Homework 3

Bayesian Network

Due: 11:59 pm, April 4

Exercise 1

Calculate the value of the queries, given the joint distribution below. (10 points)

A	B	C	P(A, B, C)
+a	+b	+c	0.30
+a	+b	-c	0.04
+a	-b	+c	0.10
+a	-b	-c	0.06
-a	+b	+c	0.11
-a	+b	-c	0.05
-a	-b	+c	0.15
-a	-b	-c	0.19

Q1.1 P(+c)

Q1.2 P(+c | -a)

Q1.3 P(+c | -a, +b)

Exercise 2

Draw three nodes of a Bayes' net with the following configurations. Shade the nodes corresponding to observed random variables in your answer. (20 points)

Q2.1. Active Causal Chain

Q2.2. Active Common Cause

Q2.3. Active Common Effect

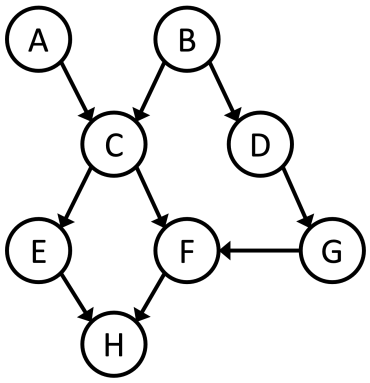
Q2.4. Inactive Causal Chain

Q2.5. Inactive Common Cause

Q2.6. Inactive Common Effect

Exercise 3

Given a Bayesian network below, write **true, false, or unknown** for the following statements. (20 points)



Q3.1. A and B are independent given C.

Q3.2. A and H are independent given F.

Q3.3. A and H are guaranteed to be independent given E.

Q3.4. E and F are guaranteed to be independent given H.

Q3.5. E and F are independent given C.

Q3.6. E and F are guaranteed to be independent given C and D.

Q3.7. A and F are independent given C and H.

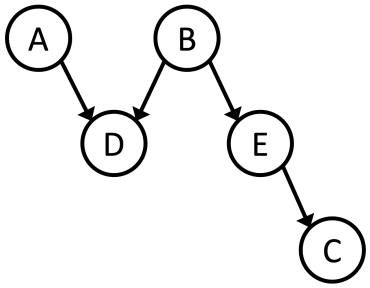
Q3.8. A and F are independent given C and D.

Q3.9. A and F are guaranteed to be independent given C and G.

Q3.10. A and F are guaranteed to be independent given C.

Exercise 4

Given a Bayesian network below, answer the following questions. (20 points)



Q4.1. Complete the equation below for calculating the joint probability distribution using the conditional probability tables of each variable.

$$P(A, B, C, D, E) = \_\_\_\_\_$$

[Variable elimination]

Complete the following steps of variable elimination to get $P(D|A = a, E = e)$. Assume that all variables are discrete. (Hint: slide 99, "Another Variable Elimination Example", of week 6)

Q4.2. Step 1) Initial factors: _____

Q4.3. Step 2) Eliminate B, this introduces the factor _____, and we are left with: _____

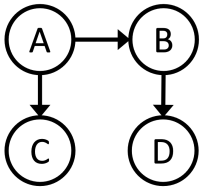
Q4.4. Step 3) Eliminate C: this introduces the factor _____, and we are left with: _____

Q4.5. Step 4) Join the remaining factors to get $f3(D, a, e)$: _____

Q4.6. Step 5) Normalize over D to get: _____

Exercise 5

Given a Bayesian network below, assume you got the following six samples using **Likelihood Weighting** with the evidence $C = +c$ and $D = -d$. (30 points)



A	P(A)
+a	0.6
-a	0.4

A	C	P(C|A)
+a	+c	0.9
+a	-c	0.1
-a	+c	0.2
-a	-c	0.8

B	D	P(D|B)
+b	+d	0.9
+b	-d	0.1
-b	+d	0.2
-b	-d	0.8

A	B	P(B|A)
+a	+b	0.7
+a	-b	0.3
-a	+b	0.5
-a	-b	0.5

A	B	C	D
+a	-b	+c	-d
-a	-b	+c	-d
+a	-b	+c	-d
+a	+b	+c	-d
+a	+b	+c	-d
-a	+b	+c	-d

Q5.1. Complete the following table using the six samples.

A	B	C	D	*Count*/N	Weight	Joint
+a	+b	+c	-d	2/6	0.09	0.03
+a	-b	+c	-d			
-a	+b	+c	-d			
-a	-b	+c	-d			

Q5.2. Based on the table of Q5.1, estimate $P(B = -b, C = +c, D = -d)$. Provide all steps to estimate it from the table.

Q5.3. Based on the table of Q5.1, estimate $P(B = -b | C = +c, D = -d)$. Provide all steps to estimate it from the table.