Math 221 Fall 2009
11/10/2009
Quiz #11

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

PICK ONE OF THE FOLLOWING:

Please indicate which one you do NOT want me to grade by putting an X through it, otherwise I will grade the first one worked on:

[10] 1. (a) Find the point on the line $y = 2x + 3$ that is closest to the origin.

This problem is very similar to an example from class. Also see example 3 on page 259.

Let (x,y) be a point on the line $y=2x+3$. so we know immediately that the y-coordinate is $y=2x+3$. ie) $(x,y) = (x, 2x+3)$.

We are trying to minimize the distance from (x,y) to $(0,0)$ so consider

$$D = \sqrt{(x-0)^2+(y-0)^2} = \sqrt{x^2+y^2} = \sqrt{x^2+(2x+3)^2}$$

instead let's minimize D^2 to make things easier (why can we do this?)

$$f(x) = D^2 = x^2+(2x+3)^2 = x^2+4x^2+12x+9 = 5x^2+12x+9 \qquad \text{domain: } \mathbb{R}$$

critical numbers of f: $f'(x) = 10x+12$

$f'(x)=0$
$10x+12=0$
$x = \frac{-12}{10} = \frac{-6}{5}$

$f'(x)$ is undefined never (since $f'(x)$ is a polynomial)

Now check that the abs min occurs when $x = \frac{-6}{5}$

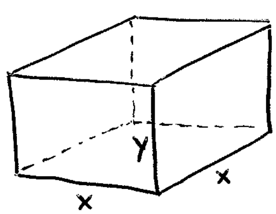
f': $-$ | $+$ at $\frac{-6}{5}$

so by Thm. on pg. 259 f has abs. min when $x = \frac{-6}{5}$ and $y = 2\left(\frac{-6}{5}\right)+3$

so $\boxed{\left(\frac{-6}{5}, \frac{3}{5}\right)}$

(b) A closed box with a square base must have a volume of 8 ft^3. Find the dimensions of the box that will minimize the amount of material used. (Note: only the base is assumed to be square!)

(I should have said only the base and the top is assumed to be square, sorry! However because I said it was a square box it is really assumed the top is also square)



given: $V = 8\text{ft}^3 = x^2y \Rightarrow y = \frac{8}{x^2}$

minimize $A = \underset{\text{top}}{x^2} + \underset{\text{bottom}}{x^2} + \underbrace{xy+xy+xy+xy}_{\text{sides}} = 2x^2+4xy$

so $A = 2x^2 + 4x\left(\frac{8}{x^2}\right) = 2x^2 + \frac{32}{x}$ Domain: $(0,\infty)$

$$A'(x) = 4x - \frac{32}{x^2} = \frac{4x^3-32}{x^2}$$

critical numbers of A:

$A'(x)=0$
$4x^3-32=0$
$x^3 = \frac{32}{4}$
$x^3 = 8$
$x = \sqrt[3]{8} = 2$

$A'(x)$ is undefined when
$x^2=0$
$\underbrace{x=0}_{\text{not in the domain}}$

A': $-$ | $+$ on $(0, \infty)$, at 2

So A has an absolute minimum when $x=2$ by Thm. on pg. 259

so $y = \frac{8}{2^2} = \frac{8}{4} = 2$

Dimensions of the box $\boxed{2\text{ft} \times 2\text{ft} \times 2\text{ft}}$