Fluidos no pascalianos y la estructura estelar relativista

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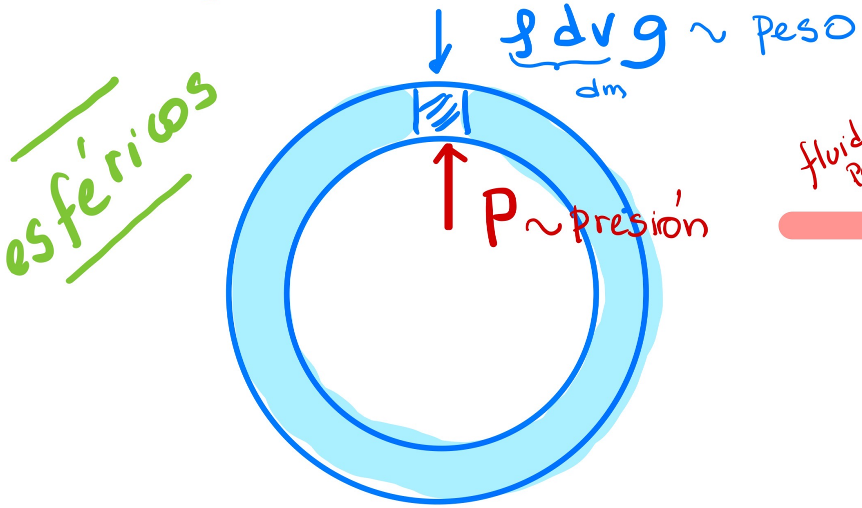
La Agenda

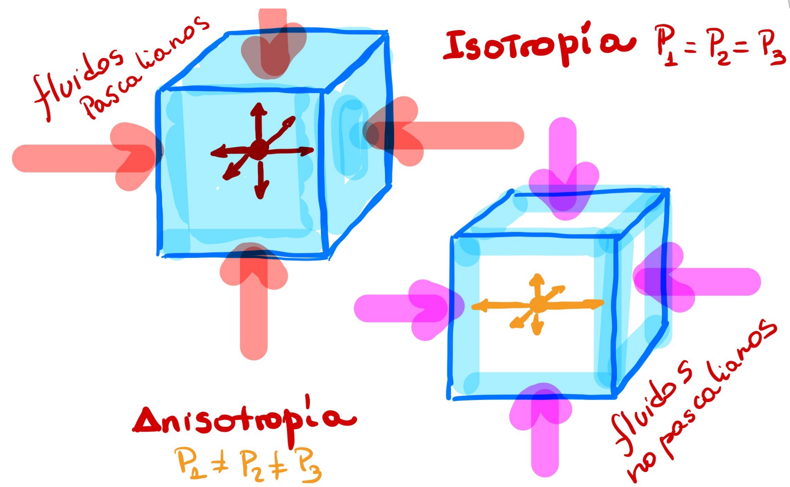
- Objetos autogravitantes anisótropos
- El gradiente de presiones
- La ecuación de estado donde la Física está
  - Las ecuaciones de estado de siempre
  - Ecuaciones de estado barótropa y no barótropa
- Las ecuaciones de Estado para materia ultradensa realista
- Varias estrategias para anisotropizar
- Estabilidad y aceptabilidad física
- Algunos polítropos
- Algunas modelos
- Conclusiones

https://www.nasa.gov/feature/goddard/2021/nasa-s-nicer-probes-the-squeezability-of-neutron-stars

Objetos autogravitantes



Isotropía *vs* anisotropía de presiones = Fluidos pascalianos *vs* no pascalianos



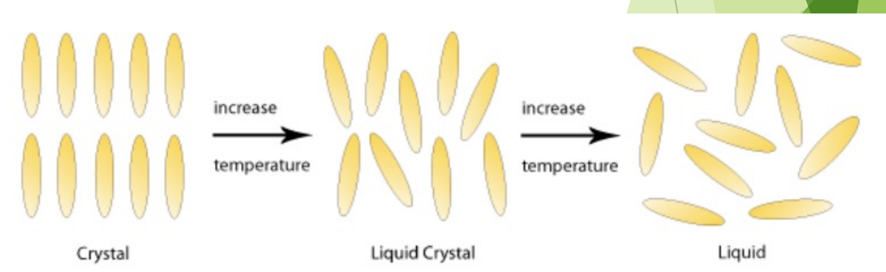
$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = \mathcal{F}\left(m(r), \rho(r), P(r), P_\perp(r), r\right),$$

$P(r)$: presión radial;

$P_\perp(r)$: presiones tangenciales;

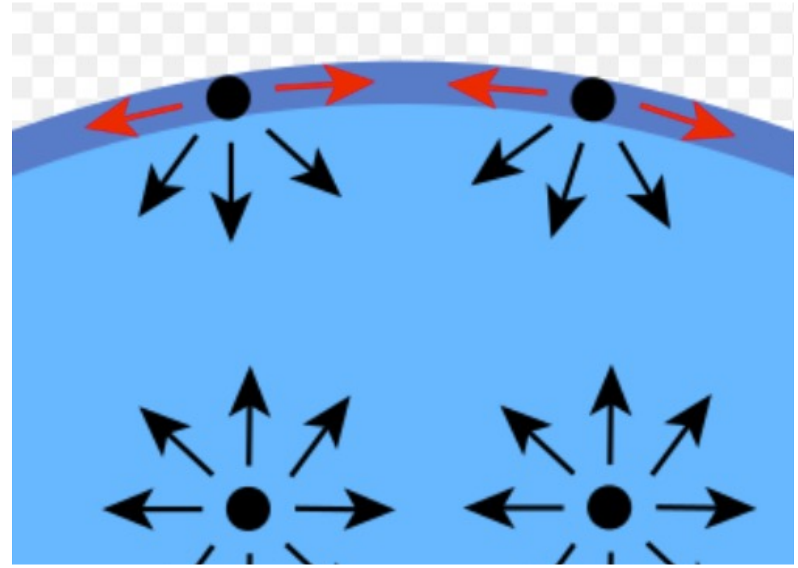
$\rho(r)$: densidad de masa;

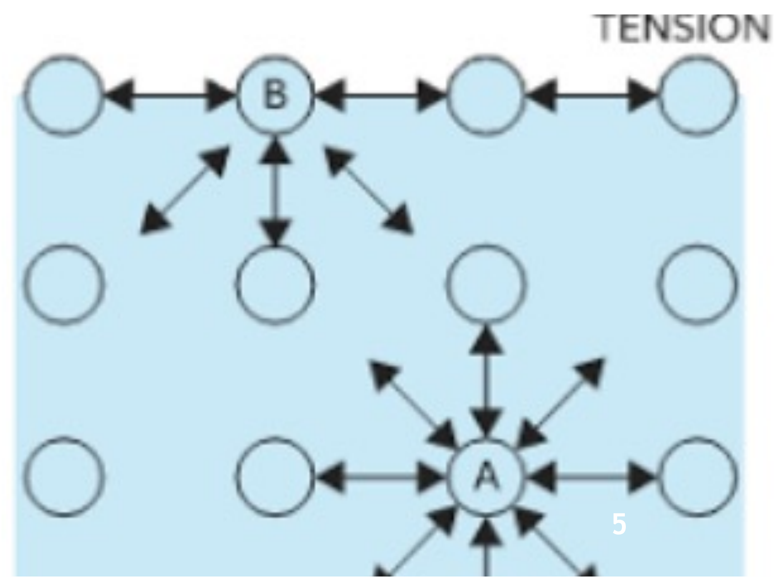
$m(r)$: masa contenida en una esfera de radio r;



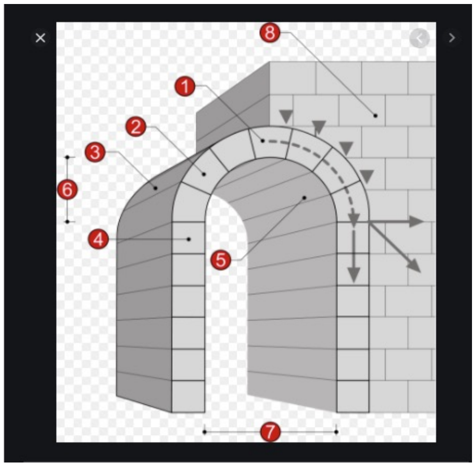
Cristales líquidos

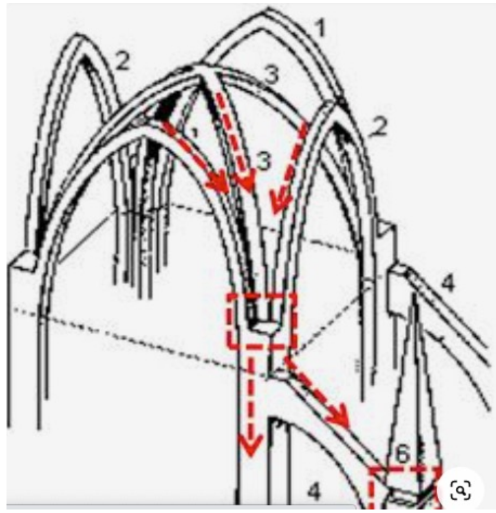
https://www.doitpoms.ac.uk/tlplib/liquid_crystals/intro.php

