

Análisis vectorial express

Luis A. Núñez

*Escuela de Física, Facultad de Ciencias,
Universidad Industrial de Santander, Santander, Colombia*



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- En general $\mathbf{a}(t) = a^k(t)\mathbf{e}_k(t) = \tilde{a}^m\tilde{\mathbf{e}}_m(t) = \bar{a}^n(t)\bar{\mathbf{e}}_n$.

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- Como siempre $\lim_{\Delta t \rightarrow 0} \frac{\mathbf{a}(t+\Delta t) - \mathbf{a}(t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{a}(t)}{\Delta t} = \frac{d\mathbf{a}(t)}{dt}$ con

$$\frac{d}{dt} [\mathbf{a}(t) + \mathbf{b}(t)] = \frac{d}{dt} \mathbf{a}(t) + \frac{d}{dt} \mathbf{b}(t) ,$$

$$\frac{d}{dt} [\alpha(t)\mathbf{a}(t)] = \left[\frac{d}{dt} \alpha(t) \right] \mathbf{a}(t) + \alpha(t) \left[\frac{d}{dt} \mathbf{a}(t) \right] ,$$

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- Pero teniendo cuidado que si $\mathbf{a}(t) = a^k(t)\mathbf{e}_k(t) \Rightarrow$

$$\frac{d\mathbf{a}(t)}{dt} = \frac{d[a^k(t)\mathbf{e}_k(t)]}{dt} = \frac{da^k(t)}{dt}\mathbf{e}_k(t) + a^k(t)\frac{d\mathbf{e}_k(t)}{dt} .$$

- En general si $\mathbf{a}(t) = |\mathbf{a}(t)| \hat{\mathbf{u}}(t) \Rightarrow$

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- Demostración:

$$\frac{d[|\mathbf{a}(t)| \hat{\mathbf{u}}(t)]}{dt} = \frac{d|\mathbf{a}(t)|}{dt} \hat{\mathbf{u}}(t) + |\mathbf{a}(t)| \frac{d\hat{\mathbf{u}}(t)}{dt}.$$

y además

$$\frac{d(|\hat{\mathbf{u}}(t)|^2)}{dt} \equiv \frac{d[\hat{\mathbf{u}}(t) \cdot \hat{\mathbf{u}}(t)]}{dt} = 0, \Rightarrow \hat{\mathbf{u}}(t) \cdot \frac{d\hat{\mathbf{u}}(t)}{dt} = 0$$

entonces

$$\hat{\mathbf{u}}(t) \perp \frac{d\hat{\mathbf{u}}(t)}{dt} \Leftrightarrow \hat{\mathbf{u}}_{\parallel} \cdot \hat{\mathbf{u}}_{\perp} = 0$$

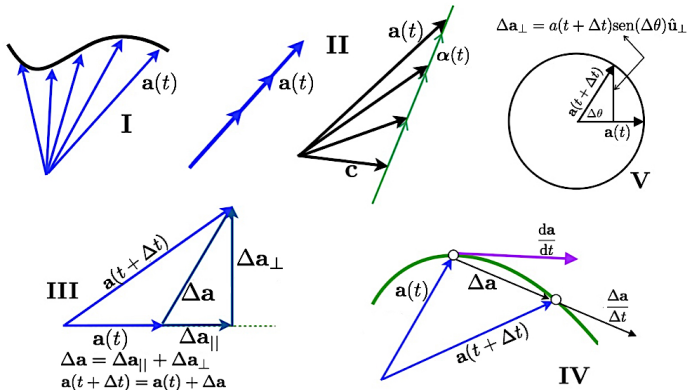


Figura: Derivación de vectores

- Como siempre, si $\mathbf{r} = \mathbf{r}(t) = r(t)\hat{\mathbf{u}}_r(t)$

$$\mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt} = v_r(t)\hat{\mathbf{u}}_r(t) + r(t)\dot{\theta}(t)\hat{\mathbf{u}}_\theta(t), \quad y$$

$$\mathbf{a}(t) = \left\{ \ddot{r}(t) - r(t)\dot{\theta}^2(t) \right\} \hat{\mathbf{u}}_r(t) + \left\{ 2\dot{r}(t)\dot{\theta}(t) + r(t)\ddot{\theta}(t) \right\} \hat{\mathbf{u}}_\theta(t).$$

Más aún, si $\mathbf{r} = \mathbf{r}(t) = r(t)\hat{\mathbf{u}}_r(t)$ y $\mathbf{v} = \mathbf{v}(t) = v(t)\hat{\mathbf{u}}_v(t)$

$$\left. \begin{aligned} \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \times \hat{\mathbf{u}}_r &= \hat{\mathbf{u}}_v \\ \hat{\mathbf{u}}_v \times \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} &= \hat{\mathbf{u}}_r \\ \hat{\mathbf{u}}_r \times \hat{\mathbf{u}}_v &= \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \end{aligned} \right\} \Rightarrow \mathbf{v}(t) = \boldsymbol{\omega} \times \mathbf{r}(t).$$

