

Homework 2

Logic

Due: 11:59 pm, March 7

Exercise 1

For each question (Q1-4), show whether a knowledge base (KB) entails or does not entail a sentence (S) by enumerating the truth values. (10 points)

Q1.1. KB: $A \vee B$, S: A KB entails a sentence S iff every interpretation

that makes KB true also makes S true.

Q1.3. KB: $\neg A \implies (B \vee C)$, S: $A \vee C$

Q1.4. KB: $(A \wedge B) \implies C$, S: $\neg A \vee \neg B \vee C$

Solution

Q1.1

A	B	$A \vee B$	S
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

KB doesn't entail a sentence S

Q1.3

A	B	C	$\neg A$	$B \vee C$	$\neg A \rightarrow (B \vee C)$	S
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	F
F	F	T	T	T	T	T
F	F	F	T	F	F	F

KB doesn't entail a sentence S

Q1.2

A	B	$A \wedge B$	S
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

KB entails a sentence S

Q1.4

A	B	C	$\neg A$	$\neg B$	$A \wedge B$	KB: $(A \wedge B) \rightarrow C$	S: $\neg A \vee \neg B \vee C$
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

KB entails a sentence S

Exercise 2

Using the following inference rules and natural deduction, prove whether a knowledge base (KB) entails or does not entail a sentence (S). Write your answer in a table format with step, formula, and derivation columns, as shown in slide 82 of the fourth-week class. (10 points)

$$\frac{\alpha \rightarrow \beta}{\frac{\alpha}{\beta}}$$

$$\frac{\alpha \rightarrow \beta}{\frac{\neg \beta}{\neg \alpha}}$$

$$\frac{\alpha}{\frac{\beta}{\alpha \wedge \beta}}$$

$$\frac{\alpha \wedge \beta}{\alpha}$$

Modus
ponensModus
tolensAnd-
introductionAnd-
elimination

Q2.1.

KB:

- $A \wedge B$
- $B \implies \neg C$
- $\neg D \implies C$

S:

D

KB entails a sentence S

step	Formula	Derivation
1	$A \wedge B$	Given
2	$B \implies \neg C$	Given
3	$\neg D \implies C$	Given
4	A	1 And-elimination
5	B	1 And-elimination
6	$\neg C$	2,5 Modus ponens
7	D	3,6 Modus tolens

Q2.2.

KB:

- $A \wedge B$
- $(B \vee C) \implies D$
- $D \implies C$

S:

 $C \wedge D$

KB entails a sentence S

1	$A \wedge B$	Given
2	$(B \vee C) \implies D$	Given
3	$D \implies C$	Given
4	B	1 And-elimination
5	$B \vee C$	4 Or-introduction
6	D	2,5 Modus ponens
7	C	3,6 Modus ponens
8	$C \wedge D$	7,6 And-introduction

Q2.3.

KB:

- $(A \implies B) \implies C$
- $B \implies D$
- $(C \vee D) \implies A$
- $B \wedge C$

S:

 $A \wedge D$

KB entails a sentence S

1	$(A \implies B) \implies C$	Given
2	$B \implies D$	Given
3	$(C \vee D) \implies A$	Given
4	$B \wedge C$	Given
5	B	4 And-elimination
6	C	4 And-elimination
7	D	2,5 Modus ponens
8	$C \vee D$	7 Or-introduction
9	A	3,8 Modus ponens
10	$A \wedge D$	9,7 And-introduction

Exercise 3

Using resolution refutation, prove that a knowledge base (KB) entails a sentence (S). Write your answer in a table format with step, formula, and derivation columns, as shown in slide 103 of the fourth-week class. (20 points)

	Step	Formula	Derivation
Q3.1.	1	$A \vee B$	Given
KB:	2	$\neg A \vee C$	Given
<ul style="list-style-type: none"> $A \vee B$ $A \implies C$ $(B \vee C) \implies D$ 	3	$\neg B \vee D$	Given
	4	$\neg C \vee D$	Given
	5	$\neg D$	Negated conclusion
	6	$\neg B$	3, 5
	7	A	1, 6
	8	C	2, 7
	9	D	4, 8
$\text{KB entails a sentence } S$			
Q3.2.	1	$A \vee B$	Given
KB:	2	$\neg A \vee C$	Given
<ul style="list-style-type: none"> $A \vee B$ $A \implies C \quad \neg A \vee C$ $B \implies D \quad \neg B \vee D$ 	3	$\neg B \vee D$	Given
	4	$\neg C \wedge \neg D$	Negated conclusion
	5	$B \vee C$	1, 2
	6	$A \vee D$	1, 3
	7	$C \vee D$	2, 6
$\text{KB entails a sentence } S$			
Q3.3.	1	A	Given
KB:	2	B	Given
<ul style="list-style-type: none"> $A \wedge B \rightarrow A, B$ $B \wedge C \iff D$ $A \implies C \rightarrow \neg A \vee C$ 	3	$\neg B \vee \neg C \vee D$	Given
	4	$\neg D \vee B$	Given
	5	$\neg D \vee C$	Given
	6	$\neg A \vee C$	Given
	7	$\neg D$	Negated conclusion
	8	C	1, 6
	9	$\neg C \vee D$	2, 3
	10	$\neg A \vee D$	6, 9
	11	D	1, 10
$\text{KB entails a sentence } S$			

Exercise 4

Convert the following natural-language sentences to first-order logic sentences. Specify the definition of your predicates and functions as shown in the following example. (23 points)

Example

Everyone loves Mary.

Answer:

$$\forall x. Love(x, Mary)$$

- Love(x, y): x loves y.

Q4.1. All students who study at Stevens Institute of Technology are smart.

