COMMENTS FOR LECTURE 11 - 2.11.2010

NATHAN REFF

We are continuing our study of the function that an $m \times n$ matrix C determinds from $\mathbb{R}^n$ to $\mathbb{R}^m$. Again we view this functions as multiplication on the left by C:

$$\boxed{\mathbf{x} \mapsto C\mathbf{x}}$$

Where $\mathbf{x}$ is some arbitary vector in $\mathbb{R}^n$. Today we showed that this function is indeed a *linear transformation*. This means that for any vectors $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$, and for any scalar k in $\mathbb{R}^n$ the following two things are satisfied:

(1) $C(\mathbf{x} + \mathbf{y}) = C(\mathbf{x}) + C(\mathbf{y})$
(2) $C(k\mathbf{x}) = kC(\mathbf{x})$

See **Theorem 2.5.1** and **Theorem 2.5.2**.
So now we can say that if C is an $m \times n$ matrix, then $\mathbf{x} \mapsto C\mathbf{x}$ is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

On Friday we will establish quite an amazing converse to this statement: suppose T is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$, then there is a unique $m \times n$ matrix B so that $T(\mathbf{x}) = B\mathbf{x}$ for every $\mathbf{x}$ in $\mathbb{R}^n$.

Department of Mathematical Sciences, Binghamton University (SUNY), Binghamton, NY 13902-6000, U.S.A.
E-mail address: reff@math.binghamton.edu