Supplementary homework problems for week 2.

1. Let $T = \begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 2 & 0 & -2 & 4 & 2 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

   Suppose that T is used to define a function in the usual way.

   (a) Compute the rank of T.

   (b) What is the domain of T?

   (c) What is the codomain of T?

   (d) Is T onto? Why or why not?

   (e) Is T one-to-one? Why or why not?

   (f) Is T a one-to-one correspondence? Why or why not?

2. Find the *rank* of each matrix from question (5)1 on page 18 of the book. You need to show work at least for those matrices where you cannot tell me the answer immediately.

Solutions:

(1) $\begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 2 & 0 & -2 & 4 & 2 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \xrightarrow{R2 \to R2 - 2R1} \begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \xrightarrow{R3 \leftrightarrow R2} \begin{bmatrix} \boxed{1} & 0 & -1 & 2 & 1 \\ 0 & \boxed{1} & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

two pivot columns

(a) rank of T = # of pivot columns of T = $\boxed{2}$

(b) T is 3×5 so domain of T = $\boxed{\mathbb{R}^5}$

(c) T is 3×5 so codomain of T = $\boxed{\mathbb{R}^3}$

(d) T is NOT onto. rank = $2 \neq 3$ = # of rows of T (see thm. 1.6.2)

(e) T is NOT one-to-one. rank = $2 \neq 5$ = # of columns of T (see thm. 1.6.2)

(f) T is NOT a one-to-one correspondence. since it is NOT one-to-one and it is NOT onto (see Cor. 1.6.3)

(2)

	REF	rank
(a)	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4/3 \end{bmatrix}$	$\boxed{2}$
(b)	$\begin{bmatrix} 2 & 0 & 8 \\ 0 & 4 & 2 \\ 0 & 0 & 2 \end{bmatrix}$	$\boxed{3}$
(c)	given	$\boxed{2}$
(d)	given	$\boxed{2}$
(e)	given	$\boxed{2}$
(f)	$\begin{bmatrix} 1 & 3 & 7 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$\boxed{2}$
(g)	given	$\boxed{2}$
(h)	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\boxed{1}$
(i)	$\begin{bmatrix} 1 & 0 & 5/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\boxed{3}$
(j)	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \pi \end{bmatrix}$	$\boxed{3}$