AAPT PHYSICS EDUCATION
AMERICAN JOURNAL of PHYSICS

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Citation: American Journal of Physics **40**, 1742 (1972); doi: 10.1119/1.1987057
View online: http://dx.doi.org/10.1119/1.1987057
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/40/12?ver=pdfcov
Published by the American Association of Physics Teachers

graduate or undergraduate tutor with only the difference in quality of instruction that that change implies.

ACKNOWLEDGMENTS

I wish to thank J. William Moore and William E. Hauck for many useful suggestions and discussions of the program and Stephen F. Becker and Ellen D. Gagne for their help in construction, grading, and administration of the delayed exam.

* This work supported in part by a grant from the Carnegie Corporation.

† Robert A. Artman began as a collaborator but died before completion of the Project.

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The Use of Computer Generated Data in Experiment Design—A Student Exercise

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(Received 12 January 1972; revised 1 May 1972)

An exercise in experiment design, envisioned for use in an undergraduate nuclear physics course, is herein described. The exercise utilizes several new methods of analysis, introduces the student to the use of the digital computer in simulating experimental data, and teaches the basic principles of experiment design.

I. INTRODUCTION

The physicist is often faced with the problem of designing an experiment which is capable of definitively testing theoretical predictions or of measuring a given quantity with a predetermined precision. Unfortunately, with some notable exceptions, this important facet of physics is often sorely neglected in undergraduate physics curricula. As one of a number of efforts aimed at bridging this gap, we have introduced into our junior year nuclear physics laboratory the exercise in experiment design described below.

The particular exercise described here was suggested to us by the interesting experiment on the successive nuclear decay of two radionuclides with nearly equal half-lives presented recently in this Journal by Scobie and Scott.[1] The authors of that paper conclude that "It is still rather difficult, however, to show that the two half-lives are different and that the second is shorter than the first." The exercise proposed here asks the student to consider the design of this experiment with particular attention devoted to establishing in advance a choice of experimental conditions for its implementation which would enable him definitively to establish the sign and magnitude of the difference in decay rates of the two radionuclides. We believe that this design exercise is of particular interest for a number of reasons:

(1) It extends the analysis of Scobie and Scott and proposes a new method of analysis which can be used to establish directly the *difference* in the two nearly equal decay rates;

(2) It introduces the student to the use of a digital computer to simulate the data which would be obtained if the experiment were performed with the chosen design parameters (this greatly facilitates the design of the experiment);

(3) It allows the student to vary the design parameters at will until a result of satisfactory accuracy can be obtained—thus helping to fill the need described above, introducing the student to an important aspect of the "real world" of physics.

II. THE "EXPERIMENT"

The "experiment" considered here is essentially that proposed by Scobie and Scott: measuring the decay of both the parent ^{101}Mo and daughter ^{101}Tc in the chain

$$^{101}\mathrm{Mo} \xrightarrow[14.6\text{ min}]{\beta^-} {}^{101}\mathrm{Tc} \xrightarrow[14.0\text{ min}]{\beta^-} {}^{101}\mathrm{Ru}.$$

The source of ^{101}Mo is obtained by irradiating MoO_3 in a thermal flux F for a time short compared with either half-life. For simplicity, we assume that both the parent and the daughter decays are simultaneously detected by a pair of NaI detectors which view the source symmetrically—each a distance h away from the source. The parent and daughter decays are distinguished by pulse-height analysis which selects γ's characteristic of ^{101}Tc and ^{101}Ru, respectively. The student has at his disposal (in theory, at least) as design parameters; M, the mass of MoO_3 irradiated; F, the thermal flux; t_I, the length of the irradiation; the size of the NaI detectors; h, the source-to-detector-face distance; the settings of the pulse-height analyzers; and T, the time interval over which counts are accumulated for each data point.

