

Name: _____

Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Write out the FORM of the partial fraction decomposition for the following (DO NOT find the numerical values for the unknown coefficients).

$$(a) \frac{4x^3 - 1}{x^3(x^2 + 9)^2} = \boxed{\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+9} + \frac{Fx+G}{(x^2+9)^2}}$$

repeated linear repeated irreducible quadratic.

$$(b) \frac{x+10}{(x^2+x+10)(x^2-4)} = \frac{x+10}{(\overset{\text{irreducible quadratic}}{x^2+x+10})(\overset{\text{linear}}{x-2})(x+2)} = \boxed{\frac{Ax+B}{x^2+x+10} + \frac{C}{x-2} + \frac{D}{x+2}}$$

2. Evaluate $\int_2^\infty \frac{1}{x\sqrt{\ln(x)}} dx$.

$$\int_2^\infty \frac{1}{x\sqrt{\ln(x)}} dx = \lim_{t \rightarrow \infty} \int_2^t \frac{1}{x\sqrt{\ln(x)}} dx$$

$$\begin{aligned} u &= \ln(x) & \frac{du}{dx} &= \frac{1}{x} \\ \downarrow & & & \\ u(2) &= \ln(2) & dx &= x du \\ u(t) &= \ln(t) & & \end{aligned}$$

$$= \lim_{t \rightarrow \infty} \int_{\ln(2)}^{\ln(t)} \frac{1}{\cancel{x} \sqrt{u}} \cdot \cancel{x} du$$

$$= \lim_{t \rightarrow \infty} \int_{\ln(2)}^{\ln(t)} u^{-1/2} du$$

$$= \lim_{t \rightarrow \infty} \left[\frac{u^{1/2}}{1/2} \right]_{\ln(2)}^{\ln(t)}$$

$$= \lim_{t \rightarrow \infty} \left[2\sqrt{u} \right]_{\ln(2)}^{\ln(t)}$$

$$= \lim_{t \rightarrow \infty} \left[2\sqrt{\ln(t)} - 2\sqrt{\ln(2)} \right]$$

$$= \boxed{\infty} \text{ diverges}$$