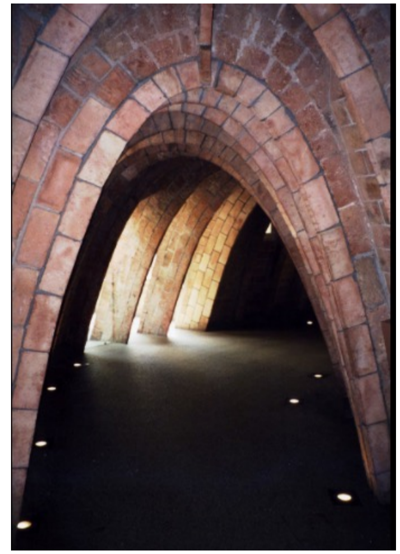


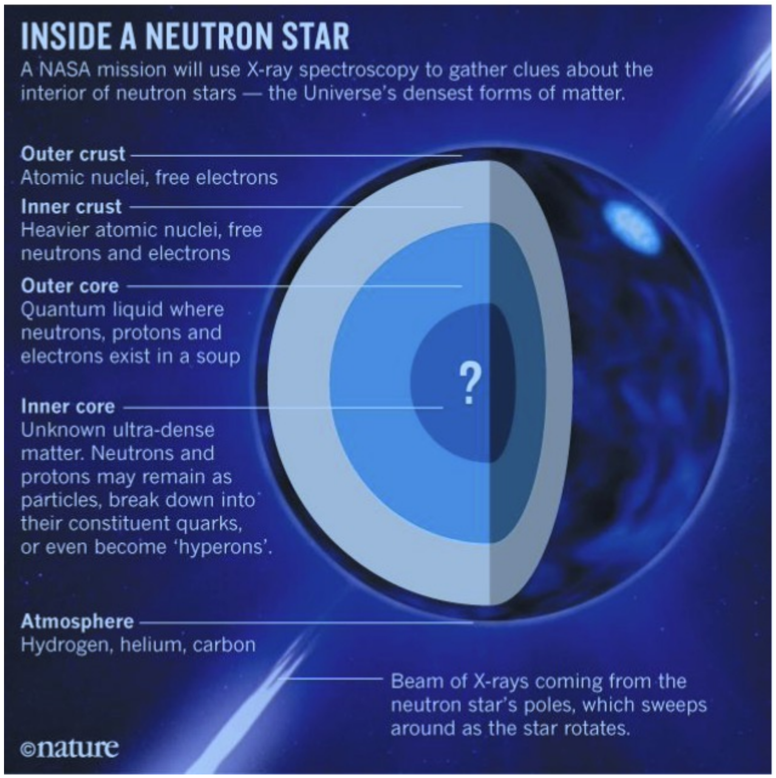

TENSION
B
A

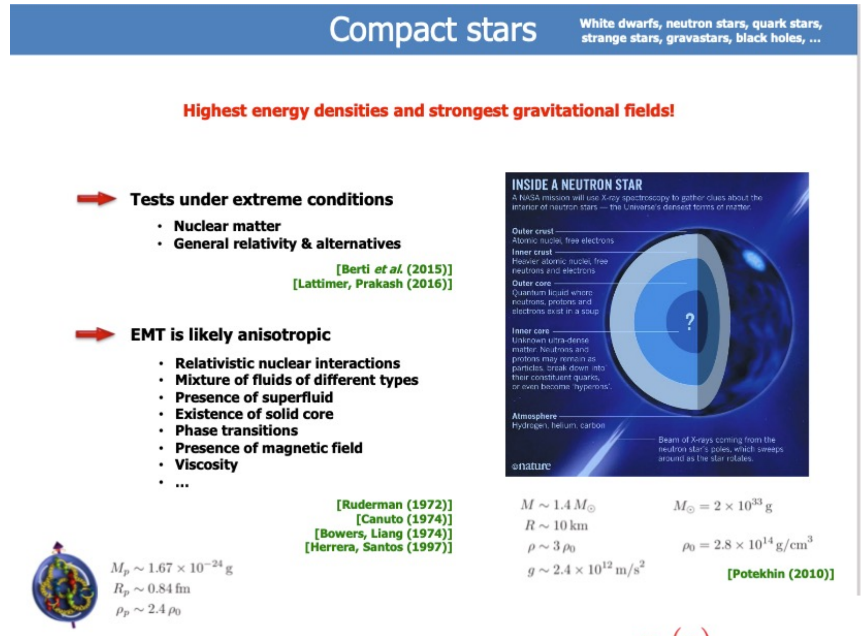
Tensiones Tangenciales en Arquitectura

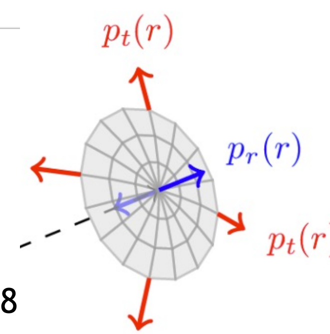












C. Lorcé (2018) Neutron stars and nucleons: Are they so different?
Correlations between partons in **nuclear** systems". **La Grande Motte**, **France**, 7-12 Oct 2018
https://ejc2018.sciencesconf.org/data/pages/trombino2018_09_24.pdf

El gradiente de presiones

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = \mathcal{F}\left(m(r), \rho(r), P(r), P_\perp(r), r\right)$$ **, En general**

y distinguiremos dos casos particulares

Caso Newtoniano

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2} + 2\frac{P_\perp(r) - P(r)}{r}$$

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2}\left(1 + \frac{P(r)}{\rho(r)c^2}\right)\left(1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right)\left(1 - 2\frac{Gm(r)}{rc^2}\right)^{-1} + 2\frac{P_\perp(r) - P(r)}{r}$$

Caso relativista

Pero las ecuaciones de estructura estelar se completan con la definición de masa

$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

Pero algo falta…..

Las ecuaciones de estado la otra parte de la Física

$$P(r) = \mathcal{W}(\rho(r), r)$$
$$P_\perp(r) = \mathcal{V}(\rho(r), P(r), r)$$

o, en su defecto

$$P(r) = \mathcal{W}(\rho(r), r)$$
$$P_\perp(r) = \mathcal{Y}(\rho(r), r)$$

Caso Newtoniano

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{m(r)\rho(r)}{r^2} + \frac{2(P_\perp(r) - P(r))}{r^2}$$

$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{m(r)\rho(r)}{r^2}\left(1 + \frac{P(r)}{\rho(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{m(r)P(r)}\right)\left(1 - \frac{2m(r)}{r}\right)^{-1} + \frac{2(P_\perp(r) - P(r))}{r^2}$$

Caso relativista

Ecuaciones de estado barótropa

$$P(r) = \mathcal{W}(\rho(r), r)$$
$$P_\perp(r) = \mathcal{Y}(\rho(r), r)$$

Estrategia: despejar y sustituir

$$\left.\begin{array}{l} P_\perp(r) = \mathcal{Y}(\rho(r), r) \\ \\ \rho(r) = \mathcal{Z}(P(r), r) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \dfrac{\mathrm{d}P}{\mathrm{d}r} = \mathcal{G}\left(m(r), P(r), \mathcal{Z}(P(r), r), r\right), \\ \\ \dfrac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \mathcal{Z}(P(r), r) \end{array}\right.$$

$$\overbrace{P_0 = P(0) \quad \mathrm{y} \quad m_0 = m(0) = 0}$$

Un ejemplo reciente

$$P(r) = \kappa_1 \rho^{\gamma_1}$$
$$P_\perp(r) = \kappa_2 \rho^{\gamma_2}$$

...y el sistema se integra, numéricamente

Abellán, G., Fuenmayor, E., & Herrera, L. (2020). The double polytrope for anisotropic matter: Newtonian case. *Physics of the Dark Universe*, *28*, 100549.

Abellán, G., Fuenmayor, E., Contreras, E., & Herrera, L. (2020). The general relativistic double polytrope for anisotropic matter. *Physics of the Dark Universe*, *30*, 100632.

THE ASTROPHYSICAL JOURNAL, 188:657–665, 1974 March 15

ANISOTROPIC SPHERES IN GENERAL RELATIVITY*

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Received 1973 June 4; revised 1973 September 25

ABSTRACT

The possible importance of locally anisotropic equations of state for relativistic spheres is discussed by generalizing the equations of hydrostatic equilibrium to include these effects. The resulting change in maximum equilibrium mass M and surface redshift z is found analytically in the case of incompressibility (ρ = const.) and a highly idealized expression for the anisotropy. Bondi's analysis of isotropic spheres is generalized to include anisotropy, and the maximum surface redshift is investigated without reference to specific equations of state. A numerical model [with $p(r) = \frac{1}{3}\rho(r)$ and a special form of anisotropy] is then solved. In general, it is found that specific models lead to increases in z typically of the same order of magnitude as the fractional anisotropy.

Subject headings: equation of state — hydrodynamics — quasi-stellar sources or objects — relativity

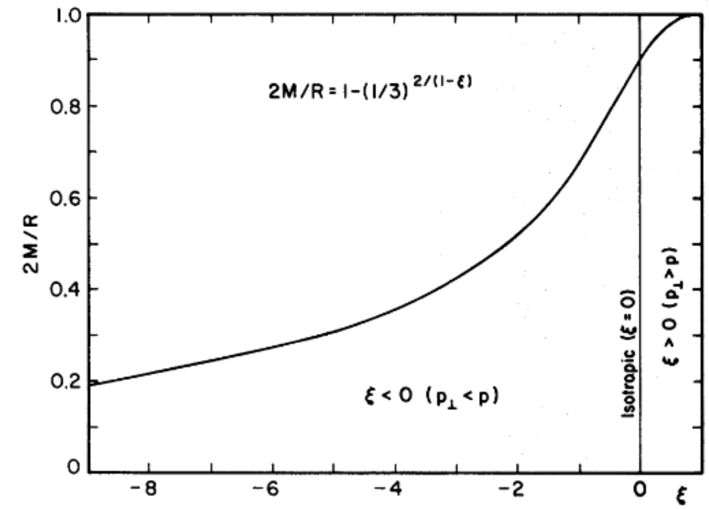

FIG. 1.—The ratio of the gravitational (Schwarzschild) radius $2M$ to stellar radius R for a spherically symmetric distribution of anisotropic matter with constant energy density $\rho = \rho_0$. The anisotropy is described by equations (3.1)–(3.2). For $\xi > 1$ the tangential pressure $p_\perp$ exceeds the radial pressure p; for $\xi < 1$, $p_\perp < p$. The isotropic case corresponds to $\xi = 0$. The assumed anisotropy can increase the maximum equilibrium mass by as much as 19% ($\xi = 1$).