III. DATA ANALYSIS

The first step is, of course, to measure λ_3—the decay constant of the parent ^{101}Mo. Since the Mo decay is a simple exponential decrease, this measurement is perfectly straightforward. The next step is to determine λ_2, the decay constant of the daughter radionuclide ^{101}Tc. As pointed out by Scobie and Scott,[1] the complexity of this decay curve and the near equality of the two decay constants make more sophisticated analysis necessary if reliable information is to be obtained. It is of interest that an analysis technique recently developed by Curtis, *et al.*[2] for use in analyzing atomic decay curves furnishes a very powerful method of approaching this problem and gives rise to an unequivocal value for λ_2 which is not subject to the ambiguities of previously suggested approaches. We present a summary of this analysis technique in Appendix I and suggest its use as an alternate to that suggested by Scobie and Scott. Once the parent and daughter mean lives are determined to be nearly equal—as determined either by the analysis of Scobie and Scott[1] or by the alternate method described here in Appendix I—it is then of interest to consider directly the difference in decay rates

$$\Delta\lambda \equiv \lambda_2 - \lambda_3. \quad (1)$$

Denoting the number of ^{101}Mo nuclei at time t by $N_3(t)$ and its decay probability per unit time by λ_3 and introducing a similar notation with subscripts 2 for ^{101}Tc and 1 for ^{101}Ru, the β decay intensities of the two radionuclides are easily shown to be

$$I_{32}(t) = \lambda_3 N_3(0)\exp(-\lambda_3 t) \quad (2)$$

$$\begin{aligned} I_{21}(t) &= N_3(0)(\lambda_2\lambda_3/\lambda_2-\lambda_3) \\ &\quad\times[\exp(-\lambda_3 t)-\exp(-\lambda_2 t)] \\ &= N_3(0)(\lambda_2\lambda_3/\Delta\lambda)\exp(-\lambda_3 t) \\ &\quad\times[1-\exp(-\Delta\lambda t)]. \quad (3) \end{aligned}$$

Thus, if one were able to measure directly $I_{32}(t)$ and $I_{21}(t)$, one could then compute the quotient

$$Q(t) \equiv I_{21}(t)/I_{32}(t) = \lambda_2\Delta\lambda^{-1}[1-\exp(-\Delta\lambda\cdot t)]$$

which, for

$$t \ll \Delta\lambda^{-1}, \tag{4}$$

can be written

$$Q(t) = \lambda_2 t[1 - \Delta\lambda \cdot (t/2)]. \tag{5}$$

A least squares fit of $Q(t)$ to a polynomial in t would then allow a measurement of $\Delta\lambda$.[3] Actually, for the present purposes it suffices to plot

$$F(t) \equiv Q(t)/t = \lambda_2[1 - \Delta\lambda(t/2)] \tag{6}$$

vs t neglecting data points near $t=0$ (where errors will be large) and data for t so large that Eq. (4) fails to be satisfied. The result should be a straight line. Determination of the slope and intercept then provides a measurement of both the sign and magnitude of $\Delta\lambda$.

In practice, the data obtained in a real experiment differs from a direct measurement of $I(t)$ in two ways:

(1) Decay branching ratios and instrumental efficiencies mean that what is measured is a multiplicative constant times the intensity; and

(2) what is actually experimentally recorded is the total number of counts in an interval of length T about the mean observation time t, proportional to the integral of the intensity over the time interval T, and the factor Q is actually constructed from these counts as

$$Q(t) \equiv C_{21}(t)/C_{32}(t), \tag{7}$$

where

$$C_{ij} = \int_{t-(T/2)}^{t+(T/2)} I_{ij}(t)\,dt.$$

The multiplicative constants introduced due to factor (1) do not in any way change the analysis suggested above (except, of course, that the intercept is no longer equal to λ_2). The integration involved in factor (2) can, in principle, distort the analysis; however, if $T \ll \lambda^{-1}$—as will surely be true here—it is easy to show that Eq. (6) remains valid with the new definition of Q.

