## SuperChic 2.0

A Monte Carlo for Central Exclusive Production

# Users guide

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#### 1 Overview

SuperChic 2 is a Fortran based Monte Carlo event generator for central exclusive production processes at parton level, for a range of Standard Model states. User—defined histograms may be output, as well as unweighted events in HEPEVT and Les Houches formats. By default the program makes use of the LHAPDF library, but otherwise the code is completely stand—alone. Relevant references are listed in Section 7.

#### 2 Installation

A compressed tar file containing all of the relevant code can be downloaded at http://projects.hepforge.org/superchic/superchic2. To extract it, simply execute tar -xzvf superchicv2.0.tar.gz and the superchicv2.0 directory will be created. This consists of:

- bin: the executables superchic and init and in the input card. input.DAT.
- bin/PDFsets: the directory where the LHAPDF grid files must be placed.
- doc: the source for this document.
- obj: the object files produced by the compiler.
- src: the Fortran source files in various subdirectories.

To compile simply run

#### > make

which will create the superchic and init executables in the bin directory as well as the object files in the obj directory. It is also possible to compile these executables separately with

#### > make superchic

and

## 3 Input parameters and runtime options

After compilation, the superchic and init executables can then be run in the bin directory, using the input.DAT file to adjust the input parameters by

#### > ./init < input.DAT

This creates the hg[intag], screening[intag] and sudakov[intag] .dat input files in the bin/inputs directory, which are used by the main superchic executable, where the [intag] is specified in the input file (see below). This must be run in the first instance, and if any of the first five parameters in the input file, corresponding to the model of soft survival, the c.m.s. energy  $\sqrt{s}$ , the PDF choice and the input [tag], are changed. After this the main MC code may be run with

#### > ./superchic < input.DAT

The adjustable parameters in the input file are described below in order:

- rts: collider energy in GeV.
- isurv: Model of soft survival, defined in [1]: must be an integer from 1 to 4.
- intag: defines the label for the input files created by executing init and read in when executing superchic.
- PDFname, PDFmember: name of PDF set, in the appropriate format for LHAPDF version 5 or 6, and set number. The corresponding grid file must be present in the bin/PDFsets directory. Alternatively, the code is set up for a limited use with the MMHT14 standalone PDF set [2], if the appropriate grid file is present in the bin/PDFsets directory. In this case the PDFINPUT flag in the makefile must be set to USER instead of the default LHAPDF. Other standalone PDF sets may be implemented,

at the user's own risk, by suitable editing of the src/init/PDFuseri.f and src/init/alphasuseri.f files.

- proc: process number, to specify the produced state. These are defined in Section 6.
- outtag: label for file output [outtag].dat, created in bin/outputs directory, and event record evrec [outtag].dat, created in bin/evrecs directory. Contains input parameter and cross section information, as well as histograms (see Section 4).
- sfaci: set to .false. to turn off soft survival effects. Note runtime significantly increased by inclusion of survival effects.
- ncall, itms: number of iterations and calls in VEGAS preconditioning run. Soft survival effects are turned off to speed up the running time. It is recommended to set the number of calls in units of 1000, and to increase the number when applying cuts.
- ncall1, inccall, itend: number of calls in VEGAS main run, and increase per iteration, as well as maximum number of iterations before run will automatically terminate. The run is automatically adjusted so that the number of calls passing cuts is approximately equal to ncall1.
- prec: the required % accuracy by the user; the main run will terminate once this level of accuracy has been reached.
- iseed: random number seed, must be a positive integer.
- s2int: determines precision of pomeron loop momentum integration performed to calculated screened amplitude. While comfortably set to high enough (sub % level) precision for most uses, if restrictive cuts are placed, in particular on the outgoing proton momenta, it is recommended that the user increases the parameter in increments of 10, and confirms that the output is stable within the required precision. If not, it should set at a sufficiently high value that this stability is reached; this will result in a longer runtime.
- genunw, nev, erec: set genunw to .true. to generate unweighted events, and specify required number of events with nev, and event

record format (lhe and hepevt for Les Houches and HEPEVT, respectively) with erec.

