

Supplementary homework problems for HW 7.

1. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix}$

- (a) Find a basis for the $\text{Col}(A)$.
- (b) Find a basis for the $\text{Null}(A)$.
- (c) Find a basis for $\text{Row}(A)$.
- (d) Find a basis for the $\text{Col}(A^T)$.
- (e) Find a basis for the $\text{Null}(A^T)$.
- (f) Find a basis for $\text{Row}(A^T)$.

2. Let T be the transformation given by the matrix (having associated/standard matrix):

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix}$$

(same as in the previous problem).

- (a) What is the domain of T ?
- (b) Find a basis for the domain of T . (Hint: You do not need to do any work, just pick a trivial basis!)
- (c) What is the codomain of T ?
- (d) Find a basis for the codomain of T . (Hint: You do not need to do any work, just pick a trivial basis!)
- (e) Find a basis for the image of T . (You do not need to do any work, just use the previous problem).
- (f) Find a basis for the kernel of T . (You do not need to do any work, just use the previous problem).

3. Let

$$X = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

- (a) Show X is an ordered basis of \mathbb{R}^2 .
- (b) Let K be the coordinate transformation defined by the ordered basis X . The vector $\mathbf{w} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ in \mathbb{R}^2 . Calculate $K(\mathbf{w})$.