Transient behaviour of a Lazy Random Walker in One Dimension

Sunip Kumar Mukherjee
Muktish Acharyya
Department of Physics, Presidency University,
86/1 College Street, Kolkata-700073, INDIA
Email: sunipkmukherjee@gmail.com, muktish.physics@presiuniv.ac.in

July 19, 2015

ABSTRACT: The motion of a lazy random walker is studied in one dimension. With a more physical definition of laziness, the diffusion of the walker is observed. The first return probability is also numerically calculated for the walker, for various laziness "levels", and a comparison is made for various laziness levels, including a normal random walker. A relation of the kind $\langle d^2 \rangle = a \times \langle t_{fr} \rangle^b$ is established, with the value of the exponent exactly 3, for all laziness levels in the transient analysis of the first return probability for the random walker. It is also established that the first return probability in the transient region is always higher for higher value of laziness, for a given time limit to the random walker, and ultimately the probability converges (asymptotically) to 1.

Keywords: Lazy random walk, first return probability, scaling

I. INTRODUCTION:

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

II. MODEL and RESULTS:

Mathematically, a lazy random walk is described where the walker has 50% probability of moving from his current position, and in one dimension has equal probability of moving in either direction. In this paper, a more physical approach is taken towards laziness. Here, laziness is defined as a continuous variable ranging from 0 to 1, where 0 laziness parameter signifies an ordinary random walk, and laziness parameter 1 signifies the case where the walker does not move at all, being completely lazy. In practise, if P_{lazy} is the laziness parameter, the walker has $P_{lazy} \times 100\%$ chance of *not* moving from that position. Once the walker decides to move, he has equal probability of moving either right or left. The diffusion of such a walker is first investigated, over a large number of samples, i.e. by releasing the random walker many times. The random walker is allowed to move for a certain amount of time, and the final position of the walker is noted after the allowed time has elapsed, and the distribution of the final position is obtained for the given sample size. For a large enough sample, the distribution is Gaussian, and in the asymptotic limit, it can be established theoretically that the distribution is indeed Gaussian. For a normal random walker, this result is already established. For a mathematical random walker, this result can also be simulated. For the definition of a lazy walker given above, the same behaviour is observed, as the reader can verify from the accompanying figures. As one intuitively expects, the "lazy walker", given the same amount of time should not travel further than the normal walker, and "lazier" walker will travel even less. So, for the same amount of allowed time, the normal walker will move around the furthest and have a rather wide distribution of displacement, and the width of the distribution will diminish with increasing laziness. But for the same laziness parameter, the walker is again expected to diffuse, i.e. to travel further, with more allowed time. This is also established in the accompanying figures. Mostly a lazy random walker behaves like a normal random walker. The "activity parameter" of a lazy random walker is defined simply by $P_{act} = 1 - P_{lazy}$. A normal random walker is thus "fully active" and an absolutely lazy random walker ($P_{lazy} = 1$) is completely inactive. In this paper, the term step is equivalent to unit time, and a step size limit determines how long a walker is allowed to walk. With a given step size limit, i.e. finite amount of time and non zero activity parameter, an ensemble of identical random walkers will end up to be at different positions after walking for the alloted "time" or step size limit, and we can obtain a distribution of the probability for the walker to be at some distance from the origin, and regardless of the newly introduced laziness parameter the distribution is Gaussian. This can be observed in Figure 1, and Figure 2. These observation prompted the assumption of a scaling behaviour involving both the "step size limit (T_{max})", as the distribution width clearly depends on it, as well as the "activity parameter (P_{act})" and equivalently the "laziness parameter". By P_r the probability of finding a random walker at a distance r from the origin is denoted. A scaling like $aP_r T_{max}^{\alpha} P_{act}^{\beta} = \exp(-bT_{max}^{\gamma} P_{act}^{\delta} r^2)$ is assumed, and the various distribution curves (all denoted by the different symbols for different sets of T_{max} and P_{act}) collapses to $\exp(-x^2)$, as is observed in Figure 3 with $a = 1/4$, $b = 0.5$, $\alpha = \beta = -0.5$, and $\gamma = \delta = -1$. It may be mentioned here that the statistics is based on $N_s = 10^7$ different random samples. That the exponents α, β and γ, δ are the same is expected, as the activity parameter simply modulates the step size limit for a statistical ensemble, i.e. a lazy walker statistically is able to move for only $P_{act} \cdot T_{max}$ amount of time.

