Revisiting Conduct Parameter Estimation in Homogeneous Goods Markets: At Least, Linear Model is Valid

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Abstract

We revisit the conduct parameter estimation in homogeneous goods markets. In contrast to the pessimistic simulation results of linear models shown in Perloff and Shen (2012), our simulation shows that the estimation becomes accurate by properly adding demand shifters in the supply estimation and increasing the sample size. We also investigate log-linear models widely used in Industrial Organization literature and recommended by Perloff and Shen (2012) and find other estimation problems. Based on the numerical investigation, at least the linear model can achieve a proper estimation of the conduct parameter.

1 Introduction

Measuring competitiveness in markets is one of the important tasks in Empirical Industrial Organization (IO) literature. Conduct parameter is regarded as a useful measure of competitiveness. However, it cannot be measured directly from data because data usually lack information about marginal cost. Therefore, the researchers have tried to identify and estimate the conduct parameter.

The literature has considered two popular specifications of the model for conduct parameter estimation in homogeneous good markets; one is the model with linear demand and linear marginal cost and the other is the model with log-linear demand and log-linear marginal cost.[1]

As for the linear model, Bresnahan (1982) considers the identification of conduct parameter in this model. Perloff and Shen (2012) show that the model in Bresnahan (1982) is suffering from the multicollinearity problem when the error terms in the demand and supply equations are zero and claim that the model parameters cannot be estimated. While the situation where the multicollinearity problem arises hardly happens not only in simulation studies but also in

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[1]As a similar strand of the literature, there is growing literature on conduct parameter estimation in differentiated goods markets. For example, see Gandhi and Nevo (2021).

practice, the paper still claims that the nearly perfect collinearity contaminates the estimation. By conducting Monte Carlo simulations, the paper shows that the marginal cost parameters and the conduct parameter cannot be accurately estimated. To avoid the multicollinearity problem and nearly perfect collinearity, the paper recommends using log-linear or some other functional form for at least one of the equations.

As for the log-linear model, which is recommended by Perloff and Shen (2012) as a valid specification for identification, the identification strategy is provided by Lau (1982). The specification is often used in the empirical papers such as Okazaki et al. (2022) and Mérel (2009).

We find that there are several problems for both the linear and log-linear model. As for the linear model, we find that the simulation in Perloff and Shen (2012) has two problems. First, their estimation of the supply equation lacks an excluded demand shifter. Second, the paper does not check the effect of increasing the sample size.

As for the log-linear model, to the best of our knowledge, there is no simulation study to justify the claim of Perloff and Shen (2012). Hyde and Perloff (1995) conduct a simulation of the model and report the result of hypothesis tests based on the simulation result. However, the paper does not show the simulation result itself. We find that the simulation still has the same problem in Perloff and Shen (2012) because it lacks the demand shifter in the supply-side estimation.

Given these problems in the simulations, we revisit the estimation of the conduct parameter in homogeneous product markets. First, we replicate the result in Perloff and Shen (2012) by complementing some details that they did not mention on instrument construction. We confirm that the accuracy of the estimation holds by including a demand shifter in the supply equation estimation properly. Given the standard deviation of the error terms fixed, when the sample size is more than 100, the accuracy of the estimation is also improved. In this sense, Perloff and Shen (2012) provides correct theoretical results, but incorrect numerical simulation results.

Second, we conduct the simulation of the model with log-linear demand and log-linear marginal cost that is suggested by Perloff and Shen (2012). We numerically show that standard regression estimates an incorrect conduct parameter out of range of $[0, 1]$ because the specification may add another identification problem of the conduct parameter and constant term of marginal costs. The nonidentification results of the log-linear model are mistakenly reported in IO literature as imprecise estimation results, for example, Okazaki et al. (2022). Thus, we conclude that the linear model is a valid specification for the conduct parameter estimation in homogeneous good markets.

2 Model

The researcher has data with T markets with homogeneous products. Assume that there are N_t firms in each market. Let $t = 1, \ldots, T$ be the index of markets and $j = 1, \ldots, N_t$ be the index of firms in market t. Firm j solves a profit maximization problem:

$$\max_{q_{jt}} \ \pi_{jt}(q_{jt}, q_{-jt}) \equiv (P_t(Q_t) - MC_{jt}(q_{jt}))q_{jt},$$

where $Q_t = \sum_{j=1}^{N_t} q_{jt}$ is the aggregate quantity, $P_t(Q_t)$ the demand function, and $MC_{jt}(q_{jt})$ the marginal cost function. From the maximization problem, we obtain the first-order condition:

$$0 = P_t(Q_t) - MCjt(q_{jt}) + \theta_j \frac{\partial P_t}{\partial Q_t} q_{jt},$$

where $\theta_{jt} = 1 + \sum_{k \neq j} \frac{\partial q_{kt}}{\partial q_{jt}}$, which is called conduct parameter. The first-order conditions for each firm are weighted by market shares and added up. Then, we obtain the supply equation:

$$P_t = -\theta_t \frac{\partial P_t}{\partial Q_t} Q_t + MC_t(Q_t), \tag{1}$$

where $\theta_t = (1/N_t) \sum_{j=1}^{N_t} s_{jt}\theta_{jt}$, $MC_t(Q_t) = (1/N_t) \sum_{j=1}^{N_t} s_{jt} MC_{jt}(q_{jt})$, and $s_{jt} = q_{jt}/Q_t$.

