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Homework 3

Bayesian Network

Due: 11:59 pm, April 4

Exercise 1

Calculate the value of the queries, given the joint distribution below. (10 points)

A	B	C	P(A, B, C)
+a	+b	+c	0.30
+a	+b	-c	0.04
+a	-b	+c	0.10
+a	-b	-c	0.06
-a	+b	+c	0.11
-a	+b	-c	0.05
-a	-b	+c	0.15
-a	-b	-c	0.19

Q1.1 P(+c)

Q1.2 P(+c | -a)

Q1.3 P(+c | -a, +b)

(1.1) $P(+c) = 0.3 + 0.1 + 0.11 + 0.15$
$= 0.66$

(1.2) $P(+c|-a) = (0.11 + 0.15)/(0.11 + 0.05 + 0.15 + 0.19)$
$= 0.52$

(1.3) $P(+c|-a,+b) = P(+c|+b)$
$= (0.3 + 0.11)/(0.3 + 0.04 + 0.11 + 0.05)$
$= 0.82$

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Exercise 2

Draw three nodes of a Bayes' net with the following configurations. Shade the nodes corresponding to observed random variables in your answer. (20 points)

Q2.1. Active Causal Chain

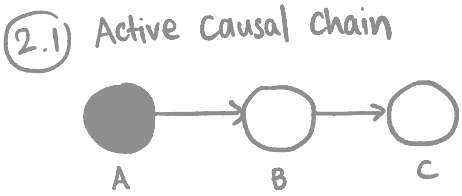
Q2.2. Active Common Cause

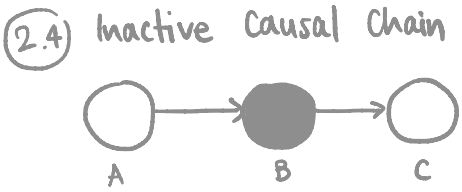
Q2.3. Active Common Effect

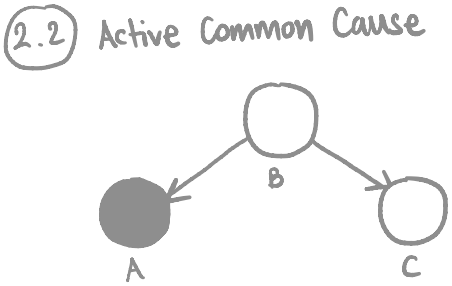
Q2.4. Inactive Causal Chain

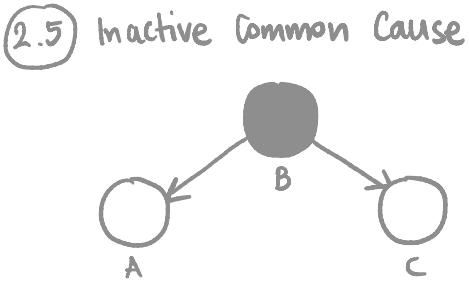
Q2.5. Inactive Common Cause

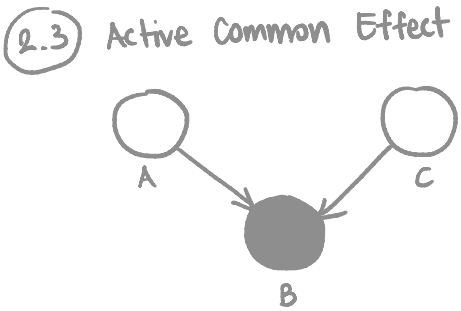
Q2.6. Inactive Common Effect

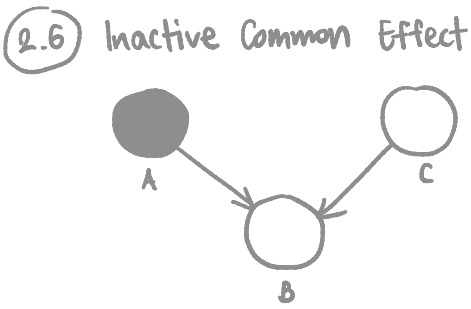












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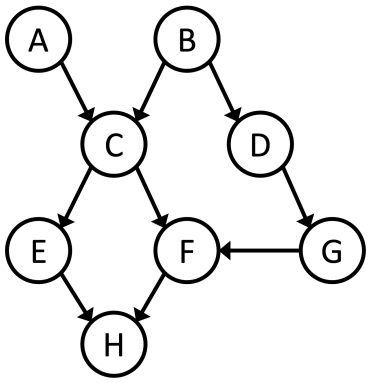
Exercise 3

Given a Bayesian network below, write **true, false, or unknown** for the following statements. (20 points)



Q3.1. A and B are independent given C. unknown

Q3.2. A and H are independent given F. unknown

Q3.3. A and H are guaranteed to be independent given E. false

Q3.4. E and F are guaranteed to be independent given H. false

Q3.5. E and F are independent given C. true

Q3.6. E and F are guaranteed to be independent given C and D. true

