

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Let $T = \begin{bmatrix} 1 & 3 & -1 & 2 & 1 \\ -2 & -6 & 3 & 4 & 2 \end{bmatrix}$

Suppose that T is used to define a function in the usual way.

- Compute the rank of T .
- What is the domain of T ?
- What is the codomain of T ?
- Is T onto? Why or why not?
- Is T one-to-one? Why or why not?
- Is T a one-to-one correspondence? Why or why not?

(a) $\begin{bmatrix} 1 & 3 & -1 & 2 & 1 \\ -2 & -6 & 3 & 4 & 2 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 + 2R_1} \begin{bmatrix} 1 & 3 & -1 & 2 & 1 \\ 0 & 0 & 1 & 8 & 4 \end{bmatrix}$
 $\uparrow \qquad \qquad \uparrow$
 two pivot columns so rank of $T = \boxed{2}$

(b) T is 2×5 so domain of T is $\boxed{\mathbb{R}^5}$

(c) T is 2×5 so codomain of T is $\boxed{\mathbb{R}^2}$

(d) T is onto because rank of $T = 2 = \#$ of rows of T

(e) T is NOT one-to-one since rank of $T = 2 \neq 5 = \#$ of columns of T

(f) T is NOT a one-to-one correspondence since T is NOT one-to-one and to be a one-to-one correspondence T must both be both one-to-one AND onto.

2. Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 3 & 5 \end{bmatrix}$$

(a) Is the product AB defined? If so compute it.

(b) Is the product BA defined? If so compute it.

(a) YES, $AB = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} (1)(-1) + (1)(0) & (1)(0) + (1)(3) & (1)(2) + (1)(5) \\ (2)(-1) + (1)(0) & (2)(0) + (1)(3) & (2)(2) + (1)(5) \end{bmatrix}$
 $= \boxed{\begin{bmatrix} -1 & 3 & 7 \\ -2 & 3 & 9 \end{bmatrix}}$

(b) No,

$$\begin{array}{cc} B & A \\ \hline 2 \times 3 & 2 \times 2 \end{array}$$

not equal hence the product BA cannot exist