Consider an econometric model of the above quantity competition. Assume that the demand function and the marginal cost function are written as:

$$P_t = f(Q_t, Y_t, \varepsilon_t^d, \alpha) \tag{2}$$

$$MC_t = g(Q_t, W_t, \varepsilon_t^c, \gamma) \tag{3}$$

where Y_t and W_t are the vector of exogenous variables, ε_t^d and ε_t^c the error terms, and α and γ are the vector of parameters. We also have the vector of the demand-side instrument variables Z_t^d and the vector of the supply-side instrument variables Z_t^c. Then we assume that the error terms satisfy the mean independence condition $E[\varepsilon_t^d \mid Y_t, Z_t^d] = E[\varepsilon_t^c \mid W_t, Z_t^c] = 0$.

3 Identification of the conduct parameter

3.1 Linear demand and linear cost

Assume that a linear demand function and a linear marginal cost function are specified as:

$$P_t = \alpha_0 - (\alpha_1 + \alpha_2 Z_t^R)Q_t + \alpha_3 Y_t + \varepsilon_t^d, \tag{4}$$

$$MC_t = \gamma_0 + \gamma_1 Q + \gamma_2 W_t + \gamma_3 R_t + \varepsilon_t^c, \tag{5}$$

where W_t and R_t are excluded cost shifter. The supply equation (1) is written as:

$$P_t = \gamma_0 + [\theta(\alpha_1 + \alpha_2 Z_t^R) + \gamma_1]Q_t + \gamma_2 W_t + \gamma_3 R_t + \varepsilon_t^c. \tag{6}$$

By substituting the linear demand equation (4) into the supply equation (6) and solving it for P_t, we can represent the aggregate quantity Q_t based on the parameters and exogenous variables as:

$$Q_t = \frac{\alpha_0 + \alpha_3 Y_t - \gamma_0 - \gamma_2 W_t - \gamma_3 R_t + \varepsilon_t^d - \varepsilon_t^c}{(1+\theta)(\alpha_1 + \alpha_2 Z_t^R) + \gamma_1}.$$

3.2 Log-linear demand and log-linear marginal cost

Consider a model with log-linear demand and log-linear marginal cost function which are given as:

$$\log P_t = \alpha_0 - (\alpha_1 + \alpha_2 Z_t^R)\log Q_t + \alpha_3 \log Y_t + \varepsilon_t^d, \tag{7}$$

$$\log MC_t = \gamma_0 + \gamma_1 \log Q_t + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c. \tag{8}$$

Since $\partial P_t/\partial Q_t = -(\alpha_1 + \alpha_2 Z^R)(P_t/Q_t)$, the supply equation (1) is written as:

$$P_t = \theta_t(\alpha_1 + \alpha_2 Z_t^R)\frac{P_t}{Q_t}Q_t + MC_t.$$

By reformulating this and taking logarithm, $\log P_t(1 - \theta_t(\alpha_1 + \alpha_2 Z_t^R)) = \log MC_t$. Then, we obtain:

$$\log P_t = -\log(1 - \theta(\alpha_1 + \alpha_2 Z_t^R)) + \gamma_0 + \gamma_1 \log Q_t + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c. \tag{9}$$

By substituting the log-linear demand equation (7) into the supply equation (9) and solving it for P_t, the log aggregate quantity is given as:

$$\log Q_t = \frac{\alpha_0 + \alpha_3 \log Y_t + \log(1 - \theta(\alpha_1 + \alpha_2 Z_t^R)) - \gamma_0 - \gamma_2 \log W_t - \gamma_3 \log R_t + \varepsilon_t^d - \varepsilon_t^c}{\gamma_1 + \alpha_1 + \alpha_2 Z_t^R}.$$

3.3 Parameter and distribution setting

We set the true parameters and distributions as in Table 1. As for the linear model, we follow the setting of Perloff and Shen (2012). The paper does not provide any details on the composition of the instrumental variables for the demand and supply estimation. We compare the model without and with a demand shifter.

As for the log-linear model, we need to use different data generating processes from the linear model to generate reasonable data through the logarithmic transformation. We confirm that both settings provide downward log-linear demand curves. A similar specification is investigated in Hyde and Perloff (1995). We change σ to see how the variation of the error terms affects the accuracy of the estimation.

[Table 1 about here.]

3.4 Simulation and estimation Procedure

To implement the simulation, we generate 1000 data sets by using R for the linear and log-linear models. The estimation of both the linear model and the log-linear model is based on the 2SLS estimation. We separately estimate the demand and supply equation. The instrument variables for the demand estimation are $Z_t^d = (Z_t^R, Y_t, H_t, K_t)$ and the instrument variables for the supply estimation are $Z_t^c = (Z_t^R, W_t, R_t, Y_t)$.

