~~An~~ MPEC Estimator for Conduct Parameter Estimation in Homogeneous Goods Markets with equilibrium existence conditions: The Case of Log-linear Specification

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Abstract

We propose a constrained ~~Generalized Method of Moments (GMM)~~ estimator for estimating conduct parameters in homogeneous goods markets. We formulate the estimation as ~~an MPEC (Mathematical Programming with Equilibrium Constraints)~~ problem incorporating theoretical conditions for the unique existence of an equilibrium price. Monte Carlo simulations confirm that the proposed estimator with equilibrium existence conditions performs better ~~for~~ estimating conduct parameters with a small ~~Root-Mean-Squared-Error,~~ for the typical specification, ~~i.e.~~, the log-linear model.

1 Introduction

Measuring competitiveness is ~~one of the important tasks~~ in empirical industrial organization literature. Conduct parameter is considered ~~to be~~ a useful measure of competitiveness. However, it cannot be directly measured from data because data usually lack information about marginal cost. Therefore, researchers ~~endeavor~~ to identify and estimate the conduct parameter.

As the simplest specification, Bresnahan (1982) considers identification of conduct parameters for the linear model. Matsumura and Otani (2023) resolves the conflict on some identification problems between Bresnahan (1982) and Perloff and Shen (2012). ~~On the other hand,~~ researchers may want to implement a different set of specifications such as the log-linear model, ~~e.g.,~~ Okazaki et al. (2022) and Mérel (2009). ~~As for~~ the log-linear model, the identification strategy is provided by Lau (1982). Estimation problems arise ~~for the~~ case, however, when searching parameters using standard solvers because the equilibrium condition given by demand and supply curves involves log-transformation. This is an obstacle to choosing better specification of the demand and supply functions.

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To overcome the problem, we propose a new estimator based on the mathematical program with equilibrium constraints (MPEC) approach advocated by Su and Judd (2012). MPEC is a constrained optimization problem whose constraint structure contains equilibrium constraints. First, we show the theoretical slope and equilibrium existence conditions under which there is a unique equilibrium price given the log-linear specification. Second, we show that MPEC estimator ~~taking into account~~ the conditions performs better ~~for~~ estimating conduct parameters than the standard approach such as ~~Two-Stage-Least-Square~~ using derivative or non-derivative solvers with or without the conditions. We also show that increasing the sample size improves the accuracy of estimation. We conclude that MPEC with slope and equilibrium existence constraints enable~~s~~ us to estimate a broader set of specifications ~~which~~ are identified theoretically shown in Lau (1982) but practically and numerically difficult to estimate.

2 Model

Consider data with T markets with homogeneous products. Assume ~~that~~ there are N firms in each market. Let $t = 1, \ldots, T$ be the index of markets. Then, we obtain the supply equation ~~as follows:~~

$$P_t = -\theta \frac{\partial P_t(Q_t)}{\partial Q_t} Q_t + MC_t(Q_t), \tag{1}$$

where Q_t is the aggregate quantity, $P_t(Q_t)$ is the demand function, $MC_t(Q_t)$ is the marginal cost function, and $\theta \in [0, 1]$, which is the conduct parameter. The equation nests perfect competition ($\theta = 0$), Cournot competition ($\theta = 1/N$), and perfect collusion ($\theta = 1$). See Bresnahan (1982).

Consider an econometric model of the above model. Assume that the demand ~~function~~ and ~~the~~ marginal cost ~~function~~ are written as follows:

$$P_t = f(Q_t, Y_t, \varepsilon_t^d, \alpha) \tag{2}$$

$$MC_t = g(Q_t, W_t, \varepsilon_t^c, \gamma) \tag{3}$$

where Y_t and W_t are the vector of exogenous variables, ε_t^d and ε_t^c are the error terms, and α and γ are the vector of parameters. We also have the demand- and supply-side instrument variables Z_t^d and Z_t^c, and assume that the error terms satisfy the mean independence condition $E[\varepsilon_t^d \mid Y_t, Z_t^d] = E[\varepsilon_t^c \mid W_t, Z_t^c] = 0$. Note that Z_t^d often is W_t and Z_t^c often is Y_t.

