# By hand calculation of band structure, TDOS, and PDOS

It is assumed that you have done the crystal field calculation and the Monte Carlo calculation at this point.

### Turn off errors

# Crystal Field Hamiltonian

#### Crystal field Hamiltonian defintion

```
In[3]:= (*Cutoff parameter *)
           t = 0.2;
            (*Functions for a Cartesian point given spherical coordinates*)
           x1[rho_, theta_, phi_] := rho * Sin[theta] * Cos[phi]
           y1[rho_, theta_, phi_] := rho * Sin[theta] * Sin[phi]
            z1[rho_, theta_, phi_] := rho * Cos[theta]
            P[rho_, theta_, phi_] :=
                  {x1[rho, theta, phi], y1[rho, theta, phi], z1[rho, theta, phi]}
            (*Pyramid and tetrahedron vertices*)
           pyr =
                         \{\{2.51559, 0., 0.\}, \{-0.518164, -1.75154, -1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, 1.8285\}, \{-0.518164, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.75154, -1.
                                \{-0.518164, 1.75154, -1.8285\}, \{-0.518164, 1.75154, 1.8285\}\};
            tetra = \{\{-1.5077737499999997, 0., -1.8285\}, \{-1.5077737499999997, 0., 1.8285\}, \}
                                {1.5259762500000003, -1.751536379154027, 0.},
                                {1.5259762500000003, 1.751536379154027, 0.}};
            (*Crystal field Hamiltonian *)
           H1Cr[rho_, theta_, phi_] :=
                  Sum [(1/Sqrt[(Norm [P[rho, theta, phi]-pyr[[i]]))^2+t^2]), \{i, 1, Length@pyr\}]
           H1Fe[rho_, theta_, phi_] := Sum [
                         (1/Sqrt[(Norm[P[rho, theta, phi]-tetra[[i]]))^2+t^2], {i, 1, Length@tetra}]
           {i, 1, Length@pyr}]
```

#### Definition of Cartesian d orbital states for Cr and Fe

```
ln[13]:= a0 = 1;
              \rho[n_{-}, r_{-}] := \frac{2r}{n = 0};
             \psi \text{nlm } \left[ \text{n}_{-}, \, \text{l}_{-}, \, \text{m}_{-}, \, \text{r}_{-}, \, \theta_{-}, \, \phi_{-} \right] := \sqrt{ \left( \frac{2}{\text{na0}} \right)^{3} \, \frac{\left( \text{n-l-l} \right) \, !}{2 \, \text{n} \, \left( \text{n+l} \right) \, !}} \, \, \text{e}^{-\rho \, [\text{n}_{+} \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] \, \text{ne}^{-\rho \, [\text{n}_{+} \, \text{r}_{-}] / 2} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \rho \, [\text{n}_{+} \, \text{r}_{-}]^{1} \, \rho \, [\text{n}_{+}]^{1} \, \rho \, [\text{n}_{+}]^
                             \texttt{LaguerreL} \Big[ \texttt{n-l-1}, \texttt{2l+1}, \rho [\texttt{n}, \texttt{r}] \Big] \, \underline{\underbrace{\texttt{sphericalHarmonicY}}_{} \, \Big[ \texttt{1,m} \; , \theta, \phi \Big] 
              \psinlmCart [n_{,1}, n_{,m_{,r}}, x_{,r}, y_{,r}] := \sqrt{\left(\frac{2}{n \, a \, 0}\right)^3 \frac{\left(n-1-1\right)!}{2 \, n \, \left(n+1\right)!}} e^{-\rho [n,r]/2} \rho [n,r]^1
                                   LaguerreL[n-1-1, 21+1, \rho[n, r]]SphericalHarmonicY[1, m, \theta, \phi]/.
                           \left\{r \to \sqrt{x^2 + y^2 + z^2} \text{, } \theta \to \text{ArcCos}\left[\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right], \ \phi \to \text{ArcCos}\left[\frac{x}{\sqrt{x^2 + y^2}}\right]\right\}
               \psi4sNorm [x_, y_, z_] := \psinlm [1, 0, 0, r, \theta, \phi] /. r \rightarrow Sqrt[x^2+y^2+z^2]
              R32[r_] := r^2E^(-r/(3a0));
              Nxy = 1 / (\sqrt{NIntegrate} [\psi xy[x, y, z]^* \psi xy[x, y, z], \{x, -\infty, \infty\}, \{y, -\infty, \infty\}, \{z, -\infty, \infty\}]);
               \psixyNorm [x_, y_, z_] := Nxy\psixy[x, y, z]
               Nxz = 1/(\sqrt{NIntegrate}[\psi xz[x, y, z]^*\psi xz[x, y, z], \{x, -\infty, \infty\}, \{y, -\infty, \infty\}, \{z, -\infty, \infty\}]);
               \psixzNorm [x_, y_, z_] := Nxz\psixz[x, y, z]
               Nyz = 1 / (\sqrt{NIntegrate} [\psi yz[x, y, z]^* \psi yz[x, y, z], \{x, -\infty, \infty\}, \{y, -\infty, \infty\}, \{z, -\infty, \infty\}]);
               \psiyzNorm [x_, y_, z_] := Nyz\psiyz[x, y, z]
               Nz2 = 1/(\sqrt{NIntegrate}[\psi z 2[x, y, z]^*\psi z 2[x, y, z], \{x, -\infty, \infty\}, \{y, -\infty, \infty\}, \{z, -\infty, \infty\}]);
               \psiz2Norm [x_, y_, z_] := Nz2\psiz2[x, y, z]
               \psi x 2 y 2 [x_, y_, z_] := (x^2 - y^2) / r^2 R 3 2 [r] /.r \rightarrow Sqrt[x^2 + y^2 + z^2]
              Nx2y2 =
                            1/\left(\sqrt{\text{NIntegrate}[\psi x 2y 2[x, y, z]^* \psi x 2y 2[x, y, z], \{x, -\infty, \infty\}, \{y, -\infty, \infty\}, \{z, -\infty, \infty\}]\right);
               \psi x 2 y 2 Norm [x_, y_, z_] := N x 2 y 2 \psi x 2 y 2 [x, y, z]
               \psiCentral[x_, y_, z_] := {\psi4sNorm [x, y, z], \psixyNorm [x, y, z],
                            \psi x z N o r m [x, y, z], \psi y z N o r m [x, y, z], \psi z 2 N o r m [x, y, z], \psi x 2 y 2 N o r m [x, y, z]}
              pyrState[x_, y_, z_] := \psi Central[x, y, z][[2;; 6]];
              pyrStateb[rho_, theta_, phi_] := pyrState[x, y, z] /.
                             \{x \to rho Sin[theta] Cos[phi], y \to rho Sin[theta] Sin[phi], z \to rho Cos[theta]\}
```