The problem facing the student, then, is to choose experimental parameters which will guarantee that the data obtained will be sufficiently accurate to determine $\Delta\lambda$. If an insufficient number of counts is collected at each point, a plot of $F(t)$ vs t will yield points so badly scattered that the existence of a slope different from zero cannot be established; in this case, the student, too, would conclude that the two half-lives were experimentally indistinguishable. The purpose of this exercise is to have the student design an experiment capable of definitively measuring $\Delta\lambda$.

IV. COMPUTER SIMULATED DATA

The primary aim of this exercise, as described earlier, is to familiarize the student with some of the considerations in experiment design. Since the effect of the analysis procedure described in Sec. III upon the accuracy of the final result is not transparent, it is very helpful to have available simulated data which can be subjected to the analysis proposed. The question of reliability, or lack thereof, of any conclusions reached concerning the sign or magnitude of $\Delta\lambda$ are, in this way, completely unambiguous. A computer utility routine, GENT, was written for this purpose several years ago. It has proved most useful both in our atomic physics research laboratory[4] and in intermediate and advanced undergraduate laboratories. GENT addresses itself to decays of the sort described in the level diagram of Fig. 1. It is a fairly general routine capable of handling up to 16 levels, each with a specified initial population $N_i(0)$. The routine computes the intensity of the decay 2→1 as well as the intensity of each transition into level 2 using the diagrammatic mnemonic of Curtis[5] and including "second order" cascade terms such as are indicated by 4→3→2 in Fig. 1. The user specifies the number of

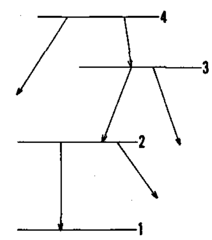

FIG. 1. Level diagram for GENT.

levels in the decay scheme NL; the initial populations $N_i(0)$, $i=2-NL$; a matrix specifying the transition probabilities per unit time between all relevant levels, A_{ij}; and the decay constants λ_i; if the user specifies, further, the detection efficiencies and counting times, GENT will add to each data point an error of random sign which is chosen randomly from a Gaussian distribution with standard deviation $\sigma(t)=(CT)^{1/2}$.[6] The result is a simulation of the data which would be obtained if an actual experiment could be carried out under the assumed conditions. It presents the student with data typical of that which would be obtained were he to carry out the experiment which he proposes.

The nuclear decay considered here is, of course, an especially simple case to which the use of GENT is ideally suited: $NL=3$, $N_2(0)=0$, $A_{31}=0$, $A_{32}=\lambda_3$, $A_{21}=\lambda_2$; only $N_3(0)$, λ_3 and λ_2 (in addition to the detection efficiencies and counting times) need be specified by the student.

V. EXPERIMENT DESIGN

As a starting point in designing an experiment to measure $\Delta\lambda$, the student may be referred to the experiment proposed by Scobie and Scott, where $M=100$ mg, $F=10^{12}$ n/cm²–sec, $t_I=5$ sec and $T=30$ sec. The student's first task, then, is to compute $N_3(0)$. To do this, he must go to the literature to find (a) the natural abundance of ^{100}Mo [7] and (b) its thermal neutron (n, γ) activation cross section.[7,8] With this information and the definition of cross section, he can proceed to determine $N_3(0)$ for the above assumptions. Next, he must study the decay scheme of the mass 101 isobars[9] in order to choose appropriate settings for the decay identification channels ($E_\gamma=1.024$ and 0.307 MeV, respectively, are good choices) and to determine what fraction of each β decay of interest is followed by a γ of the energy selected. Finally, the overall detection efficiency in each case is given by the product of this factor with the efficiency of the NaI detector and the detector–source distance selected. Again, here, the student is referred to the literature[10] to obtain the necessary information. With this information on hand, GENT may be used to construct simulated decay curves which correspond to the student's choice of the experimental parameters.

