Math 222 Spring 2011
1/28/2011
Quiz #1

Name: Key

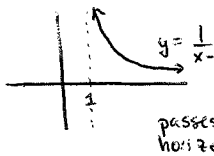
Show all work clearly and in order. Please box your answers. 10 minutes.

1. Let $f(x) = \dfrac{1}{x-1}$, $x > 1$.

[2] (a) Show $f(x)$ is one-to-one.

SOL 1: Let $f(x_1) = f(x_2)$
so $\frac{1}{x_1 - 1} = \frac{1}{x_2 - 1}$
so $x_2 - 1 = x_1 - 1$
so $x_1 = x_2$ so f is 1-1.

SOL 2:



passes the horizontal line test. Hence, f is 1-1.
(MUST have an accurate graph)

SOL 3: $f'(x) = \frac{-1}{(x-1)^2} < 0$
so f is always decreasing, hence, f is 1-1.

[2] (b) Find an explicit formula for f^{-1}.

STEP 1: $y = \frac{1}{x-1}$

STEP 2: solve for x: $x - 1 = \frac{1}{y}$
so $x = 1 + 1/y$

STEP 3: (swap x and y to get $f^{-1}(x) = y$)
$\boxed{f^{-1}(x) = 1 + 1/x}$
(I would also accept $x = 1 + 1/y$)

[2] (c) Calculate $(f^{-1})'(2)$.

SOL 1: using (b):
$(f^{-1})'(x) = 0 + \frac{-1}{x^2}$
so $(f^{-1})'(2) = \frac{-1}{2^2} = \boxed{\frac{-1}{4}}$

SOL 2: $(f^{-1})'(2) = \dfrac{1}{f'(f^{-1}(2))}$
using (b) $f^{-1}(2) = 1 + \frac{1}{2} = \frac{3}{2}$
$f'(x) = -1(x-1)^{-2} = \frac{-1}{(x-1)^2}$
so $(f^{-1})'(2) = \dfrac{1}{\frac{-1}{(\frac{3}{2}-1)^2}} = -\left(\frac{1}{2}\right)^2 = \boxed{-\frac{1}{4}}$

[2] 2. Differentiate $f(x) = \ln(\cos^2(x))$.

SOL 1: $f(x) = \ln((\cos(x))^2) = 2\ln(\cos(x))$
so $f'(x) = 2 \cdot \frac{1}{\cos(x)} \cdot \frac{d}{dx}\cos(x)$
$= 2 \cdot \frac{1}{\cos(x)} \cdot (-\sin(x)) = \boxed{-2\tan(x)}$

SOL 2: $f'(x) = \frac{1}{\cos^2(x)} \cdot \frac{d}{dx}(\cos^2(x))$
$= \frac{1}{\cos^2(x)} \cdot 2\cos(x) \cdot (-\sin(x)) = \boxed{-2\tan(x)}$

[2] 3. Pick ONE of the following. Cross out the problem you do not want graded. Otherwise I will grade the first problem worked on.

(a) Evaluate: $\displaystyle\int \frac{1}{x\ln(x)}dx$

using u-substitution:
Let $u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

so $\displaystyle\int \frac{1}{x\ln(x)}dx = \int \frac{1}{x \cdot u} \cdot x\,du = \int \frac{1}{u}\,du$
$= \ln|u| + C$
$= \boxed{\ln|\ln(x)| + C}$

(b) Use logarithmic differentiation to find the derivative of $y = \dfrac{(x^2+1)^2\cos^2(x)}{(x-1)^4}$ (you do not need to simplify). Start on the back of the quiz!

(see below)

4. Tell me some interesting facts about yourself that will help me get to know you. *Examples: What is your favorite book/movie? What is you favorite music genre? What do you like about your major? What is your favorite programming language? Did you do anything interesting over the break? Another interesting fact about yourself, etc.*

$$y = \frac{(x^2+1)^2 \cos^2(x)}{(x-1)^4}$$

$$\ln(y) = \ln\left(\frac{(x^2+1)^2 \cos^2(x)}{(x-1)^4}\right)$$

$$\ln(y) = 2\ln(x^2+1) + 2\ln(\cos(x)) - 4\ln(x-1)$$

$$\frac{d}{dx}\ln(y) = \frac{d}{dx}\left[2\ln(x^2+1) + 2\ln(\cos(x)) - 4\ln(x-1)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = 2\cdot\frac{1}{x^2+1}\cdot 2x + 2\cdot\frac{1}{\cos(x)}\cdot(-\sin(x)) - 4\,\frac{1}{x-1}\cdot 1$$

$$\frac{dy}{dx} = \left[\frac{(x^2+1)^2\cos^2(x)}{(x-1)^4}\right]\left[\frac{4x}{x^2+1} + \frac{-2\sin(x)}{\cos(x)} - \frac{4}{x-1}\right]$$