As for the linear model, we estimate the equations by using the `ivreg` package in R. Note that an important feature of the model is that we have an interaction term of the endogenous variable Q_t and the instrument variable Z_t^R.[2]

As for the log-linear model, we estimate the demand parameters as in the linear model by using R since the demand equation is linear. In contrast, for the supply side estimation, since the supply side equation is nonlinear, we use Julia and apply the generalized method of moments (GMM) with the moment condition $E[Z_t^c \varepsilon_t^c] = 0$. Note that for the supply estimation, we substitute the demand estimation result. The GMM estimator is obtained as a solution to the minimization problem.[3] To solve the minimization problem, we formulate the minimization problem via `JuMP.jl` and solve the problem by using the nonlinear solver `IPOPT.jl`. We use all starting values as zero and the tolerance is 10^{-16}.

4 Results

4.1 The linear demand and linear marginal cost model

[Table 2 about here.]

First, we replicate the result in Perloff and Shen (2012). To replicate the result, we exclude the demand shifter Y_t and assume the coefficient α_3 of Y_t is zeros, that is, there is no demand shifter for the supply e For reference, Table 2 is quoted from Perloff and Shen (2012), although we modify some notations. The sample size in each simulation data is 50 and the table shows

[2]The `ivreg` package automatically detects that the endogenous variables are Q_t and the interaction term $Z_t^R Q_t$ and runs the first stage regression for each endogenous variable by using the same instruments. To confirm this, we manually wrote R code that implements the 2SLS. When the first stage includes only the regression of Q_t, the estimation results from our code differ from the results from `ivreg`. But when we modified the code so that regress $Z_t^R Q_t$ on the instrument variables and estimate the second stage by using the predicted values of Q_t and $Z_t^R Q_t$, the result from our code and the result from `ivreg` coincided.

[3]For the detail, see Ch.14 of Wooldridge (2010).

the mean and the standard deviation of the 2SLS estimators from 1000 simulations. It shows that the demand estimation becomes more accurate as the value of the standard deviation of the error terms σ decreases. In contrast, the supply-side estimation is still biased and the standard deviation of the conduct parameter becomes larger as the value of σ increases.

Table 3 shows our replication results. Each panel shows the simulation result under different standard deviations of the error terms. This result uses the same data generation process as Perloff and Shen (2012). To see if we can correctly replicate the result in Perloff and Shen (2012), focus on the first two columns in each panel. These two columns show the mean and the standard deviation of the simulation result when the sample size is 50. While the demand parameter can be accurately estimated even though the value of σ becomes larger, the supply side parameter is biased. Especially, when σ is large and the sample size is small, the standard deviation of the parameters in the supply-side equation becomes large. Thus, we can see the patterns in Perloff and Shen (2012) which do not provide any details.

As Perloff and Shen (2012) fix the sample size to 50, we also see the effect of changing the sample size. As expected, increasing the sample size given a value of σ decreases the standard deviation of the parameter in the supply equation. However, no simulation result is close to the true values of the supply parameters and the conduct parameter. These results are consistent with Perloff and Shen (2012).

Fortunately, their pessimistic result can be easily resolved simply by adding a demand shifter and increasing the sample size. Table 4 shows the 2SLS estimation results with the demand shifter. Here, we include Y_t and assume that $\alpha_3 = 1$ as in Table 1. When $\sigma = 0.001$, Panel (a) shows that the estimation of all parameters is quite accurate. Especially, when the sample size is large, the standard deviations of all parameters are less than 0.001. Thus, this implies that a nearly perfect collinearity problem rarely occurs when a demand shifter is included.

Panel (d) shows that the supply-side estimation is still biased when $\sigma = 2$ and the sample size is 50. The value of γ_1, the coefficient of Q_t, is far from the true value $\gamma_1 = 1$ and the standard deviation is much larger than the standard deviation in the model without Y_t. However, as the sample size increases, the standard deviation improves dramatically. When the sample size is 1000, all parameters are estimated more accurately than the result in the model without Y_t. Thus, imprecise results reported in Perloff and Shen (2012) are due to the small sample size.

[Table 3 about here.]

[Table 4 about here.]

[Table 5 about here.]

[Table 6 about here.]

4.2 Log-linear demand and log-linear marginal cost

Table 5 provides the simulation results of log-linear demand specifications with a demand shifter. We calculate the mean and standard deviation only from the simulation data whose GMM minimization problems are locally solved. First, we cannot solve the log-linear model under the same data generating process. For example, when $\sigma = 2.0$, we can solve 36.6% of the 1000 simulated data. Second, we can see that the estimation of the conduct parameter fails as σ becomes large. When σ becomes larger, the mean and standard deviation of the conduct parameter estimator take unreasonable values. Only when the standard deviation $\sigma = 0.001$, the estimation results are accurate as in the linear model.

Figure 1 illustrates the histograms of the conduct parameter from all simulations. Panel (a) and (b) fix the sample size in each simulation to 1000, but each panel has different standard deviations, $\sigma = 0.5$ and 2. In each panel, the histogram has a peak around the true parameter, but we also see samples whose values are either less than zero or larger than one. Note that theoretically, the conduct parameter takes a value between zero and one. When $\sigma = 0.5$, no estimation result exceeds one, but we have values less than zero. About 25% of the simulations are out of $[0, 1]$. When $\sigma = 2$, more results are out of $[0, 1]$, which is about 40% of the simulations. Previous papers also point out that the value of the estimated conduct parameter falls out of $[0, 1]$. For example, Okazaki et al. (2022) estimate the conduct parameter with a log-linear model in their Appendix and show that estimated θ is out of $[0, 1]$. The authors state that the conduct parameter estimation is imprecise. In the next section, we will discuss why such extreme values are estimated.

