$$\mathcal{H}_{Cr-Cr} = \begin{pmatrix}
0 & t_{A,Cr}e^{i(k_1-k_2)} & t_{A,Cr}e^{ik_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
t_{A,Cr}e^{i(-k_1+k_2)} & 0 & t_{A,Cr}e^{ik_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
t_{A,Cr}e^{-ik_1} & t_{A,Cr}e^{-ik_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & t_{in,Cr}e^{-ik_2} & t_{in,Cr}e^{-ik_2} & t_{z,Cr}e^{-ik_3} & t_{out,Cr}e^{-i(k_2+k_3)} & t_{out,Cr}e^{-i(k_2+k_3)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & t_{in,Cr}e^{ik_2} & 0 & t_{in,Cr} & t_{out,Cr}e^{i(k_2-k_3)} & t_{z,Cr}e^{-ik_3} & t_{out,Cr}e^{-ik_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & t_{in,Cr}e^{ik_2} & t_{in,Cr} & 0 & t_{out,Cr}e^{i(k_2-k_3)} & t_{out,Cr}e^{-ik_3} & t_{z,Cr}e^{-ik_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & t_{z,Cr}e^{ik_3} & t_{out,Cr}e^{i(-k_2+k_3)} & t_{out,Cr}e^{i(-k_2+k_3)} & 0 & t_{in,Cr}e^{-ik_2} & t_{in,Cr}e^{-ik_2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & t_{out,Cr}e^{i(k_2+k_3)} & t_{z,Cr}e^{ik_3} & t_{out,Cr}e^{ik_3} & t_{in,Cr}e^{ik_2} & 0 & t_{in,Cr} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & t_{out,Cr}e^{i(k_2+k_3)} & t_{out,Cr}e^{ik_3} & t_{z,Cr}e^{ik_3} & t_{in,Cr}e^{ik_2} & t_{in,Cr} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{in,Cr}e^{ik_1} & t_{in,Cr} & t_{z,Cr}e^{-ik_3} & t_{out,Cr}e^{i(k_1-k_3)} & t_{out,Cr}e^{-ik_3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{in,Cr}e^{-ik_1} & 0 & t_{in,Cr}e^{-ik_1} & t_{out,Cr}e^{-i(k_1+k_3)} & t_{z,Cr}e^{-ik_3} & t_{out,Cr}e^{-i(k_1+k_3)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{in,Cr} & t_{in,Cr}e^{ik_1} & 0 & t_{out,Cr}e^{-ik_3} & t_{out,Cr}e^{i(k_1-k_3)} & t_{z,Cr}e^{-ik_3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{z,Cr}e^{ik_3} & t_{out,Cr}e^{i(k_1+k_3)} & t_{out,Cr}e^{ik_3} & 0 & t_{in,Cr}e^{ik_1} & t_{in,Cr} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{out,Cr}e^{i(-k_1+k_3)} & t_{z,Cr}e^{ik_3} & t_{out,Cr}e^{i(-k_1+k_3)} & t_{in,Cr}e^{-ik_1} & 0 & t_{in,Cr}e^{-ik_1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{out,Cr}e^{ik_3} & t_{out,Cr}e^{i(k_1+k_3)} & t_{z,Cr}e^{ik_3} & t_{in,Cr} & t_{in,Cr}e^{ik_1} & 0
\end{pmatrix}$$

$$\psi_{Cr} = \begin{pmatrix}
d_{R_{+,1}} \\ d_{R_{+,2}} \\ d_{R_{+,3}} \\ d_{R_{-,1}} \\ d_{R_{-,2}} \\ d_{R_{-,3}} \\ d_{L_{+,1}} \\ d_{L_{+,2}} \\ d_{L_{+,3}} \\ d_{L_{-,1}} \\ d_{L_{-,2}} \\ d_{L_{-,3}} \\ d_{z,1} \\ d_{z,2} \\ d_{z,3}
\end{pmatrix}$$