The next interesting property of such walkers that has been investigated, is the first return probability. After the random walker sets out, in one dimension, given infinite time, a normal walker returns with a certainty to the place where it started from. Which implies that the return probability is 1. But, given finite amount of time, the probability is not exactly 1, but is somewhat less, and this number increases with the amount of time, asymptotically, to one, as Figure 4 clearly shows. In fact, as *Poyla* showed in his papers, the convergence is as $1 - \frac{1}{\sqrt{2t}}$, a result which is re-established in the accompanying figure. Figure 4 contains the plot of P_{fr} *vs.* t_{max}, for various laziness parameters, and all the curves show the same behaviour as of the normal random walker, the only difference being that P_{fr} is less for the same t_{max} with increasing laziness. Which is expected, as with increasing laziness parameter, the walker has less chance of moving about. So, from Figure 4, it is expected that the P_{fr} *vs.* t_{max} plots for various laziness parameters can be scaled to a single curve. Since $P_{act} \times t_{max}$ gives (probalistically) the time utilized by the lazy walker, this form of scaling is expected, and the requisite scaling produces the data collapse described in Figure 5, and all the points collapse to the curve $1 - \frac{1}{\sqrt{1.65 \times t_{max}}}$. As a cross check, $P_{fr} = 0.45$ for $t_{max} = 2$ from this curve, which is close to the actual probability, 0.5. This discrepancy is of course an inevitable consequence of having a finite size of the ensemble in numerical simulations.
Next, the two quantities $\langle t_{fr} \rangle$ and $\langle d^2_{max} \rangle$ are studied. $\langle t_{fr} \rangle$ is defined as the time taken for a walker (either lazy or non lazy) to return to the origin, within a given limit t_{max}. It must be understood that, for **1**, the walker must be displaced from the origin at first. This is unimportant for an ordinary random walker, but for a lazy random walker it may be the case that the walker never moves withing the given time limit. Such cases are of course not treated as a successful return to the origin, as the walker had not moved out in the first place. In the cases the walker *does* move out of the starting point, the total time that the walker takes to get back to the origin after the walker is allowed to move is counted, i.e. say the walker does not move at the first few steps, but does move out of the origin and returns. Then the first few steps are **1** discarded. d_{max} is the maximum distance the walker moves away from the origin **1**. d^2_{max} is averaged over an ensemble of random walkers. Figure 6 contains the plot of $\langle d^2_{max} \rangle$ vs $\langle t_{fr} \rangle$, for laziness parameters ranging from 0 to 0.9, and different values of t_{max}. It is evident, that the points for different laziness parameters but with same t_{max} group up, and for these point groups, a P_{act}^{-1} behaviour is observed. However, $\langle d^2_{max} \rangle$ vs $\langle t_{fr} \rangle$ shows a behaviour like $\langle d^2_{max} \rangle = A \times P_{act} \times \langle t_{fr} \rangle$. Again, we observe the similar scaling behavior, as plotted in Figure 7.