[Table 7 about here.]

[Table 8 about here.]

[Figure 1 about here.]

5 A problem in log-linear demand specification

[Figure 2 about here.]

First, note that in Table 5, the mean of γ_0, the constant term in the log-linear marginal cost function, is far from the true parameter and the standard deviation is large compared with other marginal cost parameters. Recall that the first and second term in the supply equation (9) is $-\log(1-\theta(\alpha_1+\alpha_2 Z_t^R))+\gamma_0$. As the first term is always negative, for any value of θ that satisfies $1-\theta(\alpha_1+\alpha_2 Z_t^R)$, we can find a value of γ_0 that cancel out the first and the second term each other. Especially when the other estimated parameters are close to the true value, by adjusting the value of θ and γ_0 to cancel out the first and second term, we can make the value of the GMM objective function under the estimated parameters close to the value of the GMM objective function under the true parameter.

Figure 2 illustrates the contour map of the GMM objective function with respect to γ_0 and θ for one typical data. To draw the map, we fix the other demand and cost parameter values to the true values and set the grid size to 0.01. We can see that the contour map has a flat region around the true parameters. Especially, even though we vary the value of θ, the GMM value does not change around the true value of γ_0.

As we reported in Section 4.2, the value of the estimator of the conduct parameter can take very large or small values. Do such values minimize the value of the GMM objective function? To check this, for each simulation, we compute (1) the absolute differences between the true and estimated values of θ and γ_0 and (2) the absolute differences between the value of the GMM objective function under the true and estimated parameter. Figure 3 shows the plot of these differences. Each dot corresponds to each simulation and the color of each dot represents the value of the difference in the GMM objective function. The darker a dot is, the smaller the difference is. We expect that when the difference between the true parameter value and the estimation result is small, the difference in the value of the GMM function is small, which implies that the color of the dot is darker. However, we can see dark dots even when the difference in the conduct parameter is large. For example, when $\sigma = 0.5$, dark dots appear when the difference is more than 5000. When $\sigma = 2$, dark dots appear even when the difference is more than 100,000. While Lau (1982) shows the joint identification result of the conduct parameter and the cost parameter in a general case, Figures 2 and 3 imply that the estimation of the conduct parameter and the cost parameter becomes imprecise.

[Figure 3 about here.]

The next question is whether we can obtain accurate results by imposing the parameter constraint, that is, $\theta \in [0, 1]$. Our conclusion is no.

Figure 4 overlaps the histograms of the result of the conduct parameter estimation with the parameter constraint on Figure 1. For the values in $(0, 1)$, the shape of the histograms is similar, but we can see the spike around $\theta = 0$ and 1. This is simply because all estimation results out of the bound in the estimation without constraint are gathered at the corner points of the constraint. The literature interprets $\theta = 0$ and 1 as perfect competition and collusion, respectively. Thus, putting the parameter constraint on the conduct parameter makes researchers misinterpret that the markets are perfectly competitive or collusive. For example, Mérel (2009) uses log-linear models and the estimated θ is 0.002 with 0.007 standard error, which means perfect competitive markets. The estimation may fail in the boundary as in Figure 4.

In summary, the suggestion of the log-linear model in Perloff and Shen (2012) adds another problem in its estimation. We conclude that at least the linear model can achieve a proper estimation of the conduct parameter in homogeneous good markets.

[Figure 4 about here.]

6 Conclusion

We revisit the conduct parameter estimation in homogeneous goods markets. In contrast to the pessimistic simulation results shown in Perloff and Shen (2012), our simulation shows that the estimation becomes accurate by properly adding demand shifters in the supply estimation and increasing the sample size. We also show that the log-linear model recommended by Perloff and Shen (2012) has another estimation problem. Based on the numerical investigation, we conclude that at least the linear model can achieve a proper estimation of the conduct parameter in homogeneous good markets.

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Table 1: True parameters and distributions

(a) Parameters

	linear	log-linear
α_0	10.0	10.0
α_1	1.0	1.0
α_2	1.0	0.1
α_3	1.0	1.0
γ_1	1.0	1.0
γ_2	1.0	1.0
γ_3	1.0	1.0
θ	0.5	0.3

(b) Distributions

	linear	log-linear
Demand shifter		
Y_t	$N(0,1)$	$N(0,1)$
Rotation demand shifter		
Z_t^R	$N(10,1)$	$U(0,1)$
Cost shifter		
W_t	$N(3,1)$	$U(1,3)$
R_t	$N(0,1)$	$U(0,1)$
H_t	$W_t + N(0,1)$	$W_t + U(0,1)$
K_t	$R_t + N(0,1)$	$R_t + U(0,1)$
Error		
ε_t^d	$N(0,\sigma)$	$N(0,\sigma)$
ε_t^c	$N(0,\sigma)$	$N(0,\sigma)$

Note: $\sigma = \{0.001, 0.5, 1.0, 2.0\}$. N : Normal distribution. U : Uniform distribution.