$$\mathcal{H}_{Fe-Fe} = \begin{pmatrix}
0 & 0 & 0 & t_{z,Fe}e^{ik_3} & t_{out,Fe}e^{ik_3} & t_{out,Fe}e^{i(-k_1-k_2+k_3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & t_{out,Fe}e^{ik_3} & t_{z,Fe}e^{ik_3} & t_{out,Fe}e^{i(-k_1-k_2+k_3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & t_{out,Fe}e^{i(k_1+k_2+k_3)} & t_{out,Fe}e^{i(k_1+k_2+k_3)} & t_{z,Fe}e^{ik_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
t_{z,Fe}e^{ik_3} & t_{out,Fe}e^{ik_3} & t_{out,Fe}e^{i(-k_1-k_2+k_3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
t_{out,Fe}e^{ik_3} & t_{z,Fe}e^{ik_3} & t_{out,Fe}e^{i(-k_1-k_2+k_3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
t_{out,Fe}e^{i(k_1+k_2+k_3)} & t_{out,Fe}e^{i(k_1+k_2+k_3)} & t_{z,Fe}e^{ik_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{in,Fe}e^{-ik_1} & t_{in,Fe}e^{-ik_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & t_{in,Fe}e^{ik_1} & 0 & t_{in,Fe} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & t_{in,Fe}e^{ik_1} & t_{in,Fe} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{in,Fe}e^{ik_2} & t_{in,Fe} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{in,Fe}e^{-ik_2} & 0 & t_{in,Fe}e^{-ik_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{in,Fe} & t_{in,Fe}e^{ik_2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$\psi_{Fe} = \begin{pmatrix}
d_{z_{+,1}} \\ d_{z_{+,2}} \\ d_{z_{+,3}} \\ d_{z_{-,1}} \\ d_{z_{-,2}} \\ d_{z_{-,3}} \\ d_{R_1} \\ d_{R_2} \\ d_{R_3} \\ d_{L_1} \\ d_{L_2} \\ d_{L_3} \\ d_{\pi,1} \\ d_{\pi,2} \\ d_{\pi,3}
\end{pmatrix}$$

$$\mathcal{H}_{Cr-Fe} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{i(k_1+k_3)} & t'_{Cr-Fe}e^{ik_3} & t'_{Cr-Fe}e^{ik_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{i(k_1+k_3)} & t'_{Cr-Fe}e^{ik_3} & t'_{Cr-Fe}e^{ik_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{i(k_1+k_3)} & t'_{Cr-Fe}e^{ik_3} & t'_{Cr-Fe}e^{ik_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{ik_1} & t'_{Cr-Fe} & t'_{Cr-Fe} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{ik_1} & t'_{Cr-Fe} & t'_{Cr-Fe} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{ik_1} & t'_{Cr-Fe} & t'_{Cr-Fe} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{-ik_1} & t'_{Cr-Fe}e^{-ik_1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{ik_1} & 0 & t'_{Cr-Fe} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{ik_1} & t'_{Cr-Fe} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{ik_2} & t'_{Cr-Fe} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe}e^{-ik_2} & 0 & t'_{Cr-Fe}e^{-ik_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t'_{Cr-Fe} & t'_{Cr-Fe}e^{ik_2} & 0 & 0 & 0 & 0 \\
t_{Cr-Fe} & t_{Cr-Fe} & t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe} & t_{Cr-Fe} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe} & t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe} & t_{Cr-Fe} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe}e^{ik_3} & t_{Cr-Fe} & t_{Cr-Fe} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$\mathcal{H}_{NM} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \bigotimes \mathcal{H}_{Cr-Cr} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \bigotimes \mathcal{H}_{Cr-Fe} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \bigotimes \mathcal{H}^{\dagger}_{Cr-Fe} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bigotimes \mathcal{H}_{Fe-Fe}$$