- readwt, wtmax: set readwt to .true. to read in maximum weight wtmax input by the user. This should be given by the output 'maximum weight' in output[outtag].dat of a run with genunw=.true., with all other physics inputs left unchanged. Setting readwt to .true. bypasses the maximum weight evaluation stage in the run. This allows for a faster runtime, and is useful when setting multiple runs with genunw=.true. (i.e. each with a different iseed, in order to produce multiple event records in parallel).
- ymin, ymax, mmim, mmax: general cuts on the central system rapidity and invariant mass.
- gencuts: flag to determine if further cuts will be placed.
- spincorr: flag to determine if spin correlations are included in the particle decays. The processes for which these may be included are described in Section 5.
- ptamin, etamin, etamax (...): cuts on two, three and four body final states: see Section 5 for momentum assignments.
- rjet, jalg: jet algorithm parameters: jet radius R and algorithm (antikt, kt or durham) used for 3-jet events.
- m2b, pdgid1, pdgid2:  $\chi_{c,b}$  two-body decay parameters: set mass and PDG numbers for decays, see Section 5.

## 4 Output, histograms and cuts

The input parameters for a run, the output cross section, and the generated histograms are stored in the output[outtag].dat file in the bin/ouputs directory.

By default, the system rapidity and invariant mass (where appropriate) distributions are produced, however the user may define their own histograms in the src/user/histo.f. In addition, further user-defined cuts may be placed in src/user/cuts.f. In both cases the particle momenta are stored

in the array q(i,j), where i=1,2,3 corresponds to the x,y,z components of the 3-momentum and i=4 the energy component, while j specifies the particle number. The outgoing protons correspond to j=3,4 and the central system to j=5, while the remaining final-state particle are defined for each process in Table 1.

## 5 Notes on processes

In this Section further details are given about the generated processes: see Section 6 for a complete list, along with the corresponding momentum labelling. We note that the run–time is typically much shorter for photoproduction and two–photon induced processes, due to the simpler underlying theoretical calculation.

#### 5.1 Process 1 : SM Higgs boson production

The production of SM Higgs boson of mass  $m_H = 126$  GeV, decaying to bb, with mass  $m_b = 4.75$  GeV, with branching ratio  $\text{Br}(H \to b\bar{b}) = 57.7\%$ . Two body cuts may be placed using the input file with b and  $\bar{b}$  momenta given by p(a) and p(b), respectively.

## 5.2 Process 2 : $\gamma \gamma$ production

The production of photon pairs via the  $gg \to \gamma \gamma$  quark-loop induced process. Two body cuts may be placed using the input file on the final-state photon pairs of momenta p(a) and p(b).

#### 5.3 Process 3–5: dijet production

The production of gg, massless  $q\overline{q}$  and  $b\overline{b}$  dijets, with mass  $m_b = 4.75$  GeV, is given by processes 3, 4 and 5, respectively. Two body cuts may be placed using the input file with q(b) and  $\overline{q}(\overline{b})$  momenta given by p(a) and p(b), respectively.

#### 5.4 Processes 6–7: trijet production

The production of ggg and  $q\bar{q}g$  trijets, with massless quarks, is given by processes 6 and 7, respectively. Three body cuts may be placed using the

input file with the q,  $\overline{q}$  and g having momenta given by p(a), p(b) and p(c), respectively.

#### 5.5 Processes 8–16: light meson pair production

The production of light meson pairs, following the formalism described in [3, 4], where further details and definitions can be found. Two body cuts may be placed using the input file with the momenta of positive and negatively charged mesons, and the  $\eta$  and  $\eta'$  meson (for the case of  $\eta\eta'$  production), given by p(a) and p(b), respectively.