Table 2: Estimation results in Table 2 of from Perloff and Shen (2012)

	$\sigma = 0.001$	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 2$
α_0	10.00 (0.001)	9.96 (0.33)	9.86 (0.65)	9.46(1.20)
α_1	1.00 (0.004)	0.99 (1.98)	0.97 (3.96)	0.88(7.80)
α_2	1.00 (0.004)	0.99 (0.21)	0.97 (0.42)	0.87 (0.82)
γ_1	0.46 (0.88)	0.46 (0.91)	0.47 (0.93)	0.49 (1.04)
γ_2	5.85 (7.89)	5.85 (8.15)	5.78 (8.21)	5.73 (8.66)
θ	-0.31 (1.31)	-0.29 (1.34)	0.09 (11.48)	-1.53 (30.41)

Note: True parameters: $\alpha_1 = \alpha_2 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_0 = 10, \alpha_3 = 0, \theta = 0.5$. Perloff and Shen (2012) exclude Y_t. We change the parameter notations from the original paper. Note that Perloff and Shen (2012) do not provide γ_0 and γ_3.

Table 3: Estimation results of the linear model without demand shifter

(a) $\sigma = 0.001$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	10.000	0.001	10.000	0.001	10.000	0.000	10.000	0.000
α_1	1.000	0.004	1.000	0.003	1.000	0.002	1.000	0.001
α_2	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
γ_0	5.446	6.981	5.388	7.986	5.423	7.825	5.063	6.801
γ_1	0.506	0.775	0.512	0.888	0.509	0.869	0.549	0.756
γ_2	0.506	0.776	0.512	0.887	0.509	0.869	0.549	0.756
θ	-0.241	1.164	-0.231	1.331	-0.237	1.304	-0.177	1.134
R^2 (demand)	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
R^2 (supply)	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
Sample size (n)		50		100		200		1000

(b) $\sigma = 0.5$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	9.993	0.466	9.993	0.312	10.001	0.215	10.002	0.093
α_1	0.963	2.138	0.965	1.484	1.012	1.023	0.991	0.441
α_2	1.002	0.243	1.002	0.168	0.999	0.118	1.002	0.049
γ_0	5.332	10.459	5.227	11.592	5.112	15.871	5.470	7.476
γ_1	0.405	3.214	0.434	1.989	0.474	1.744	0.516	1.102
γ_2	0.517	1.157	0.528	1.222	0.546	1.816	0.504	0.830
θ	-0.210	1.879	-0.206	1.951	-0.186	2.705	-0.247	1.238
R^2 (demand)	0.720	0.088	0.725	0.061	0.726	0.041	0.728	0.018
R^2 (supply)	0.160	7.674	-0.119	19.529	-0.724	30.775	0.491	2.041
Sample size (n)		50		100		200		1000

Note: True parameters: $\alpha_1 = \alpha_2 = \gamma_0 = \gamma_1 = \gamma_2 = 1, \alpha_0 = 10, \theta = 0.5.$ and $\alpha_3 = 0$

Table 3: Estimation results of the linear model without demand shifter (Continued)

(c) $\sigma = 1.0$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	9.975	0.964	9.953	0.636	10.007	0.441	9.991	0.189
α_1	1.120	4.491	0.942	2.885	0.883	2.055	1.035	0.902
α_2	0.981	0.492	0.993	0.326	1.015	0.227	0.993	0.101
γ_0	5.631	9.410	5.520	7.580	5.161	9.226	5.556	7.424
γ_1	-0.107	19.285	0.129	5.240	0.488	3.541	0.489	1.210
γ_2	0.476	1.043	0.495	0.835	0.540	1.030	0.494	0.820
θ	-0.201	3.603	-0.217	1.478	-0.183	1.528	-0.260	1.229
R^2 (demand)	0.205	0.357	0.234	0.221	0.232	0.150	0.245	0.060
R^2 (supply)	-0.920	17.898	-0.395	5.271	-0.904	12.486	-0.421	12.047
Sample size (n)		50		100		200		1000

(d) $\sigma = 2.0$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	9.515	6.752	9.912	1.479	9.987	0.943	9.987	0.396
α_1	0.362	19.344	0.710	6.192	1.154	4.363	0.986	1.728
α_2	0.934	1.092	1.004	0.743	0.981	0.494	0.998	0.204
γ_0	5.658	6.892	5.464	8.387	5.695	8.243	5.572	10.796
γ_1	0.956	52.166	1.715	42.062	-0.056	11.467	0.388	3.140
γ_2	0.479	0.827	0.496	0.907	0.486	0.902	0.497	1.185
θ	-0.296	5.941	-0.439	5.106	-0.235	2.034	-0.256	1.771
R^2 (demand)	-3.456	87.362	-0.513	1.557	-0.436	0.563	-0.376	0.185
R^2 (supply)	-1.104	5.881	-2.311	26.606	-1.993	26.973	-3.591	49.060
Sample size (n)		50		100		200		1000

Note: True parameters: $\alpha_1 = \alpha_2 = \gamma_0 = \gamma_1 = \gamma_2 = 1, \alpha_0 = 10, \theta = 0.5.$ and $\alpha_3 = 0$

Table 4: Estimation results of the linear model with demand shifter

(a) $\sigma = 0.001$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	10.000	0.001	10.000	0.001	10.000	0.000	10.000	0.000
α_1	1.000	0.004	1.000	0.003	1.000	0.002	1.000	0.001
α_2	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
α_3	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
γ_0	1.000	0.001	1.000	0.001	1.000	0.001	1.000	0.000
γ_1	1.000	0.005	1.000	0.004	1.000	0.002	1.000	0.001
γ_2	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
γ_3	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
θ	0.500	0.001	0.500	0.000	0.500	0.000	0.500	0.000
R^2 (demand)	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
R^2 (supply)	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
Sample size (n)		50		100		200		1000