Astrophysics > High Energy Astrophysical Phenomena

[Submitted on 1 Nov 2017 (v1), last revised 3 Jan 2018 (this version, v2)]

Using gravitational-wave observations and quasi-universal relations to constrain the maximum mass of neutron stars

Luciano Rezzolla, Elias R. Most, Lukas R. Weih

Combining the GW observations of merging systems of binary neutron stars and quasi-universal relations, we set constraints on the maximum mass that can be attained by nonrotating stellar models of neutron stars. More specifically, exploiting the recent observation of the GW event GW 170817 and drawing from basic arguments on kilonova modeling of GRB 170817A, together with the quasi-universal relation between the maximum mass of nonrotating stellar models $M_{\rm TOV}$ and the maximum mass supported through uniform rotation $M_{\rm max} = \left(1.20^{+0.02}_{-0.05}\right) M_{\rm TOV}$ we set limits for the maximum mass to be $2.01^{+0.04}_{-0.04} \leq M_{\rm TOV}/M_\odot \lesssim 2.16^{+0.17}_{-0.15}$, where the lower limit in this range comes from pulsar observations. Our estimate, which follows a very simple line of arguments and does not rely on the modeling of the electromagnetic signal in terms of numerical simulations, can be further refined as new detections become available. We briefly discuss the impact that our conclusions have on the equation of state of nuclear matter.

La anisotropía de presiones aumenta la masa máxima de equilibrio

JMAPA,801200801212

Some models of anisotropic spheres in general relativity

M. Cosenza and L. Herrera [a)]
Departamento de Fisica, Facultad de Ciencias, Universidad Central de Venezuela, Caracas, Venezuel

M. Esculpi
Departamento de Fisica Aplicada, Facultad de Ingenieria, Universidad Central de Venezuela, Caracc Venezuela

L. Witten [b)]
Physics Department, University of Cincinnati, Cincinnati, Ohio 45221

(Received 20 May 1980; accepted for publication 20 August 1980)

A heuristic procedure is developed to obtain interior solutions of Einstein's equations for anisotropic matter from known solutions for isotropic matter. Five known solutions are generalized to give solutions with anisotropic sources.

PACS numbers: 04.20.Cv

Start New Search

full:"Anisotropic pressure" full:"General Relativity" full:"star" year:1974-2022

Your search returned **1,255** results

Date

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	Totals	Refereed
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Normalized paper count	621.3	482.0

Total Normalized

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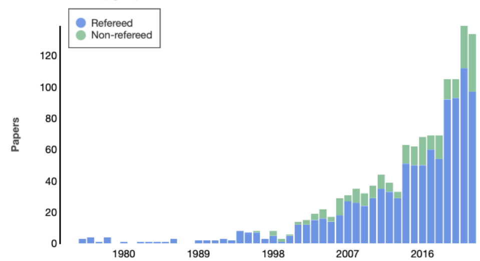

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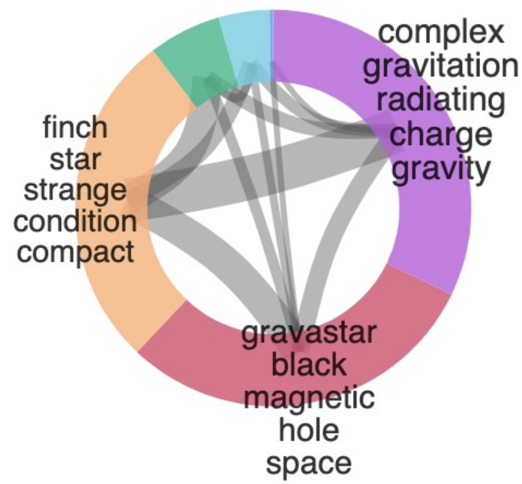

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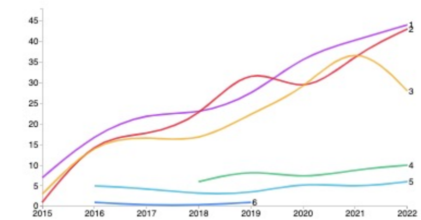

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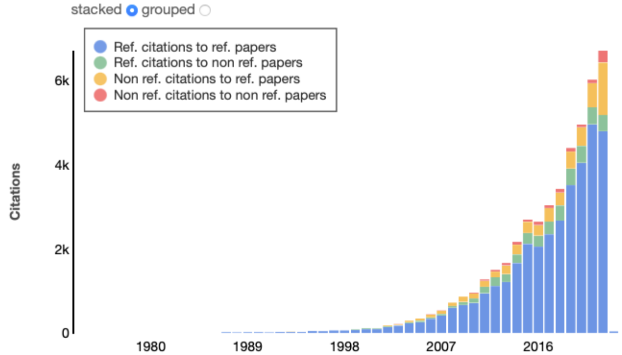
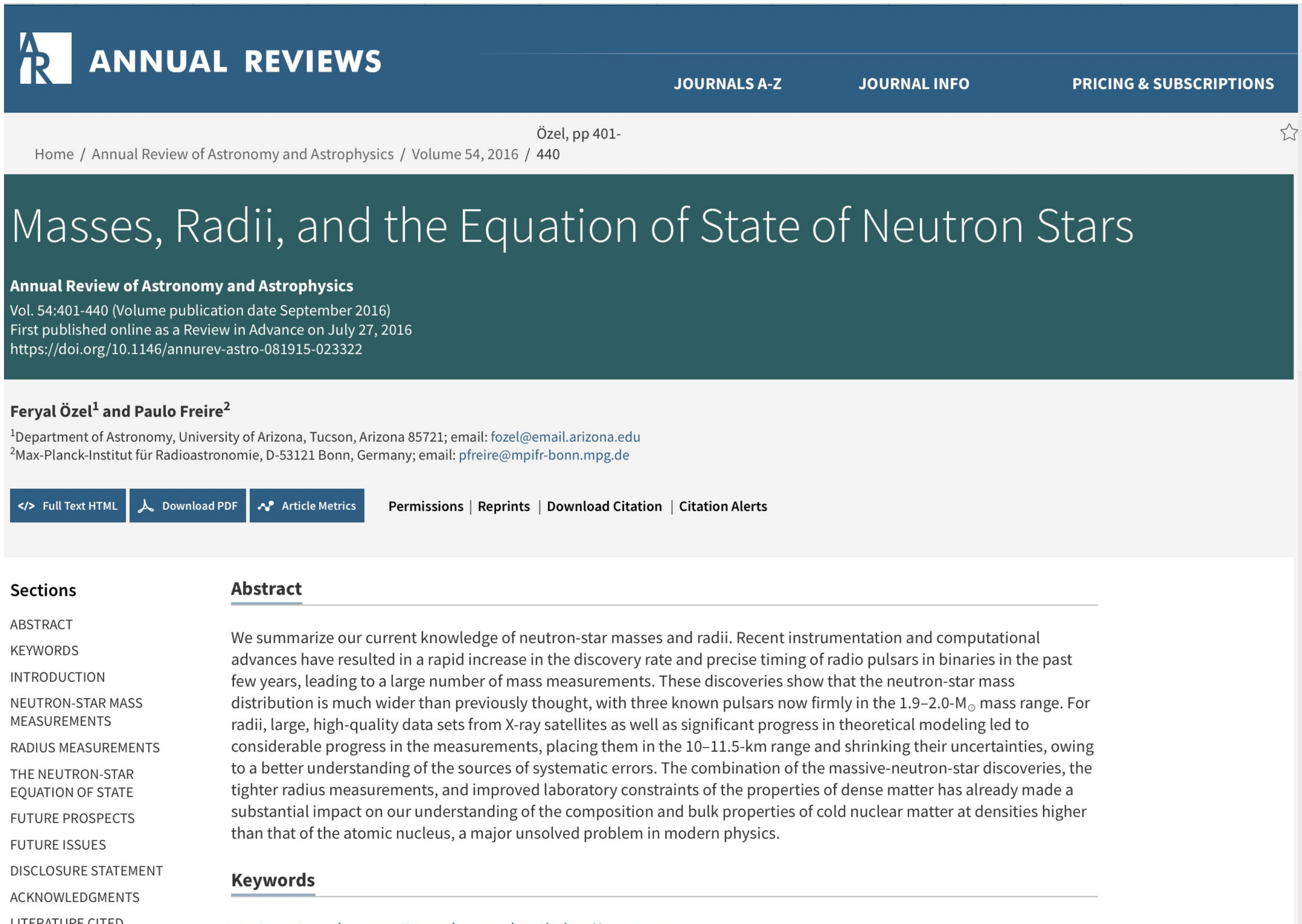
Citations

	Totals	Refereed
Number of citing papers	25036	21513
Total citations	46151	41068
Number of self-citations	7203	6984
Average citations	36.8	40.4
Median citations	10	14
Normalized citations	19509.7	17942.1
Refereed citations	39813	35853
Average refereed citations	31.7	35.3
Median refereed citations	9	12
Normalized refereed citations	16886.6	15692.7

Total Normalized

stacked grouped

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Home / Annual Review of Astronomy and Astrophysics / Volume 54, 2016 / Özel, pp 401-440

Masses, Radii, and the Equation of State of Neutron Stars

Annual Review of Astronomy and Astrophysics
Vol. 54:401-440 (Volume publication date September 2016)
First published online as a Review in Advance on July 27, 2016
https://doi.org/10.1146/annurev-astro-081915-023322

