System of Linear Equations

$$x_1 + x_2 + 3x_3 = 2$$

$$-x_1 + 3x_2 + x_3 = 0$$

$$2x_1 - x_2 + x_3 = 1$$

Add equations 1 to equation 2:

$$x_1 + x_2 + 3x_3 = 2$$

$$0x_1 + 4x_2 + 4x_3 = 2$$

$$2x_1 - x_2 + x_3 = 1$$

Add -2 times equation 1 to equation 3:

$$x_1 + x_2 + 3x_3 = 2$$

$$0x_1 + 4x_2 + 4x_3 = 2$$

$$0x_1 - 3x_2 - 5x_3 = -3$$

Add 3/4 times equation 2 to equation 3:

$$x_1 + x_2 + 3x_3 = 2$$

 $0x_1 + 4x_2 + 4x_3 = 2$
 $0x_1 + 0x_2 - 2x_3 = -3/2$

Multiply equation 2 by 1/4:

$$x_1 + x_2 + 3x_3 = 2$$

$$0x_1 + x_2 + x_3 = 1/2$$

$$0x_1 + 0x_2 - 2x_3 = -3/2$$

Multiply equation 3 by -1/2:

$$x_1 + x_2 + 3x_3 = 2$$

 $0x_1 + x_2 + x_3 = 1/2$
 $0x_1 + 0x_2 + x_3 = 3/4$

Add -1 times equation 3 to equation 2:

$$x_1 + x_2 + 3x_3 = 2$$

 $0x_1 + x_2 + 0x_3 = -1/4$
 $0x_1 + 0x_2 + x_3 = 3/4$

Augmented Matrix

$$\left[\begin{array}{ccc|c}
1 & 1 & 3 & 2 \\
-1 & 3 & 1 & 0 \\
2 & -1 & 1 & 1
\end{array} \right]$$

Add row 1 to row 2:

$$\left[\begin{array}{ccc|c}
1 & 1 & 3 & 2 \\
0 & 4 & 4 & 2 \\
2 & -1 & 1 & 1
\end{array}\right]$$

Add -2 times row 1 to row 3:

$$\begin{bmatrix}
1 & 1 & 3 & 2 \\
0 & 4 & 4 & 2 \\
0 & -3 & -5 & -3
\end{bmatrix}$$

Add 3/4 times row 2 to row 3:

$$\left[\begin{array}{ccc|ccc}
1 & 1 & 3 & 2 \\
0 & 4 & 4 & 2 \\
0 & 0 & -2 & -3/2
\end{array} \right]$$

Multiply row 2 by 1/4:

$$\left[\begin{array}{ccc|ccc}
1 & 1 & 3 & 2 \\
0 & 1 & 1 & 1/2 \\
0 & 0 & -2 & -3/2
\end{array}\right]$$

Multiply row 3 by -1/2:

$$\left[\begin{array}{ccc|c}
1 & 1 & 3 & 2 \\
0 & 1 & 1 & 1/2 \\
0 & 0 & 1 & 3/4
\end{array}\right]$$

Add -1 times row 3 to row 2:

$$\left[\begin{array}{ccc|c}
1 & 1 & 3 & 2 \\
0 & 1 & 0 & -1/4 \\
0 & 0 & 1 & 3/4
\end{array}\right]$$

System of Linear Equations

Add -3 times equation 3 to equation 1:

$$x_1 + x_2 + 0x_3 = -1/4$$

 $0x_1 + x_2 + 0x_3 = -1/4$
 $0x_1 + 0x_2 + x_3 = 3/4$

Add -1 times equation 2 to equation 1:

$$x_1 + 0x_2 + 0x_3 = 0$$

 $0x_1 + x_2 + 0x_3 = -1/4$
 $0x_1 + 0x_2 + x_3 = 3/4$

So
$$x_1 = 0$$
, $x_2 = -1/4$ and $x_3 = 3/4$.

Augmented Matrix

Add -3 times row 3 to row 1:

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -1/4 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array}\right]$$

Add -1 times row 2 to row 1:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 3/4 \end{array}\right]$$

So
$$x_1 = 0$$
, $x_2 = -1/4$ and $x_3 = 3/4$.