#### 5.5.1 Processes 8–9: $\pi\pi$ production

Charged  $\pi^+\pi^-$  and neutral  $\pi^0\pi^0$  production are given by processes 8 and 9, respectively. The 'Chernyak–Zhitnisky' (CZ) wave function for the pion with  $f_{\pi} = 93$  MeV, i.e. with  $a_2^1(\mu_0^2) = 2/3$  and the rest zero.

#### 5.5.2 Processes 10-11: KK production

Charged  $K^+K^-$  and neutral  $K_0K_0$  production are given by processes 8 and 9, respectively. The CZ wave function is again taken for the Kaon, with  $f_K = f_{\pi} = 93$  MeV set in order to approximately account for corrections due to the strange quark mass and deviations from the wave function choice.

#### 5.5.3 Process 12: $\rho_0\rho_0$ production

The CZ wave function is again taken, with  $f_{\rho} = 141$  MeV.

#### 5.5.4 Processes 13–15: $\eta(')\eta(')$ production

 $\eta\eta$ ,  $\eta\eta'$  and  $\eta'\eta'$  production are given by processes 13, 14 and 15, respectively. A general two–angle mixing scheme is taken for the  $\eta$  and  $\eta'$  decay constants, with the values given in [4]. The CZ wave function is taken for the flavour–singlet and octet  $q\bar{q}$  components, and the gluonic component  $a_2^G(\mu_0^2)$  is set to zero by default (although a non–zero contribution will enter at higher scales due to the wave function evolution). A non–zero starting value may be taken by adjust the variable a2g in the appropriate part of scr/main/process.f.

#### 5.5.5 Process 16 : $\phi\phi$ production

The  $\phi$  wave function and decay constant is given as described in Appendix B of [5].

#### 5.6 Processes 17–19: $J/\psi$ and $\psi(2S)$ pair production

 $J/\psi J/\psi$ ,  $J/\psi \psi(2S)$  and  $\psi(2S)\psi(2S)$  production, via the  $\mu^+\mu^-$  decay channel, are given by processes 17, 18 and 19, respectively; see [5] for theory discussion. The branching ratios are given by  ${\rm Br}(J/\psi \to \mu^+\mu^-) = 5.961\%$  and  ${\rm Br}(\psi(2S) \to \mu^+\mu^-) = 0.79\%$ . Spin correlations in these decays may be included with the spincorr flag. Four body cuts on the final–state muons may be placed using the input file, with the momenta  $\mu^+(8)$ ,  $\mu^-(9)$ ,  $\mu^+(10)$  and  $\mu^-(11)$ , as defined in Table 1 given by p(a), p(b), p(c) and p(d), respectively.

## 5.7 Processes 20–22: $\chi_{cJ} \to J/\psi \gamma \to \mu^+ \mu^- \gamma$ production

 $\chi_{c0}$ ,  $\chi_{c1}$  and  $\chi_{c2}$  production are given by processes 20, 21 and 22, respectively. The  $\chi_{cJ} \to J/\psi \gamma$  branching ratios are given by 1.27%, 33.9% and 19.2%, respectively, and Br $(J/\psi \to \mu^+\mu^-)$  = 5.961%. Spin correlations in these decays may be included with the spincorr flag. Three body cuts may be placed using the input file, with the  $\gamma$ ,  $\mu^+$  and  $\mu^-$  momenta given by p(a), p(b) and p(c), respectively.

## 5.8 Processes 23–27: $\chi_{cJ}$ two-body decays

Two-body decays of the  $\chi_{cJ}$  state into arbitrary states are included, where the mass and PDG numbers may be specified in the input file.  $\chi_{c1}$  and  $\chi_{c2}$  decays into a pair of scalar states S are given by processes 24 and 25, respectively, while  $\chi_{c1}$  and  $\chi_{c2}$  decays into a pair of fermion states f are given by processes 26 and 27, respectively. Spin correlations in these decays may be included with the **spincorr** flag. The  $\chi_{c0}$  decay (for which scalar and fermion final-states are treated in the same way) is given by process 23. Two body cuts may be placed using the input file, with the  $S^+$  (f) and  $S^-$  ( $\overline{f}$ ) momenta given by p(a) and p(b), respectively.

