Math 314 Summer 2011
6/29/2011 Quiz #17

Name: Key

Show all work clearly and in order. Please box your answers. 10 minutes.

1. Express each of the following sums in closed form (without using summation notation and without using an ellipsis $\cdots$).

(a) $\sum_{k=0}^{n} \binom{n}{k} \frac{1}{2^k} = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{1}{2}\right)^k 1^{n-k} \underset{\substack{\uparrow \\ \text{binomial thm.}}}{=} \left(1+\frac{1}{2}\right)^n = \boxed{\left(\frac{3}{2}\right)^n}$

(b) $\sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{1}{2^k} = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{-1}{2}\right)^k 1^{n-k} \underset{\substack{\uparrow \\ \text{binomial thm.}}}{=} \left(1-\frac{1}{2}\right)^n = \boxed{\left(\frac{1}{2}\right)^n}$

2. Find the coefficient of $u^{16}v^4$ in $(u^2 - v^2)^{10}$.

using the binomial thm. we have $(u^2-v^2)^{10} = \sum_{k=0}^{10} \binom{10}{k} (u^2)^k (-v^2)^{10-k}$

OR $= \sum_{k=0}^{10} \binom{10}{k} (u^2)^{10-k} (-v^2)^k$

for this version, if $k=8$ the term is

$\binom{10}{8} u^{16} (-v^2)^2 = \binom{10}{8} u^{16} v^4$ so $\binom{10}{8} = \frac{10!}{8!(2!)} = \frac{10 \cdot 9}{2} = \boxed{45}$

for this version if $k=2$ then the term is

$\binom{10}{2} u^{16} (-v^2)^2$

$= \binom{10}{2} u^{16} v^4$

so

$\binom{10}{2} = \boxed{45}$

SAME answer.

3. Prove: for all integers $n \geq 1$,

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0.$$

proof:

notice that $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = \sum_{k=0}^{n} \binom{n}{k} (-1)^k 1^{n-k}$

$= (1+(-1))^n$ by binomial thm.

$= 0^n$

$= 0$

$\square$