Q3.7. A and F are independent given C and H. unknown

Q3.8. A and F are independent given C and D. true

Q3.9. A and F are guaranteed to be independent given C and G. true

Q3.10. A and F are guaranteed to be independent given C. false

3.1 $A \perp\!\!\!\perp B \mid C$

A → C ← B active

Inactive A → C → F ← G ← D ← B

Inactive A → C → E → H ← F ← G ← D ← B

3.2 $A \perp\!\!\!\perp H \mid F$

A → C → E → H active

A → C → F → H inactive

A → C ← D → D → G → F → H

Inactive Inactive

3.3 $A \perp\!\!\!\perp H \mid E$

A → C → E → H inactive

A → C → F → H active

A → C ← B → D → G → F → H

Inactive

3.4 $E \perp\!\!\!\perp F \mid H$

E → H ← F active

E ← C → F active

E ← C ← B → D → G → F active

3.5 $E \perp\!\!\!\perp F \mid C$

E ← C → F Inactive

E → H ← F Inactive

E ← C ← B → D → G → F

Inactive

3.6 $E \perp\!\!\!\perp F \mid C, D$

E → H ← F Inactive

E ← C → F Inactive

E ← C ← B → D → G → F

Inactive Inactive

3.7 $A \perp\!\!\!\perp F \mid C, H$

A → C → F Inactive

A → C → E → H ← F Inactive

A → C ← B → D → G → F active

3.8 $A \perp\!\!\!\perp F \mid C, D$

A → C → F Inactive

A → C → E → H ← F Inactive

A → C ← B → D → G → F

Inactive

3.9 $A \perp\!\!\!\perp F \mid C, G$

A → C → F Inactive

A → C → E → H ← F Inactive

A → C ← B → D → G → F

Inactive

3.10 $A \perp\!\!\!\perp F \mid C$

A → C → F Inactive

A → C → E → H ← F Inactive

A → C ← B → D → G → F active

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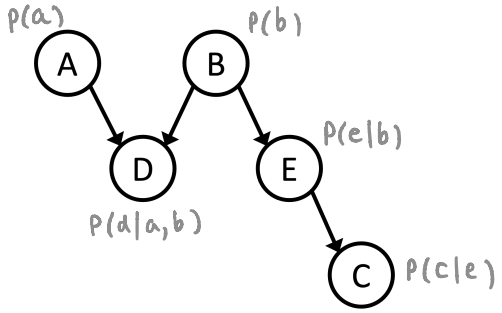
Exercise 4

Given a Bayesian network below, answer the following questions. (20 points)



Q4.1. Complete the equation below for calculating the joint probability distribution using the conditional probability tables of each variable.

$$P(A, B, C, D, E) = \underline{P(D)\,P(B)\,P(D|A,B)\,P(E|B)\,P(C|E)}$$

[Variable elimination]

