

# Posterior Predictive Distribution

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## Posterior Predictive (Formal Derivation)

**Definition 1: Conditional Distribution**

$$p(Y_1, Y_2, Y_3 | X_1, X_2, X_3) = \frac{p(Y_1, Y_2, Y_3, X_1, X_2, X_3)}{p(X_1, X_2, X_3)}$$

**Definition 2: Marginal Distribution**

$$p(Y_1, Y_2) = \int p(Y_1, Y_2, Y_3) dY_3$$

**Derivation of Posterior Predictive Distribution**  $P(Y|D)$

$$\begin{aligned} P(Y|D) &= \frac{P(Y, D)}{P(D)} \quad (\text{by definition}) \\ \Rightarrow P(Y|D) &= \int \frac{P(Y, D, \theta)}{P(D)} d\theta \quad (\text{marginalization over } \theta) \\ &= \int P(Y|D, \theta) \frac{P(D|\theta)P(\theta)}{P(D)} d\theta \quad (\text{chain rule: } P(Y, D, \theta) = P(Y|D, \theta)P(D|\theta)P(\theta)) \\ &= \int P(Y|D, \theta)P(\theta|D) d\theta \quad (\text{Bayes' rule for } P(\theta|D)) \\ &\quad (\text{Assuming all data relevant to } Y \text{ is captured by } \theta, P(Y|D, \theta) = P(Y|\theta)) \\ P(Y|D) &= \int P(Y|\theta)P(\theta|D) d\theta \quad (\text{posterior predictive distribution}) \end{aligned}$$