#### 5.9 Process 28: $\eta_c$ production

Currently included without any further decay. comment on widths, including for b

## 5.10 Processes 29–31: $\chi_{bJ} \rightarrow \Upsilon \gamma \rightarrow \mu^+ \mu^- \gamma$ production

 $\chi_{b0}$ ,  $\chi_{b1}$  and  $\chi_{b2}$  production are given by processes 29, 30 and 31, respectively. The  $\chi_{bJ} \to \Upsilon \gamma$  branching ratios are given by 1.76%, 33.9% and 19.1%, respectively, and the Br( $\Upsilon \to \mu^+ \mu^-$ ) is given by 2.48%. Spin correlations in these decays may be included with the **spincorr** flag. Three body cuts may be placed using the input file, with the  $\gamma$ ,  $\mu^+$  and  $\mu^-$  momenta given by p(a), p(b) and p(c), respectively.

#### 5.11 Processes 32–36 : $\chi_{bJ}$ two-body decays

Two-body decays of the  $\chi_{bJ}$  state into arbitrary states are included, where the mass and PDG numbers may be specified in the input file.  $\chi_{b1}$  and  $\chi_{b2}$  decays into a pair of scalar states S are given by processes 33 and 34, respectively, while  $\chi_{b1}$  and  $\chi_{b2}$  decays into a pair of fermion states f are given by processes 35 and 36, respectively. Spin correlations in these decays may be included with the **spincorr** flag. The  $\chi_{b0}$  decay (for which scalar and fermion final-states are treated in the same way) is given by process 32. Two body cuts may be placed using the input file, with the  $S^+$  (f) and  $S^-$  ( $\overline{f}$ ) momenta given by p(a) and p(b), respectively.

## 5.12 Process 37: $\eta_b$ production

Currently included without further decay.

## 5.13 Process 38: $J/\psi \to \mu^+\mu^-$ photoproduction

Included with a simple power–law fit to the  $\gamma p \to J/\psi p$  cross section, with the parameters given in [6]. Spin correlations for the  $J/\psi \to \mu^+\mu^-$  decay may be included with the spincorr flag. The branching ratio is given by  ${\rm Br}(J/\psi \to \mu^+\mu^-) = 5.961\%$ . Two body cuts may be placed using the input file, with the  $\mu^+$  and  $\mu^-$  momenta given by p(a) and p(b), respectively.

### 5.14 Process 39: $\Upsilon(1S) \to \mu^+ \mu^-$ photoproduction

Included with a simple power-law fit to the  $\gamma p \to \Upsilon p$  cross section, with the parameters given in [6]. Spin correlations for the  $\Upsilon \to \mu^+\mu^-$  decay may be included with the spincorr flag. The branching ratio is given by  $\text{Br}(J/\psi \to \mu^+\mu^-) = 2.48\%$ . Two body cuts may be placed using the input file, with the  $\mu^+$  and  $\mu^-$  momenta given by p(a) and p(b), respectively.

#### 5.15 Processes 40: $\psi(2S) \to \mu^+ \mu^-$ photoproduction

Included with a simple power–law fit to the  $\gamma p \to \psi(2S)p$  cross section, assumed to have the same energy scaling parameter  $\delta$  as in the case of  $J/\psi$  production, but with  $\sigma(\psi(2S))/\sigma(J/\psi)=16.6\%$ , prior to branching. Spin correlations for the  $\psi(2S)\to \mu^+\mu^-$  decay may be included with the spincorr flag. The branching ratio is given by  ${\rm Br}(\psi(2S)\to\mu^+\mu^-)=0.79\%$ . Two body cuts may be placed using the input file, with the  $\mu^+$  and  $\mu^-$  momenta given by  ${\bf p}({\bf a})$  and  ${\bf p}({\bf b})$ , respectively.

