Análisis vectorial express

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Derivación de Vectores 1/2

- En general $\mathbf{a}(t) = a^k(t)\mathbf{e}_k(t) = \tilde{a}^m\tilde{\mathbf{e}}_m(t) = \bar{a}^n(t)\bar{\mathbf{e}}_n$.

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- En general $\mathbf{a}(t) = a^k(t)\mathbf{e}_k(t) = \tilde{a}^m\tilde{\mathbf{e}}_m(t) = \bar{a}^n(t)\bar{\mathbf{e}}_n$.
- Como siempre $\lim_{\Delta t\to 0} \frac{\mathbf{a}(t+\Delta t)-\mathbf{a}(t)}{\Delta t} = \lim_{\Delta t\to 0} \frac{\Delta\mathbf{a}(t)}{\Delta t} = \frac{\mathrm{d}\mathbf{a}(t)}{\mathrm{d}t}$ con

$$
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\frac{\mathrm{d}}{\mathrm{d}t}\left[\mathbf{a}(t)+\mathbf{b}(t)\right] &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{a}(t)+\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{b}(t) \,, \\
\frac{\mathrm{d}}{\mathrm{d}t}\left[\alpha(t)\mathbf{a}(t)\right] &= \left[\frac{\mathrm{d}}{\mathrm{d}t}\alpha(t)\right]\mathbf{a}(t)+\alpha(t)\left[\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{a}(t)\right] \,, \\
\frac{\mathrm{d}}{\mathrm{d}t}\left[\mathbf{a}(t)\cdot\mathbf{b}(t)\right] &= \left[\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{a}(t)\right]\cdot\mathbf{b}(t)+\mathbf{a}(t)\cdot\left[\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{b}(t)\right] \,, \\
\frac{\mathrm{d}}{\mathrm{d}t}\left[\mathbf{a}(t)\times\mathbf{b}(t)\right] &= \left[\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{a}(t)\right]\times\mathbf{b}(t)+\mathbf{a}(t)\times\left[\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{b}(t)\right] \,.
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\end{aligned}
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- Pero teniendo cuidado que si $\mathbf{a}(t) = a^k(t)\mathbf{e}_k(t) \Rightarrow$

$$
\frac{\mathrm{d}\mathbf{a}(t)}{\mathrm{d}t} = \frac{\mathrm{d}\left[a^k(t)\mathbf{e}_k(t)\right]}{\mathrm{d}t} = \frac{\mathrm{d}a^k(t)}{\mathrm{d}t}\mathbf{e}_k(t) + a^k(t)\frac{\mathrm{d}\mathbf{e}_k(t)}{\mathrm{d}t} \,.
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Derivación de Vectores 2/2

- En general si $\mathbf{a}(t) = |\mathbf{a}(t)|\,\hat{\mathbf{u}}(t) \Rightarrow$

$$\frac{\mathrm{d}\mathbf{a}(t)}{\mathrm{d}t} = \frac{\mathrm{d}\left[|\mathbf{a}(t)|\,\hat{\mathbf{u}}(t)\right]}{\mathrm{d}t} = \frac{\mathrm{d}\,|\mathbf{a}(t)|}{\mathrm{d}t}\hat{\mathbf{u}}_{\parallel} + |\mathbf{a}(t)|\,\hat{\mathbf{u}}_{\perp}\,, \text{ con } \hat{\mathbf{u}}_{\parallel}\cdot\hat{\mathbf{u}}_{\perp} = 0\,.$$

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- Demostración:

$$\frac{\mathrm{d}\left[|\mathbf{a}(t)|\,\hat{\mathbf{u}}(t)\right]}{\mathrm{d}t} = \frac{\mathrm{d}\,|\mathbf{a}(t)|}{\mathrm{d}t}\hat{\mathbf{u}}(t) + |\mathbf{a}(t)|\,\frac{\mathrm{d}\hat{\mathbf{u}}(t)}{\mathrm{d}t}\,.$$

y además

$$\frac{\mathrm{d}\left(|\hat{\mathbf{u}}(t)|^2\right)}{\mathrm{d}t} \equiv \frac{\mathrm{d}\left[\hat{\mathbf{u}}(t)\cdot\hat{\mathbf{u}}(t)\right]}{\mathrm{d}t} = 0\,, \Rightarrow \hat{\mathbf{u}}(t)\cdot\frac{\mathrm{d}\hat{\mathbf{u}}(t)}{\mathrm{d}t} = 0$$

entonces

$$\hat{\mathbf{u}}(t) \perp \frac{\mathrm{d}\hat{\mathbf{u}}(t)}{\mathrm{d}t} \Leftrightarrow \hat{\mathbf{u}}_{\parallel}\cdot\hat{\mathbf{u}}_{\perp} = 0$$

