

8.4

$$\begin{aligned}
 \textcircled{3} \textcircled{a} [T]_B &= \begin{bmatrix} [T(1)]_B & [T(x)]_B & [T(x^2)]_B \end{bmatrix} \\
 &= \begin{bmatrix} [1]_B & [x-1]_B & [(x-1)^2]_B \end{bmatrix} \\
 &= \begin{bmatrix} [1]_B & [-1+x]_B & [1-2x+x^2]_B \end{bmatrix} \\
 &= \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

8(a)

$$\begin{aligned}
 [T]_{B', B^2} &= \begin{bmatrix} [T(1)]_{B'} & [T(x)]_{B'} & [T(x^2)]_{B'} \end{bmatrix} \\
 &= \begin{bmatrix} [x(1)]_{B'} & [x(1(x-3))]_{B'} & [x(1(x-3)^2)]_{B'} \end{bmatrix} \\
 &= \begin{bmatrix} [x]_{B'} & [x^2-3x]_{B'} & [x(x^2-6x+9)]_{B'} \end{bmatrix} \\
 &= \begin{bmatrix} [x]_{B'} & [x^2-3x]_{B'} & [x^3-6x^2+9x]_{B'} \end{bmatrix} \\
 &= \begin{bmatrix} [x]_{B'} & [-3x+x^2]_{B'} & [9x-6x^2+x^3]_{B'} \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 & 0 \\ 1 & -3 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

8.3 Using $T: P_2 \rightarrow \mathbb{R}^2$, FIND $[T]_{B',B}$ if:

(17) ~~17~~ $B = \{1, x\} \leftarrow \text{basis for } P_1$
 $B' = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \leftarrow \text{basis for } \mathbb{R}^2$

$$\begin{aligned} [T]_{B',B} &= \left[[T(1)]_{B'}, [T(x)]_{B'} \right] \\ &= \left[\begin{bmatrix} 1 \\ 1 \end{bmatrix}_{B'}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{B'} \right] \\ &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

Check: $p(x) = a_0 + a_1 x \in P_1 \rightarrow [p(x)]_B = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_0 + a_1 \end{bmatrix} = \begin{bmatrix} p(0) \\ p(1) \end{bmatrix} \quad \checkmark$$