A Star Algorithm

1st Approach Used : Manhattan distance heuristic

Description : Consider the relaxed problem of 15 puzzle problem where tiles can be stacked in the same position. In this case the minimum distance required for a tile to go to its correct position is given by :

$$\text{distance} = |x1-x2| + |y1-y2|$$

2nd Approach Used : Linear Conflict heuristic

Description : We get a tighter condition with linear conflict in addition with Manhattan distance. Two tiles (a,b) are in a linear conflict if both the tiles lie on the same row/column and their goal states lie on the same row/column and if a is ahead of b and if goal state of a is behind of b. There will be two additional steps required in this case. This can be demonstrated by an example.

	11	9	

In this case, the tile 9 has to move up and down (2 times) in order to avoid collision with tile 11. This adds upto 2 extra moves to the Manhattan distance.

	11		
		9	

	11		
	9		

		11	
	9		

			11
	9		

	9		11

Optimality : In the case of Manhattan distance we are considering the relaxed state space where the blocks can stack on top of each other thus making it an admissible heuristic.

In a similar way, Linear Conflict + Manhattan distance we can stack tiles with the restriction of them belonging to the same row/ same column as their goal state thus making it an admissible heuristic since its part of a relaxed problem.

Source for Linear Conflict : Criticizing Solutions to Relaxed Models Yields Powerful Admissible Heuristics by ANDREW MAYER and MOT1 YUNG

Table of comparison :

Path cost:	Manhattan Distance		Manhattan Distance + Linear Conflict	
14	Nodes: 31	Time: 0.001s	Nodes: 31	Time: 0.0012s
36	Nodes: 83,824	Time: 3.2538s	Nodes: 16,712	Time: 0.7372s
40	Nodes: 1,137,025	Time: 48.9593s	Nodes: 83,354	Time: 4.056s
44	Nodes: 770,660	Time: 31.8107s	Nodes: 155,733	Time: 7.6728s