# 5.16 Processes 41: $\psi(2S) \to J/\psi(\to \mu^+\mu^-)\pi^+\pi^-$ photoproduction

Included as described above. Decay performed according to phase space, without any spin correlations, and with  ${\rm Br}(\psi(2S)=J/\psi\pi^+\pi^-)=34.45\%$  and  ${\rm Br}(J/\psi\to\mu^+\mu^-)=5.961\%$ . Four body cuts on the final–state muons may be placed using the input file, with the momenta  $\pi^+(7)$ ,  $\pi^-(8)$ , $\mu^+(9)$  and  $\mu^-(10)$ , as defined in Table 1 given by p(a), p(b), p(c) and p(d), respectively.

## 5.17 Processes 42–43: $W^+W^-$ production (pp)

Two-photon induced W boson pair production. Process 41 and 42 correspond to the muon and electron decay channels respectively, with  $\text{Br}(W \to \nu \mu) = 10.63\%$  and  $\text{Br}(W \to \nu e) = 10.71\%$ . Spin correlations may be included with the spincorr flag. Four body cuts on the final-state muons may be placed using the input file, with the momenta  $\nu_{\mu}(8)$ ,  $\mu^{+}(9)$ ,  $\overline{\nu}_{\mu}(10)$  and  $\mu^{-}(11)$ , as defined in Table 1 given by p(a), p(b), p(c) and p(d), respectively, and similarly for the election decay channel.

## 5.18 Processes 44–46: $l^+l^-$ production (pp)

Two-photon induced lepton pair production. Processes 44 and 45 correspond to  $e^+e^-$ ,  $\mu^+\mu^-$  and  $\tau^+\tau^-$  production. Two body cuts may be placed using the input file, with the  $l^+$  and  $l^-$  momenta given by p(a) and p(b), respectively.

#### 5.19 Processes 48–52: $e^+e^-$ initial state

Two-photon processes as above, but with initial-state  $e^+e^-$ .

## 6 Summary of processes

The generated process are summarised in Table 1.

Table 1: Generated processes, with momenta numbering specified by p(j), as stored in array q(i, j).

QCD-induced (Durham model)				
Number	Final-State			
1	$H(5) \rightarrow b(6) + \overline{b}(6)$			
2	$\gamma(6) + \gamma(7)$			
3	g(6) + g(7)			
4	$q(6) + \overline{q}(7)$			
5	$b(6) + \bar{b}(7)$			
6	g(6) + g(7) + g(8)			
7	$q(6) + \overline{q}(7) + g(8)$			
8	$\pi^+(6) + \pi^-(7)$			
9	$\pi^0(6) + \pi^0(7)$			
10	$K^{+}(6) + K^{-}(7)$			
11	$K_0(6) + K_0(7)$			
12	$\rho_0(6) + \rho_0(7)$			
13	$\eta(6) + \eta(7)$			
14	$\eta(6) + \eta'(7)$			
15	$\eta'(6) + \eta'(7)$			
16	$\phi(6) + \phi(7)$			
17	$J/\psi(6)(\to \mu^+(8) + \mu^-(9)) + J/\psi(7)(\to \mu^+(10) + \mu^-(11))$			
18	$J/\psi(6)(\to \mu^+(8) + \mu^-(9)) + \psi_{2S}(7)(\to \mu^+(10) + \mu^-(11))$			