2.1 Log-linear demand and log-linear marginal cost

Consider the most typical specification which is nonlinear-in-parameters as the log-linear model. Assume that log-linear demand and cost functions are specified as~~:~~

$$\log P_t = \alpha_0 - (\alpha_1 + \alpha_2 Z_t^R) \log Q_t + \alpha_3 \log Y_t + \varepsilon_t^d, \tag{4}$$

$$\log MC_t = \gamma_0 + \gamma_1 \log Q_t + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c \tag{5}$$

where Y_t is an excluded demand shifter, W_t and R_t are excluded cost shifters, and Z_t^R is Bresnahan's demand rotation instrument. Since $\partial P_t / \partial Q_t = -(\alpha_1 + \alpha_2 Z_t^R)(P_t/Q_t)$, Equation (1) is written as~~:~~

$$P_t = \theta(\alpha_1 + \alpha_2 Z_t^R) P_t + MC_t. \tag{6}$$

By reformulating this and taking logarithm~~,~~ $\log P_t(1 - \theta(\alpha_1 + \alpha_2 Z_t^R)) = \log MC_t$ ~~Then,~~ we obtain~~:~~

$$\log P_t = -\log(1 - \theta(\alpha_1 + \alpha_2 Z_t^R)) + \gamma_0 + \gamma_1 \log Q_t + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c. \tag{7}$$

An obstacle in estimation is log-transformation in Equation (7). The standard numerical algorithm stops with invalid errors when the inside of the log function, $1 - \theta(\alpha_1 + \alpha_2 Z_t^R)$, becomes negative during the search for an optimizer. When the term is negative, equilibrium prices do not exist. This problem is not specific to the above model and happens in other specifications such as a model with linear demand and log-linear marginal cost.

Explicitly, we derive the conditions for the number of equilibrium prices:

Proposition 1. *Let* $\Xi = \gamma_0 + \gamma_1 \frac{\alpha_0 + \alpha_3 \log Y_t + \varepsilon_t^d}{\alpha_1 + \alpha_2 Z_t^R} + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c$. *The number of equilibrium prices* $P_t^* > 0$ *is determined as follows:*

- *When* $1 - \theta(\alpha_1 + \alpha_2 Z_t^R) \leq 0$, *there is no equilibrium price,*
- *When* $1 - \theta(\alpha_1 + \alpha_2 Z_t^R) > 0$,
  - *If* $-\gamma_1/(\alpha_1 + \alpha_2 Z^R) = 1$, *there are infinitely many equilibrium prices when* $\exp(\Xi) = 1 - \theta(\alpha_1 + \alpha_2 Z_t^R)$, *but there is no equilibrium price otherwise,*
  - *If* $-\gamma_1/(\alpha_1 + \alpha_2 Z^R) \neq 1$, *there is a unique equilibrium price.*

See the online appendix for the proof and the detail.

3 Estimation

To estimate parameters in the demand and supply equations, a standard approach is to use a GMM estimation. Let $\xi = (\alpha_0, \alpha_1, \ldots, \alpha_3, \gamma_0, \gamma_1, \ldots, \gamma_3, \theta)$ be the vector of parameters in the demand and supply equations. See the nonlinear system ~~two-stage least square~~ approach (N2SLS) in online appendix.

3.1 MPEC estimator for the conduct parameter model

We propose a novel and simple estimator for the log-linear conduct parameter model by utilizing the MPEC procedure advocated by Su and Judd (2012). Our setting is similar to Dubé et al. (2012) in which the GMM objective function with equilibrium constraints is constructed.