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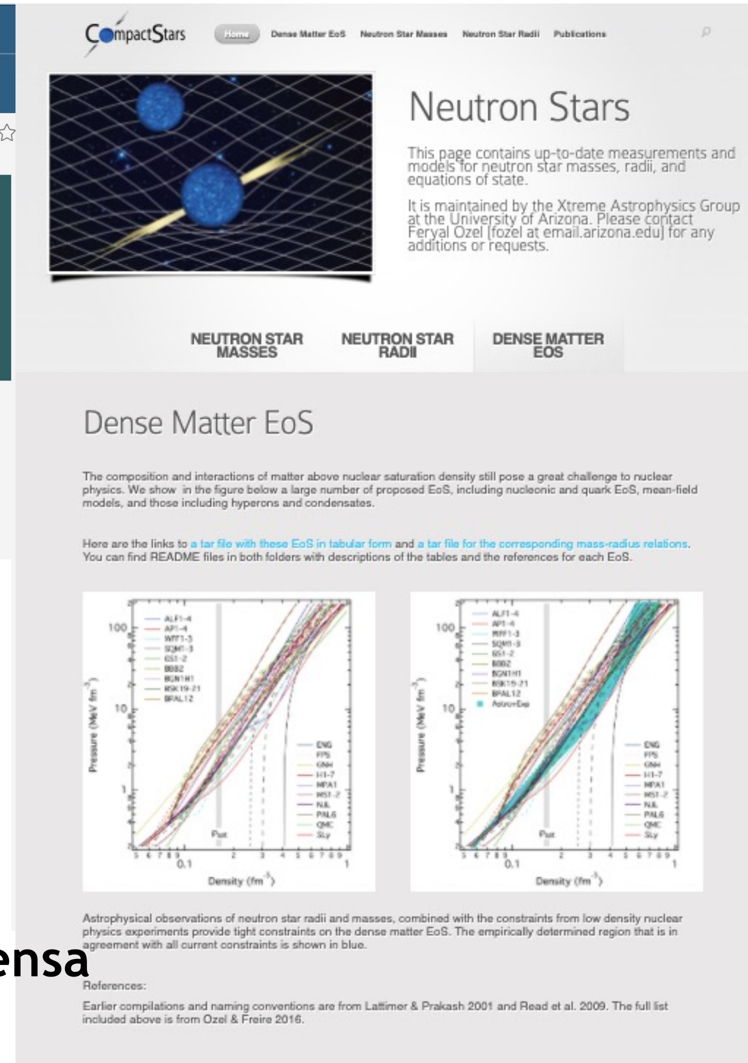
Sections

Abstract

We summarize our current knowledge of neutron-star masses and radii. Recent instrumentation and computational advances have resulted in a rapid increase in the discovery rate and precise timing of radio pulsars in binaries in the past few years, leading to a large number of mass measurements. These discoveries show that the neutron-star mass distribution is much wider than previously thought, with three known pulsars now firmly in the 1.9–2.0-$M_{\odot}$ mass range. For radii, large, high-quality data sets from X-ray satellites as well as significant progress in theoretical modeling led to considerable progress in the measurements, placing them in the 10–11.5-km range and shrinking their uncertainties, owing to a better understanding of the sources of systematic errors. The combination of the massive-neutron-star discoveries, the tighter radius measurements, and improved laboratory constraints of the properties of dense matter has already made a substantial impact on our understanding of the composition and bulk properties of cold nuclear matter at densities higher than that of the atomic nucleus, a major unsolved problem in modern physics.

Keywords


CompactStars
Home | Dense Matter EoS | Neutron Star Masses | Neutron Star Radii | Publications

Neutron Stars

This page contains up-to-date measurements and models for neutron star masses, radii, and equations of state.

It is maintained by the Xtreme Astrophysics Group at the University of Arizona. Please contact Feryal Ozel [fozel at email.arizona.edu] for any additions or requests.

NEUTRON STAR MASSES | NEUTRON STAR RADII | DENSE MATTER EOS

Dense Matter EoS

The composition and interactions of matter above nuclear saturation density still pose a great challenge to nuclear physics. We show in the figure below a large number of proposed EoS, including nucleonic and quark EoS, mean-field models, and those including hyperons and condensates.

Here are the links to a tar file with these EoS in tabular form and a tar file for the corresponding mass-radius relations. You can find README files in both folders with descriptions of the tables and the references for each EoS.

Astrophysical observations of neutron star radii and masses, combined with the constraints from low density nuclear physics experiments provide tight constraints on the dense matter EoS. The empirically determined region that is in agreement with all current constraints is shown in blue.

References:

Earlier compilations and naming conventions are from Lattimer & Prakash 2001 and Read et al. 2009. The full list included above is from Ozel & Freire 2016.

Ecuaciones de estado realistas para materia ultradensa

http://xtreme.as.arizona.edu/NeutronStars/

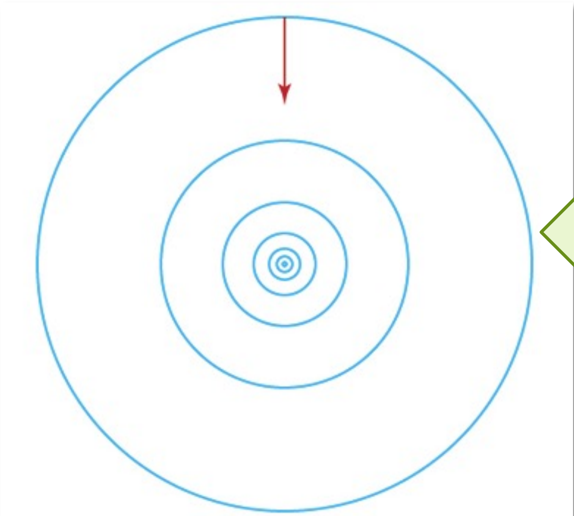
Ozel, F., & Freire, P. (2016). ***Masses, Radii, and Equation of State of Neutron Stars***. *ArXiv*. https://doi.org/10.1146/annurev-astro-081915-023322

Caso Newtoniano

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2} + 2\frac{P_\perp(r) - P(r)}{r}$$

Peroooo
¿ y la anisotropía?

$$P(r) = 0 \quad \Rightarrow 0 = -\frac{Gm(r)\rho(r)}{r^2} + 2\frac{P_\perp(r)}{r} \quad \Rightarrow P_\perp = \frac{Gm(r)\rho(r)}{2r}$$



Caso extremo: Cáscaras esféricas sostenidas únicamente por tensiones tangenciales
Las cáscaras no pesan

G. Lemaitre. L'Univers en expansion *Ann. Sot. Sci. Bruxelles* **A53** (1933) 51.

$$P(r) = 0 \quad \Rightarrow P_\perp = \frac{Gm(r)\rho(r)}{2r}\left(1 - 2\frac{Gm(r)}{rc^2}\right)^{-1}$$

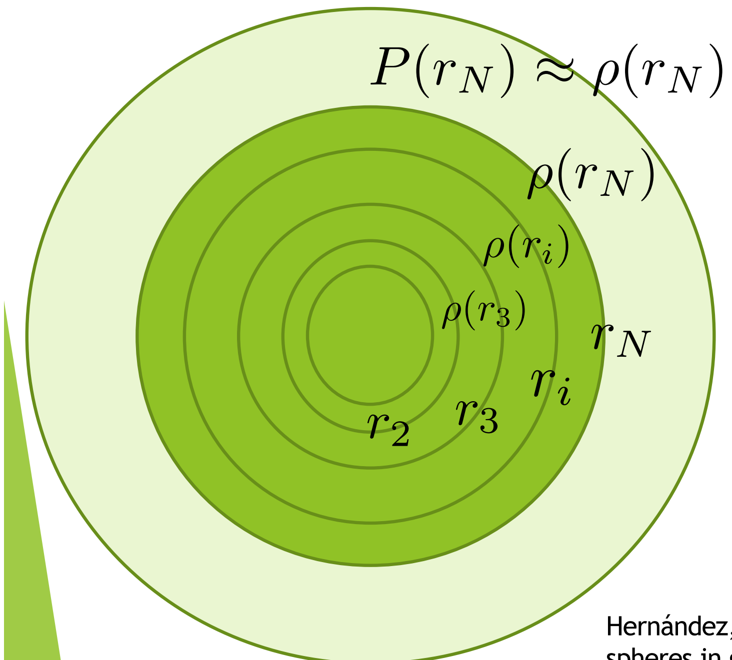
Caso relativista

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2}\left(1 + \frac{P(r)}{\rho(r)c^2}\right)\left(1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right)\left(1 - 2\frac{Gm(r)}{rc^2}\right)^{-1} + 2\frac{P_\perp(r) - P(r)}{r}$$

A lo largo de varios años hemos propuesto algunas ecuaciones de estado

$$P(r) = \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2 \rho(\bar{r})\, \mathrm{d}\bar{r} \approx \rho(r) - \frac{2}{r^3}\sum_i^N \rho(r_i)\,\Delta r_i$$

$$P(r_N) \approx \rho(r_N) - \frac{2}{r_N^3}\sum_i^N \rho(r_i)\,\Delta r_i$$



Una ecuación de estado no-local: La presión es función de las densidades sumadas
en todos los puntos interiores

Son las ecuaciones que modelan algunas grietas en las paredes

Estudiamos esferas con esta ecuación de estado para modelar posible
Objetos compactos relativistas

Hernández, H., & Núñez, L. A. (2004). Nonlocal equation of state in anisotropic static fluid spheres in general relativity. *Canadian journal of physics*, *82*(1), 29-51. Hernández, H., Nunez, L. A., & Percoco, U. (1999). Non-local equation of state in general relativistic radiating spheres. *Classical and Quantum Gravity*, *16*(3), 871.

Estrategias para anisotropizar

Para el caso newtoniano $\frac{\mathrm{d}\tilde{P}}{\mathrm{d}\tilde{r}} = -\frac{\mu}{\kappa}\frac{\tilde{m}\tilde{\rho}}{\tilde{r}^2} + 2\frac{\tilde{P}_\perp - \tilde{P}}{\tilde{r}}$

Podemos suponer

$$\Delta_{Newton} = \tilde{P}_\perp - \tilde{P} = C_{Newton}\frac{\tilde{m}\tilde{\rho}}{r} \quad \Rightarrow \frac{\mathrm{d}\tilde{P}}{\mathrm{d}\tilde{r}} = \left(2C_{Newton} - \frac{\mu}{\sigma}\right)\frac{\tilde{m}\tilde{\rho}}{\tilde{r}^2}$$

Pero también es posible

$$\Delta_{CH} = \tilde{P}_\perp - \tilde{P} = -\frac{\tilde{r}}{2}\left(\frac{1-h}{h}\right)\frac{\mathrm{d}\tilde{P}}{\mathrm{d}\tilde{r}}$$

Cosenza, M., Herrera, L., Esculpi, M., & Witten, L. (1981). Some models of anisotropic spheres in general relativity. *Journal of Mathematical Physics*, *22*(1), 118-125.

o también

$$\Delta_{HIM} = \tilde{P}_\perp - \tilde{P} = C_{HIM}\frac{\tilde{P}}{\tilde{r}}$$

Horvat, D., Ilijić, S., & Marunović, A. (2010). Radial pulsations and stability of anisotropic stars with a quasi-local equation of state. *Classical and quantum gravity*, *28*(2), 025009.