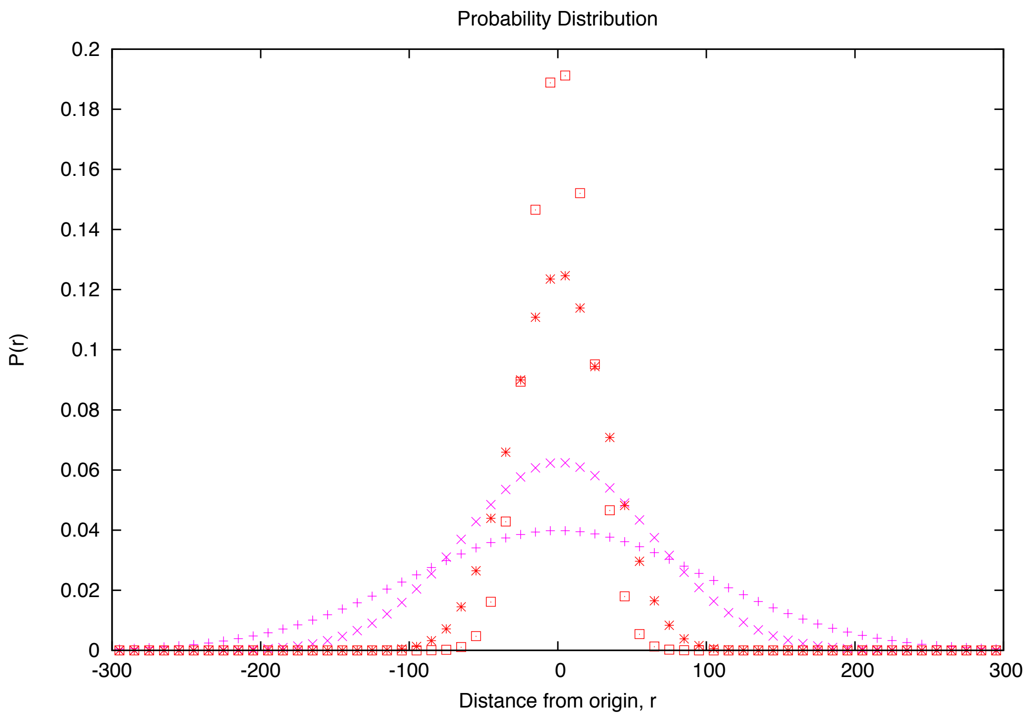

Figure 1: P_r vs. r plot for 1000 and 10000 step size limits, and laziness 0 and 0.6. "+" denotes random walk with 10000 steps, 0 laziness. "$\times$" denotes the same with 10000 steps, 0.6 laziness, "$*$" denotes 1000 stepsize, 0.0 laziness and "$\Box$" denotes 1000 stepsize and 0.6 laziness.

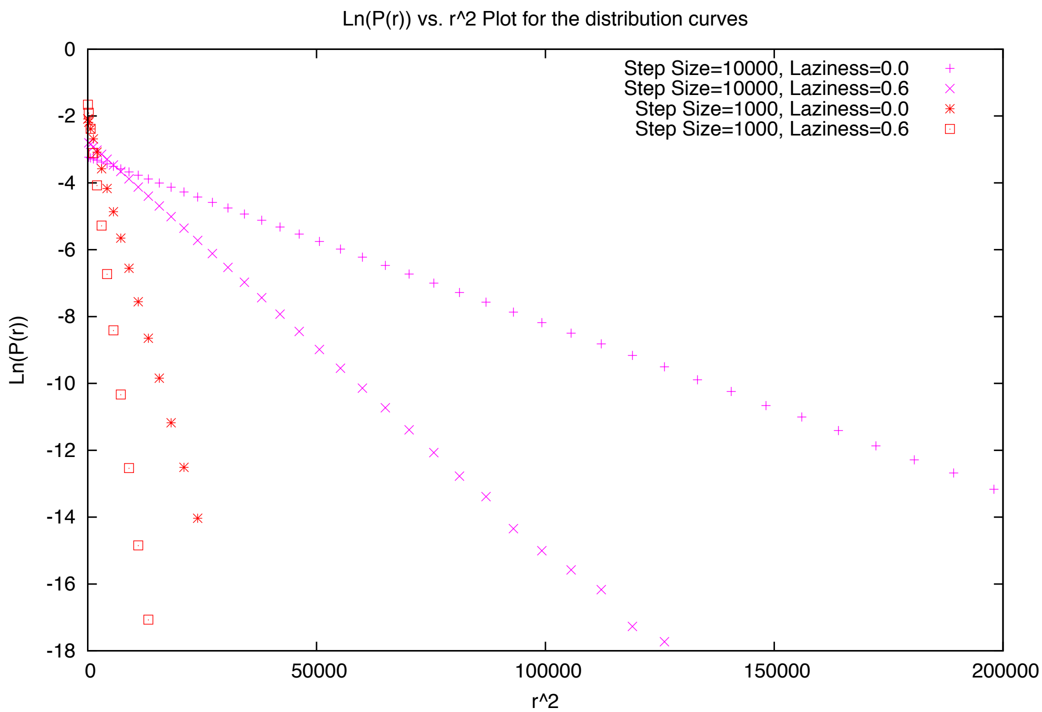

Figure 2: P_r vs. r plot for 1000 and 10000 step size limits, and laziness 0 and 0.6. "+" denotes random walk with 10000 steps, 0 laziness. "×" denotes the same with 10000 steps, 0.6 laziness, "∗" denotes 1000 stepsize, 0.0 laziness and "□" denotes 1000 stepsize and 0.6 laziness.

