

Hi all,

I like the simplicity of the paper. It is clear and you wrote it very well. I looked for a mistake really hard but it seems that you got this!! In fact, I am inclined to believe that my input is not necessary but I comment below anyway.

- I think where you present your contribution (more specifically) is the last paragraph of section 2.1. This is a nice place to present your assumptions and proposition (without the proof ofc).

An obstacle in the estimation is log-transformation in Equation (5). The standard numerical algorithm stops with invalid errors when the inside of the log function, $1 - \theta(\alpha_1 + \alpha_2 Z_t^R)$, becomes negative during the search for an optimizer. **Here put something like** [In proposition 1, we show that a positive equilibrium price does not exist when this term is negative. This problem is not specific to the above model and happens in other specifications, such as a model with linear demand and log-linear marginal cost. In the proposition, we also set conditions where positive equilibrium price exists and is unique....]

Now talk about your focus on equilibrium price [We define an equilibrium price as P_t^* , which is the price at the point of intersection between the demand and marginal cost functions. It's important to note that we consider only positive prices moving forward. This assumption ensures that if an equilibrium price exists, it must be strictly greater than zero.] (If you state something like this you do not have to repeat yourself saying $P_t > 0$ all the time. Also, specifically defining it like this helps you to not make easy mistakes. For instance, "when the term is negative, equilibrium prices do not exist" might be a "half"-false statement without mentioning your equilibrium price is positive.)

Now you are ready for your proposition. [Therefore, Proposition 1 proposes the conditions for the unique existence of P_t^* solving the demand equation (2) and supply equation (5) for P_t under $\theta \in [0, 1]$. [Proposition]]

- Regarding the proposition. The only missing detail is to assume $\alpha_1 + \alpha_2 Z^R$ not equal to 0. You assume this in MPEC section but your proposition and MPEC sections are different than each other.

~~I am revising the proposition following the feedback that I usually get from Prof Eraslan. There are two important things in your proposition. The first is that when that term is negative (which is the problem in the estimation) there is no equilibrium. The second important thing is conditions to have a unique equilibrium. The rest of it is for the completeness of your proof and is rather technical, and distracting. That is why I would choose to put everything in the following order. (I would also try not to define Ξ in the beginning of the proposition. However, I could not find a smart way to integrate somewhere else. One possibility would be defining it just after equation 5 without explicitly showing for $\log Q$: $\log P_t = -\log(1 - \theta(\alpha_1 + \alpha_2 Z_t^R)) + \gamma_0 + \gamma_1 \log Q_t + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c$~~

Proposition 1. Let $\Xi := \gamma_0 + \gamma_1 \frac{\alpha_0 + \alpha_3 \log Y_t + \varepsilon_t^d}{\alpha_1 + \alpha_2 Z_t^R} + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c$. The conditions for the existence and uniqueness of equilibrium prices P_t^* are as follows:

- When $1 - \theta(\alpha_1 + \alpha_2 Z_t^R) \leq 0$, no equilibrium price exists,
- When $1 - \theta(\alpha_1 + \alpha_2 Z_t^R) > 0$,
- If $-\gamma_1/(\alpha_1 + \alpha_2 Z_t^R) \neq 1$, a unique equilibrium price exists,
- If $-\gamma_1/(\alpha_1 + \alpha_2 Z_t^R) = 1$ and $\exp(\Xi) = 1 - \theta(\alpha_1 + \alpha_2 Z_t^R)$, there are infinitely many equilibrium prices, but no equilibrium price exists otherwise.

- In MPEC constraints, one additional constraint is $P_t > 0$ must be written there? Also, note the following. If you write the proposition at the end of section 2.1 you could relate your proposed estimation to it pretty easily in this section. Now, the audience would be ready to make sense of components like $1 - \theta(\alpha_1 + \alpha_2 Z_t^R)$ and $-\gamma_1/(\alpha_1 + \alpha_2 Z_t^R)$ when you present (11), (12), (13). Then you can specifically emphasize that with the way you set MPEC, now only possibility is that we are finding a unique equilibrium. You can say look dude this corresponds to a unique eq bc I claimed this and this in my proposition above and proof is in the appendix feel free to check.
- Regarding proof. After the sentence “any price that satisfies (14)...” start employing star notation make sure that you distinguish between the price that satisfies certain equations and just the generic element. Then, at the beginning of pg 9, you should state that any price satisfying (14) satisfies $\Delta(P_t) = 0$.

While defining $\Delta(P_t)$ put paranthesis under two terms and label them like I III whatever you pick. Then, you do not have to refer them second term first term etc.

On page 9, you say $P_t \geq 0$ this is a violation remember we assumed positive.

To argue the proof, just give me the general outline first. like there are two cases we discuss, the first case corresponds to this term being nonpositive. Then explain as you did on that page. Then, state the second case, which is when the term is positive. (quick note: you say we now assume this term is positive. Do not write like that because it seems you restrict yourself. Not at all. This is just another case not an assumption.)

When you explain figure 1a and say that there is a unique eq price because one term is increasing and one is decreasing. Make sure you talk about the values of these terms at the price 0 and at the infinity. Unique equilibrium is not there because one term is increasing and other is decreasing. It is there because additional to these monotonicities, the starting point and ending points are in a specific way.

- On page 1, consider revising the sentence “However, researchers may want to implement a different set of specifications such as the log-linear model ...” to “However, researchers often implement alternative specifications, such as the log-linear model ...” This change helps to convey the idea that the use of alternative specifications is indeed practice among researchers, rather than a mere possibility in space-time continuum.

- On page 2, consider representing the generic demand and marginal cost functions as $P_t(Q_t)$ and $MC_t(Q_t)$, respectively, to maintain consistency with the notation used in Equation 1.
- In section 2.1, there is an additional parameter (excluded cost shifter) pops up (R). Do you need mean independence condition on it too?
- On page 3, defining $\xi = (\alpha_0, \alpha_1, \dots, \alpha_3, \gamma_0, \gamma_1, \dots, \gamma_3, \theta)$ instead of $\xi = (\alpha_0, \alpha_1, \alpha_2, \alpha_3, \gamma_0, \gamma_1, \gamma_2, \gamma_3, \theta)$ is a bit cringe for my taste :P. Also, instead of conditional moment conditions, we can write conditional moment restrictions, requirements, constraints etc.