f'(0) = 1$
- , and
- $f(0) = 4$
- .

$$f'(x) = e^x + \sin(x) + C$$

$$f'(0) = e^0 + \sin(0) + C = 1$$

$$1 + 0 + C = 1$$

$$\underline{C = 0}$$

$$f'(x) = e^x + \sin(x)$$

$$f(x) = e^x + (-\cos(x)) + D$$

$$f(0) = e^0 - \cos(0) + D = 4$$

$$1 - 1 + D = 4$$

$$\underline{D = 4}$$

$$\boxed{f(x) = e^x - \cos(x) + 4}$$

2. Find the most general antiderivative of the following:

$$(a) f(x) = \frac{5x^2 + x - \sqrt{x} + 1}{x}$$

$$= \frac{5x^2}{x} + \frac{x}{x} - \frac{x^{1/2}}{x} + \frac{1}{x}$$

$$= 5x + 1 - x^{-1/2} + \frac{1}{x}$$

$$F(x) = \frac{5x^2}{2} + x - \frac{x^{1/2}}{1/2} + \ln|x| + C = \boxed{\frac{5}{2}x^2 + x - 2\sqrt{x} + \ln|x| + C}$$

$$(b) f(x) = 4x^2(x-3)^2 = 4x^2(x^2 - 6x + 9)$$

$$= 4x^4 - 24x^3 + 36x^2$$

$$F(x) = \frac{4x^5}{5} - \frac{24x^4}{4} + \frac{36x^3}{3} + C = \boxed{\frac{4}{5}x^5 - 6x^4 + 12x^3 + C}$$

3. Find
- $f(x)$
- if
- $f'''(x) = 2e^x$
- .

$$f''(x) = 2e^x + C$$

$$f'(x) = 2e^x + Cx + D$$

$$\boxed{f(x) = 2e^x + \frac{Cx^2}{2} + Dx + E}$$