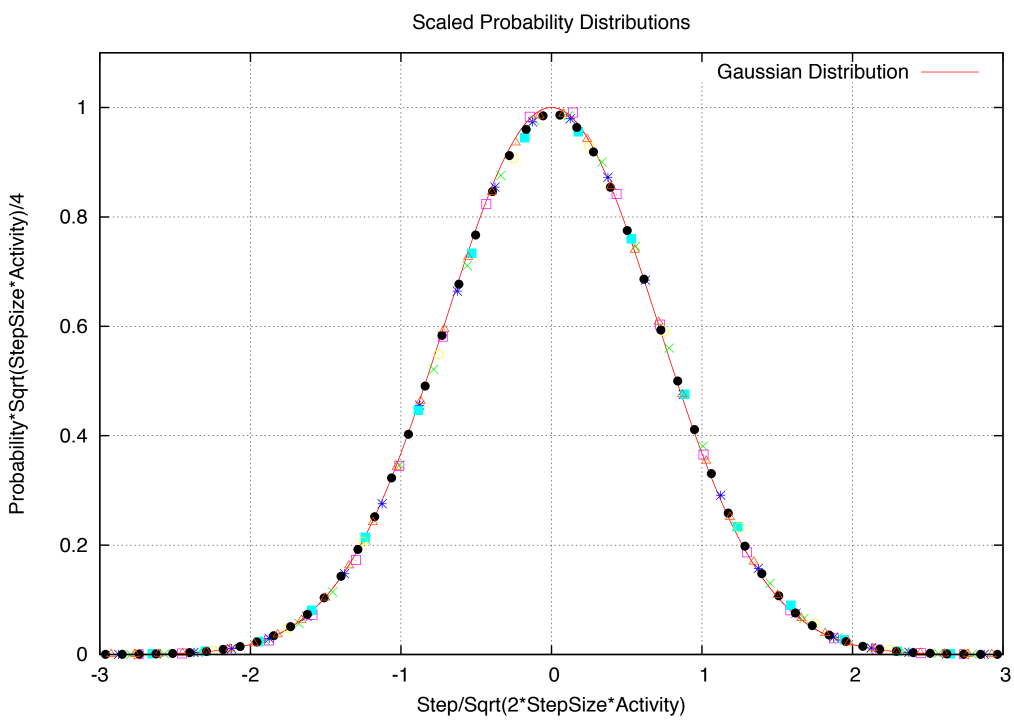

Figure 3: $P_r \cdot \sqrt{P_{act} \cdot T_{max}}/4$ vs. $\sqrt{2 \cdot P_{act} \cdot T_{max}} r$ plot for all step size limit (T_{max}) and all laziness parameters (P_{lazy}), and the scaling is apparent from the data collapse.

