TEST 4 PRACTICE PROBLEMS CALCULUS II (MATH 152) FALL 2013

(1) Use the **Ratio Test** to determine whether the series converges or diverges. If the test is inconclusive then say so.

(a)
$$\sum_{k=1}^{\infty} \frac{7^k}{k!}$$

(c)
$$\sum_{k=1}^{\infty} \frac{k^2}{(2k+1)!}$$

(b)
$$\sum_{n=1}^{\infty} \frac{3^n 4^{2n}}{(2n)!}$$

$$(d) \sum_{n=1}^{\infty} \frac{7^n}{n2^{2n}}$$

(2) Use the **Root Test** to determine whether the series converges or diverges. If the test is inconclusive then say so.

(a)
$$\sum_{k=1}^{\infty} \left(\frac{7k^2 - 5}{5k^2 + k - 1} \right)^k$$

(c)
$$\sum_{k=1}^{\infty} \left(\frac{\tan^{-1}(k)}{\pi} \right)^k$$

(b)
$$\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)^k$$

(d)
$$\sum_{h=1}^{\infty} \frac{n^{2n}}{3^{3n}}$$

(3) Use the **Alternating Series Test** to determine whether the series converges. If the test is inconclusive then say so.

(a)
$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2+e^k}$$

(c)
$$\sum_{k=2}^{\infty} \frac{(-1)^{k+1} \ln(k)}{k}$$

(b)
$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 + 1}$$

(d)
$$\sum_{k=0}^{\infty} \frac{\cos(n\pi)}{k\sqrt{k}}$$

(4) Determine whether the following series converges or diverges.

(a)
$$\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{2n}}$$

(c)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n + 1}$$

(b)
$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

(d)
$$\sum_{n=1}^{\infty} \frac{\sin(n)\sin(2n)\sin(3n)\cdots\sin(2013n)}{n^{2013}}$$

(5) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a)
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2 + 1}$$

(d)
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

(b)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$$

(e)
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^3}{2n^3+1}$$

(c)
$$\sum_{n=1}^{\infty} \frac{\sin(n)\cos(3n)}{n^8}$$

(f)
$$\sum_{n=1}^{\infty} \sin(n\pi/2)$$

(6) Using the formula, set up a table and find the first FOUR nonzero terms of the Maclaurin series for

(a)
$$f(x) = \frac{5}{1+2x} = 5(1+2x)^{-1}$$
 (c) $f(x) = \cos(2x-1)$
(d) $f(x) = \tan^{-1}(x)$

(c)
$$f(x) = \cos(2x - 1)$$

(b)
$$f(x) = e^{-2x^2}$$

(7) Using the formula, set up a table and find the first THREE nonzero terms of the Taylor series about the given x_0 for

(a)
$$f(x) = \tan^{-1}(x), x_0 = 1.$$

(c)
$$f(x) = \cos(x), x_0 = \pi/2.$$

(b)
$$f(x) = e^{-x}, x_0 = 5.$$

(c)
$$f(x) = \cos(x), x_0 = \pi/2.$$

(d) $f(x) = \sin\left(\frac{\pi}{2}x\right), x_0 = 1.$

(8) Find the radius and interval of convergence for the power series

(a)
$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n^2}$$
(b)
$$\sum_{n=0}^{\infty} \frac{x^n}{n+1}$$

(e)
$$\sum_{n=1}^{\infty} (n+2)!(x-5)^n$$
(f)
$$\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$$
(g)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$$

$$\text{(b) } \sum_{n=0}^{\infty} \frac{x^n}{n+1}$$

(f)
$$\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$$

(c)
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{\sqrt{n}}$$

(g)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$$

(d)
$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n} (n!)^2}$$

(h)
$$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{3^n \ln n}$$

NATHAN REFF, DIVISION OF MATHEMATICS, ALFRED UNIVERSITY, ALFRED, NY 14802, U.S.A. $E ext{-}mail\ address: reff@alfred.edu}$