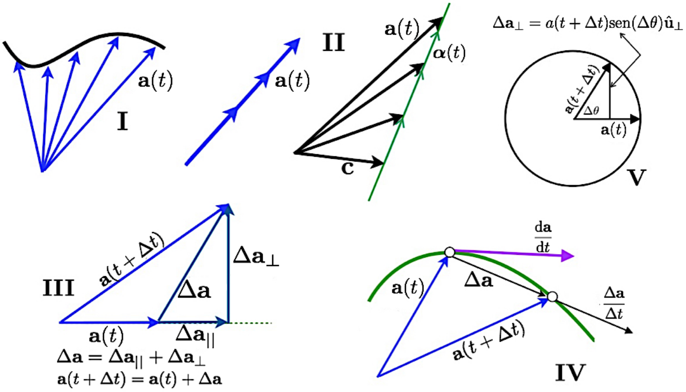

Figura: Derivación de vectores

Velocidades y aceleraciones

- Como siempre, si $\mathbf{r} = \mathbf{r}(t) = r(t)\hat{\mathbf{u}}_r(t)$

$$\mathbf{v}(t) = \frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t} = v_r(t)\hat{\mathbf{u}}_r\,(t) + r(t)\dot{\theta}(t)\hat{\mathbf{u}}_\theta(t)\,, \quad \text{y}$$

$$\mathbf{a}(t) = \left\{\ddot{r}(t) - r(t)\dot{\theta}^2(t)\right\}\hat{\mathbf{u}}_r(t) + \left\{2\,\dot{r}(t)\dot{\theta}(t) + r(t)\ddot{\theta}(t)\right\}\hat{\mathbf{u}}_\theta(t)\,.$$

Más aún, si $\mathbf{r} = \mathbf{r}(t) = r(t)\hat{\mathbf{u}}_r(t)$ y $\mathbf{v} = \mathbf{v}(t) = v(t)\hat{\mathbf{u}}_v(t)$

$$\left.\begin{array}{c} \dfrac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \times \hat{\mathbf{u}}_r = \hat{\mathbf{u}}_v \\[2em] \hat{\mathbf{u}}_v \times \dfrac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} = \hat{\mathbf{u}}_r \\[2em] \hat{\mathbf{u}}_r \times \hat{\mathbf{u}}_v = \dfrac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \end{array}\right\} \quad \Rightarrow \quad \mathbf{v}(t) = \boldsymbol{\omega} \times \mathbf{r}(t)\,.$$

- Funciones (campos) escalares $\phi = \phi(t) = \phi(x, y, z)$
  Funciones (campos) vectoriales
  $\mathbf{A}(x, y, z) =$
  $A_x(x, y, z)\mathbf{i} + A_y(x, y, z)\mathbf{j} + A_z(x, y, z)\mathbf{k}\,.$

Gradiente

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- $\phi\left(x^i\right)$ (Campo escalar) y $\mathbf{V}\left(x^i\right)$ (Campo Vectorial)

$$\mathbf{r} = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k} \quad \Rightarrow \left\{ \begin{array}{c} \phi = \phi(x, y, z) \equiv \phi\left(\mathbf{r}\right) \\ \\ \mathbf{V} = \mathbf{V}(x, y, z) \equiv \mathbf{V}\left(\mathbf{r}\right) \end{array} \right.$$

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- Si $\phi(\mathbf{r}(t)) = \phi(x(t), y(t), z(t))$, entonces

$$\frac{\mathrm{d}\phi(\mathbf{r}(t))}{\mathrm{d}t} = \left[\frac{\partial\phi(x,y,z)}{\partial x}\mathbf{i} + \frac{\partial\phi(x,y,z)}{\partial y}\mathbf{j} + \frac{\partial\phi(x,y,z)}{\partial z}\mathbf{k}\right] \cdot \left[\frac{\mathrm{d}x(t)}{\mathrm{d}t}\mathbf{i} + \frac{\mathrm{d}y(t)}{\mathrm{d}t}\mathbf{j} + \frac{\mathrm{d}z(t)}{\mathrm{d}t}\mathbf{k}\right]$$

$$\frac{\mathrm{d}\phi(\mathbf{r}(t))}{\mathrm{d}t} = \boldsymbol{\nabla}\phi(x(t), y(t), z(t)) \cdot \frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t} ,$$

y a $\boldsymbol{\nabla}\phi(x(t), y(t), z(t))$ lo llamaremos el **gradiente** de la función $\phi(\mathbf{r}(t))$:

