Math 152 - Fall 2013 - Quiz #5 **Score:** _____ out of 10.

Name: Key

Show all work clearly and in order. Please box your answers.

SOLVE 1 SIDE:

Please indicate which side you do NOT want me to grade by putting a GIANT X through it, otherwise I will grade the first side worked on:

1. Evaluate $\int \sqrt{49 - x^2}\,dx$.

$x = 7\sin\theta \Rightarrow \frac{dx}{d\theta} = 7\cos\theta \Rightarrow dx = 7\cos\theta\,d\theta$

$$\int \sqrt{49 - x^2}\,dx = \int \sqrt{49 - (7\sin\theta)^2}\, 7\cos\theta\,d\theta$$

$$= \int \sqrt{49 - 49\sin^2\theta}\; 7\cos\theta\,d\theta$$

$$= \int \sqrt{49(1 - \sin^2\theta)}\; 7\cos\theta\,d\theta$$

$$= \int 7\sqrt{\cos^2\theta}\; 7\cos\theta\,d\theta$$

$$= \int 7\cos\theta\; 7\cos\theta\,d\theta = \int 49\cos^2\theta\,d\theta$$

$$= 49\int \tfrac{1}{2}(1 + \cos(2\theta))\,d\theta$$

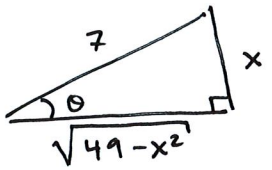
$$= \frac{49}{2}\int [1 + \cos(2\theta)]\,d\theta$$

$$= \frac{49}{2}\left[\theta + \frac{\sin(2\theta)}{2}\right] + C$$

$$= \frac{49}{2}\left[\theta + \frac{2\sin\theta\cos\theta}{2}\right] + C$$

$$= \frac{49}{2}\left[\theta + \cos\theta\sin\theta\right] + C$$

$x = 7\sin\theta \Rightarrow \sin\theta = \frac{x}{7} = \frac{\text{opp}}{\text{hyp}}$



$\theta = \sin^{-1}\left(\frac{x}{7}\right)$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{49 - x^2}}{7}$

$$= \boxed{\frac{49}{2}\left[\sin^{-1}\left(\frac{x}{7}\right) + \left(\frac{\sqrt{49 - x^2}}{7}\right)\left(\frac{x}{7}\right)\right] + C}$$

2. Write out the FORM of the partial fraction decomposition for the following (DO NOT find the numerical values for the unknown coefficients).

(a) $\dfrac{4x^2+3x-1}{x(x-2)(x+5)} = \dfrac{A}{x} + \dfrac{B}{x-2} + \dfrac{C}{x+5}$

(b) $\dfrac{x}{(x^2+x+20)(x^2-4)} = \dfrac{x}{(x^2+x+20)(x-2)(x+2)} = \dfrac{Ax+B}{x^2+x+20} + \dfrac{C}{x-2} + \dfrac{D}{x+2}$

(c) $\dfrac{2x+10}{x(x-1)^2(x^2+1)^2} = \dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2} + \dfrac{Dx+E}{(x^2+1)} + \dfrac{Fx+G}{(x^2+1)^2}$

3. Evaluate $\displaystyle\int \frac{1}{x^2+8x+12}\,dx$.

Step 1: deg(NUM) = 1, deg(DENOM) = 2 } DONE (No long division)

Step 2/3: $\dfrac{1}{x^2+8x+12} = \dfrac{1}{(x+6)(x+2)} = \dfrac{A}{x+6} + \dfrac{B}{x+2}$

$= \dfrac{A(x+2)}{(x+6)(x+2)} + \dfrac{B(x+6)}{(x+2)(x+6)}$

$= \dfrac{A(x+2) + B(x+6)}{(x+2)(x+6)}$

$1 = A(x+2) + B(x+6)$

$1 = Ax + 2A + Bx + 6B$

$0 = A + B$ $\qquad$ $1 = 2A + 6B$

$A = -B \longrightarrow 1 = -2B + 6B$

$1 = 4B$

$A = -1/4 \longleftarrow B = 1/4$

$\displaystyle\int \frac{1}{x^2+8x+12}\,dx = \int \frac{-1/4}{x+6} + \frac{1/4}{x+2}$

$= \boxed{-\tfrac{1}{4}\ln|x+6| + \tfrac{1}{4}\ln|x+2| + C}$