#### Express crystal field Hamiltonian in basis of Cartesian d orbital states

```
In[37]:= range = ω;
      (*Matrix Elements for Crystal Field Hamiltonian *)
      VCr[i_, j_] := NIntegrate[pyrStateb[rho, theta, phi][[i]] *
              {\tt H1Cr[rho, theta, phi]*pyrStateb[rho, theta, phi][[j]]rho^2Sin[theta],}
            \{\text{rho}, 0, \text{range}\}, \{\text{theta}, 0, \pi\}, \{\text{phi}, 0, 2\pi\}\}
     VFe[i_, j_] := NIntegrate[pyrStateb[rho, theta, phi][[i]] *
              H1Fe[rho, theta, phi]*pyrStateb[rho, theta, phi][[j]]rho^2Sin[theta],
           \{\text{rho}, 0, \text{range}\}, \{\text{theta}, 0, \pi\}, \{\text{phi}, 0, 2\pi\}\}
```

#### Calculate all matrix elements and put them in a table

```
In[43]:= HCFCr = Monitor[Table[VCr[i, j], \{i, 5\}, \{j, 5\}], \{i, j\}];
    HCFFe = Monitor[Table[VFe[i, j], {i, 5}, {j, 5}], {i, j}];
     eigsCr=Chop@Eigensystem @HCFCr;
     eigsFe=Chop@Eigensystem @HCFFe;
```

#### Output the results of crystal field splitting

```
In[47]:= eigsCr[[2]] >> "~/pyramid /CFSResults/eigsCr.txt";
     eigsFe[[2]] >> "~/tetrahedron/CFSResults/eigsFe.txt";
```

#### Definition of unitary matrices from MC

```
| In[40]:= unitary | overlapMatrix_, m_ , n_, θ_ | :=
         unitaryMatrix=IdentityMatrix@Length@overlapMatrix;
         unitaryMatrix[[m , m ]] = Cos[\theta];
         unitaryMatrix[[m , n]] = Sin[\theta];
         unitaryMatrix[[n, m]] = -Sin[\theta];
         unitaryMatrix[[n, n]] = Cos[\theta];
         unitaryMatrix
    unitaryMatricesoverlapMatrix_, changesArrayVar_] :=
         unitary overlapMatrix, changesArrayVar[[#, 1, 1]], changesArrayVar[[#, 1, 2]],
                changesArrayVar[[#, 1, 3]] & /@Range@Length@changesArrayVar;
     constructUtot[overlapMatrix_, changesArrayVar_] :=
         unitaryMatricesList=
           unitary[overlapMatrix, changesArrayVar[[#, 1, 1]], changesArrayVar[[#, 1, 2]],
                   changesArrayVar[[#, 1, 3]]] & /@Range@Length@changesArrayVar;
         Utot = IdentityMatrix Length@overlapMatrix ;
         Do [Utot = unitaryMatricesList[i]].Utot;
           , {i, Length@unitaryMatricesList];
         utot
```