(b) $\sigma = 0.5$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	9.982	0.465	10.007	0.323	9.992	0.213	9.994	0.097
α_1	0.955	2.257	1.024	1.523	1.018	1.016	0.969	0.454
α_2	0.999	0.255	0.999	0.176	0.996	0.115	1.001	0.051
α_3	0.995	0.108	1.003	0.075	0.999	0.050	0.999	0.022
γ_0	0.939	0.730	0.995	0.474	0.979	0.345	0.995	0.152
γ_1	0.689	3.438	0.876	1.925	0.919	1.302	0.997	0.548
γ_2	1.009	0.109	0.999	0.071	1.003	0.051	1.000	0.023
γ_3	1.001	0.108	1.003	0.075	1.003	0.052	1.000	0.022
θ	0.547	0.351	0.517	0.208	0.514	0.134	0.503	0.058
R^2 (demand)	0.762	0.083	0.763	0.057	0.765	0.036	0.764	0.016
R^2 (supply)	0.768	0.075	0.768	0.050	0.766	0.035	0.763	0.016
Sample size (n)		50		100		200		1000

Note: True parameters: $\alpha_1 = \alpha_2 = \alpha_3 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_0 = 10, \theta = 0.5$.

Table 4: Estimation results of the linear model with demand shifter (Continued)

(c) $\sigma = 1.0$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	9.973	1.023	9.998	0.641	9.996	0.448	9.984	0.187
α_1	0.976	4.398	0.831	2.962	1.061	2.060	1.011	0.905
α_2	0.994	0.495	1.016	0.324	0.993	0.234	0.994	0.100
α_3	0.994	0.223	1.002	0.153	0.999	0.099	0.997	0.045
γ_0	0.682	1.741	0.909	1.055	0.914	0.709	0.996	0.308
γ_1	6.859	210.877	0.321	6.247	0.662	2.954	0.950	1.109
γ_2	1.035	0.244	1.011	0.157	1.011	0.103	1.000	0.045
γ_3	1.045	0.246	1.012	0.150	1.010	0.104	1.002	0.045
θ	0.101	18.455	0.598	0.732	0.554	0.303	0.509	0.113
R^2 (demand)	0.301	0.409	0.305	0.199	0.310	0.134	0.319	0.054
R^2 (supply)	0.267	0.369	0.308	0.186	0.306	0.128	0.315	0.052
Sample size (n)		50		100		200		1000

(d) $\sigma = 2.0$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	9.737	2.584	10.071	1.669	9.960	0.947	9.998	0.412
α_1	0.729	10.822	1.008	6.495	1.236	4.259	1.021	1.810
α_2	0.956	1.253	1.023	0.779	0.969	0.482	0.997	0.210
α_3	0.976	0.584	1.008	0.343	0.996	0.225	1.003	0.092
γ_0	-1.074	19.523	0.449	2.994	0.829	1.507	0.949	0.631
γ_1	59.209	1750.596	-1.416	56.886	-2.617	38.895	0.897	2.333
γ_2	1.242	2.420	1.065	0.404	1.020	0.219	1.006	0.093
γ_3	1.230	2.318	1.055	0.400	1.010	0.219	1.008	0.092
θ	-6.168	233.873	0.872	6.326	0.918	3.799	0.524	0.244
R^2 (demand)	-0.653	3.934	-0.513	1.300	-0.363	0.518	-0.319	0.191
R^2 (supply)	-10.545	132.912	-0.670	1.930	-0.401	0.505	-0.326	0.174
Sample size (n)		50		100		200		1000

Note: True parameters: $\alpha_1 = \alpha_2 = \alpha_3 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_0 = 10, \theta = 0.5$.

Table 5: Estimation results of the log-linear model with demand shifter

(a) $\sigma = 0.001$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	10.000	0.007	10.000	0.004	10.000	0.002	10.000	0.001
α_1	1.000	0.001	1.000	0.001	1.000	0.001	1.000	0.000
α_2	0.100	0.000	0.100	0.000	0.100	0.000	0.100	0.000
α_3	1.000	0.001	1.000	0.000	1.000	0.000	1.000	0.000
γ_0	1.000	0.004	1.000	0.003	1.000	0.002	1.000	0.001
γ_1	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
γ_2	1.000	0.001	1.000	0.000	1.000	0.000	1.000	0.000
γ_3	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
θ	0.300	0.002	0.300	0.002	0.300	0.001	0.300	0.001
Solved (%)		0.641		0.609		0.638		0.598
Sample size (n)		50		100		200		1000

(b) $\sigma = 0.5$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	9.012	2.571	9.687	1.673	9.923	1.220	10.019	0.541
α_1	0.775	0.569	0.928	0.371	0.984	0.272	1.005	0.120
α_2	0.097	0.077	0.103	0.047	0.098	0.033	0.100	0.014
α_3	0.886	0.285	0.962	0.186	0.994	0.142	1.002	0.063
γ_0	1.023	1.301	1.068	1.235	1.173	1.051	1.178	0.586
γ_1	0.997	0.149	0.997	0.110	0.995	0.074	0.995	0.032
γ_2	0.982	0.249	0.992	0.180	0.999	0.118	0.996	0.052
γ_3	0.997	0.109	1.000	0.077	0.996	0.052	0.997	0.023
θ	-1.512	12.373	-3.353	42.000	-0.876	7.016	-0.042	2.013
Solved (%)		0.448		0.51		0.582		0.636
Sample size (n)		50		100		200		1000

Note: True parameters: $\alpha_1 = \alpha_3 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_0 = 10, \alpha_2 = 0.1, \theta = 0.3$. We calculate the mean and standard deviation only from the simulation data whose GMM minimization problems are locally solved.