Rahmansyah, A., Sulaksono, A., Wahidin, A. B., & Setiawan, A. M. (2020). Anisotropic neutron stars with hyperons: implication of the recent nuclear matter data and observations of neutron stars. *The European Physical Journal C*, *80*(8), 1-17.

Igual para el caso relativista, tenemos una oferta variada

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}\tilde{r}} = -\frac{\mu}{\kappa}\frac{\tilde{m}\tilde{\rho}}{\tilde{r}^2}\left(1+\kappa\frac{\tilde{P}}{\tilde{\rho}}\right)\left(1+3\eta\kappa\frac{\tilde{P}\tilde{r}^3}{\tilde{m}}\right)\left(1-2\mu\frac{\tilde{m}}{\tilde{r}}\right)^{-1}+2\frac{\tilde{P}_\perp-\tilde{P}}{\tilde{r}}$$

Es possible suponer

$$\Delta_{CH} = \tilde{P}_\perp - \tilde{P} = -\frac{\tilde{r}}{2}\left(\frac{1-h}{h}\right)\frac{\mathrm{d}\tilde{P}}{\mathrm{d}\tilde{r}}$$

Cosenza, M., Herrera, L., Esculpi, M., & Witten, L. (1981). Some models of anisotropic spheres in general relativity. *Journal of Mathematical Physics*, *22*(1), 118-125.

o también

$$\Delta_{HIM} = \tilde{P}_\perp - \tilde{P} = C_{HIM}\frac{\tilde{P}}{\tilde{r}}$$

Horvat, D., Ilijić, S., & Marunović, A. (2010). Radial pulsations and stability of anisotropic stars with a quasi-local equation of state. *Classical and quantum gravity*, *28*(2), 025009.

o también

$$T^{\mu\nu} = \hat{T}^{\mu\nu} + \alpha\,\theta^{\mu\nu} \qquad \Rightarrow \Delta_{OV} = P_\perp - P_r = -\alpha\left(\theta^2_2 - \theta^1_1\right).$$

Ovalle, J. (2017). Decoupling gravitational sources in general relativity: from perfect to anisotropic fluids. *Physical Review D*, *95*(10), 104019.

o también

$$\Delta_{RP} = C_{RP}\; f(\rho)K^\mu(P_r)_{;\mu}\,,$$

Raposo, G., Pani, P., Bezares, M., Palenzuela, C., & Cardoso, V. (2019). Anisotropic stars as ultracompact objects in general relativity. *Physical Review D*, *99*(10), 104072.

o también

$$P_r = P_r(\rho) \quad \Rightarrow \Delta_{BA} = \frac{1}{4}\left(1-\frac{2m}{r}\right)(\rho+P_r(\rho))\left(8\,\pi\,r^2P_r(\rho)+\frac{2m}{r}\right)+\frac{r}{2}v_s^2\rho'\,,$$

Hernández, H., Suárez-Urango, D., & Núñez, L. A. (2021). Acceptability conditions and relativistic barotropic equations of state. *The European Physical Journal C*, *81*(3), 1-17.

Otra vez, a partir de la ecuación de la ecuación de equilibrio hidrostático

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -(\rho + P)\frac{m + 4\pi r^3 P}{r(r-2m)} + \frac{2}{r}\left(P_\perp - P\right) \ ,$$

Reinterpretamos la anisotropía como una densidad

$$\tilde{\rho}_{ani} = \frac{2(r-2m)}{(m + 4\pi r^3 P)}\left(P_\perp - P\right) \ ,$$

y verificamos la contribución de ese componente de la densidad de energía

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -(\rho - \tilde{\rho}_{ani} + P)\frac{m + 4\pi r^3 P}{r(r-2m)},$$

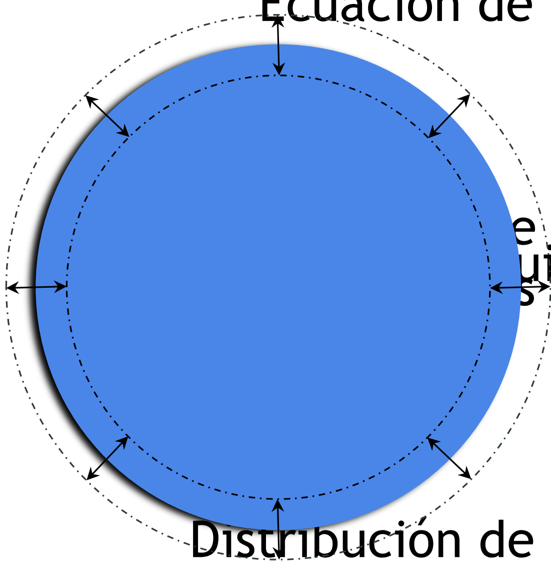
Ospino, J, Hernández, H. & Núñez, L. A. (2023) Revisiting non Pascalian fluid in General Relativity, *en preparación*

Estabilidad

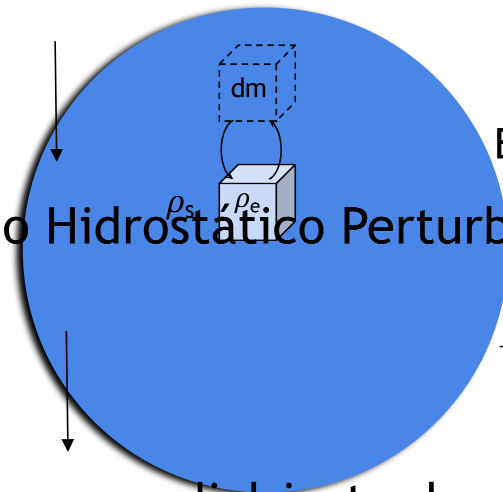
Perturbaciones radiales

Estabilidad convectiva

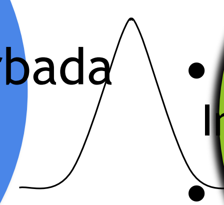
Ecuación de Equilibrio Hidrostático = 0



S. Chandrasekhar, Phys. Rev. Lett., 1964.



uilibrio Hidrostático Perturbada

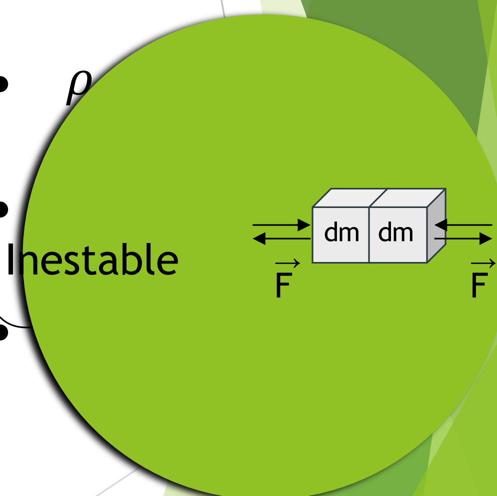


Distribución de la Fuerza radial justo después de la Perturbación

H. Bondi, Proc. R. Soc. London, Ser. A., 1964.

Fracturas

- ρ
-
-



L. Herrera, Physics Letters A., 1992.

Sobre las fracturas

Para formalizar el concepto de fractura construimos la cantidad

$$\mathcal{R} \equiv \frac{\mathrm{d}P}{\mathrm{d}r} + (\rho + P)\frac{m + 4\pi r^3 P}{r(r-2m)} - \frac{2}{r}\left(P_\perp - P\right) ,$$

Expandimos e identificamos la perturbación al equilibrio

$$\mathcal{R} \approx \mathcal{R}_0(\rho, P, P_\perp, m, P') + \frac{\partial \mathcal{R}}{\partial \rho}\delta\rho + \frac{\partial \mathcal{R}}{\partial P}\delta P + \frac{\partial \mathcal{R}}{\partial P_\perp}\delta P_\perp + \frac{\partial \mathcal{R}}{\partial m}\delta m + \frac{\partial \mathcal{R}}{\partial P'}\delta \mathcal{R}_p ,$$

$$\delta\mathcal{R} \equiv \delta \underbrace{P'}_{\mathcal{R}_p} + \delta \underbrace{\left[(\rho + P)\frac{m + 4\pi r^3 P}{r(r-2m)}\right]}_{\mathcal{R}_g} + \delta \underbrace{\left(2\frac{P}{r} - 2\frac{P_\perp}{r}\right)}_{\mathcal{R}_a} = \delta\mathcal{R}_p + \delta\mathcal{R}_g + \delta\mathcal{R}_a ,$$

$$\delta\mathcal{R}_p = \left(\frac{P''}{\rho'}\right)\delta\rho = \left((v^2)' + v^2\frac{\rho''}{\rho'}\right)\delta\rho$$

$$\delta\mathcal{R}_g = \left(\frac{\partial \mathcal{R}_g}{\partial \rho} + \frac{\partial \mathcal{R}_g}{\partial P}v^2 + \frac{\partial \mathcal{R}_g}{\partial m}\frac{4\pi r^2 \rho}{\rho'}\right)\delta\rho$$

$$\delta\mathcal{R}_a = \left(\frac{v^2 - v_\perp^2}{r}\right)\delta\rho,$$

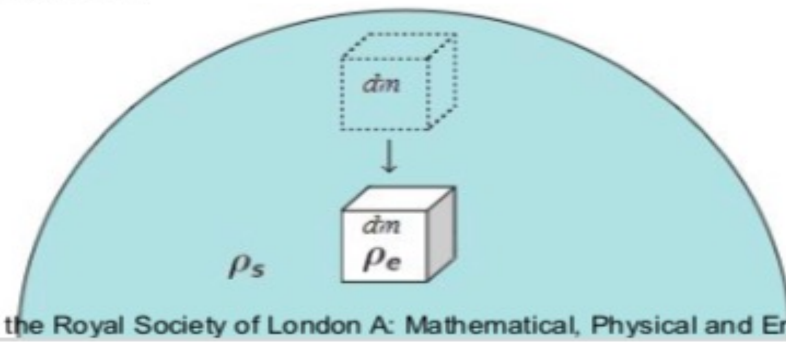
- En Abreu, H., Hernandez, H., & Nunez, L. A. (2007). Sound speeds, cracking and the stability of self-gravitating anisotropic compact objects. *Classical and Quantum Gravity*, *24*(18), 4631. **Consideramos perturbaciones de densidad constante que no afecten el gradiente de presiones**
- En González, G. A., Navarro, A., & Núnez, L. A. (2015, March). Cracking of anisotropic spheres in general relativity revisited. In *Journal of Physics: Conference Series* (Vol. 600, No. 1, p. 012014). IOP Publishing. **Consideramos perturbaciones acotadas y que afecten el gradiente de presiones**

Las perturbaciones acotadas que afecten el gradiente de presiones estabilizan la configuración

Estabilidad convectiva

Criterio de estabilidad utilizando el principio de Arquímedes.