Table 1: (continued)

```
\psi_{2S}(6)(\rightarrow \mu^{+}(8) + \mu^{-}(9)) + \psi_{2S}(7)(\rightarrow \mu^{+}(10) + \mu^{-}(11))
19
20
            \chi_{c0}(5) \to \gamma(6) + J/\psi(7) (\to \mu^{+}(8) + \mu^{-}(9))
            \chi_{c1}(5) \to \gamma(6) + J/\psi(7) (\to \mu^{+}(8) + \mu^{-}(9))
21
            \chi_{c2}(5) \to \gamma(6) + J/\psi(7) (\to \mu^{+}(8) + \mu^{-}(9))
22
23
            \chi_{c0}(5) \to S(6) + S(7)
24
            \chi_{c1}(5) \to S(6) + S(7)
25
            \chi_{c2}(5) \to S(6) + S(7)
26
            \chi_{c1}(5) \rightarrow f(6) + \overline{f}(7)
27
            \chi_{c2}(5) \rightarrow f(6) + \overline{f}(7)
28
            \eta_c(5)
            \chi_{b0}(5) \to \gamma(6) + \Upsilon_{1S}(7)(\to \mu^{+}(8) + \mu^{-}(9))
29
            \chi_{b1}(5) \to \gamma(6) + \Upsilon_{1S}(7)(\to \mu^{+}(8) + \mu^{-}(9))
30
            \chi_{b2}(5) \to \gamma(6) + \Upsilon_{1S}(7)(\to \mu^{+}(8) + \mu^{-}(9))
31
32
            \chi_{b0}(5) \to S(6) + S(7)
33
            \chi_{b1}(5) \to S(6) + S(7)
34
            \chi_{b2}(5) \to S(6) + S(7)
35
            \chi_{b1}(5) \rightarrow f(6) + \overline{f}(7)
           \chi_{b2}(5) \to f(6) + \overline{f}(7)
36
37
            \eta_b(5)
```

#### Photoproduction

```
38 J/\psi(5) \to \mu^{+}(6)\mu^{-}(7)

39 \Upsilon_{1S}(5) \to \mu^{+}(6)\mu^{-}(7)

40 \psi_{2S}(5) \to \mu^{+}(6)\mu^{-}(7)

41 \psi_{2S}(5) \to J/\psi(6)(\to \mu^{+}(9) + \mu^{-}(10)) + \pi^{+}(7) + \pi^{-}(8)
```

#### Two-photon collisions

pp				
42	$W^{+}(\to \nu_{\mu}(8) + \mu^{+}(9)) + W^{-}(\to \overline{\nu}_{\mu}(10) + \mu^{-}(11))$			
43	$W^{+}(\to \nu_e(8) + e^{+}(9)) + W^{-}(\to \overline{\nu}_e(10) + e^{-}(11))$			
44	$e^+(6) + e^-(7)$			
45	$\mu^{+}(6) + \mu^{-}(7)$			
46	$\tau^{+}(6) + \tau^{-}(7)$			
47	Empty			
$e^+e^-$				
48	$W^{+}(\rightarrow \nu_{\mu}(8) + \mu^{+}(9)) + W^{-}(\rightarrow \overline{\nu}_{\mu}(10) + \mu^{-}(11))$			
49	$W^{+}(\rightarrow \nu_{e}(8) + e^{+}(9)) + W^{-}(\rightarrow \overline{\nu}_{e}(10) + e^{-}(11))$			
50	$e^{+}(6) + e^{-}(7)$			

#### Table 1: (continued)

51	$\mu^+(6) + \mu^-(7)$
52	$ au^+(6) +  au^-(7)$
53	Empty

## 7 References

Physics details and selected results for jet and  $\chi_{c,b}$  production as well photoproduction and two-photon induced processes from SuperChic 2 may be found in [6], while a summary of the Durham model can be found in e.g. [7–9]. Further references for other specific processes are listed below:

- SM Higgs boson: [10].
- $\gamma \gamma$ : [11, 12].
- heavy  $\chi_{c,b}$ ,  $\eta_{c,b}$ , quarkonia: [11, 13, 14].
- Light meson pairs: [3, 4, 15].
- $J/\psi$  and  $\psi(2S)$  pair production: [5].

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