Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Compute the first four terms in each of the following sequences:

   (a) $\forall n \geq 0$, $s_n = 5 - 3n$.

   $s_0 = 5 - 3(0) = 5$
   $s_1 = 5 - 3(1) = 2$
   $s_2 = 5 - 3(2) = -1$
   $s_3 = 5 - 3(3) = -4$

   (b) $\forall n \geq 0$, $s_n = 3 \cdot 2^n$.

   $s_0 = 3 \cdot 2^0 = 3 \cdot 1 = 3$
   $s_1 = 3 \cdot 2^1 = 3 \cdot 2 = 6$
   $s_2 = 3 \cdot 2^2 = 3 \cdot 4 = 12$
   $s_3 = 3 \cdot 2^3 = 3 \cdot 8 = 24$

   (c) $s_0 = 1$ and $\forall n \geq 1$, $s_n = 1 + n - s_{n-1}$.

   $s_0 = 1$
   $s_1 = 1 + 1 - 1 = 1$
   $s_2 = 1 + 2 - 1 = 2$
   $s_3 = 1 + 3 - 2 = 2$

2. Find a closed formula for each of the following sequences:

   (a) $4, 6, 8, 10, 12, \ldots$

   $\forall n \geq 0, \ s_n = 4 + 2n$

   (b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$

   $\forall n \geq 1, \ s_n = \frac{1}{2^n}$

   (c) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

   $\forall n \geq 1, \ s_n = \frac{1}{n}$

3. ♠ $\binom{\lfloor \frac{n}{2} \rfloor}{2} = \dfrac{(\lfloor \frac{n}{2} \rfloor)!}{2!(\lfloor \frac{n}{2} \rfloor - 2)!}$.

if n is even, $\lfloor \frac{n}{2} \rfloor = \frac{n}{2} \Rightarrow \binom{\lfloor \frac{n}{2} \rfloor}{2} = \dfrac{(\frac{n}{2})!}{2!(\frac{n}{2} - 2)!} = \dfrac{(\frac{n}{2})!}{2!(\frac{n-4}{2})!} = \dfrac{(\frac{n}{2})(\frac{n}{2}-1)\cancel{(\frac{n}{2}-2)!}}{2!\cancel{(\frac{n}{2}-2)!}} = \dfrac{n(n-2)}{8}$.

if n is odd, $\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2} \Rightarrow \binom{\lfloor \frac{n}{2} \rfloor}{2} = \dfrac{(\frac{n-1}{2})!}{2!(\frac{n-1}{2} - 2)!} = \dfrac{(\frac{n-1}{2})(\frac{n-1}{2}-1)\cancel{(\frac{n-1}{2}-2)!}}{2!\cancel{(\frac{n-1}{2}-2)!}} = \dfrac{(n-1)(n-3)}{8}$