Comments for Lecture 13
2.17.2010

Here is another example problem using methods from section 2.6.

Example: Consider the linear transformation T:$\mathbb{R}^2 \to \mathbb{R}^2$ defined as:

$$T(\mathbf{x}) = T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$$

where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is in $\mathbb{R}^2$.

Consider also the linear transformation S:$\mathbb{R}^2 \to \mathbb{R}$ defined as:

$$S(\mathbf{y}) = S\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right) = y_2$$

where $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ is in $\mathbb{R}^2$.

Questions:

(1) Find the associated matrix A such that $T\mathbf{x} = A\mathbf{x}$ for any $\mathbf{x}$ in $\mathbb{R}^2$.
(2) Find the associated matrix B such that $S\mathbf{y} = B\mathbf{y}$ for any $\mathbf{y}$ in $\mathbb{R}^2$.
(3) Does the composition $S(T(\mathbf{x}))$ exist? If so find the associated matrix C such that $S(T(\mathbf{x})) = C\mathbf{x}$ for any $\mathbf{x}$ in $\mathbb{R}^2$.
(4) Does the composition $T(S(\mathbf{y}))$ exist? If so find the associated matrix D such that $T(S(\mathbf{y})) = D\mathbf{y}$ for any $\mathbf{y}$ in $\mathbb{R}^2$.

Solutions:

(1) We did this in the comments from lecture 12.

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(2) We did this in the comments from lecture 12.

$$B = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

(3) Using **Theorem 2.6.2** the composition $S(T(\mathbf{x}))$ exists and we have $C = BA$.

$$C = BA = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

(Notice that you could also find this by the method used to solve (1) and (2) but I wanted you to see how we can use theorem 2.6.2 to solve this problem.)

(4) This composition does not exist. The domain of T is $\mathbb{R}^2$ and the codomain of S is $\mathbb{R}$ (so anything in the image of S will be in $\mathbb{R}$ and cannot be an "input" for T). To convince even more look at what would happen:

$$T(S(\mathbf{y})) = T\left(S\left(\left[\begin{array}{c} y_1 \\ y_2 \end{array}\right]\right)\right) = T(\underbrace{y_2}_{\text{in } \mathbb{R}}) = \text{Does not exist}$$

The domain of T is $\mathbb{R}^2$ and y_2 is just a real number and not a vector in $\mathbb{R}^2$ so we cannot make sense of $T(y_2)$.