To construct the sample analog of the unconditional moments, we use the demand equation (4) and the marginal cost function (5) directly. Then the residuals in the demand and supply equations are

$$\varepsilon_t^d(\xi) = \log P_t - \alpha_0 + (\alpha_1 + \alpha_2 Z_t^R) \log Q_t - \alpha_3 \log Y_t \tag{8}$$

$$\varepsilon_t^c(\xi) = \log MC_t - \gamma_0 - \gamma_1 \log Q_t - \gamma_2 \log W_t - \gamma_3 \log R_t. \tag{9}$$

Note that we ~~don't~~ know the value of MC_t. In the nonlinear system 2SLS, the value of MC_t is recovered ~~by~~ using the first-order condition (6) and we derived the supply equation based on the log-transformation of the first-order condition. An MPEC estimator avoids this log-transformation problem by putting the first-order condition as an equality constraint and regarding MC_t as a variable for the minimization problem. ~~We emphasize that~~ while we focus on the log-linear specification, MPEC allows for more flexible specifications on the demand and supply equations. ~~Also,~~ the programming cost of MPEC is lower than the standard approach because MPEC does not need analytical equilibrium expressions for constructing moment conditions.

By using the residuals in (8) and (9), we define the MPEC estimator as the vector ξ^* that solves the problem

$$\min_{\xi, \{MC_t\}_{t=1}^T} \quad g(\xi)^\top W g(\xi) \tag{10}$$

$$\text{s.t.} \quad P_t = \theta(\alpha_1 + \alpha_2 Z_t^R) P_t + MC_t, \quad t = 1, \ldots, T \tag{11}$$

$$0 \leq MC_t, \quad t = 1, \ldots, T. \tag{12}$$

Constraint (12) is necessary because $\log MC_t$ becomes invalid when MC_t is negative. We can also use the same weight matrix (23) in the N2SLS.

3.2 Other parameter constraints

Based on the assumptions in Proposition 1, we also impose the following constraints for the estimation of the conduct parameter model:

$$0 \leq \theta \leq 1, \tag{13}$$

$$\alpha_1 + \alpha_2 Z_t^R > 0, \quad \gamma_1 > 0, \quad t = 1, \ldots, T \tag{14}$$

$$1 - \theta(\alpha_1 + \alpha_2 Z_t^R) > 0, \quad t = 1, \ldots, T. \tag{15}$$

Constraint (13) is a standard assumption on the conduct parameter. Constraint~~s~~ (14) ~~imply~~ the downward slope of the demand curve and the upward slope of the marginal cost. As we explained in online appendix, they are not necessary for the existence of an equilibrium price. However, we impose them to make the demand ~~equation~~ and ~~the~~ supply ~~equation~~ realistic. Constraint~~s~~ (15) ~~are~~ necessary to avoid searching parameters over the domain in which equilibrium prices do not exist.

4 Simulation results

We compare MPEC estimation with the N2SLS with or without Constraints (14) and (15). Table 1 presents the results. First, as the sample size increases, the ~~RMSE~~ and bias decrease for both models. These magnitudes show comparable levels of the results of the linear model, shown in online appendix. Second, we focus on estimating the conduct parameter because the results in the other parameters are comparable. When the sample size is $T = 1500$, MPEC estimation with the slope and the equilibrium constraint has smaller bias (0.014) and RMSE (0.217) than those under N2SLS without these constraints (-0.058 and 0.281) and those under N2SLS with the constraint (-0.148 and 0.333). These differences ~~come~~ only from whether we solve log-transformation as a constraint ~~or not~~. Third, the percentage of convergence in MPEC estimation is larger than N2SLS. This is the practical advantage of MPEC over the standard approach. With additional experiments under various σ, these results are robust, shown in online appendix. ~~That is,~~ MPEC is a better estimation approach with slope and equilibrium existence constraints than N2SLS in estimating the conduct parameter.

5 Conclusion

We propose a constrained GMM estimator for conduct parameter estimation in homogeneous goods markets by formulating the estimation as an MPEC problem. Our approach avoids the complex transformation within the equilibrium conditions and allows a broader set of specifications. To use MPEC approach, we first derive equilibrium conditions for the unique existence of an equilibrium price. Second, we conduct Monte Carlo simulations and confirm that MPEC estimator with slope and equilibrium existence constraints performs better than N2SLS with or without slope and equilibrium existence constraints for estimating conduct parameters.

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