Elementary row operations (Definition 8, p10)

1. Switch two equations (rows).
2. Multiply one equation (row) by a non-zero constant.
3. Add a multiple of one equation (row) to a different equation (row). Specifically, replace a row by the sum of that row and a multiple of some different row.

IMPORTANT! On tests you will be required to identify the row operation that you are using in each step. The way to describe this is by annotating each step in the following style:

1. $R2 \leftrightarrow R5$ (switch rows 2 and 5).
2. $R4 \to \frac{1}{13}R4$ (divide row 4 by 13).
3. $R1 \to R1 - 3R2$ (replace row 1 with row 1 minus 3 times row 2).

WARNING! The operation of type 3 is *ALWAYS* 'replace a row by itself plus or minus a multiple of a different row'. So $R2 \to R1 - 3R2$ is <u>*NOT*</u> a legal row operation.

The previous example using the above convention:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ -1 & 3 & 1 & 0 \\ 2 & -1 & 1 & 1 \end{array}\right] \xrightarrow{R2 \to R2+R1} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 2 & -1 & 1 & 1 \end{array}\right] \xrightarrow{R3 \to R3-2R1} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 0 & -3 & -5 & -3 \end{array}\right]$$

$$\xrightarrow{R3 \to R3+\frac{3}{4}R2} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 4 & 4 & 2 \\ 0 & 0 & -2 & -3/2 \end{array}\right] \xrightarrow{R2 \to \frac{1}{4}R2} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & -2 & -3/2 \end{array}\right] \xrightarrow{R3 \to \frac{-1}{2}R3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & 3/4 \end{array}\right] \xrightarrow{R2 \to R2-R3} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array}\right] \xrightarrow{R1 \to R1-3R3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & -1/4 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array}\right]$$

$$\xrightarrow{R1 \to R1-R2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array}\right]$$

So $x_1 = 0$, $x_2 = -1/4$ and $x_3 = 3/4$.