- El operador nabla y la imaginación nos provee:

el gradiente $\boldsymbol{\nabla}\phi(x,y,z) = \partial^i \phi(x^j)\mathbf{e}_i \quad \Leftrightarrow \quad \boldsymbol{\nabla}(\circ) = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)(\circ) = \mathbf{e}_i \partial^i (\circ)\,.$

el rotacional $\boldsymbol{\nabla} \times \mathbf{E} = \varepsilon^{ijk}\partial_j E_k\ \mathbf{e}_i \quad \Leftrightarrow \quad \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right) \times (\circ) = \varepsilon^{ijk}\partial_j E_k\ \mathbf{e}_i \mathbf{e}_i \partial^i (\circ)\,.$

la divergencia $\boldsymbol{\nabla} \cdot \mathbf{A} = \frac{\partial A^i(x^j)}{\partial x^i} \quad \Leftrightarrow \quad \boldsymbol{\nabla} \cdot (\circ) = \partial_j (\circ)^j$

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- La derivada total de un campo vectorial será:

$$\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} = \frac{\mathrm{d}A_x(x,y,z)}{\mathrm{d}t}\mathbf{i} + \frac{\mathrm{d}A_y(x,y,z)}{\mathrm{d}t}\mathbf{j} + \frac{\mathrm{d}A_z(x,y,z)}{\mathrm{d}t}\mathbf{k} = \frac{\mathrm{d}A^i\left(x^j\right)}{\mathrm{d}t}\mathbf{e}_i\,,$$

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- Entonces cada componente:

$$\frac{\mathrm{d}\left(A^i(x(t),y(t),z(t))\right)}{\mathrm{d}t} = \frac{\mathrm{d}\left(A^i\left(x^j(t)\right)\right)}{\mathrm{d}t} = \frac{\partial\left(A^i\left(x^j\right)\right)}{\partial x^k}\frac{\mathrm{d}x^k(t)}{\mathrm{d}t} = \left(\frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t}\cdot\boldsymbol{\nabla}\right)A^i(x,y,z)\,.$$

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$$\frac{\mathrm{d}\left(A^i(x(t),y(t),z(t))\right)}{\mathrm{d}t} = \frac{\mathrm{d}\left(A^i\left(x^j(t)\right)\right)}{\mathrm{d}t} = \frac{\partial\left(A^i\left(x^j\right)\right)}{\partial x^k}\frac{\mathrm{d}x^k(t)}{\mathrm{d}t} = \left(\frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t} \cdot \boldsymbol{\nabla}\right) A^i(x,y,z)\,.$$

- En términos vectoriales es:

$$\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} = \left(\frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t} \cdot \boldsymbol{\nabla}\right)\mathbf{A} \equiv (\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{A} \ \Rightarrow\ \frac{\mathrm{d}(\circ)}{\mathrm{d}t} = (\mathbf{v} \cdot \boldsymbol{\nabla})\,(\circ) \equiv v^i \partial_i\,(\circ)\,,$$

con $\mathbf{v}$ la derivada del radio vector posición $\mathbf{r}(t)$,

Integración vectorial

1. El primer caso es el trivial: $\int \mathbf{A}(u)\,\mathrm{d}u = \mathbf{i}\int A_x(u)\mathrm{d}u + \mathbf{j}\int A_y(u)\mathrm{d}u + \mathbf{k}\int A_z(u)\mathrm{d}u = \left(\int A^i(u)\mathrm{d}u\right)\mathbf{e}_i$.

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2. Un campo escalar a lo largo de un vector: $\int_C \phi\,(\mathbf{r})\,\mathrm{d}\mathbf{r}$
   $\int_C \phi(x^i)\,(\mathrm{d}x\mathbf{i} + \mathrm{d}y\mathbf{j} + \mathrm{d}z\mathbf{k}) =$
   $\mathbf{i}\int_C \phi(x^i)\mathrm{d}x + \mathbf{j}\int_C \phi(x^i)\mathrm{d}y + \mathbf{k}\int_C \phi(x^i)\mathrm{d}z\,.$

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3. Un vector a lo largo de otro vector: $\int_C \mathbf{F}\,(\mathbf{r}) \cdot \mathrm{d}\mathbf{r}$
$\int_C \mathbf{F}\,(\mathbf{r}) \cdot \mathrm{d}\mathbf{r} = \int_C F_x(x,y,z)\,\mathrm{d}x + \int_C F_y(x,y,z)\,\mathrm{d}y + \int_C F_z(x,y,z)\,\mathrm{d}z =$
$\int_C F^i(x^j)\,\mathrm{d}x_i\,.$

En presentación consideramos

1. Vectores variables $\mathbf{a}(t) = a^k(t)\mathbf{e}_k(t) = \tilde{a}^m\tilde{\mathbf{e}}_m(t) = \bar{a}^n(t)\bar{\mathbf{e}}_n$.