#### Import results from file to define crystal field effect in Hamiltonian

```
In[117]:= CFSUCr = << "~/pyramid /CFSResults/eigsCr.txt";
     CFSUFe = << "~/tetrahedron/CFSResults/eigsFe.txt";</pre>
     pyrData = << "~/pyramid /ChangesArray/changesArray pyramid .txt";</pre>
     tetraData = << "~/tetrahedron/ChangesArray/changesArray tetrahedron.txt";</pre>
     pyrU = constructUtot[IdentityMatrix[5], pyrData];
      tetra\mathcal{U} = construct\mathcal{U}tot | IdentityMatrix\mathbb{Q}5, tetraData | .
             \{\{1,0,0,0,0\},\{0,1,0,0,0\},\{0,0,0,0,1\},\{0,0,0,1,0\},\{0,0,1,0,0\}\};
     UCr = CFSUCr.Transpose@pyrU;
     UFe = CFSUFe.Transpose@tetraU;
      eigvals=<< "~/pyramid /CFSResults/eigvalsCr.txt";
      eigvals2= << "~/tetrahedron/CFSResults/eigvalsFe.txt";
     offDiagonal[matrixTest_ , unitaryU_] :=
        Chop [unitary u]. Diagonal Matrix [matrix Test]. unitary u]
     defineCrystalFields[splitCr_, splitFe_] :=
           splitMatCr = offDiagonal[splitCr(eigvals-Mean@eigvals), UCr];
             \{\{0,0,0,0,1\},\{0,0,0,1,0\},\{0,0,1,0,0\},\{0,1,0,0,0\},\{1,0,0,0,0\}\};
          CFSCr = U2Cr<sup>†</sup>.splitMatCr.U2Cr;
          splitMatFe=offDiagonal[splitFe(eigvals2-Mean@eigvals2), UFe];
             \{\{0, 1, 0, 0, 0\}, \{1, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0\}, \{0, 0, 1, 0, 0\}, \{0, 0, 0, 0, 1\}\};
          CFSFe = U2Fe^{\dagger}.splitMatFe.U2Fe;
In[129]:= defineCrystalFields[1000, 1000]
```

#### Define primitive vectors and reciprocal vectors for k path definition

```
In[61]:= a = 6.0675;
       c = 3.6570;
       primvecs = \left\{a\{1,0,0\}, a\left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right\}, c\{0,0,1\}\right\};
       origin= {0, 0, 0};
       reciprocalVectors[primitiveVectors_] :=
              a1 = primitiveVectors[[1]];
              a2 = primitiveVectors[[2]];
              a3 = primitiveVectors[[3]];
              2\pi\left\{\frac{\text{Cross[a2,a3]}}{\text{a1.Cross[a2,a3]}}, \frac{\text{Cross[a3,a1]}}{\text{a2.Cross[a3,a1]}}, \frac{\text{Cross[a1,a2]}}{\text{a3.Cross[a1,a2]}}\right\}
       recVectors = reciprocalVectors@primvecs;
       point1=.5recVectors[[1]];
       point2={.25Norm @recVectors[[1]], .5Norm @recVectors[[1]], 0};
       nPoints = 100;
       kPath[t_{-}] := Piecewise \left\{ \left\{ point1t, 0 \le t \le 1 \right\}, \left\{ \left( point2 - point1 \right) (t-1) + point1, 1 \le t \le 2 \right\}, \right\}
                  { (origin-point2) (t-2) + (point2), 2 \le t \le 3}, 

{\frac{\pi}{2c} \{0, 0, 1\} (t-3), 3 \le t \le 4\}, \{point1(t-4) + \frac{\pi}{2c} \{0, 0, 1\}, 4 \le t \le 5\},
                  \left\{ \left( point2-point1 \right) (t-5) + point1 + \frac{\pi}{2c} \{0, 0, 1\}, 5 \le t \le 6 \right\}
                  \left\{ \left( \text{origin-point2} \right) (t-6) + \left( \text{point2} \right) + \frac{\pi}{2c} \{0, 0, 1\}, 6 \le t \le 7 \right\} \right\}
       kPoints=kPath[#] &/@Range[0,7,\frac{1}{nPoints}];
```

## Definition of sectors and total Hamiltonian

#### Angles

```
ln[73]:= angle1 = (\pi/180) (180 - 90.0604);
     angle2 = (\pi/180) (180 - 75.4757);
     angle3 = (\pi/180) (180 - 138.6132);
     angle4 = (\pi/180) (180 - 120);
     angle5 = (\pi/180) (180 - 100.9823);
     angle6 = (\pi/180) (180 - 142.9040);
     ang1 = (\pi/180) (180 - 67.7109);
     ang2 = (\pi/180) (180 - 70.925);
     ang3 = (\pi/180) (180 - 134.9172);
     ang4 = (\pi/180) (180 - 71.452);
     ang5 = (\pi/180) (180 - 138.6132);
```