Complete the following steps of variable elimination to get $P(D|A = a, E = e)$. Assume that all variables are discrete. (Hint: slide 99, "Another Variable Elimination Example", of week 6)

Q4.2. Step 1) Initial factors: $P(a)\,P(B)\,P(D|a,B)\,P(e|B)\,P(C|e)$

Q4.3. Step 2) Eliminate B, this introduces the factor $f_1(D,e) = \Sigma_b P(B)P(D|a,B)P(e|B)$, and we are left with: $P(a)\,f_1(D,e)\,P(C|e)$

Q4.4. Step 3) Eliminate C: this introduces the factor $f_2(e) = \Sigma_c P(C|e)$, and we are left with: $P(a), f_1(D,e), f_2(e)$

Q4.5. Step 4) Join the remaining factors to get $f3(D, a, e)$: $f_3(D,a,e) = P(a)\,f_1(D,e)\,f_2(e)$

Q4.6. Step 5) Normalize over D to get: $P(D|a,e)$

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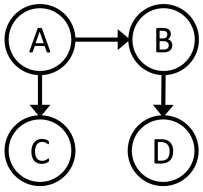
Exercise 5

Given a Bayesian network below, assume you got the following six samples using **Likelihood Weighting** with the evidence $C = +c$ and $D = -d$. (30 points)



A	P(A)
+a	0.6
-a	0.4

A	C	P(C|A)
+a	+c	0.9
+a	-c	0.1
-a	+c	0.2
-a	-c	0.8

B	D	P(D|B)
+b	+d	0.9
+b	-d	0.1
-b	+d	0.2
-b	-d	0.8

A	B	P(B|A)
+a	+b	0.7
+a	-b	0.3
-a	+b	0.5
-a	-b	0.5

A	B	C	D
+a	-b	+c	-d
-a	-b	+c	-d
+a	-b	+c	-d
+a	+b	+c	-d
+a	+b	+c	-d
-a	+b	+c	-d

Q5.1. Complete the following table using the six samples.

A	B	C	D	*Count*/*N*	Weight	Joint
+a	+b	+c	-d	2/6	0.09	0.03
+a	-b	+c	-d	2/6	0.72	0.24
-a	+b	+c	-d	1/6	0.02	0.0033
-a	-b	+c	-d	1/6	0.16	0.0267

Q5.2. Based on the table of Q5.1, estimate $P(B = -b, C = +c, D = -d)$. Provide all steps to estimate it from the table.

Q5.3. Based on the table of Q5.1, estimate $P(B = -b | C = +c, D = -d)$. Provide all steps to estimate it from the table.

(5.2)
$$P(-b,+c,-d) = w(+a,-b,+c,-d)\cdot P(+a,-b,+c,-d) + w(-a,-b,+c,-d)\cdot P(-a,-b,+c,-d)$$
$$= P(-d|-b)P(+c|+a)\cdot P(+a,-b,+c,-d) + P(-d|-b)P(+c|-a)\cdot P(-a,-b,+c,-d)$$
$$= (0.8)(0.9)(2/6) + (0.8)(0.2)(1/6)$$
$$= 0.267$$

(5.3)
$$P(+c,-d) = w(+a,+b,+c,-d)\cdot P(+a,+b,+c,-d) + w(+a,-b,+c,-d)\cdot P(+a,-b,+c,-d)$$
$$+ w(-a,+b,+c,-d)\cdot P(-a,+b,+c,-d) + w(-a,-b,+c,-d)\cdot P(-a,-b,+c,-d)$$
$$= (0.09)(2/6) + (0.72)(2/6) + (0.02)(1/6) + (0.16)(1/6)$$
$$= 0.3$$

$$P(-b|+c,-d) = P(-b,+c,-d)\,/\,P(+c,-d)$$
$$= 0.267\,/\,0.3$$
$$= 0.89$$