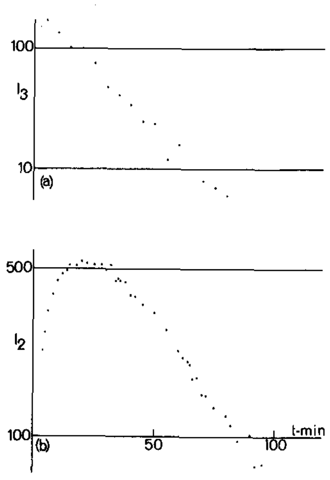

FIG. 2. (a) Computer generated observed decay curve for ^{101}Mo parent nuclide. (b) Computer generated observed decay curve for ^{101}Tc daughter nuclide.

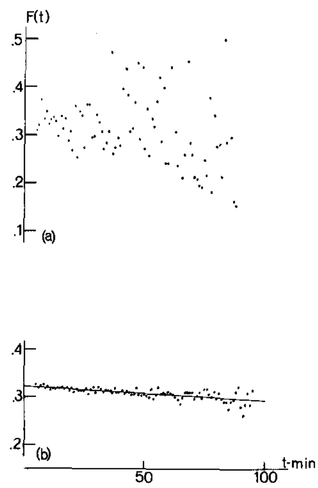

FIG. 3. (a) $F(t)$ vs t (min) for data of Fig. 2. (b) $F(t)$ vs t (min) for improved statistics; the straight line corresponds to $\Delta=0.0019\pm0.003$ min^{-1}.

VI. SAMPLE RESULTS

To illustrate the sort of results obtained by the student, representative but rather arbitrary detector parameters are here chosen and the preceding analysis is carried out. Assuming the use of $1\frac{7}{8}$ in.$\times$2 in. cylindrical detectors which are available in our undergraduate laboratory and assuming $h=10$ cm, the simulated decay curves shown in Fig. 2 are obtained by use of GENT. The function $F(t)$ defined in Eq. (6) is then computed and the result obtained is shown in Fig. 3(a). It is clear that the data points are badly scattered and that it is difficult to establish definitive conclusions about Δ. While careful inspection of the graph already does seem to indicate the sign of Δ, it is clear that obtaining a reliable numerical value of Δ requires more accurate data. On the other hand, if the experimental parameters are changed in such a way as to increase the total number of counts at each data point by a factor of 100, the results appear as in Fig. 3(b). From these improved data, a value of $\Delta\lambda = +0.0019 \pm 0.0003\ \mathrm{min}^{-1}$ is obtained, in good agreement with the assumed value of $\Delta\lambda = 0.0020\ \mathrm{min}^{-1}$.[8] Of course, it is the student's task to suggest methods of obtaining the required increase in statistical accuracy and his choice among the various options should reflect a number of considerations. For example, if an increased thermal flux is suggested, its availability must be considered; if larger detectors are required, their availability and cost must be a factor; if longer counting times are suggested, their effect on the approximations made in analyzing the data must be taken into account. The overall result should be a design, within the limits of technology and economic feasibility, which gives rise to an unambiguous determination of the quantities of interest. The power of the technique of computer simulation as a pedagogic tool for understanding proper experiment design is clearly illustrated in this exercise.

VII. SUMMARY

An exercise in experiment design for a nuclear physics laboratory which introduces the use of a digital computer to simulate experimental results has been outlined. It also introduces several new analysis techniques to supplement those suggested in the experiment of Scobie and Scott.

ACKNOWLEDGMENTS

We wish to thank Mr. Dennis Chojnacki for coding the program GENT and Dr. L. J. Curtis for his careful reading of this manuscript and his comments thereon.

