Homework 2

Logic

Due: 11:59 pm, March 7

Exercise 1

For each question (Q1-4), show whether a knowledge base (KB) entails or does not entail a sentence (S) by enumerating the truth values. (10 points)

Q1.1. KB: $A \vee B$, S: A

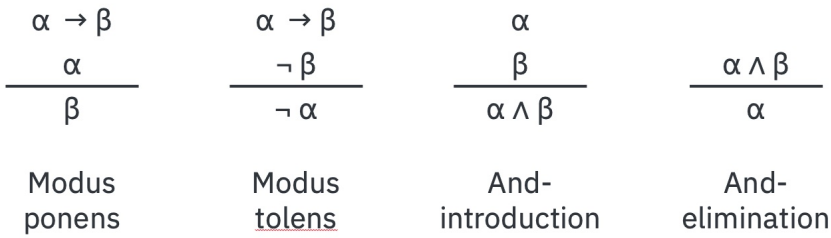
Q1.2. KB: $A \wedge B$, S: A

Q1.3. KB: $\neg A \implies (B \vee C)$, S: $A \vee C$

Q1.4. KB: $(A \wedge B) \implies C$, S: $\neg A \vee \neg B \vee C$

Exercise 2

Using the following inference rules and natural deduction, prove whether a knowledge base (KB) entails or does not entail a sentence (S). Write your answer in a table format with step, formula, and derivation columns, as shown in slide 82 of the fourth-week class. (10 points)



Q2.1.

KB:

- $A \land B$
- $B \implies \neg C$
- $\neg D \implies C$

S:

D

Q2.2.

KB:

- $A \land B$
- $(B \lor C) \implies D$
- $D \implies C$

S:

$C \land D$

Q2.3.

KB:

- $(A \implies B) \implies C$
- $B \implies D$
- $(C \lor D) \implies A$
- $B \land C$

S:

$A \land D$

Exercise 3

Using resolution refutation, prove that a knowledge base (KB) entails a sentence (S). Write your answer in a table format with step, formula, and derivation columns, as shown in slide 103 of the fourth-week class. (20 points)

Q3.1.

KB:

- $A \vee B$
- $A \implies C$
- $(B \vee C) \implies D$

S:

D

Q3.2.

KB:

- $A \vee B$
- $A \implies C$
- $B \implies D$

S:

$C \vee D$

Q3.3.

KB:

- $A \wedge B$
- $B \wedge C \iff D$
- $A \implies C$

S:

D

Exercise 4

Convert the following natural-language sentences to first-order logic sentences. Specify the definition of your predicates and functions as shown in the following example. (23 points)

Example

Everyone loves Mary.

Answer:

$\forall x.Love(x, Mary)$

- Love(x, y): x loves y.

Q4.1. All students who study at Stevens Institute of Technology are smart.

Q4.2. Some people who are good at math like chess.

Q4.3. If all employees work hard, the company will make more profit this year.

Q4.4. There is a singer who everyone loves.

Q4.5. The singer loves everyone.

Exercise 5

Check whether there is the most general unifier (MGU) of two clauses ω_1 and ω_2. Write MGU if the clauses have it. (20 points)

Q5.1. ω_1: A(B, C), ω_2: A(x, y)

Q5.2. ω_1: A(x, f(D, x)), ω_2: A(E, f(D, y))

Q5.3. ω_1: A(x, y), ω_2: A(f(C, y), z)

Q5.4. ω_1: P(A, x, f(g(y))), ω_2: P(y, f(z), f(z))

Q5.5. ω_1: P(x, g(f(A)), f(x)), ω_2: P(f(y), z, y)

Q5.6. ω_1: P(x, f(y)), ω_2: P(z, g(w))

Exercise 6

Using resolution refutation, prove that a knowledge base (KB) entails a sentence (S). (17 points)

KB:

- $Mother(Jane, Emma)$
- $Alive(Jane)$
- $\forall x, y. Mother(x, y) \implies Parent(x, y)$
- $\forall x, y. Parent(x, y) \land Alive(x) \implies Older(x, y)$

S:

$Older(Jane, Emma)$

Q6.1 Write KB and S in the clausal form.

Q6.2 Derive S from KB using the resolution rule. Write your answer in a table format shown in slide 72 of the fifth-week lecture. (specify the unifier in each row).