#### Chromium subsector and full sector

```
In[84]:= hamiltonian1 [kx_, ky_, kz_, tz_] := tz Cos[angle4]
                  \left\{\left\{0\,,\,e^{-i\,k\text{vec}\left[kx,ky,kz\right].\left(\text{primvecs}\,\left[\left[1\right]\right]+\text{primvecs}\,\left[\left[2\right]\right]\right)},\,e^{-i\,k\text{vec}\left[kx,ky,kz\right].\text{primvecs}\,\left[\left[1\right]\right]}\right\},\right.\right\}
                       \begin{cases} e^{i \text{ kvec}[kx,ky,kz].(\text{primvecs }[[1]]+\text{primvecs }[[2]])}, 0, e^{i \text{ kvec}[kx,ky,kz].\text{primvecs }[[2]]} \end{cases}, \\ e^{i \text{ kvec}[kx,ky,kz].\text{primvecs }[[1]]}, e^{-i \text{ kvec}[kx,ky,kz].\text{primvecs }[[2]]}, 0 \end{cases} ; 
       hamiltonian2 [kx_, ky_, kz_, tPerp_, tzp_, t2p_] :=
               \left\{\left\{\text{0, tPerpCos}\left[\text{angle2}\right], \text{ tPerpCos}\left[\text{angle2}\right], \text{ tzpCos}\left[\text{angle1}\right]e^{i\,\text{kvec}\left[\text{kx,ky,kz}\right].\text{primvecs}}\right.\right.\right.\right.\right\}
                      t2pCos[angle3] e kvec[kx,ky,kz].primvecs [[3]],
                      t2pCos[angle3] e<sup>i kvec[kx,ky,kz].primvecs</sup> [[3]]}, {tPerpCos[angle2],
                      0, tPerpCos[angle2], t2pCos[angle3] e i kvec[kx,ky,kz].primvecs [[3]],
                      tzpCos[angle1] e kvec[kx,ky,kz].primvecs [[3]],
                      t2pCos[angle3] e<sup>i kvec[kx,ky,kz].primvecs</sup> [[3]]}, {tPerpCos[angle2],
                      tPerpCos[angle2], 0, t2pCos[angle3] e<sup>i kvec[kx,ky,kz].primvecs</sup> [[3]],
                      t2pCos[angle3] e kvec[kx,ky,kz].primvecs [[3]],
                      tzpCos[angle1] e kvec[kx,ky,kz].primvecs [[3]]},
                  \left\{ \texttt{tzpCos} \left[ \underbrace{\texttt{angle1}}_{\texttt{e}^{-\texttt{i}}} \underbrace{\texttt{kvec}[\texttt{kx,ky,kz}].\texttt{primvecs}}_{\texttt{primvecs}} \right. \left[ [3] \right] \right.
                      t2pCos[angle3]e^{-ikvec[kx,ky,kz].primvecs} [[3]]
                      t2pCos[angle3] e-ikvec[kx,ky,kz].primvecs [[3]], 0, tPerpCos[angle2],
                      tPerpCos[angle2]}, {t2pCos[angle3]e<sup>-ikvec[kx,ky,kz].primvecs [[3]]</sup>,
                      tzpCos[angle1] e-ikvec[kx,ky,kz].primvecs [[3]],
                      t2pCos[angle3] e<sup>-ikvec[kx,ky,kz].primvecs</sup> [[3]], tPerpCos[angle2], 0,
                      tPerpCos[angle2]}, {t2pCos[angle3]e-ikvec[kx,ky,kz].primvecs [[3]],
                      t2pCos[angle3]e^{-ikvec[kx,ky,kz].primvecs} [[3]], tzpCos[angle1]
                          e^{-i \, k vec \, [kx, ky, kz] \, .primvecs} \, [[3]], tPerpCos[angle2], tPerpCos[angle2], 0}};
       hamiltonian3 [kx_, ky_, kz_, tPerp_, tzp_, t2p_] :=
               {{0, tPerpCos[angle2]e<sup>-ikvec[kx,ky,kz].primvecs [[1]]</sup>, tPerpCos[angle2]
                          eikvec[kx,ky,kz].primvecs [[2]], tzpCos[angle1]eikvec[kx,ky,kz].primvecs [[3]],
                      t2pCos[angle3] e kvec[kx,ky,kz].(-primvecs [[1]]+primvecs [[3]]),
                      t2pCos[angle3] eikvec[kx,ky,kz].(primvecs [[2]]+primvecs [[3]])},
