TEST 1

Math 104

Score: _____ out of 100

2/7/2013

Name: _____ Key _____

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.

- Show all work (if necessary), clearly and in order, if you want to recieve full credit. Please make sure you read the directions for each problem. If the problem requires work I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- Please box/circle or otherwise indicate your final answers.

- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.

- This test has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!

- Good luck!

1. Simplify the following. Your simplification should have **no negative exponents**.

   (a) $x^4 \cdot x^7 =$ $x^{4+7} = \boxed{x^{11}}$

   (b) $\dfrac{x^5}{x^{-4}} =$ $x^{5-(-4)} = \boxed{x^9}$

   (c) $\left(\dfrac{x^2 y}{z^3}\right)^3 = \boxed{\dfrac{x^6 y^3}{z^9}}$

2. Simplify the following.

   (a) $(30x - 7z) - (12x - 2z) =$

   $30x - 7z - 12x + 2z = \boxed{18x \quad -5z}$

   (b) $(3x^2 + 10xy + y^2) - (x^2 + 5xy + y^2) =$

   $3x^2 + 10xy + y^2 - x^2 - 5xy - y^2 =$

   $\boxed{2x^2 + 5xy}$

(c) $(2x - 3y)(4x - 5y) =$

$= (2x)(4x) + (2x)(-5y) + (-3y)(4x) + (-3y)(-5y)$

$= 8x^2 - 10xy - 12xy + 15y^2$

$= \boxed{8x^2 - 22xy + 15y^2}$

(d) $(x^2 - 2x + 3)(x - 1) =$

$= (x^2)(x) + (x^2)(-1) + (-2x)(x) + (-2x)(-1) + (3)(x) + (3)(-1)$

$= x^3 - x^2 - 2x^2 + 2x + 3x - 3$

$= \boxed{x^3 - 3x^2 + 5x - 3}$

3. Factor the following polynomials.

(a) $x^3 + 2x^4 = \boxed{x^3(1 + 2x)}$

(b) $x^2 - x - 6 = \boxed{(x - 3)(x + 2)}$

(c) $6x^2 - 5x - 6 =$

$\Bigg\{$ $ac = 6(-6) = -36.$ 2 factors of -36 that sum to -5:

$\qquad\qquad\qquad\qquad -9 \quad$ and $+4$

$= 6x^2 - 9x + 4x - 6$

$= 3x(2x - 3) + 2(2x - 3)$

$= \boxed{(2x - 3)(3x + 2)}$

4. Simplify the following fractions.

(a) $\dfrac{2x^2 + 5x - 12}{2x^2 - 11x + 12} = \dfrac{(x+4)(2x-3)}{(x-4)(2x-3)} = \boxed{\dfrac{x+4}{x-4}}$

$\left(\text{assuming } 2x-3 \neq 0 \atop \text{i.e., } x \neq 3/2\right)$

(b) $\dfrac{3x^2y^2}{6y^3z} \cdot \dfrac{2xz^4}{4z} = \dfrac{6x^3y^2z^4}{6\cdot 4 \cdot y^3 z^2} = \boxed{\dfrac{x^3 z^2}{4y}}$

(c) $\dfrac{3x^2}{y^5} \div \dfrac{10x^4z}{y^4} = \dfrac{3x^2}{y^5} \cdot \dfrac{y^4}{10x^4z} = \boxed{\dfrac{3}{10yx^2z}}$

(d) $\dfrac{10}{x+3} - \dfrac{5x}{x-2} = \dfrac{10(x-2)}{(x+3)(x-2)} - \dfrac{5x(x+3)}{(x-2)(x+3)}$

$= \dfrac{10x - 20 - 5x(x+3)}{(x+3)(x-2)}$

$= \dfrac{10x - 20 - 5x^2 - 15x}{(x+3)(x-2)}$

$= \boxed{\dfrac{-5x^2 - 5x - 20}{(x-2)(x+3)}}$

5. Simplify the following radicals.

(a) $\sqrt{32}\sqrt{2} = \sqrt{32 \cdot 2} = \sqrt{64} = \boxed{8}$

(b) $\sqrt{100} \div \sqrt{4} = \sqrt{100/4} = \sqrt{25} = \boxed{5}$

(c) $\sqrt{169x^{10}y^4z^8} = \sqrt{169}\sqrt{x^{10}}\sqrt{y^4}\sqrt{z^8}$

$= 13\, x^{10/2}\, y^{4/2}\, z^{8/2}$

$= \boxed{13\, x^5 y^2 z^4}$

(d) If $\sqrt{80x^5y^6z^{11}} = A\sqrt{B}$, then $A = \underline{4x^2y^3z^5}$ and $B = \underline{5xz}$

$\overset{11}{}$

$\sqrt{80}\sqrt{x^5}\sqrt{y^6}\sqrt{z^{11}}$

$= \sqrt{16 \cdot 5}\sqrt{x x^4}\sqrt{y^6}\sqrt{z z^{10}}$

$= \sqrt{16}\sqrt{5}\sqrt{x}\sqrt{x^4}\sqrt{y^6}\sqrt{z}\sqrt{z^{10}}$

$= 4\sqrt{5}\sqrt{x}\; x^2\, y^3\sqrt{z}\; z^5$

$= 4x^2y^3z^5\sqrt{5xz}$

6. ♠ Extra Credit: If $\sqrt[4]{\dfrac{256x^{20}y^6z^{21}}{z^5w^8}} = A\sqrt[4]{B}$,

then $A = \dfrac{4x^5yz^4}{w^2}$ and $B = y^2$

$= \dfrac{\sqrt[4]{256}\ \sqrt[4]{x^{20}}\ \sqrt[4]{y^6}\ \sqrt[4]{z^{21}}}{\sqrt[4]{z^5}\ \sqrt[4]{w^8}}$

$= \dfrac{4\,x^5\ \sqrt[4]{y^4y^2}\ \sqrt[4]{z^{20}z^1}}{\sqrt[4]{z^4z}\ w^2}$

$= \dfrac{4x^5y\ \sqrt[4]{y^2}\ z^5\ \sqrt[4]{z}}{z\ \sqrt[4]{z}\ w^2}$

$= \dfrac{4x^5yz^5}{zw^2}\ \sqrt[4]{\dfrac{y^2z}{z}}$

$= \dfrac{4x^5yz^4}{w^2}\ \sqrt[4]{y^2}$