- Log-log demand : $\log P_t = \alpha_0 + (\alpha_1 + \alpha_2 Z_t)\log Q_t + \varepsilon_{Dt}$
- Log marginal cost : $\log mc_t = \gamma_0 + \gamma_1 Q_t + \varepsilon_{St}$
- The supply side equation is $\log P_{mt}\left(1 + \frac{\theta}{(\alpha_1+\alpha_2 Z_t)N_t}\right) = \gamma_0 + \gamma_1 Q_t + \varepsilon_{St}$
- There are $2, 4, 8, 10$ firms in each market. No entry is assumed. The researcher can also observe T markets.
- The equilibrium quantity is determined by the supply relation.

Assume that N firms compete in a homogeneous product market. The demand function is given as $\log P_t = \alpha_0 + (\alpha_1 + \alpha_2 Z_t)\log Q_t + \varepsilon_{Dt}$ where $Q_t = \sum_{i=1\dots N}$

1 Replication of Perloff and Shen (2012)

This paper points out that the conduct parameter can not be estimated accurately when the demand and marginal cost are linear due to multicolinearity. Note that we change the notation from the original paper

- Demand relation: $p = \alpha_0 - [\alpha_1 + \alpha_2 Z]Q + \alpha_3 Y + \varepsilon_d$
- Marginal cost: $MC = \gamma_0 + \gamma_1 Q + \gamma_2 w + \gamma_3 r + \varepsilon_c$
- Supply relation: $p = \gamma_0 + [\theta(\alpha_1 + \alpha_2 Z) + \gamma_1]Q + \gamma_2 w + \gamma_3 r + \varepsilon_c$
- $w \sim N(3,1), r \sim N(0,1), Z \sim N(10,1)$
- $\alpha_1 = \alpha_2 = \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_3 = 0, \alpha_0 = 10, \theta = 0.5.$
- $\varepsilon_c \sim N(0, \sigma_d)$, $\varepsilon_d \sim N(0, \sigma_c)$, and $\sigma = \sigma_d = \sigma_c$

From the supply relation and the demand relation, we have

$$\alpha_0 - [\alpha_1 + \alpha_2 Z]Q + \alpha_3 Y + \varepsilon_d = \gamma_0 + [\theta(\alpha_1 + \alpha_2 Z) + \gamma_1]Q + \gamma_2 w + \gamma_3 r + \varepsilon_c, \tag{1}$$

which gives us the equilibrium aggregate quantity Q such that

$$Q_t = \frac{\alpha_0 + \alpha_3 Y - \gamma_0 - \gamma_2 w - \gamma_3 r + (\varepsilon_d - \varepsilon_c)}{(1+\theta)(\alpha_1 + \alpha_2 Z) + \gamma_1}.$$

Consider a profit maximization problem of firm j such that

$$\max_{q_{jt}} \ \pi_{jt}(q_{jt}, q_{-jt}) \equiv (P(Q_t) - mc_{jt})q_{jt},$$

The first-order condition is given as

As $\alpha_3 = 0$, Y can be ignored.

To generate simulation data, after calculating the equilibrium aggregate quantity Q for market t, the equilibrium price is obtained from the demand relation. To estimate the parameters, we use the two-step least squares estimation. Note that we can separately estimate the parameters in the demand and supply relation. First, regress the exogenous variable Z and the cost shifters w, r on the aggregate quantity Q, which gives a predicted value of Q denoted as $\hat{Q}$. Next, estimate the parameters in the demand relation. By regressing $(\alpha_1 + \alpha_2 Z_t)Q, Q, w$ and r on the price p, we estimate the parameters in the supply relation.

The two-stage least square estimation

- Regress $Q_t = \pi_0 + \pi_1 Z + \pi_2 w + \pi_3 r + \nu$ and obtain $\hat{Q}$
- Estimate the demand parameters by regressing $P_t = \alpha_0 - [\alpha_1 + \alpha_2 Z]\hat{Q} + \alpha_3 Y + \varepsilon_d$
- Given the demand parameter, regress $p = \gamma_0 + \theta(\hat{\alpha}_1 + \hat{\alpha}_2 Z)Q + \gamma_1 Q + \gamma_2 w + \gamma_3 r + \varepsilon_c$