EXAM 3

Math 221 - 09 - Calculus I
4/30/2009

Name: Key

When you are finished please sign the following:

Signature: ____________________

By signing my name I pledge that I have not broken the Student Academic Honesty Code at any point during this examination.

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please box/circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 6 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

Part B. Problems solving. (70% of the total points) **You need to show your work!!**

2. (*10 points*)

a. (*5 pts*) $\int (2\sec(\theta)\tan(\theta) - \theta^6 + 5)d\theta =$ $\int 2\sec(\theta)\tan\theta\, d\theta - \int \theta^6 d\theta + \int 5\, d\theta$

$$= \boxed{2\sec\theta - \frac{\theta^7}{7} + 5\theta + C}$$

b. (*5 pts*) $\int 3x^2\sqrt{x^3 + 15}\,dx =$

Let $u = x^3 + 15 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2}$

So $\int 3x^2\sqrt{x^3+15}\,dx = \int \cancel{3x^2}\sqrt{u}\,\frac{du}{\cancel{3x^2}}$

$$= \int \sqrt{u}\,du$$

$$= \int u^{1/2}\,du$$

$$= \frac{u^{3/2}}{3/2} + C$$

$$= \boxed{\frac{2}{3}(x^3+15)^{3/2} + C}$$

3. (*15 points*)

1st Solution :

a. (*5 pts*) $\dfrac{d}{dx}\displaystyle\int_0^x \cos(t)dt = \boxed{\cos(x)}$ by FTC 1

2nd Solution :

$$\frac{d}{dx}\int_0^x \cos(t)dt = \frac{d}{dx}\Big[\sin(t)\Big]_0^x$$
$$= \frac{d}{dx}\Big[\sin(x) - \sin(0)\Big]$$
$$= \boxed{\cos(x)}$$

b. (*5 pts*) $\dfrac{d}{dx}\displaystyle\int_2^{100\pi} \cos(t)dt =$

1st solution : $\displaystyle\int_2^{100\pi}\cos(t)dt$ is a number (constant) and therefore $\dfrac{d}{dx}\left(\displaystyle\int_2^{100\pi}\cos(t)dt\right) = \boxed{0}$

2nd solution : $\dfrac{d}{dx}\left(\displaystyle\int_2^{100\pi}\cos(t)dt\right) = \dfrac{d}{dx}\Big[\sin(t)\Big]_2^{100\pi}$
$$= \frac{d}{dx}\Big[\sin(100\pi) - \sin(2)\Big]$$
$$= \boxed{0}$$

c. (*5 pts*) $\dfrac{d}{dx}\displaystyle\int_0^{\sin(x)} \cos(t)dt =$

1st Solution (method from the book) : Let $u = \sin(x)$ then

$$\frac{d}{dx}\int_0^{\sin(x)}\cos(t)dt = \frac{d}{dx}\int_0^u \cos(t)\,dt \underset{\substack{\uparrow \\ \text{By Chain Rule}}}{=} \left(\frac{d}{du}\int_0^u \cos(t)\,dt\right)\frac{du}{dx} = \cos(u)\frac{d}{dx}u$$
$$= \cos(\sin(x))\frac{d}{dx}\sin(x)$$
$$= \boxed{\cos(\sin(x))\cos(x)}$$

2nd solution : If we let $g(x) = \int_0^x \cos(t)dt$ then $g(\sin(x)) = \int_0^{\sin(x)}\cos(t)\,dt$

So now we need to find $\dfrac{d}{dx}g(\sin(x)) \underset{\substack{\uparrow \\ \text{Chain Rule}}}{=} g'(\sin(x))\cos(x) \underset{\substack{\uparrow \\ \text{using part (a) above.}}}{=} \boxed{\cos(\sin(x))\cos(x)}$

3rd solution : $\dfrac{d}{dx}\displaystyle\int_0^{\sin(x)}\cos(t)dt = \dfrac{d}{dx}\Big[\sin(t)\Big]_0^{\sin(x)} = \dfrac{d}{dx}\Big[\sin(\sin(x)) - \sin(0)\Big] \underset{\substack{\uparrow \\ \text{by Chain Rule.}}}{=} \boxed{\cos(\sin(x))\cos(x)}$

4. (*15 points*)

a. (*5 pts*) $\displaystyle\int_{500}^{500} \sin(\theta)\sin(\cos(\theta))d\theta = \boxed{0}$

The limits of integration are the same!

b. (*5 pts*) Suppose $\displaystyle\int_2^4 f(x)dx = 3 - \sqrt{13}$. Find the value of $\displaystyle\int_4^2 5f(y)dy$:

$$\int_4^2 5f(y)dy = 5\int_4^2 f(y)dy = -5\underbrace{\int_2^4 f(y)dy}$$

this is the given integral. Note that the variable does not matter!

$$= -5\left(3 - \sqrt{13}\right)$$

$$= \boxed{-15 + 5\sqrt{13}}$$

c. (*5 pts*) $\displaystyle\int_{-\pi}^{\pi} \sin(x)\sqrt{1+\cos(x)}\,dx =$

1st solution (quick) : notice $f(x) = \sin(x)\sqrt{1+\cos(x)}$ is ODD

since $f(-x) = \sin(-x)\sqrt{1+\cos(-x)} \underset{\uparrow}{=} -\sin(x)\sqrt{1+\cos(x)} = f(x)$

since $\sin(x)$ is ODD and $\cos(x)$ is EVEN

hence, $\displaystyle\int_{-\pi}^{\pi} \sin(x)\sqrt{1+\cos(x)}\,dx = \boxed{0}$

2nd solution : Let $u = 1+\cos(x) \implies u(-\pi) = 1+\cos(-\pi) = 0$ and $\dfrac{du}{dx} = -\sin(x)$