- Bondi (1964)
  - Si $\rho_e > \rho_s$, la gravedad tenderá a atraer al elemento de fluido hacia el centro de la esfera y el sistema será inestable.
  - Si $\rho_e = \rho_s$, el sistema será metaestable.
  - Si $\rho_e < \rho_s$, el elemento de fluido retornará a su posición inicial y el sistema será estable.



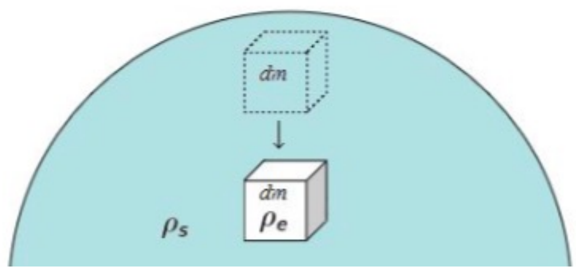
H. Bondi, Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. 1964.

Estabilidad convectiva

Considerando un cubo infinitesimal con densidad $\rho(r_p)$ que es desplazado hacia el centro de la esfera y cuya posición inicial es denotada como r_p se tiene:

$$\rho(r_p) \to \rho(r_p)+\delta\rho(r)\,, \quad \text{siendo} \quad \delta\rho(r) = \rho'(r)(-\delta r) \quad \text{y} \quad r = r_p - \delta r$$

donde r es la posición desplazada del cubo y -□r el desplazamiento hacia el centro. Por lo tanto, la densidad del cubo luego de ser desplazado siempre será mayor que al inicio.

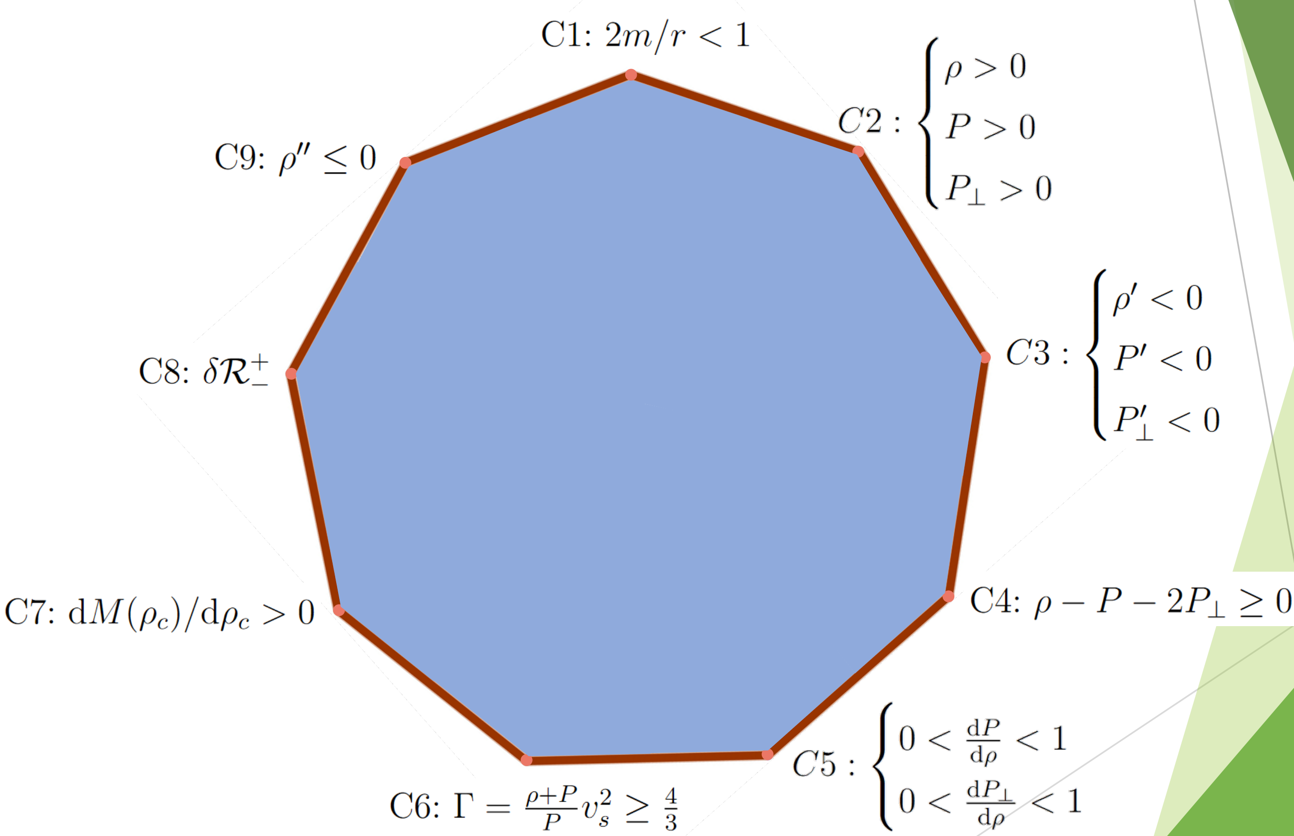


Finalmente, expandiendo $\rho'(r)$ alrededor de r_p se tiene:

$$\rho'(r_p) + \rho''(r_p)\delta r \leq \rho'(r_p) \quad \Rightarrow \quad \rho''(r) \leq 0$$

H. Hernández, L. Núñez, A. Vásquez-Ramírez, The European Physical Journal C, 2018

Condiciones de aceptabilidad física



B. Ivanov, Eur. Phys. J. C, 2017.

¿Qué hemos hecho recientemente?

- Hernández, H., Suárez-Urango, D., & Núñez, L. A. (2021). Acceptability conditions and relativistic barotropic equations of state. *The European Physical Journal C*, *81*(3), 1-17.
- Suárez-Urango, D., Ospino, J., Hernández, H., & Núñez, L. A. (2022). Acceptability conditions and relativistic anisotropic generalized polytropes. *The European Physical Journal C*, *82*(2), 1-22.

Propusimos una ecuación barótropa $P = \kappa \rho^{1+\frac{1}{n}} + \alpha\rho - \beta$

con parámetros dependientes $\beta = \kappa {\rho_b}^{1+\frac{1}{n}} + \alpha \rho_b$

$$\kappa = \frac{\sigma - \alpha\left[1 - \varkappa\right]}{{\rho_c}^{\frac{1}{n}}\left[1 - \varkappa^{1+\frac{1}{n}}\right]} \qquad \text{con} \qquad \sigma = \frac{P_c}{\rho_c} \quad \text{y} \quad \varkappa = \frac{\rho_b}{\rho_c}$$

Las velocidades del sonido serán

$$v_s^2 = \frac{\partial P}{\partial \rho} = \kappa\left[1 + \frac{1}{n}\right]\rho^{\frac{1}{n}} + \alpha$$

$$v_{s_\perp}^2 = \frac{\partial P_\perp}{\partial \rho} = \frac{1}{2}\left[3 + r\frac{\rho''}{\rho'}\right]v_s^2 + \frac{1}{\rho'}\left[\frac{e^{2\lambda}}{4}(\rho + P)\left(8\,\pi\, r^2 P - e^{-2\lambda} + 1\right)\right]' + \frac{\kappa r(n+1)\rho^{\frac{1}{n}}}{2n^2\rho}\rho',$$

Propusimos dos perfiles de densidad

$$\textbf{Propuesta 1:} \qquad e^{2\lambda} = \left[1 + Ar^2 + Br^4\right]^{-1} \;\Rightarrow\; \rho = -\frac{3A + 5Br^2}{8\pi}$$

y,

$$\textbf{Propuesta 2:} \qquad e^{2\lambda} = \frac{K(1 + Ar^2)}{K + Br^2} \;\Rightarrow\; \rho = \frac{(KA - B)\left(3 + Ar^2\right)}{8\pi K \left(1 + Ar^2\right)^2}\,.$$

Integramos analíticamente dos modelos

$$\frac{\mathrm{d}P}{\mathrm{d}r} + (\rho + P)\frac{m + 4\pi r^3 P}{r(r - 2m)} - \frac{2}{r}\left(P_\perp - P\right) = 0\,,$$

Para una ecuación politropa generalizada

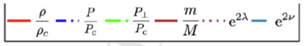
$$P = \kappa \rho^{1+\frac{1}{n}} + \alpha\rho - \beta$$