- Funciones (campos) escalares $\phi = \phi(t) = \phi(x, y, z)$

Funciones (campos) vectoriales

$$\mathbf{A}(x, y, z) =$$

$$A_x(x, y, z)\mathbf{i} + A_y(x, y, z)\mathbf{j} + A_z(x, y, z)\mathbf{k}.$$

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- $\phi(x^i)$ (Campo escalar) y $\mathbf{V}(x^i)$ (Campo Vectorial)

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Rightarrow \begin{cases} \phi = \phi(x, y, z) \equiv \phi(\mathbf{r}) \\ \mathbf{V} = \mathbf{V}(x, y, z) \equiv \mathbf{V}(\mathbf{r}) \end{cases}$$

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- Si $\phi(\mathbf{r}(t)) = \phi(x(t), y(t), z(t))$, entonces

$$\frac{d\phi(\mathbf{r}(t))}{dt} = \left[\frac{\partial\phi(x,y,z)}{\partial x}\mathbf{i} + \frac{\partial\phi(x,y,z)}{\partial y}\mathbf{j} + \frac{\partial\phi(x,y,z)}{\partial z}\mathbf{k} \right] \cdot \left[\frac{dx(t)}{dt}\mathbf{i} + \frac{dy(t)}{dt}\mathbf{j} + \frac{dz(t)}{dt}\mathbf{k} \right]$$

$$\frac{d\phi(\mathbf{r}(t))}{dt} = \nabla\phi(x(t), y(t), z(t)) \cdot \frac{d\mathbf{r}(t)}{dt},$$

y a $\nabla\phi(x(t), y(t), z(t))$ lo llamaremos el **gradiente** de la función $\phi(\mathbf{r}(t))$:

- El operador nabla y la imaginación nos provee:

el gradiente $\nabla \phi(x, y, z) = \partial^i \phi(x^j) \mathbf{e}_i \Leftrightarrow \nabla(\circ) = \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) (\circ) = \mathbf{e}_i \partial^i (\circ).$

el rotacional $\nabla \times \mathbf{E} = \varepsilon^{ijk} \partial_j E_k \mathbf{e}_i \Leftrightarrow \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) \times (\circ) = \varepsilon^{ijk} \partial_j E_k \mathbf{e}_i \partial^i (\circ).$

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- La derivada total de un campo vectorial será:

$$\frac{d\mathbf{A}}{dt} = \frac{dA_x(x,y,z)}{dt} \mathbf{i} + \frac{dA_y(x,y,z)}{dt} \mathbf{j} + \frac{dA_z(x,y,z)}{dt} \mathbf{k} = \frac{dA^i(x^j)}{dt} \mathbf{e}_i,$$

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- Entonces cada componente:

$$\frac{d(A^i(x(t), y(t), z(t)))}{dt} = \frac{d(A^i(x^j(t)))}{dt} = \frac{\partial(A^i(x^j))}{\partial x^k} \frac{dx^k(t)}{dt} = \left(\frac{d\mathbf{r}(t)}{dt} \cdot \nabla \right) A^i(x, y, z).$$

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- En términos vectoriales es:

$$\frac{d\mathbf{A}}{dt} = \left(\frac{d\mathbf{r}(t)}{dt} \cdot \nabla \right) \mathbf{A} \equiv (\mathbf{v} \cdot \nabla) \mathbf{A} \Rightarrow \frac{d(\circ)}{dt} = (\mathbf{v} \cdot \nabla) (\circ) \equiv v^i \partial_i (\circ),$$

con \mathbf{v} la derivada del radio vector posición $\mathbf{r}(t)$,

- ① El primer caso es el trivial: $\int \mathbf{A}(u) \, du =$
 $\mathbf{i} \int A_x(u) du + \mathbf{j} \int A_y(u) du + \mathbf{k} \int A_z(u) du = \left(\int A^i(u) du \right) \mathbf{e}_i.$

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- ② Un campo escalar a lo largo de un vector: $\int_C \phi(\mathbf{r}) \, d\mathbf{r} = \int_C \phi(x^i) (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}) = \mathbf{i} \int_C \phi(x^i) dx + \mathbf{j} \int_C \phi(x^i) dy + \mathbf{k} \int_C \phi(x^i) dz.$

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- 3 Un vector a lo largo de otro vector: $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$
 $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = \int_C F_x(x, y, z) dx + \int_C F_y(x, y, z) dy + \int_C F_z(x, y, z) dz = \int_C F^i(x^j) dx_i.$

En presentación consideramos

- 1 Vectores variables $\mathbf{a}(t) = a^k(t)\mathbf{e}_k(t) = \tilde{a}^m\tilde{\mathbf{e}}_m(t) = \bar{a}^n(t)\bar{\mathbf{e}}_n$.

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6 Integración vectorial $\int \mathbf{A}(u) du$, $\int_C \phi(\mathbf{r}) d\mathbf{r}$ y $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$