                  {tPerpCos[angle2]eikvec[kx,ky,kz].primvecs [[1]],0,
```

```
\texttt{tPerpCos[angle2]} \, e^{i \, kvec \, [kx, ky, kz] \, . \, (primvecs \ [[1]] + primvecs \ [[2]])} \, ,
              t2pCos[angle3] e kvec[kx,ky,kz].(primvecs [[1]]+primvecs [[3]]),
              tzpCos[angle1]eikvec[kx,ky,kz].primvecs [[3]],
               t2p \cos \left[ angle 3 \right] e^{i k vec[kx,ky,kz].(primvecs [[1]] + primvecs [[2]] + primvecs [[3]])} \right\}, 
          \{tPerpCos[angle2]e^{-ikvec[kx,ky,kz].primvecs} [[2]],
              \texttt{tPerpCos[angle2]} \, e^{-i \, k vec \, [kx, ky, kz] \, \cdot \, (prim vecs \, [[1]] + prim vecs \, [[2]])} \, ,
              0, t2pCos[angle3]e^{ikvec[kx,ky,kz].(-primvecs [[2]]+primvecs [[3]])},
               t2p \cos \left[ angle 3 \right] e^{i \cdot k vec \left[ kx, ky, kz \right] \cdot \left( -prim vecs \quad \left[ \left[ 1 \right] \right] - prim vecs \quad \left[ \left[ 2 \right] \right] + prim vecs \quad \left[ \left[ 3 \right] \right] \right) }, 
              tzpCos[angle1] eikvec[kx,ky,kz].primvecs [[3]]},
          {tzpCos[angle1]e-ikvec[kx,ky,kz].primvecs [[3]],
              t2p\cos\left[angle3\right]e^{-ikvec\left[kx,ky,kz\right].\left(primvecs\left[[1]\right]+primvecs\left[[3]\right]\right)},
              t2p \cos \left[angle3\right] e^{i k vec[kx,ky,kz].(primvecs [[2]]-primvecs [[3]])},
              0, tPerpCos[angle2]e<sup>-ikvec[kx,ky,kz].primvecs [[1]]</sup>,
              tPerpCos[angle2] e kvec[kx,ky,kz].primvecs [[2]]},
          \label{eq:cos_angle3} \left\{ \texttt{t2pCos} \left[ \texttt{angle3} \right] \, \texttt{e}^{\texttt{i} \, \texttt{kvec} \left[ \texttt{kx}, \texttt{ky}, \texttt{kz} \right] \cdot \left( \texttt{primvecs} \, \, \left[ \left[ 1 \right] \right] - \texttt{primvecs} \, \, \left[ \left[ 3 \right] \right] \right) \right. ,
              tzpCos[angle1] e-ikvec[kx,ky,kz].primvecs [[3]],
               t2p cos[angle3] e^{i kvec[kx,ky,kz].(primvecs [[1]]+primvecs [[2]]-primvecs [[3]])}, 
              tPerpCos[angle2] eikvec[kx,ky,kz].primvecs [[1]], 0,
              tPerpCos[angle2] eikvec[kx,ky,kz].(primvecs [[1]]+primvecs [[2]])},
           \{t2p\cos[angle3]e^{-ikvec[kx,ky,kz].(primvecs [[2]]+primvecs [[3]])},
              tzpCos[angle1] e-ikvec[kx,ky,kz].primvecs [[3]],
              tPerpCos[angle2]e-ikvec[kx,ky,kz].primvecs [[2]],
               \mathsf{tPerpCos} \left[ \mathsf{angle2} \right] e^{-i\,\mathsf{kvec}\left[\mathsf{kx},\mathsf{ky},\mathsf{kz}\right]\,.\, (\mathsf{primvecs} \quad [[1]] + \mathsf{primvecs} \quad [[2]])}\,,\,\,0 \right\} \right\}; 
hamiltonianCrCr [kx_, ky_, kz_, tz_, tPerp_, tzp_, t2p_, CFS_] :=
   ArrayFlatten[{{hamiltonian2 [kx, ky, kz, tPerp, tzp, t2p], ConstantArray[0, {6, 6}],
                  ConstantArray[0, {6, 3}]}, {ConstantArray[0, {6, 6}],
                  hamiltonian3 [kx, ky, kz, tPerp, tzp, t2p], ConstantArray[0, {6, 3}]},
              {ConstantArray[0, {3, 6}], ConstantArray[0, {3, 6}],
                 hamiltonian1 | kx, ky, kz, tz | \} | +
       \label{lem:arrayFlatten} ArrayFlatten[Table[IdentityMatrix[3]CFS[[i,j]], \{i,5\}, \{j,5\}]]
```