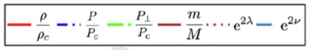
Propuesta 1

Propuesta 2

Table 1 Physical parameters for analytic polytropic solutions for EoS-1 and EoS-2, which fulfil the acceptability conditions. For EoS-1 the other α values correspond to: -0.10 ($n = 1.0$), -0.15 ($n = 1.5$) and -0.25 ($n = 2.0$). The parameter σ can take values between 0.10 and 0.18 approximately. The model EoS-1 could describe the mass of the millisecond pulsar in SR J1738+0333. The mass for the EoS-2 compact object is close to the lowest-mass-pulsar J0453+1559 companion ($1.174 \pm 0.004\ M_\odot$)

Input parameters	EoS-1	EoS-2
$\mu = 2M/r_b$	0.43	0.34
$\varkappa = \rho_b/\rho_c$	0.60	0.10
α	0.05 ($n = 0.5$)	0.05
Output parameters	**EoS-1**	**EoS-2**
$\rho_c \times 10^{15}$ (g/cm^3)	0.91	2.59
$\rho_b \times 10^{14}$ (g/cm^3)	5.46	2.59
M ($M_\odot$)	1.46	1.15



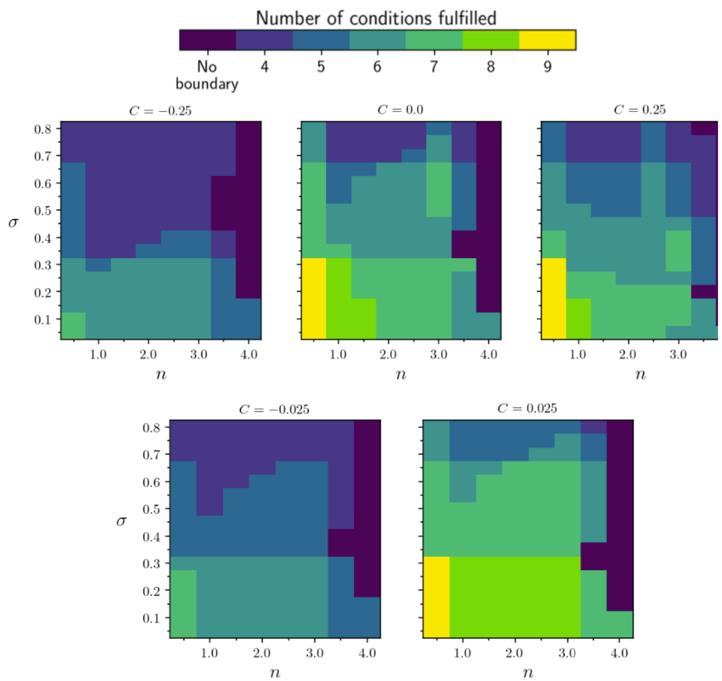


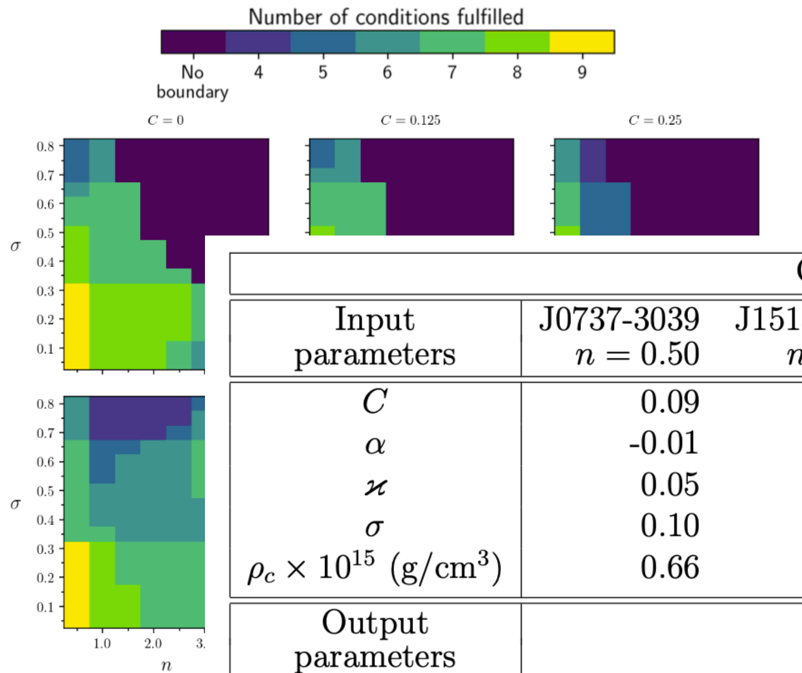
Y seguimos…

Suárez-Urango, D., Ospino, J., Hernández, H., & Núñez, L. A. (2022). Acceptability conditions and relativistic anisotropic generalized polytropes. *The European Physical Journal C*, *82*(2), 1-22

Nos fuimos full numérico para la misma ecuación de estado barótropa

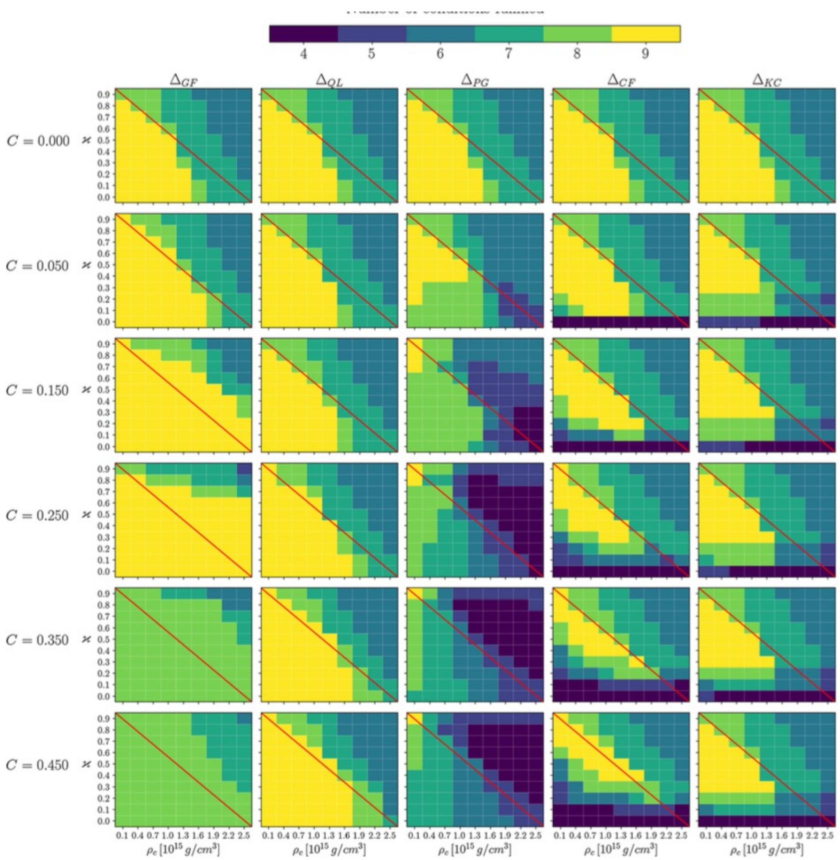
$$P = \kappa \rho^{1+\frac{1}{n}} + \alpha\rho - \beta$$

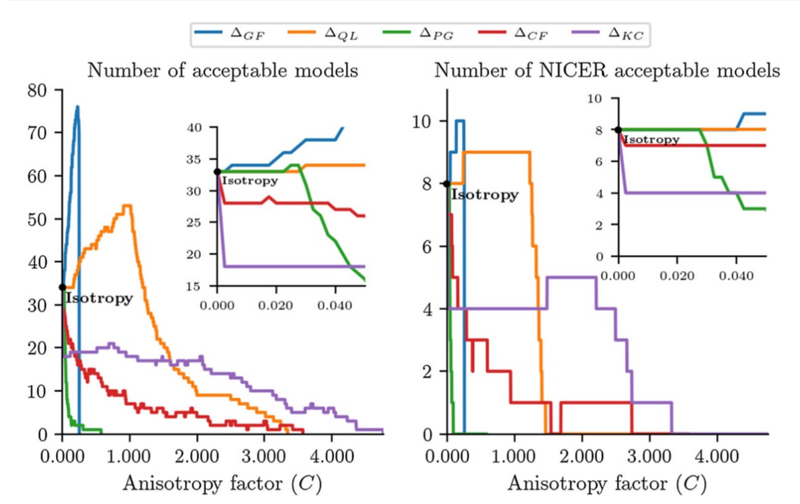


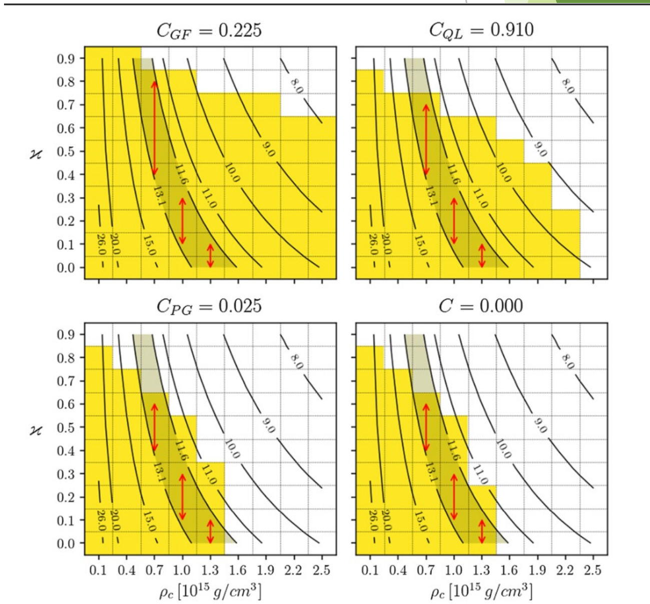


$2: \begin{cases} \rho > 0 \\ P > 0 \end{cases}$

	Object			
Input parameters	J0737-3039 $n = 0.50$	J1518+4904 $n = 1.00$	GMn075 $n = 0.75$	PMn075 $n = 0.75$
C	0.09	0.125	0.05	0.05
α	-0.01	0.01	-0.01	0.0
$\varkappa$	0.05	0.15	0.17	0.0
σ	0.10	0.15	0.18	0.18
$\rho_c \times 10^{15}$ (g/cm^3)	0.66	1.79	1.41	1.41
Output parameters				
M ($M_\odot$)	1.33	1.56	1.50	1.56
R (km)	11.49	9.88	10.0	10.9
$2\mathcal{C}_\star$	0.34	0.47	0.44	0.42
$\rho_b \times 10^{14}$ (g/cm^3)	0.33	2.69	2.4	0.0
$k_{2\star}$	0.06	0.03	0.04	0.04
$\tilde{\Lambda}_{(1.4)*}$	165.30	50.80	63.60	70.40







Suárez-Urango, D., Becerra, L. M., Ospino, J., & Núñez, L. A. (2023). The physical acceptability conditions and the strategies to obtain anisotropic compact objects. *The European Physical Journal C*, *83*(11), 1018.