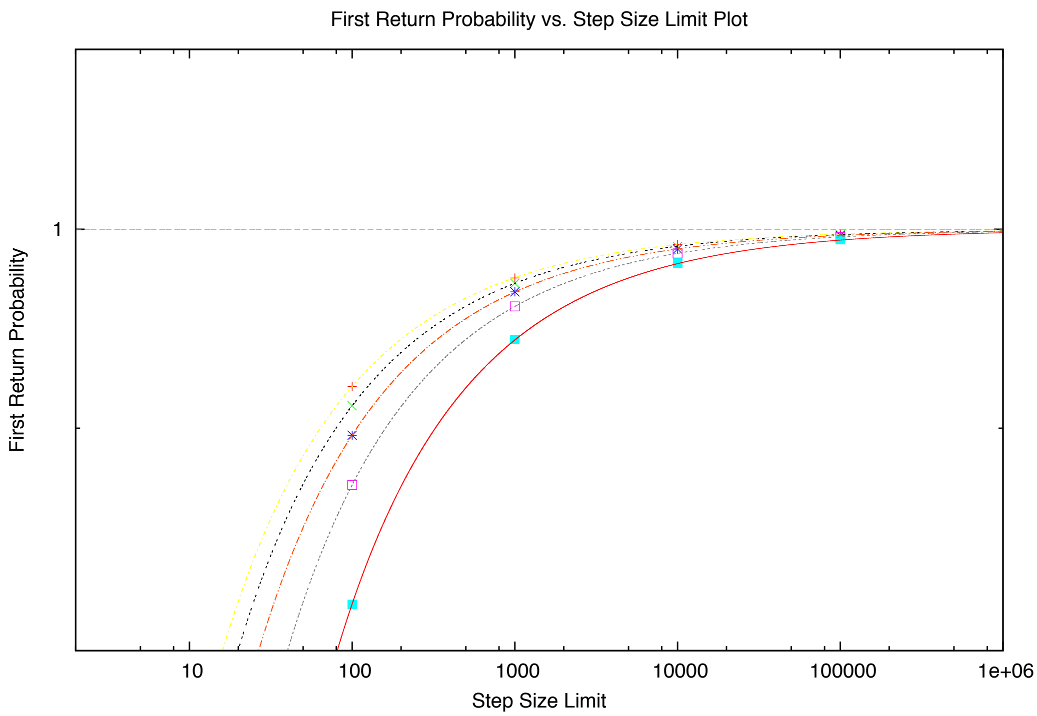

Figure 4: P_{FR} vs. t_{max} plot with laziness from 0 to 0.8. "+" denotes random walk 0 laziness. "×" denotes the same 0.2 laziness, "∗" denotes 0.4 laziness, "□" denotes0.6 laziness, and ■ denotes 0.8 laziness.

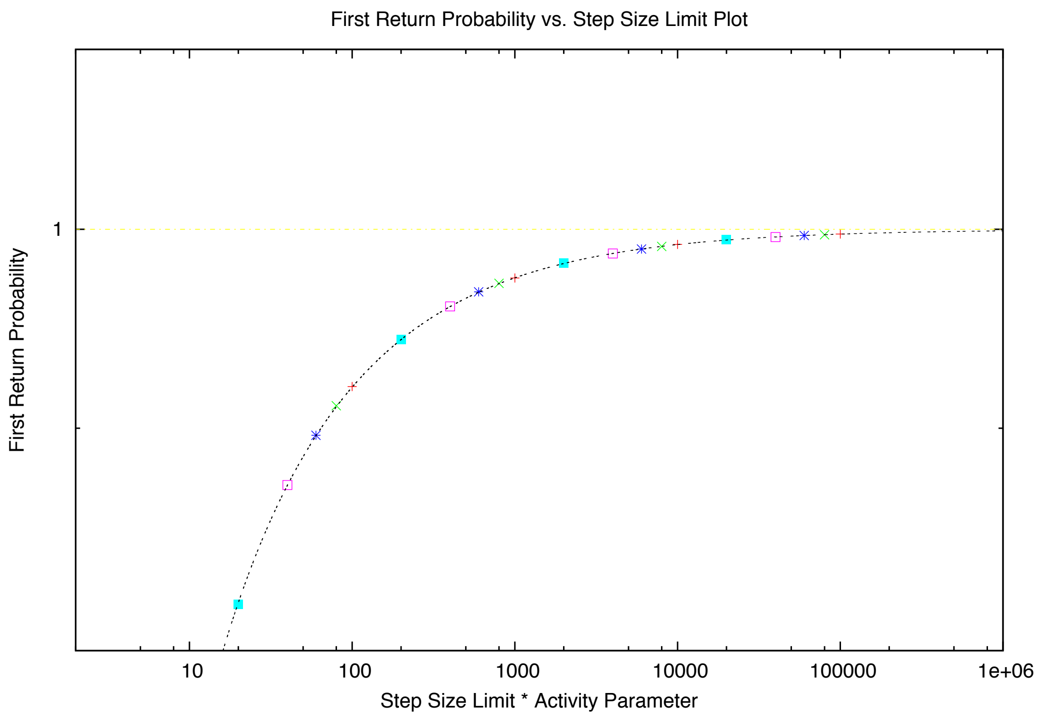

Figure 5: P_{FR} vs. $(a/P_{act})t_{max}$ plot for laziness from 0 to 0.8. "+" denotes random walk 0 laziness. "$\times$" denotes the same 0.2 laziness, "$*$" denotes 0.4 laziness, "$\square$" denotes0.6 laziness, and $\blacksquare$ denotes 0.8 laziness.

