

11. Logaritmické rovnice

Úloha 1. Určete, za jakých podmínek jsou definovány tyto výrazy (tj. definiční obory):

- (a) $\log(-3x)$
- (b) $\log_7(2x + 7)$
- (c) $\log_3\left(\frac{x-1}{3-x}\right)$
- (d) $\ln(-2x^2 - 7x + 4)$
- (e) $\log_5 \sqrt{x+2}$
- (f) $\log(|x+3| - 4)$
- (g) $\frac{1}{\log x}$

Úloha 2. Řešte následující rovnice s neznámou $x \in \mathbb{R}$:

- (a) $\log_2(x+1) = 3$
- (b) $\log_{\frac{1}{2}}(2-x) = -2$
- (c) $\log_4(5x-4) = 2$
- (d) $\log_2 \frac{3-x}{x+3} = -2$
- (e) $\log_2 \log_3 \log_{\frac{1}{2}} x = 0$
- (f) $\log_2(14 + 2\log_7(1 + 2\log_{\frac{1}{2}} x)) = 4$
- (g) $\log(2x-3) = \log(3x-5)$
- (h) $\log_3(3-4x) = \log_3(2x-3)$
- (i) $\log_5(x^2+2x) = \log_5(-3x)$
- (j) $\log_2(x^2-x) = \log_2 x$
- (k) $\log x = 2\log 5 + \log 4$
- (l) $\frac{\log_3 x}{1 + \log_3 2} = 2$
- (m) $\log_6(x+1) + \log_6 x = 1$
- (n) $\log_2(x+7) - \log_2 x = 3$
- (o) $\log(x+3) = \log x + \log 3$
- (p) $\log_8 \sqrt{x+30} + \log_8 \sqrt{x} = 1$
- (q) $\log x^5 - \log x^4 + \log x^3 = 12$
- (r) $\log \sqrt{x} + \log \frac{1}{x^2} - \log x^3 + \frac{11}{2} = \frac{\log x^2}{1 + \log 10}$
- (s) $3\log 2x^2 + 2\log 3x^3 = 5\log x + 2\log 6x^3$
- (t) $\log 100x + \log 10x = 7$
- (u) $\frac{3}{2}\log \frac{x^2}{10} + \log \frac{100}{x^3} - \log \frac{\sqrt{10}}{x} = -2$
- (v) $\frac{\log_3(6x-2)}{\log_3(x-3)} = 2$
- (w) $\log_5\left(x - \frac{1}{4}\right) = -\log_5\left(x + \frac{7}{2}\right)$
- (x) $\log_2^2 x + 2\log_2 x - 3 = 0$
- (y) $\log_{\frac{1}{2}}^2(x+1) + 5\log_{\frac{1}{2}}(x+1) = 6$
- ★ (z) $\log_{\frac{1}{2}} x \cdot \log_{\frac{1}{2}} 4x = \frac{\log_{\frac{1}{2}} 16x}{\log_{\frac{1}{2}} 8} + \log_{\frac{1}{2}} 4$
- ★ (aa) $x^{\log x} = 100x$
- ★ (ab) $1000x^2 = x^{\log x}$
- ★ (ac) $(\sqrt{x})^{1+\log_2 x} = 2$

1. (a) $x \in (-\infty; 0)$ (b) $x \in (-\frac{7}{2}; \infty)$ (c) $x \in (1; 3)$ (d) $x \in (-4; \frac{1}{2})$

(e) $x \in (-2; \infty)$ (f) $x \in (-\infty; -7) \cup (1; \infty)$ (g) $x \in (0; 1) \cup (1; \infty)$

2. (a) $\{7\}$ (b) $\{-2\}$ (c) $\{4\}$ (d) $\{\frac{9}{5}\}$ (e) $\{\frac{1}{8}\}$ (f) $\{\frac{1}{8}\}$ (g) $\{2\}$ (h) \emptyset

(i) $\{-5\}$ (j) $\{2\}$ (k) $\{100\}$ (l) $\{36\}$ (m) $\{2\}$ (n) $\{1\}$ (o) $\{\frac{3}{2}\}$ (p) $\{2\}$

(q) $\{1000\}$ (r) $\{10\}$ (s) $\{\frac{1}{2}\}$ (t) $\{100\}$ (u) $\{\frac{1}{100}\}$ (v) $\{11\}$ (w) $\{\frac{1}{2}\}$

(x) $\{2; \frac{1}{8}\}$ (y) $\{-\frac{1}{2}; 63\}$ (z) $\{\frac{1}{2}; \frac{\sqrt[3]{2}}{2}\}$ (aa) $\{\frac{1}{10}; 100\}$ (ab) $\{\frac{1}{10}; 1000\}$

(ac) $\{\frac{1}{4}; 2\}$