APPENDIX I

It is always of interest to point out to the student of physics examples of techniques and methods which have been developed in one area of physics which then find application in a different—though perhaps related—area. In this light, it is of interest to note that an analysis technique developed recently for use in atomic transition probability measurements by Curtis *et al.*[2] is ideally suited to the determination of the nuclear decay constant of the radioactive daughter nucleus ^{101}Tc in the experiment being considered here. It therefore furnishes an alternate approach to that described in the letter of Scobie and Scott.[1]

The method, which will be referred to as the arbitrarily normalized decay curve (ANDC) method, can be easily understood as follows:

In the notation developed in Secs. III and V of the text, the differential equation describing the rate of change of the population of ^{101}Tc is

$$(dN_2/dt) = A_{32}N_3 - A_{21}N_2. \qquad \text{(A1)}$$

Moreover, the intensity of either observed transition can be written

$$I_{ij} = E_{ij}A_{ij}N_i, \qquad \text{(A2)}$$

where E_{ij} denotes the overall detection efficiency for the transition $i \to j$. Thus, Eq. (A1) can be recast in terms of the observed intensities as

$$\tau_2(dI_{21}/dt) = (E_{21}I_{32}/E_{32}) - I_{21} \qquad \text{(A3)}$$

where $\tau_2 = A_{21}^{-1}$ denotes the mean life $(T_{1/2}/\ln 2)$ of ^{101}Tc. Finally, denoting the relative detection efficiency as $\xi \equiv (E_{21}/E_{32})$ and integrating Eq.

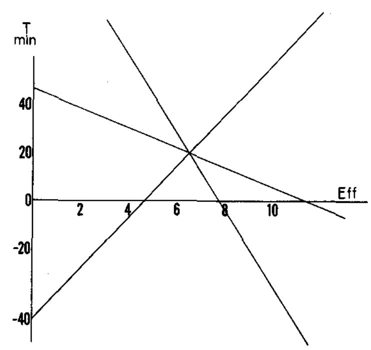

Fig. 4. Partial ANDC analysis of computer generated decay data of Fig. 2(a) (poorer statistics).

(A3) between two arbitrarily chosen times, t_i and t_f, gives

$$\tau_2 = a(t_i, t_f) - b(t_i, t_f)\xi, \tag{A4}$$

where

$$a \equiv \int_{t_i}^{t_f} dt I_{21} \Big/ [I_{21}(t_i) - I_{21}(t_f)]$$

and

$$b \equiv \int_{t_i}^{t_f} dt I_{32} \Big/ [I_{21}(t_i) - I_{21}(t_f)].$$

For any choice of t_i and t_f, a and b can be obtained by numerical integration of the measured decay curves $I_{ij}(t)$ (a simple summation of data points usually suffices). It is important to note that the intensity may be measured in arbitrary units, i.e., the detection efficiency need not be known. Equation (4) then determines a linear relationship between τ_2 and ξ which may be graphed. If a second choice of t_i and t_f is made, a second straight line results and the intersection of the two lines determines both τ_2 and ξ. Further choices of t_i and t_f overdetermine the problem and the sharpness of the mutual intersection of all such lines is a measure of the uncertainty in the values of τ_2 and ξ obtained.

This analysis procedure has been applied to the simulated data of Fig. 2(a). The result is shown in Fig. 4 where, for clarity, only three representations of Eq. (4) are displayed. The result

$$\tau_2 = 20.0 \pm 0.5 \text{ min}$$

is in excellent agreement with the expected result, $\tau_2 = 20.2$ min. The relative detection efficiency $\xi = 6.5$ is also in excellent agreement with that used in generating the data.

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[6] The random choices are performed with the aid of a pseudo random number generator which, in our case, is part of the FORTRAN Scientific Subroutines provided by The University of Toledo computation center. Typical schemes for pseudo random number generation and the use of Monte Carlo techniques in providing variables with appropriate distributions are discussed, e.g., in Herman Cahn, *Atomic Energy Report AECU-3259, 1954* (unpublished).

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[8] *Neutron Cross Sections*, BNL 325, 2nd ed., 1958 (unpublished).

[9] *Nuclear Data Sheets*, Nuclear Data Group, 1959–1965, Reprinted by Academic, New York, 1966.

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