$u(\pi) = 1+\cos(\pi) = 0$

so $\displaystyle\int_{-\pi}^{\pi} \sin(x)\sqrt{1+\cos(x)}\,dx = \int_0^0 \cancel{\sin(x)}\sqrt{u}\,\frac{du}{-\cancel{\sin(x)}} = -\int_0^0 \sqrt{u}\,du = \boxed{0}$

5. (*20 points*)

a. (*10 pts*) Write $\int_0^{10} \sin(x^2)dx$ as a limit of Riemann sums taking the sample points to be the right endpoints on the subintervals. **DO NOT EVALUATE THE LIMIT**

$$\Delta x = \frac{b-a}{n} = \frac{10-0}{n} = \frac{10}{n}$$

$$x_i = a + i\Delta x = 0 + i\left(\frac{10}{n}\right) = \frac{10i}{n}$$

$$\int_0^{10} \sin(x^2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{, here } f(x) = \sin(x^2)$$

$$= \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\left(\frac{10i}{n}\right)^2\right)\left(\frac{10}{n}\right)}$$

b. (*10 pts*) Find f given $f''(x) = 1 + 6x$, $f'(0) = 1$, $f(0) = 216$.

$$f'(x) = x + \frac{6x^2}{2} + C = x + 3x^2 + C$$

$$f'(0) = 1 = 0 + 3(0)^2 + C \implies C = 1$$

$$f'(x) = x + 3x^2 + 1$$

$$f(x) = \frac{x^2}{2} + \frac{3x^3}{3} + x + D$$

$$f(0) = 216 = \frac{0^2}{2} + 0^3 + 0 + D \implies D = 216$$

$$\boxed{f(x) = \frac{x^2}{2} + x^3 + x + 216}$$

PICK ONE OF THE FOLLOWING:

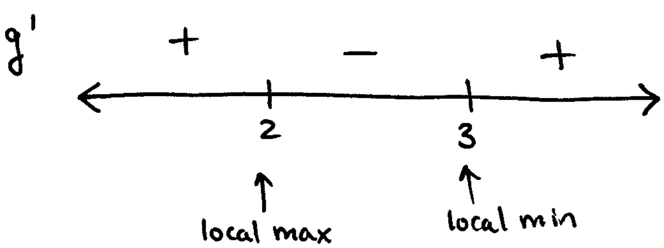
Please indicate which one you do NOT want me to grade by putting an X through it, otherwise I will grade the first one worked on:

6. (*10 points*)

a. (*10 pts*) Suppose $g(x) = \int_0^x \frac{t^2-5t+6}{t^2+4}dt$ for $x \geq 0$. At what values of x does g have a local maximum or minimum?

$$g'(x) = \frac{d}{dx}\left(\int_0^x \frac{t^2-5t+6}{t^2+4}\,dt\right) = \frac{x^2-5x+6}{x^2+4} = \frac{(x-2)(x-3)}{x^2+4}$$

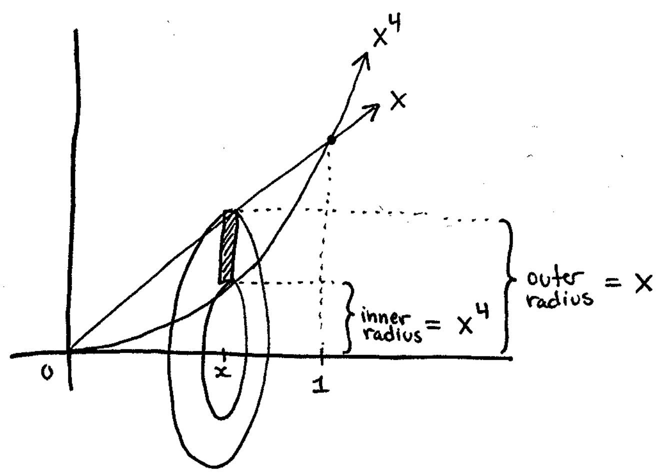
$$g'(x) = 0 = \frac{(x-2)(x-3)}{x^2+4} \implies x = 2 \quad \text{and} \quad x = 3$$



So g has a local max or min at x-coordinates

$$\boxed{x = 2 \quad \text{and} \quad x = 3}$$

b. (*10 pts*) Find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^4$ about the x-axis.



<u>intersection points</u>:

$$x = x^4$$
$$0 = x^4 - x$$
$$0 = x(x^3-1)$$
$$x = 0 \quad \underline{\text{OR}} \quad x = 1$$

<u>inner radius</u>: x^4

<u>outer radius</u>: x

$$A(x) = \pi x^2 - \pi (x^4)^2$$
$$= \pi(x^2 - x^8)$$

so $$V = \int_a^b A(x)dx = \int_0^1 \pi(x^2-x^8)dx = \pi \int_0^1 (x^2-x^8)dx = \pi\left[\frac{x^3}{3} - \frac{x^9}{9}\right]_0^1$$
$$= \pi\left[\frac{1}{3} - \frac{1}{9}\right]$$
$$= \boxed{\frac{2\pi}{9}}$$