Iron sector

```
|n[88]:= hamiltonianFeFeA [kx_, ky_, kz_, tzFe_, tzFep_, tperpFe_] := ArrayFlatten@
           \left\{\left\{\text{ConstantArray[0, \{3,3\}], }\left\{\left\{\text{tzFeCos[angle5]}\,e^{i\,k\text{vec[kx,ky,kz].primvecs}}\right.\right.\right.\right.\right.\right.\right.\right.\right\}
                      tzFepCos[angle6] eikvec[kx,ky,kz].primvecs [[3]],
                      {tzFepCos[angle6] e kvec[kx,ky,kz].primvecs [[3]],
                      tzFeCos[angle5]eikvec[kx,ky,kz].primvecs [[3]],
                      \label{eq:tzfepcos} \left[ \text{angle6} \right] e^{i \, \text{kvec} \left[kx, ky, kz\right] \, . \, \left( \text{primvecs} \quad [[1]] + \text{primvecs} \quad [[2]] + \text{primvecs} \quad [[3]] \right)} \, ,
                      tzFeCos[angle5]e^{ikvec[kx,ky,kz].primvecs}[[3]]}, ConstantArray[0, {3,3}],
                ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}]},
              \{\{\{tzFeCos[angle5]e^{ikvec[kx,ky,kz].primvecs}[[3]],
                         tzFepCos[angle6]e^{ikvec[kx,ky,kz].primvecs} [[3]],
                         tzFepCos[angle6] eikvec[kx,ky,kz].(-primvecs [[1]]-primvecs [[2]]+primvecs [[3]])},
                      {tzFepCos[angle6] e^{i kvec[kx,ky,kz].primvecs [[3]]},
                         tzFeCos[angle5] e<sup>ikvec[kx,ky,kz].primvecs [[3]]</sup>,
                         tzFepCos[angle6]e^{ikvec[kx,ky,kz].(-primvecs [[1]]-primvecs [[2]]+primvecs [[3]])},
                      \label{eq:tzfepcos} \left[ \text{angle6} \right] e^{i \, \text{kvec} \left[kx, ky, kz\right] \, . \, \left( \text{primvecs} \quad [[1]] + \text{primvecs} \quad [[2]] + \text{primvecs} \quad [[3]] \right)} \, ,
                         tzFeCos[angle5] eikvec[kx,ky,kz].primvecs [[3]]}},
                ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}],
                ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}]},
              {ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}], tperpFeCos[angle4]
                    \left\{ \left\{ 0 \text{, } e^{-i \text{kvec}[kx,ky,kz].primvecs [[1]]} \right\} \right\}, 
                      \left\{e^{i \text{kvec}[kx,ky,kz].primvecs} \left[\begin{bmatrix}1\end{bmatrix},0,1\right\},\left\{e^{i \text{kvec}[kx,ky,kz].primvecs} \left[\begin{bmatrix}1\end{bmatrix},1,0\right\}\right\},
                ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}]},
              \{ ConstantArray[0, \{3,3\}], ConstantArray[0, \{3,3\}], ConstantArray[0, \{3,3\}], 
                tperpFeCos[angle4] {{ 0, e^{i \text{ kvec}[kx,ky,kz].primvecs}} [[2]], 1},
                      {e-ikvec[kx,ky,kz].primvecs [[2]], 0, e-ikvec[kx,ky,kz].primvecs [[2]]},
                      {1, e<sup>i kvec[kx,ky,kz].primvecs</sup> [[2]], 0}}, ConstantArray[0, {3, 3}]},
              {ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}],
                ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}]}}
     hamiltonianFeFe [kx_, ky_, kz_, tzFe_, tzFep_, tperpFe_, CFS_] :=
        hamiltonianFeFeA [kx, ky, kz, tzFe, tzFep, tperpFe] +
           ArrayFlatten[Table[IdentityMatrix[3]CFS[[i,j]], {i,5}, {j,5}]]
```