Q4.2. Some people who are good at math like chess.

Q4.3. If all employees work hard, the company will make more profit this year.

Q4.4. There is a singer who everyone loves.

Q4.5. The singer loves everyone.

Solution

(Q4.1)

$$\forall x \text{ Student}(x) \wedge \text{Study}(x, \text{Stevens Institute of Technology}) \rightarrow \text{Smart}(x)$$

$\text{Student}(x)$: x is a student

$\text{Study}(x, y)$: x studies at y

$\text{Smart}(x)$: x is smart

(Q4.2)

$$\exists x P(x) \wedge G(x, \text{Math}) \wedge C(x, \text{chess})$$

$P(x)$: x is a person

$G(x, y)$: x is good at y

$C(x, z)$: x likes z

(Q4.3)

$$\forall x E(x) \wedge W(x) \rightarrow PF(\text{Company})$$

$E(x)$: x is an employee

$W(x)$: x works hard

$PF(\text{Company})$: Company will make more profit this year.

(Q4.4)

$$\exists y \text{ Singer}(y) \wedge [\forall x. Love(x, y)]$$

$\text{Singer}(y)$: y is a singer

$\text{Love}(x, y)$: x loves y

(Q4.5)

$$\forall x \text{ singer}(x) \rightarrow \forall y \text{ love}(x, y)$$

$\text{Singer}(x)$: x is a singer

$\text{love}(x, y)$: x loves y

Exercise 5

Check whether there is the most general unifier (MGU) of two clauses ω_1 and ω_2 . Write MGU if the clauses have it. (20 points)

Q5.1. $\omega_1: A(B, C)$, $\omega_2: A(x, y)$

Q5.2. $\omega_1: A(x, f(D, x))$, $\omega_2: A(E, f(D, y))$

Q5.3. $\omega_1: A(x, y)$, $\omega_2: A(f(C, y), z)$

Q5.4. $\omega_1: P(A, x, f(g(y)))$, $\omega_2: P(y, f(z), f(z))$

Q5.5. $\omega_1: P(x, g(f(A)), f(x))$, $\omega_2: P(f(y), z, y)$

Q5.6. $\omega_1: P(x, f(y))$, $\omega_2: P(z, g(w))$

Solution

questions	ω_1	ω_2	MGU	$\omega_1 s = \omega_2 s$
Q5.1	$A(B, C)$	$A(x, y)$	$\{B/x, C/y\}$	$A(x, y)$
Q5.2	$A(x, f(D, x))$	$A(E, f(D, y))$	$\{x/E, f(D, E)/f(D, y)\}$ $\{x/E, D/D, E/y\}$ $\{x/E, E/y\}$	$A(y, f(D, y))$
Q5.3	$A(x, y)$	$A(f(C, y), z)$	$\{x/f(C, y), y/z\}$	$A(f(C, z), z)$
Q5.4	$P(A, x, f(g(y)))$	$P(y, f(z), f(z))$	$\{A/y, x/f(z), f(g(y))/f(z)\}$ $\{A/y, x/f(z), g(y)/z\}$	$P(y, f(z), f(z))$
Q5.5	$P(x, g(f(A)), f(x))$	$P(f(y), z, y)$	$\{x/f(y), g(f(A))/z, f(x)/y\}$ No MGU	-
Q5.6	$P(x, f(y))$	$P(z, g(w))$	$\{x/z, f(y)/g(w)\}$	$P(z, g(w))$

Exercise 6

Using resolution refutation, prove that a knowledge base (KB) entails a sentence (S). (17 points)

KB:

- $Mother(Jane, Emma)$
- $Alive(Jane)$
- $\forall x, y. Mother(x, y) \implies Parent(x, y)$
- $\forall x, y. Parent(x, y) \wedge Alive(x) \implies Older(x, y)$

S:

$Older(Jane, Emma)$

Q6.1 Write KB and S in the clausal form.

Q6.2 Derive S from KB using the resolution rule. Write your answer in a table format shown in slide 7/24 of the fifth-week lecture. (specify the unifier in each row).

Solution

Q6.1

KB:

$Mother(Jane, Emma)$

$Alive(Jane)$

$\forall x, y. Mother(x, y) \rightarrow Parent(x, y)$

$\forall x, y. Parent(x, y) \wedge Alive(x) \rightarrow Older(x, y)$

S:

$Older(Jane, Emma)$

Clausal Form

$Mother(Jane, Emma)$

$Alive(Jane)$

$\forall x, y [\neg Mother(x, y) \vee Parent(x, y)]$
 $\neg Mother(x, y) \vee Parent(x, y)$

$\forall x, y [\neg (Parent(x, y) \wedge Alive(x)) \vee Older(x, y)]$
 $\forall x, y [\neg Parent(x, y) \vee \neg Alive(x) \vee Older(x, y)]$
 $\forall w, z [\neg Parent(w, z) \vee \neg Alive(w) \vee Older(w, z)]$
 $\neg Parent(w, z) \vee \neg Alive(w) \vee Older(w, z)$

$Older(Jane, Emma)$

Q6.2

1	$Mother(Jane, Emma)$	Given
2	$Alive(Jane)$	Given
3	$\neg Mother(x, y) \vee Parent(x, y)$	Given
4	$\neg Parent(w, z) \vee \neg Alive(w) \vee Older(w, z)$	Given
5	$\neg Older(Jane, Emma)$	Negated S

6	$Parent(Jane, Emma)$	$1, 3 \{ x/Jane, y/Emma \}$
7	$\neg Alive(Jane) \vee Older(Jane, Emma)$	$4, 6 \{ w/Jane, z/Emma \}$

8	$Older(Jane, Emma)$	$2, \neg 5$
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KB entails a sentence S