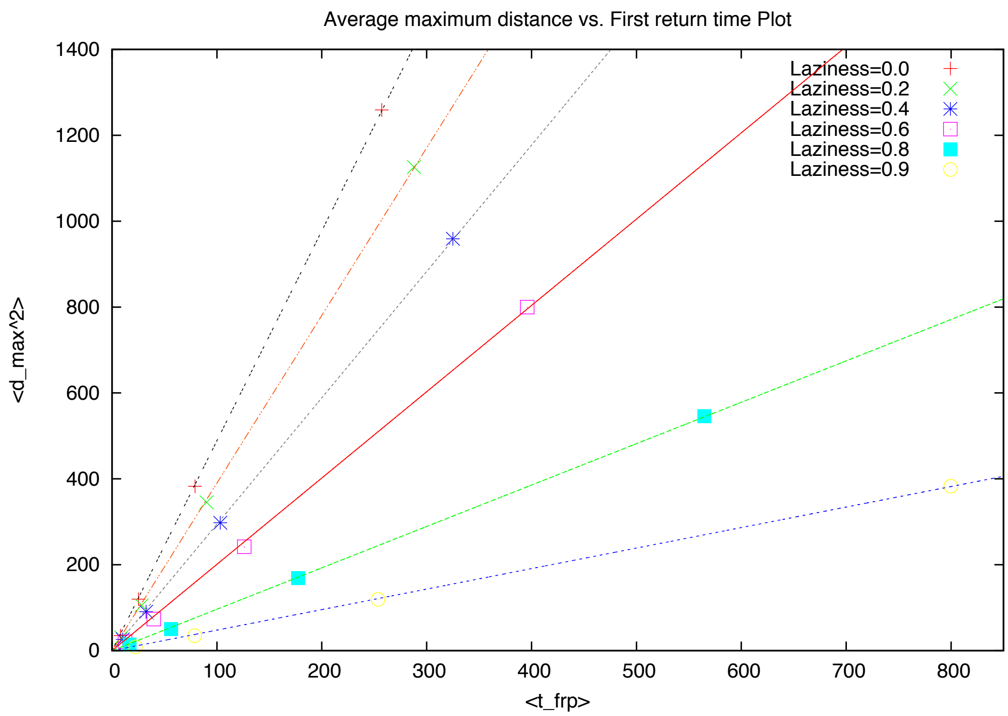

Figure 6: $\langle d_{max}\rangle^2$ vs. $\langle t_{FR}\rangle$ plot for laziness from 0 to 0.9, and t_{max} ranging from 100 to 100000 in a geometric progression. Each t_{max} has a group of points, and in each group the points are arranged in decreasing order of laziness.

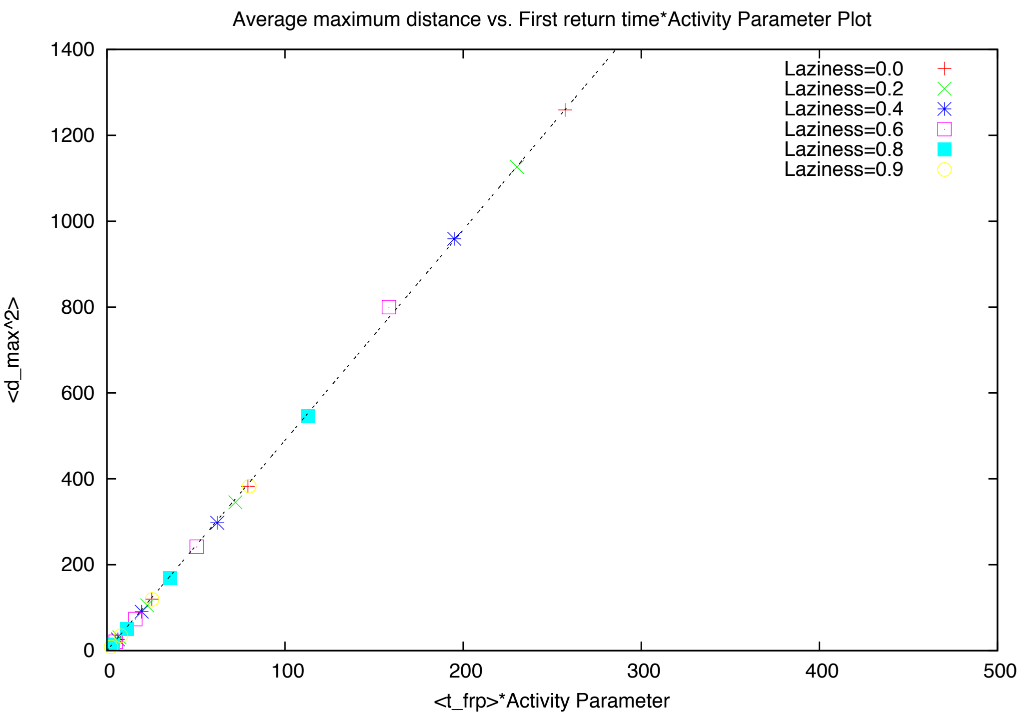

Figure 7: $\langle d_{max} \rangle^2$ vs. $\langle t_{FR} \rangle \times P_{act}$ plot for laziness from 0 to 0.9, and t_{max} ranging from 100 to 100000 in a geometric progression. Each t_{max} has a group of points, and in each group the points are arranged in decreasing order of laziness.