## Chromium-Iron sector defintion

```
In[90]:= hamiltonianCrFe [kx_, ky_, kz_, tCF_, tCFp_] :=
               ArrayFlatten@{{ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}],
                                \left\{ \left\{ \texttt{tCFCos[ang3]} \, e^{i \, k \text{vec}[kx, ky, kz] \, . \, (\text{primvecs [[1]]} + \text{primvecs [[3]]}) \, , \, \text{tCFCos[ang2]} \right. \right. 
                                              e^{i\,k\text{vec}[kx,ky,kz].\text{primvecs}\quad [[3]]}\,,\,\text{tCFCos}[\text{ang1}]\,e^{i\,k\text{vec}[kx,ky,kz].\text{primvecs}\quad [[3]]}\,\big\}\,,
                                    \left\{ \texttt{tCFCos[ang1]} \, e^{i \, k \text{vec}[kx, ky, kz] \, . \, (\text{primvecs} \quad [[1]] + \text{primvecs} \quad [[3]])} \, , \, \, \texttt{tCFCos[ang3]} \right\}
                                              e<sup>i kvec[kx,ky,kz].primvecs [[3]]</sup>, tCFCos[ang2]e<sup>i kvec[kx,ky,kz].primvecs [[3]]</sup>},
                                    \label{eq:cos_ang2} \left\{ \texttt{tCFCos[ang2]} \, e^{i \, (kvec[kx,ky,kz] \, \cdot \, (primvecs \ [[1]] + primvecs \ [[3]]))} \, , \, \texttt{tCFCos[ang1]} \right.
                                              e^{i \text{kvec}[kx,ky,kz].primvecs} [[3]], tCFCos[ang3] e^{i \text{kvec}[kx,ky,kz].primvecs} [[3]]}},
                               ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}]},
                          {ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}],
                               \{\{tCFCos[ang3]e^{ikvec[kx,ky,kz].primvecs}[[1]], tCFCos[ang2], tCFCos[ang1]\},
                                     \left\{ \texttt{tCFCos[ang1]} \, e^{i \, k vec \, [kx, ky, kz] \, .primvecs} \, \, \, ^{[[1]]} \, , \, \, \texttt{tCFCos[ang3]} \, , \, \, \texttt{tCFCos[ang2]} \right\},
                                    {tCFCos[ang2] e ikvec[kx,ky,kz].primvecs [[1]], tCFCos[ang1], tCFCos[ang3]}},
                               ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}]},
                          {ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}],
                               \texttt{ConstantArray[0, \{3,3\}], \{ \texttt{tCFCos[ang3]} \, e^{i \, kvec \, [kx,ky,kz] \, .primvecs \quad [[3]]} \, ,}
                                          tCFCos[ang2] e kvec[kx,ky,kz].(primvecs [[2]]+primvecs [[3]])
                                          tCFCos[ang1] e<sup>i kvec[kx,ky,kz].primvecs [[3]]</sup>},
                                    \left\{ \texttt{tCFCos[ang1]} \, e^{i \, (k \text{vec}[kx, ky, kz] \, . \, (prim \text{vecs} \quad [[1]] + prim \text{vecs} \quad [[3]]))} \, , \right.
                                          tCFCos[ang3] e ikvec[kx,ky,kz].(primvecs [[1]]+primvecs [[2]]+primvecs [[3]]),
                                          tCFCos[ang2] e kvec[kx,ky,kz].(primvecs [[1]]+primvecs [[3]])},
                                    {tCFCos[ang2] e<sup>i (kvec[kx,ky,kz].(-primvecs [[2]]+primvecs [[3]]))</sup>,
                                          tCFCos[ang1] e kvec[kx,ky,kz].primvecs [[3]],
                                          tCFCos[ang3] eikvec[kx,ky,kz].(-primvecs [[2]]+primvecs [[3]])}},
                               ConstantArray[0, {3, 3}]}, {ConstantArray[0, {3, 3}],
                               ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}],
                               \left\{\left\{\,\texttt{tCFCos[ang3], tCFCos[ang2]}\,e^{i\,kvec[kx,ky,kz].primvecs}\,\,^{[[2]]},\,\mathsf{tCFCos[ang1]}\right\},
                                    \{tCFCos[ang1] e^{ikvec[kx,ky,kz].primvecs} [[1]],
                                          tCFCos[ang3] e kvec[kx,ky,kz].(primvecs [[1]]+primvecs [[2]]),
                                          tCFCos[ang2] e ikvec[kx,ky,kz].primvecs [[1]] },
                                    \left\{ \texttt{tCFCos[ang2]} \, e^{-i \, k \text{vec}[kx, ky, kz] \cdot \text{primvecs}} \,\, \text{[[2]]} \,, \, \text{tCFCos[ang1]} \,, \right.
                                          tCFCos[ang3] e<sup>-ikvec[kx,ky,kz].primvecs [[2]]</sup>}},
                               ConstantArray[0, \{3, 3\}], \{\{\{tCFpCos[ang4], tCFpCos[ang4], tCFpC
                                          tCFpCos[ang5]e-ikvec[kx,ky,kz].(primvecs [[1]]+primvecs [[2]])},
                                    \{ \texttt{tCFpCos[ang5]} \, e^{i \, kvec[kx, ky, kz] \, . \, (primvecs \ [[1]] + primvecs \ [[2]])} \, ,
                                          tCFpCos[ang4] e<sup>ikvec[kx,ky,kz].(primvecs [[1]]+primvecs [[2]])</sup>, tCFpCos[ang4]},
                                    \left\{ \texttt{tCFpCos[ang4]} \, e^{i \, kvec \, [kx, ky, kz] \, .primvecs} \, \, \text{[[1]]} \, , \, \texttt{tCFpCos[ang5]} \right.
                                               e<sup>i kvec[kx,ky,kz].primvecs [[1]]</sup>, tCFpCos[ang4]e<sup>-ikvec[kx,ky,kz].primvecs [[2]]</sup>}},
                               \{ \{ tCFpCos[ang4] e^{i kvec[kx,ky,kz].primvecs [[3]]} .
                                          tCFpCos[ang4] e kvec[kx,ky,kz].primvecs [[3]],
                                          \mathsf{tCFpCos[ang5]}\,e^{i\,kvec\,[kx,ky,kz]\,.\,(-primvecs\ [[1]]-primvecs\ [[2]]+primvecs\ [[3]])}\,,
                                    \left\{ \texttt{tCFpCos[ang5]} \, e^{i \, k \text{vec} \left[kx, ky, kz\right] \, . \, \left( \text{primvecs} \quad [[1]] + \text{primvecs} \quad [[2]] + \text{primvecs} \quad [[3]] \right) \right. ,
                                          tCFpCos[ang4]eikvec[kx,ky,kz].(primvecs [[1]]+primvecs [[2]]+primvecs [[3]]),
                                          tCFpCos[ang4]e<sup>ikvec[kx,ky,kz].primvecs [[3]]</sup>},
                                    \label{eq:cos_ang4} \left\{ \texttt{tCFpCos[ang4]} \, e^{i\, kvec[kx,ky,kz] \, . \, (primvecs \ [[1]] + primvecs \ [[3]])} \, , \right.
                                          tCFpCos[ang5]eikvec[kx,ky,kz].(primvecs [[1]]+primvecs [[3]]),
                                          tCFpCos[ang4]e<sup>ikvec[kx,ky,kz].(-primvecs [[2]]+primvecs [[3]])</sup>},
                               ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}], ConstantArray[0, {3, 3}]}
```

