Math 304 Spring 2010
3/25/2010
Quiz #9

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. What is the dimension of a vector space?
   A. The number of possible bases for the vector space.
   B. The set of all vectors in the vector space.
   C. The number of vectors in the vector space.
   D. The span of all the vectors in the vector space.
   E. The number of vectors in the span of any set of vectors in the vector space.
   F. The number of vectors in a basis of the vector space.
   G. None of the above.

2. ~~The dimension of the kernel of a linear transformation $T:\mathbb{R}^{90} \to \mathbb{R}^{10}$ is equal to 5. What is the rank(T)?~~
   ~~A. 5~~
   ~~B. 95~~
   ~~C. 80~~
   ~~D. 85~~
   ~~E. 0~~
   ~~F. 10~~

This is not possible since the rank(T) cannot be greater than 10.

3. The dimension of the image of a linear transformation $T:\mathbb{R}^{90} \to \mathbb{R}^{10}$ is equal to 5. What is the dimension of the kernel of T?
   A. 5
   B. 95
   C. 80
   D. 85
   E. 0
   F. 10

Recall: $\underbrace{\dim(\mathrm{im}(T))}_{=5} + \dim(\ker(T)) = \underbrace{\dim(\mathrm{Domain}(T))}_{=\dim(\mathbb{R}^{90}) = 90}$

so $\dim(\ker(T)) = 90 - 5 = 85$

4. Let B be a $n \times n$ matrix. Suppose $\mathbf{x}$ is in the null space of B then
   A. $B\mathbf{x} = \mathbf{0}$
   B. $\mathbf{x}$ must be the zero vector
   C. $B\mathbf{x} = \mathbf{y}$ where $\mathbf{y} \neq 0$
   D. None of the above.

5. Suppose that A and B are $n \times n$ matrices. Show that if $\mathbf{x}$ is in the null space of B then $\mathbf{x}$ is in the null space of AB.

$\vec{x}$ is in the nullspace of $B \implies B\vec{x} = \vec{0}$

notice

$$(AB)\vec{x} = A(B\vec{x}) = A\vec{0} = \vec{0}$$

so $\vec{x}$ is in the nullspace of AB.