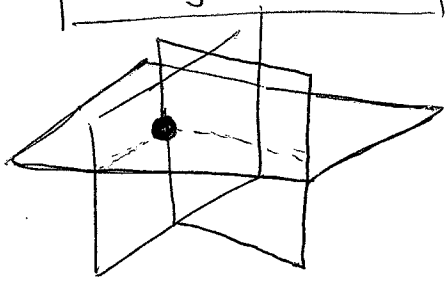
Linear systems with three unknowns (geometrically)

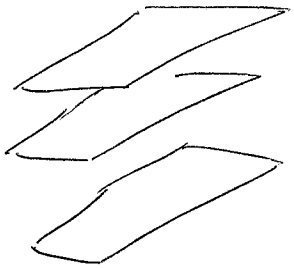
Each linear equation in three variables: $c_1x_1+c_2x_2+c_3x_3=k$ represents a plane in (x_1,x_2,x_3)-space.

The Good
Exactly One Solution

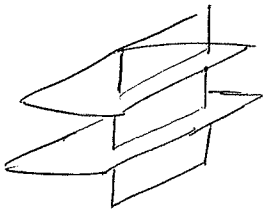


(intersection is a point)

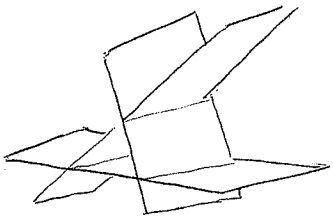
The Bad
No Solutions



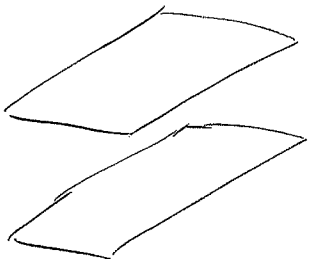
(3 parallel planes)
no common intersection



(2 parallel planes)
no common intersection

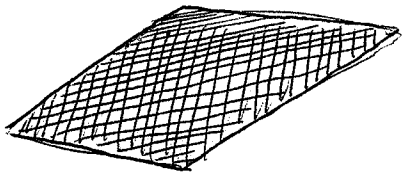


(no common intersection)

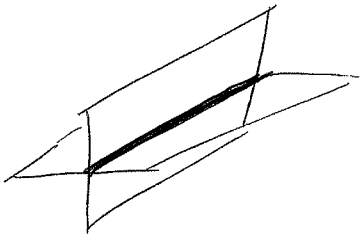


(two coincident planes
parallel to the third:
no common intersection)

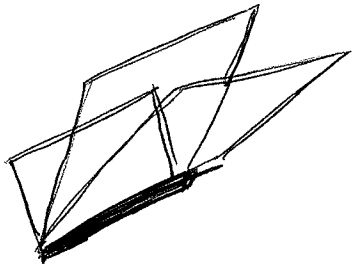
The Ugly
Infinitely many solutions



all planes are coincident
intersection is a plane



(two coincident planes;
intersection is a line)



(intersection is a line)
(think of pages in a book)