Math 222 Spring 2011
3/4/2011
Quiz #6

Name: Key

Please box your answers. Show all work clearly and in order.

3 1. Suppose you want to approximate $\int_a^b f(x)dx$. You subdivide the interval $[a, b]$ into n subintervals: $[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]$. Assume these subintervals are of equal length $\Delta x = \frac{b-a}{n}$. Write out the formulas for each of the following approximations.

(a) Midpoint Rule: $M_n = \Delta x \left[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n) \right]$ where $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$

(b) Trapezoidal Rule: $T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n) \right]$

(c) Simpson's Rule: $S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$

7 2. Pick ONE of the following. Cross out the problem you do not want graded. Otherwise I will grade the first problem worked on.

(a) Evaluate: $\int \frac{x^3 - 4x - 10}{x^2 - x - 6} dx$.

STEP 1:

$$\begin{array}{r|l} & x + 1 \\ x^2 - x - 6 & x^3 \quad - 4x - 10 \\ & -(x^3 - x^2 - 6x) \\ & 0 + x^2 + 2x - 10 \\ & -(x^2 - x - 6) \\ & 0 \quad 3x - 4 \end{array}$$

so $\frac{x^3-4x-10}{x^2-x-6} = x + 1 + \frac{3x-4}{x^2-x-6}$

STEP 2: $x^2 - x - 6 = (x-3)(x+2)$

STEP 3:

$$\frac{3x-4}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2} = \frac{A(x+2) + B(x-3)}{(x-3)(x+2)}$$

So $3x - 4 = Ax + 2A + Bx - 3B = (A+B)x + 2A - 3B$

So

$A + B = 3 \Rightarrow A = 3 - B$

$2A - 3B = -4 \Rightarrow 2(3 - B) - 3B = -4$

$\Rightarrow 6 - 2B - 3B = -4$

$6 - 5B = -4$

$-5B = -10$

$B = 2$ and $A = 3 - 2 = 1$

(b) Evaluate: $\int \frac{x^2 - x + 6}{x^3 + 3x} dx$.

STEP 1: $2 < 3$, DONE

STEP 2: $x^3 + 3x = x(x^2 + 3)$

STEP 3:

$$\frac{x^2 - x + 6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3} = \frac{A(x^2+3) + (Bx + C)x}{x(x^2+3)}$$

so $x^2 - x + 6 = Ax^2 + 3A + Bx^2 + Cx = (A+B)x^2 + Cx + 3A$

so
$A + B = 1$
$C = -1$
$3A = 6$
(by comparing coefficients)

so
$A = 6/3 = 2$
$B = 1 - A = 1 - 2 = -1$
$C = -1$

so

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx = \int \left(\frac{2}{x} + \frac{-1x - 1}{x^2 + 3} \right) dx$$

$$= \int \left(\frac{2}{x} - \frac{x}{x^2+3} - \frac{1}{x^2+3} \right) dx$$

$$= 2\int \frac{1}{x} dx - \int \frac{x}{x^2+3} dx - \int \frac{1}{x^2+3} dx$$

So

$$\int \frac{x^3-4x-10}{x^2-x-6}\,dx = \int\left(x+1+\frac{1}{x-3}+\frac{2}{x+2}\right)dx$$

$$= \boxed{\frac{x^2}{2}+x+\ln|x-3|+2\ln|x+2|+C}$$

$$= 2\ln|x| - \int \frac{x}{x^2+3}\,dx - \int \frac{1}{3\left(\frac{x^2}{3}+1\right)}\,dx$$

Let $u = x^2+3$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$$= 2\ln|x| - \int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}} - \frac{1}{3}\int \frac{1}{\left(\frac{x}{\sqrt{3}}\right)^2+1}\,dx$$

Let $u = \frac{x}{\sqrt{3}} \Rightarrow \frac{du}{dx} = \frac{1}{\sqrt{3}}$

$\Rightarrow dx = \sqrt{3}\,du$

$$= 2\ln|x| - \frac{1}{2}\int \frac{1}{u}\,du - \frac{1}{3}\int \frac{1}{u^2+1}\cdot\sqrt{3}\,du$$

$$= \boxed{2\ln|x| - \frac{1}{2}\ln|x^2+3| - \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C}$$