#### Total Hamiltonian defintion

```
In[102]:= hamiltonianFinal kx_, ky_, kz_, tz_, tPerp_,
          tzp_, t2p_, tCF_, tCFp_, tzFe_, tzFep_, tperpFe_] :=
       ArrayFlatten@{{hamiltonianCrCr [kx, ky, kz, tz, tPerp, tzp, t2p, CFSCr],
              hamiltonianCrFe [kx, ky, kz, tCF, tCFp]},
            {hamiltonianCrFe [kx, ky, kz, tCF, tCFp],
              hamiltonianFeFe [kx, ky, kz, tzFe, tzFep, tperpFe, CFSFe]}}
```

#### Plot and export band structure

```
In[130]:= nElectrons = 33;
     plotBandStructure[ham_ , title_, tz_,
           tPerp_, tzp_, t2p_, tCF_, tCFp_, tzFe_, tzFep_, tperpFe_| :=
           hams = ham [kPoints[[#, 1]], kPoints[[#, 2]], kPoints[[#, 3]], tz, tPerp, tzp,
                     t2p, tCF, tCFp, tzFe, tzFep, tperpFe &/@Range@Length@kPoints;
           eigsList=Sort/@Chop@Eigenvalues/@hams ;
           sorted = Sort@Flatten[eigsList];
           \mu = \text{sorted}[[Round[(nElectrons/2)Length@eigsList]]};
           data = Transpose [eigsList-\mu];
           eigsys=Eigensystem /@hams ;
           ListPlot data, Joined → True,
             PlotRange \rightarrow \{-10, 10\}, PlotLabel \rightarrow title, AxesLabel \rightarrow \{"", "Energy"\},
             Ticks \rightarrow \left\{ \left\{ \left\{ 0\,,\, "\Gamma" \right\},\, \left\{ nPoints,\, "M" \right\},\, \left\{ 2\,nPoints,\, "K" \right\},\, \left\{ 3\,nPoints,\, "\Gamma" \right\},\, \right\} \right\}
                     {4 nPoints, "A"}, {5 nPoints, "L"}, {6 nPoints, "H"},
                     {7nPoints, "A"}}, All}(*,PlotStyle→Black*)]
     plotMagneticBandStructure[ham_ , title_, tz_, tPerp_, tzp_, t2p_,
           tCF_, tCFp_, tzFe_, tzFep_, tperpFe_, JH_, SzCrvar_, SzFevar_] :=
           hams = ham [1, 1, 1, kPoints[[#, 1]], kPoints[[#, 2]],
                     kPoints[[#, 3]], tz, tPerp, tzp, t2p, tCF, tCFp, tzFe, tzFep,
                     tperpFe, 0, 0.05, JH, SzCrvar, SzFevar & /@Range@Length@kPoints;
           eigsList=Sort/@Chop@Eigenvalues/@hams ;
           sorted = Sort@Flatten | eigsList ;
           \mu = \text{sorted}[[Round[(nElectrons) Length@eigsList]]};
           data = Transpose [eigsList-\mu];
           eigsys=Eigensystem /@hams ;
           ListPlot data, Joined → True,
             PlotRange \rightarrow \{-8, 8\}, PlotLabel \rightarrow title, AxesLabel \rightarrow \{"", "Energy"\},
             Ticks \rightarrow \{\{\{0, "\Gamma"\}, \{nPoints, "M"\}, \{2nPoints, "K"\}, \{3nPoints, "\Gamma"\}, \}\}
                     {4 nPoints, "A"}, {5 nPoints, "L"}, {6 nPoints, "H"}, {7 nPoints, "A"}}, All},
              (*PlotStyle→Black,*)ImageSize → 800]
      exportBandStructure[ham_ , title_, tz_, tPerp_,
           tzp_, t2p_, tCF_, tCFp_, tzFe_, tzFep_, tperpFe_, name_ | :=
           bandStruct = plotBandStructure [ham , title,
                tz, tPerp, tzp, t2p, tCF, tCFp, tzFe, tzFep, tperpFe;
           Export [name , bandStruct]
     blackPlot = plotBandStructure [hamiltonianFinal,
```

In[134]:= blackPlot = plotBandStructure [hamiltonianFinal, "Band structure of optimal overlap hopping model ", 1, 1, 1, 1, 1, 1, 1, 1]

