Math 222 Spring 2011
2/4/2011
Quiz #2 Name: ________________________________

Show all work clearly and in order. Please box your answers. 10 minutes.

$\boxed{3}$ 1. Evaluate the following. No work is needed.

(a) $\frac{d}{dx}e^x = e^x$

(b) $\frac{d}{dx}a^x = a^x \ln(a)$

(c) $\frac{d}{dx}(\log_a x) = \frac{1}{x\ln(a)}$

(d) $\int e^x dx = e^x + C$

(e) $\int a^x dx = \frac{a^x}{\ln(a)} + C,\ a \neq 1$

(f) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

$\boxed{2}$ 2. Differentiate: $y = e^{1/x}$.

Solution:

$$y' = e^{1/x}\cdot\frac{d}{dx}\left(\frac{1}{x}\right) = e^{1/x}\cdot\left(\frac{-1}{x^2}\right) = \frac{-e^{1/x}}{x^2}.$$

$\boxed{3}$ 3. Evaluate: $\displaystyle\int_1^2 x3^{x^2}\,dx$.

Solution: Let $u = x^2 \implies \dfrac{du}{dx} = 2x \implies dx = \dfrac{du}{2x}$. Also $u(1) = 1^2 = 1$ and $u(2) = 2^2 = 4$.

Therefore,

$$\int_1^2 x3^{x^2}dx = \int_1^4 x3^u\frac{du}{2x} = \frac{1}{2}\int_1^4 3^u du = \frac{1}{2}\left[\frac{3^u}{\ln(3)}\right]_1^4 = \frac{3^4-3}{2\ln(3)}.$$

$\boxed{2}$ 4. Pick ONE of the following. Cross out the problem you do not want graded. Otherwise I will grade the first problem worked on.

(a) Evaluate: $\displaystyle\int\frac{3^x}{3^x+1}dx$

(b) Differentiate: $y = x^{\sin(x)}$.

Solution: **(a)** Let $u = 3^x + 1 \implies \dfrac{du}{dx} = 3^x\ln(3) \implies dx = \dfrac{du}{3^x\ln(3)}$.

Therefore,

$$\int\frac{3^x}{3^x+1}dx = \int\frac{3^x}{u}\frac{du}{3^x\ln(3)} = \frac{1}{\ln(3)}\int\frac{1}{u}du = \frac{1}{\ln(3)}\ln|u| + C = \frac{1}{\ln(3)}\ln|3^x+1| + C.$$

Also, $\frac{1}{\ln(3)}\ln(3^x+1)+C$ is an acceptable answer since $3^x+1 \geq 0$ for all x.

Solution: (b)
Solution 1: Using logarithmic differentiation we can solve this problem.

$$\begin{aligned}
y &= x^{\sin(x)} \\
\ln(y) &= \ln(x^{\sin(x)}) = \sin(x)\ln(x) \\
\frac{d}{dx}\ln(y) &= \frac{d}{dx}(\sin(x)\ln(x)) \\
\frac{1}{y}y' &= \sin(x)\left(\frac{d}{dx}\ln(x)\right) + \left(\frac{d}{dx}\sin(x)\right)\ln(x) \\
\frac{1}{y}y' &= \sin(x)\frac{1}{x} + \cos(x)\ln(x) \\
y' &= y\left(\frac{\sin(x)}{x} + \cos(x)\ln(x)\right) \\
y' &= x^{\sin(x)}\left(\frac{\sin(x)}{x} + \cos(x)\ln(x)\right)
\end{aligned}$$

Solution 2: Notice that by the cancellation equation $z = e^{\ln(z)}$ the following is true:

$$y = x^{\sin(x)} = e^{\ln(x^{\sin(x)})} = e^{\sin(x)\ln(x)}.$$

Now differentiating:

$$\begin{aligned}
y' &= e^{\sin(x)\ln(x)}\left(\frac{d}{dx}\sin(x)\ln(x)\right) \\
&= e^{\sin(x)\ln(x)}\left(\sin(x)\left(\frac{d}{dx}\ln(x)\right) + \left(\frac{d}{dx}\sin(x)\right)\ln(x)\right) \\
&= e^{\sin(x)\ln(x)}\left(\sin(x)\frac{1}{x} + \cos(x)\ln(x)\right) \\
&= e^{\sin(x)\ln(x)}\left(\frac{\sin(x)}{x} + \cos(x)\ln(x)\right) \\
&= x^{\sin(x)}\left(\frac{\sin(x)}{x} + \cos(x)\ln(x)\right).
\end{aligned}$$