Math 221 Fall 2009
10/12/2009
Quiz #7

Name: key

Show all work clearly and in order. Please box your answers. 10 minutes.

[4] 1. Find $\frac{dy}{dx}$ if $x^3 + y^3 = 1$.

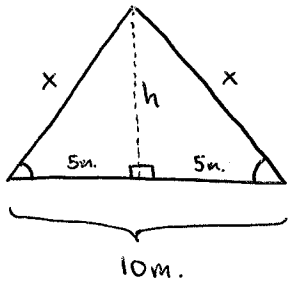
$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(1)$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-3x^2}{3y^2} = \boxed{\frac{-x^2}{y^2}}$$

[6] 2. An isosceles triangle has base length 10 m and height that is changing at a rate of 1 m/sec. At what rate is the perimeter of the triangle increasing when the height is 1 m?

picture:



by pythag. thm. $x = \sqrt{h^2 + 5^2} = \sqrt{h^2 + 25}$

Let P be the perimeter of the triangle

given: $\frac{dh}{dt} = +1$ m/sec

unknown: $\frac{dP}{dt}$ when $h = 1$m.

equation relating h and P: $P = 10 + \sqrt{h^2+25} + \sqrt{h^2+25}$

$$P = 10 + 2\sqrt{h^2+25} = 10 + 2(h^2+25)^{1/2}$$

differentiation: $\frac{dP}{dt} = \frac{d}{dt}\left(10 + 2(h^2+25)^{1/2}\right)$

$$\frac{dP}{dt} = 0 + 2\left(\tfrac{1}{2}\right)(h^2+25)^{-1/2} 2h \frac{dh}{dt}$$

substitution: $\left.\frac{dP}{dt}\right|_{h=1m.} = (1^2+25)^{-1/2} 2(1)(1) = \boxed{\frac{2}{\sqrt{26}} \frac{m}{sec.}}$