Math 314 Summer 2010
6/15/2010 Quiz #11 Name: 110011101011010

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Show that $0.\overline{65}$ is rational by writing it in the form $0.\overline{65} = \frac{a}{b}$ where $a, b \in \mathbb{Z}$ and $b \neq 0$.

Solution: Let $x = 0.\overline{65}$, so $100x = 65.\overline{65}$. This gives

$$\begin{aligned} 100x - x &= 65.\overline{65} - 0.\overline{65} \\ 99x &= 65 \\ x &= \frac{65}{99} \end{aligned}$$

2. Compute $(3214567 + 234514) \bmod 10$ without using the value of $3214567{+}234514$.

Solution: $3214567 \equiv 7 (\text{mod} 10)$ and $234514 \equiv 4 (\text{mod} 10)$. So $(3214567 + 234514) \equiv 7 + 4 \equiv 1 (\text{mod} 10)$. Therefore $(3214567 + 234514) \bmod 10 = 1$.

3. Compute $5^{55} \bmod 19$

Solution: Since 19 is prime and $19 \nmid 5$ so Fermat's little theorem gives us the following congrunce:

$$5^{18} \equiv 1 (\text{mod} 19)$$

So now we can take powers to obtain:

$$(5^{18})^3 \equiv 1^3 (\text{mod} 19)$$

That is,

$$5^{54} \equiv 1 (\text{mod} 19)$$

So multiplying through by 5 we obtain:

$$5^{55} \equiv 5 (\text{mod} 19)$$

Therefore $5^{55} \bmod 19 = 5$

4. Compute $\binom{5}{3}$

Solution:

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = \frac{5 \cdot 4}{2 \cdot 1} = 10$$