Math 314 Summer 2011
6/24/2011 Quiz #14

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes. Due on 6/27/2011

1. Prove that $\mathbb{Z}$ and $4\mathbb{Z} = \{n \in \mathbb{Z} \mid n = 4k \text{ for some } k \in \mathbb{Z}\}$ have the same cardinality.

Let $f: \mathbb{Z} \to 4\mathbb{Z}$ be defined as follows: For an integer n, $f(n) = 4n$

f is one-to-one: Let $n_1, n_2 \in \mathbb{Z}$. Suppose $f(n_1) = f(n_2) \Rightarrow 4n_1 = 4n_2 \Rightarrow n_1 = n_2$. Hence f is one-to-one. ✓

f is onto: Suppose $m \in 4\mathbb{Z}$. $\Rightarrow \exists k \in \mathbb{Z}$ such that $m = 4k$. Notice $f(k) = 4k = m$. Hence f is onto. ✓

Therefore f is a one-to-one correspondence. Hence $\mathbb{Z}$ and $4\mathbb{Z}$ have the same cardinality.

2. Let $X = \{a, b, c\}$. Define a relation R on $\mathcal{P}(X)$ as follows:

$$\forall A, B \in \mathcal{P}(X), ARB \iff A \text{ has the same number of elements as } B.$$

(a) Is $\{a\}R\{b\}$? Yes

(b) Is $\{a\}R\{a\}$? Yes

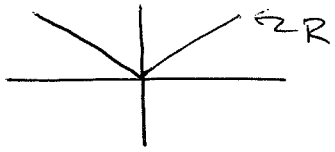
(c) Is $\{a\}R\{d\}$? NO, $\{d\} \notin \mathcal{P}(X)$

(d) Is $\{a, a\}R\{a, b\}$? No, $\{a, a\} = \{a\}$

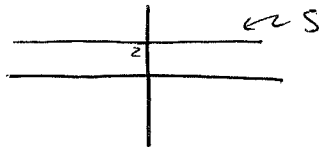
3. Define relations R and S on $\mathbb{R}$ as follows:

$$R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = |x|\}, \text{ and}$$
$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = 2\}.$$

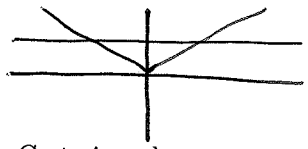
(a) Draw R in the Cartesian plane.



(b) Draw S in the Cartesian plane.



(c) Draw $R \cup S$ in the Cartesian plane.



(d) Draw $R \cap S$ in the Cartesian plane.