Table 5: Estimation results of the log-linear model with demand shifter (Continued)

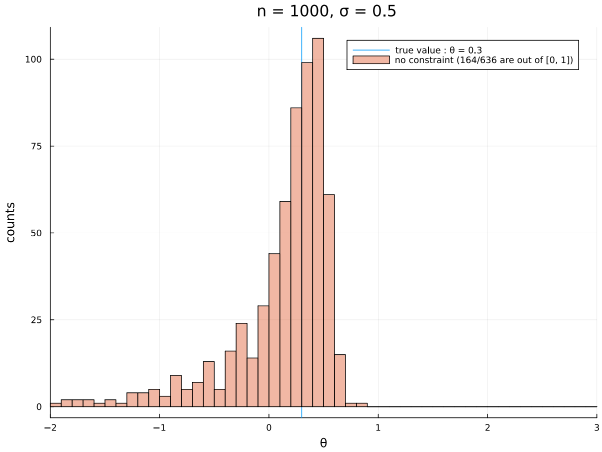
(c) $\sigma = 1.0$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	7.273	5.968	8.403	3.392	9.258	2.266	10.001	1.115
α_1	0.379	1.301	0.633	0.759	0.829	0.502	0.999	0.247
α_2	0.093	0.217	0.104	0.092	0.101	0.064	0.103	0.029
α_3	0.680	0.752	0.820	0.400	0.918	0.274	1.000	0.129
γ_0	1.165	1.786	1.129	1.543	1.170	1.405	1.273	0.995
γ_1	0.955	0.301	0.987	0.208	1.009	0.148	0.996	0.065
γ_2	0.939	0.491	0.975	0.344	1.005	0.258	0.992	0.105
γ_3	0.980	0.211	0.992	0.146	1.005	0.108	0.997	0.046
θ	-3.627	33.508	-2.726	18.209	-0.542	34.688	-0.825	5.156
Solved (%)		0.366		0.391		0.47		0.649
Sample size (n)		50		100		200		1000

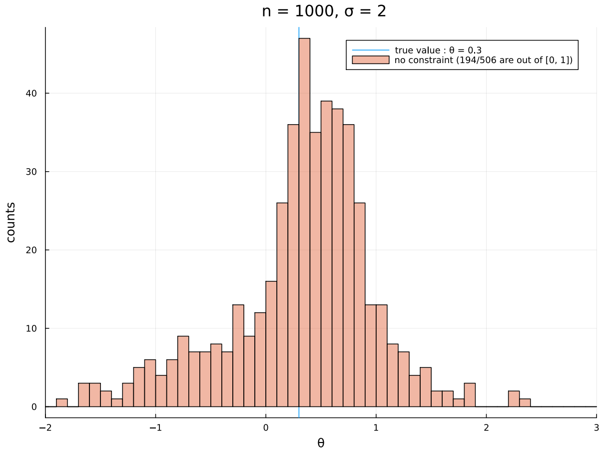
(d) $\sigma = 2.0$

	Mean	SD	Mean	SD	Mean	SD	Mean	SD
α_0	5.544	3.842	6.702	3.615	7.573	3.340	9.659	2.046
α_1	0.015	0.859	0.260	0.796	0.452	0.734	0.920	0.455
α_2	0.059	0.256	0.078	0.197	0.085	0.120	0.104	0.053
α_3	0.500	0.532	0.634	0.442	0.727	0.405	0.966	0.237
γ_0	1.019	2.795	1.020	2.112	0.976	1.773	1.001	1.172
γ_1	0.966	0.562	1.012	0.408	1.005	0.292	0.993	0.138
γ_2	0.992	1.012	0.968	0.691	0.993	0.486	0.993	0.236
γ_3	0.970	0.413	0.997	0.305	1.006	0.207	0.997	0.092
θ	-2.256	11.987	-2.278	39.411	-1.954	12.612	-0.545	5.290
Solved (%)		0.321		0.331		0.373		0.506
Sample size (n)		50		100		200		1000

Note: True parameters: $\alpha_1 = \alpha_3 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_0 = 10, \alpha_2 = 0.1, \theta = 0.3$. We calculate the mean and standard deviation only from the simulation data whose GMM minimization problems are locally solved.



(a)



(b)

Figure 1: Histograms of the estimation result of the conduct parameter.

Note: True parameters: $\alpha_1 = \alpha_3 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_0 = 10, \alpha_2 = 0.1, \theta = 0.3$.