Conclusiones

- La anisotropía en presiones es un candidato razonable para modelar objetos compactos
- Los parámetros n, α, σ y C son los más importantes.
- Los índices politrópos bajos, n<1, conducen a modelos físicamente aceptables
- Las anisotropías positivas pequeñas producen mejores modelos que las anisotropías negativas o grandes. **Cuanto más pequeña es la anisotropía, más aceptables son los modelos.**
- Las configuraciones de materia politrópica son más viables cuando se considera la densidad de energía total
- Los modelos inestables frente a los movimientos convectivos no presentan fracturas. Son inestabilidades independientes
- Los modelos podrían representar objetos astrofísicos físicamente interesantes. Mostramos cuatro de estos candidatos a NS. La deformaciones de marea, $\tilde{\Lambda}$ (1,4)⋆ para todos estos modelos son menores que el límite superior crítico estimado por LIGO
- En cuanto al púlsar masivo recientemente observado J0740+6620, encontramos que podemos modelarlo como un model anisótropo, para los valores de los parámetros α, $\varkappa$ y n considerados.

Gracias

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nunezluis

Ecuaciones de estado barótropa

$$P(r) = \mathcal{W}(\rho(r), r)$$
$$P_\perp(r) = \mathcal{Y}(\rho(r), r)$$

Estrategia 2: derivar y sustituir

$$\left.\begin{aligned} \frac{\mathrm{d}P(\rho,r)}{\mathrm{d}r} &= \frac{\partial \mathcal{W}}{\partial \rho}\frac{\mathrm{d}\rho(r)}{\mathrm{d}r} \\ \frac{\mathrm{d}P_\perp(\rho,r)}{\mathrm{d}r} &= \frac{\partial \mathcal{Y}}{\partial \rho}\frac{\mathrm{d}\rho(r)}{\mathrm{d}r} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \frac{\mathrm{d}P}{\mathrm{d}r} &= \mathcal{F}(m(r), \rho(r), P(r), P_\perp(r), r), \\ \frac{\mathrm{d}m}{\mathrm{d}r} &= 4\pi r^2 \rho(r) \\ \frac{\mathrm{d}\rho}{\mathrm{d}r} &= \frac{1}{v_s}\frac{\mathrm{d}P}{\mathrm{d}r} \\ \frac{\mathrm{d}P_\perp(\rho,r)}{\mathrm{d}r} &= v_{s\perp}\frac{\mathrm{d}\rho(r)}{\mathrm{d}r} \end{aligned}\right.$$

$$\text{donde, } v_s = \frac{\partial \mathcal{W}}{\partial \rho} \quad \text{y} \quad v_{s\perp} = \frac{\partial \mathcal{Y}}{\partial \rho}$$

$$\overbrace{P_0 = P(0) = P_\perp(0), \quad \rho_0 = \rho(0) \text{y} \quad m_0 = m(0) = 0}$$

…y el sistema se integra, numéricamente

Ecuaciones de estado no barótropa

$$P(r) = \mathcal{W}(\rho(r), r)$$

$$P_\perp(r) = \mathcal{V}(\rho(r), P(r), r)$$

Estrategia 1: despejar y sustituir

$$\left.\begin{array}{l} P_\perp(r) = \mathcal{V}(\rho(r), P(r), r) \\ \\ \rho(r) = \mathcal{Z}(P(r), r) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \dfrac{\mathrm{d}P}{\mathrm{d}r} = \mathcal{G}\left(m(r), P(r), \mathcal{Z}(P(r), r), r\right), \\ \\ \dfrac{\mathrm{d}m}{\mathrm{d}r} = \underbrace{4\pi r^2 \mathcal{Z}(P(r), r)}_{P_0 = P(0) \quad \text{y} \quad m_0 = m(0) = 0} \end{array}\right.$$

…y el sistema se integra, numéricamente

Ecuaciones de estado no-barótropa

$$P(r) = \mathcal{W}(\rho(r), r)$$
$$P_{\perp}(r) = \mathcal{V}(\rho(r), P(r), r)$$

Estrategia 2: derivar y sustituir

$$\left.\begin{aligned} \frac{\mathrm{d}P(\rho,r)}{\mathrm{d}r} &= \frac{\partial \mathcal{W}}{\partial \rho}\frac{\mathrm{d}\rho(r)}{\mathrm{d}r} + \frac{\partial \mathcal{W}}{\partial r} \\ \frac{\mathrm{d}P_{\perp}(\rho,r)}{\mathrm{d}r} &= \frac{\partial \mathcal{V}}{\partial \rho}\frac{\mathrm{d}\rho(r)}{\mathrm{d}r} + \frac{\partial \mathcal{V}}{\partial P}\frac{\mathrm{d}P(r)}{\mathrm{d}r} + \frac{\partial \mathcal{V}}{\partial r} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \frac{\mathrm{d}P}{\mathrm{d}r} &= \mathcal{F}(m(r), \rho(r), P(r), P_{\perp}(r), r), \\ \frac{\mathrm{d}m}{\mathrm{d}r} &= 4\pi r^2 \rho(r), \\ \frac{\mathrm{d}\rho}{\mathrm{d}r} &= \frac{1}{v^2}\left(\frac{\mathrm{d}P}{\mathrm{d}r} - \frac{\partial \mathcal{W}}{\partial r}\right), \\ \frac{\mathrm{d}P_{\perp}}{\mathrm{d}r} &= v_{\perp}^2 \frac{\mathrm{d}\rho(r)}{\mathrm{d}r} + \frac{\partial \mathcal{V}}{\partial P}\frac{\mathrm{d}P(r)}{\mathrm{d}r} + \frac{\partial \mathcal{V}}{\partial r} \end{aligned}\right.$$

$$\text{donde, } v_s = \frac{\partial \mathcal{W}}{\partial \rho} \quad \text{y} \quad v_{s\perp} = \frac{\partial \mathcal{V}}{\partial \rho}$$

$$\overbrace{P_0 = P(0) = P_{\perp}(0), \quad \rho_0 = \rho(0) \text{y} \quad m_0 = m(0) = 0}$$

…y el sistema se integra, numéricamente

…y el sistema se integra, numéricamente

Caso, newtoniano isótropo

$$\rho(r) = f(r) \Rightarrow m(r) = \int_0^r \mathrm{d}r\, 4\pi r^2 f(r) \Rightarrow \frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2} \Rightarrow P(r)$$

Caso relativista isótropo

$$\rho(r) = f(r) \Rightarrow m(r) = \int_0^r \mathrm{d}r\, 4\pi r^2 f(r) \Rightarrow$$

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2}\left(1 + \frac{P(r)}{\rho(r)c^2}\right)\left(1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right)\left(1 - 2\frac{Gm(r)}{rc^2}\right)^{-1} \Rightarrow$$

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -F(r) - G(r)P(r) + H(r)P^2(r)$$

Ecuación de Riccatti

WIKIPEDIA
The Free Encyclopedia

Riccati equation

From Wikipedia, the free encyclopedia

In mathematics, a **Riccati equation** in the narrowest sense is any first-order ordinary differential equation that is quadratic in the unknown function. In other words, it is an equation of the form

$$y'(x) = q_0(x) + q_1(x)\,y(x) + q_2(x)\,y^2(x)$$

where $q_0(x) \neq 0$ and $q_2(x) \neq 0$. If $q_0(x) = 0$ the equation reduces to a Bernoulli equation, while if $q_2(x) = 0$ the equation becomes a first order linear ordinary differential equation.

The equation is named after Jacopo Riccati (1676–1754).[1]

More generally, the term **Riccati equation** is used to refer to matrix equations with an analogous quadratic term, which occur in both continuous-time and discrete-time linear-quadratic-Gaussian control. The steady-state (non-dynamic) version of these is referred to as the algebraic Riccati equation.

Contents [hide]

Reduction to a second order lir

Obtaining solutions by quadrature [edit]

The correspondence between Riccati equations and second-order linear ODEs has other consequences. For example, if one solution of a 2nd order ODE is known, then it is known that another solution can be obtained by quadrature, i.e., a simple integration. The same holds true for the Riccati equation. In fact, if one particular solution y_1 can be found, the general solution is obtained as

$$y = y_1 + u$$

Substituting

$$y_1 + u$$

in the Riccati equation yields

$$y_1' + u' = q_0 + q_1 \cdot (y_1 + u) + q_2 \cdot (y_1 + u)^2,$$

and since

$$y_1' = q_0 + q_1\,y_1 + q_2\,y_1^2,$$

it follows that

$$u' = q_1\,u + 2\,q_2\,y_1\,u + q_2\,u^2$$

or

$$u' - (q_1 + 2\,q_2\,y_1)\,u = q_2\,u^2,$$

which is a Bernoulli equation. The substitution that is needed to solve this Bernoulli equation is

$$z = \frac{1}{u}$$

Substituting

$$y = y_1 + \frac{1}{z}$$

directly into the Riccati equation yields the linear equation

$$z' + (q_1 + 2\,q_2\,y_1)\,z = -q_2$$

A set of solutions to the Riccati equation is then given by

$$y = y_1 + \frac{1}{z}$$

where z is the general solution to the aforementioned linear equation.

See also [edit]