Recapitulando

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4. El operador nabla imaginativo:

el gradiente $\boldsymbol{\nabla}\phi(x, y, z) = \partial^i\phi(x^j)\mathbf{e}_i \quad \Leftrightarrow \quad \boldsymbol{\nabla}(\circ) = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)(\circ) = \mathbf{e}_i\partial^i(\circ)$.

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5. La derivada de un campo vectorial $\mathbf{A}(x(t), y(t), z(t))$
$\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} = \left(\frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t} \cdot \boldsymbol{\nabla}\right)\mathbf{A} \equiv (\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{A} \;\Rightarrow\; \frac{\mathrm{d}(\circ)}{\mathrm{d}t} = (\mathbf{v} \cdot \boldsymbol{\nabla})\,(\circ) \equiv v^i\partial_i\,(\circ)$,

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$\mathbf{A}(x, y, z) = A_x(x, y, z)\mathbf{i} + A_y(x, y, z)\mathbf{j} + A_z(x, y, z)\mathbf{k}$.
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el gradiente $\boldsymbol{\nabla}\phi(x, y, z) = \partial^i\phi(x^j)\mathbf{e}_i \quad \Leftrightarrow \quad \boldsymbol{\nabla}(\circ) = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)(\circ) = \mathbf{e}_i\partial^i(\circ)$.

el rotacional $\boldsymbol{\nabla} \times \mathbf{E} = \varepsilon^{ijk}\partial_j E_k\ \mathbf{e}_i \quad \Leftrightarrow \quad \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right) \times (\circ) = \varepsilon^{ijk}\partial_j E_k\ \mathbf{e}_i\mathbf{e}_i\partial^i(\circ)$.

la divergencia $\boldsymbol{\nabla} \cdot \mathbf{A} = \frac{\partial A^i(x^j)}{\partial x^i} \quad \Leftrightarrow \quad \boldsymbol{\nabla} \cdot (\circ) = \partial_j(\circ)^j$

5. La derivada de un campo vectorial $\mathbf{A}(x(t), y(t), z(t))$
$\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} = \left(\frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t} \cdot \boldsymbol{\nabla}\right)\mathbf{A} \equiv (\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{A} \ \Rightarrow\ \frac{\mathrm{d}(\circ)}{\mathrm{d}t} = (\mathbf{v} \cdot \boldsymbol{\nabla})\,(\circ) \equiv v^i\partial_i\,(\circ)$,
6. Integración vectorial $\int \mathbf{A}(u)\, \mathrm{d}u$, $\int_C \phi\,(\mathbf{r})\,\mathrm{d}\mathbf{r}$ y $\int_C \mathbf{F}\,(\mathbf{r}) \cdot \mathrm{d}\mathbf{r}$

Para la discusión

1. Tal vez, uno de los problemas que ilustra mejor el uso del álgebra vectorial y la derivación de vectores es el movimiento bajo fuerzas centrales. La ley de gravitación de Newton nos dice que para un sistema de dos masas, m y M se tiene:

$$\sum \mathbf{F} = m\,\mathbf{a} \;\Rightarrow\; mG\frac{M}{r_{mM}^2}\hat{\mathbf{u}}_r = m\,\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \;\Rightarrow\; \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{GM}{r_{mM}^2}\hat{\mathbf{u}}_r\,.$$

   A partir de la *velocidad areolar*, $\mathbf{v}_a$, (área barrida por unidad de tiempos por el radio vector posición, $\mathbf{r}(t)$) encuentre la trayectoria de la partícula $r = r(\theta)$

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CONSTRUIMOS FUTURO

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2. Consideramos $\mathbf{F}(\mathbf{r}) = \left(3x^2 + 2xy^3\right)\hat{\mathbf{i}} + 6xy\,\hat{\mathbf{j}}$.

   Queremos evaluar la siguiente integral: $\int_{(0,0)}^{\left(1,\frac{3}{4}\sqrt{2}\right)} \mathbf{F}(\mathbf{r})\cdot\mathrm{d}\mathbf{r}$
   Donde la curva que une esos puntos viene parametrizada por:

$$x = 2\tau^2\,,\;\; y = \tau^3 + \tau \;\Rightarrow\; \frac{\partial x(\tau)}{\partial \tau} = 4\tau\,, \frac{\partial y(\tau)}{\partial \tau} = 3\tau^2 + 1\,,$$