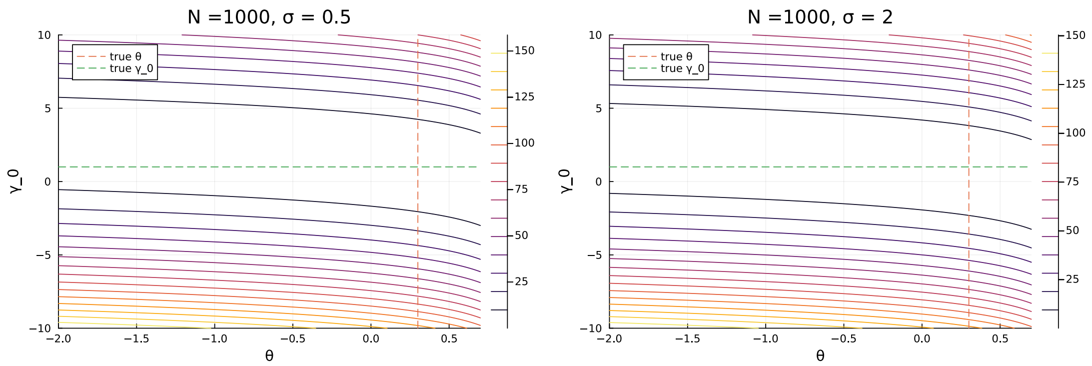


Figure 2: Contour map of GMM objective function

Note: True parameters: $\alpha_1 = \alpha_3 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_0 = 10, \alpha_2 = 0.1, \theta = 0.3$.

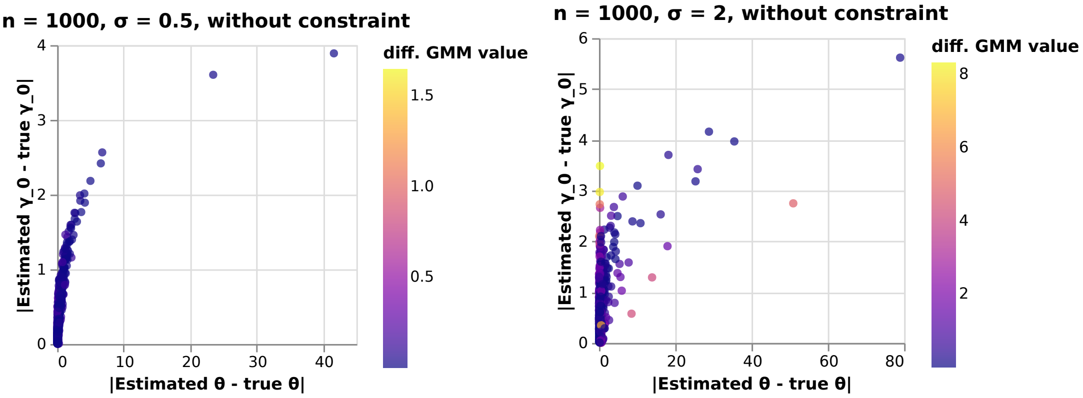
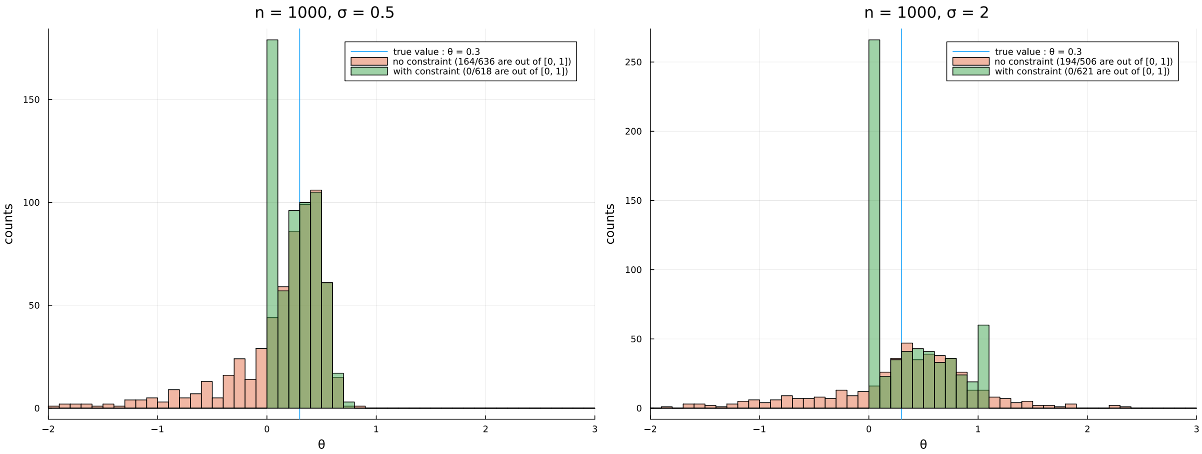


Figure 3: The difference of the GMM objective function under true parameter and the estimation result

Note: The x-axis is the difference between the values of the true and estimated conduct parameter. The y-axis is the difference between the values of estimated and true γ_0. Each dot represents the value of the difference of the GMM objective function under the true and estimated values of the parameters.



(a) (b)

Figure 4: Histograms of the conduct parameter with and without constraint.

Note: True parameters: $\alpha_1 = \alpha_3 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_0 = 10, \alpha_2 = 0.1, \theta = 0.3$.