Bayesian statistics for ecology
1. Motivation and simple example

Olivier Gimenez

March 2021

Credit where credit's due

- Ruth King, Byron Morgan, Steve Brooks (our workshops and Bayesian analysis for population ecology book).
- Richard McElreath (Statistical rethinking book and lecture videos).
- Jim Albert and Jingchen Hu (Probability and Bayesian modelling book).
- Materials shared by Tristan Marh, Jason Matthiopoulos, Francisco Rodriguez Sanchez, Kerrie Mengersen and Mark Lai.

Slides codes and data

- All material prepared with `R`.
- `R Markdown` used to write reproducible material.
- Slides available on FigShare here.
- Material available on Github there.

Objectives

- Try and demystify Bayesian statistics, and what we call MCMC.
- Make the difference between Bayesian and Frequentist analyses.
- Understand the Methods section of ecological papers doing Bayesian stuff.
- Run Bayesian analyses, safely hopefully.

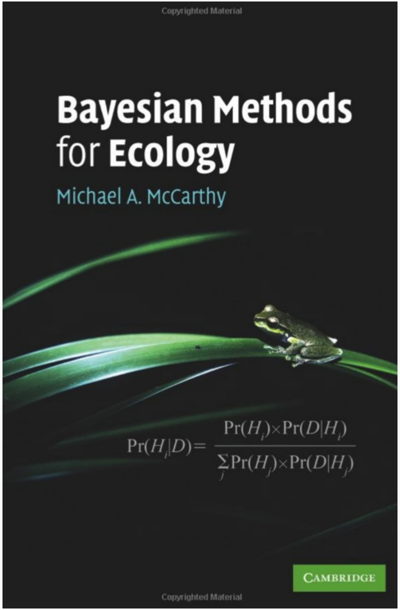

BRACE YOURSELF

What is on our plate?

1. Bayesian inference: Motivation and examples.
2. The likelihood.
3. A detour to explore priors.
4. Markov chains Monte Carlo methods (MCMC).
5. Bayesian analyses in R with the Jags software.
6. Contrast ecological hypotheses with model selection.
7. Heterogeneity and multilevel models (aka mixed models).

I want moooooore


I ONLY LIKE TWO THINGS:
THEY'RE BOTH BOOKS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Bayesian Methods for Ecology
Michael A. McCarthy
$\Pr(H_i|D) = \dfrac{\Pr(H_i) \times \Pr(D|H_i)}{\sum_j \Pr(H_j) \times \Pr(D|H_j)}$
CAMBRIDGE
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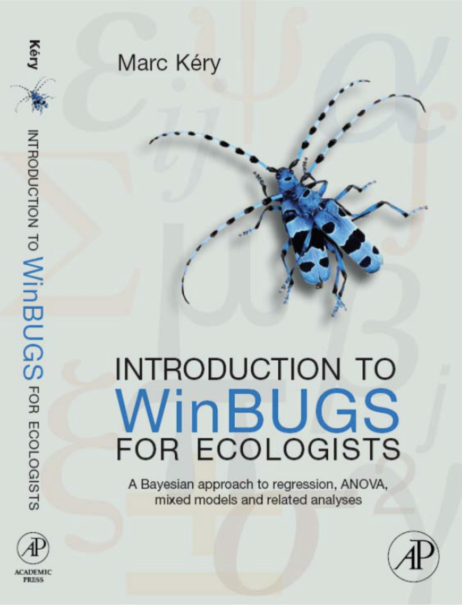
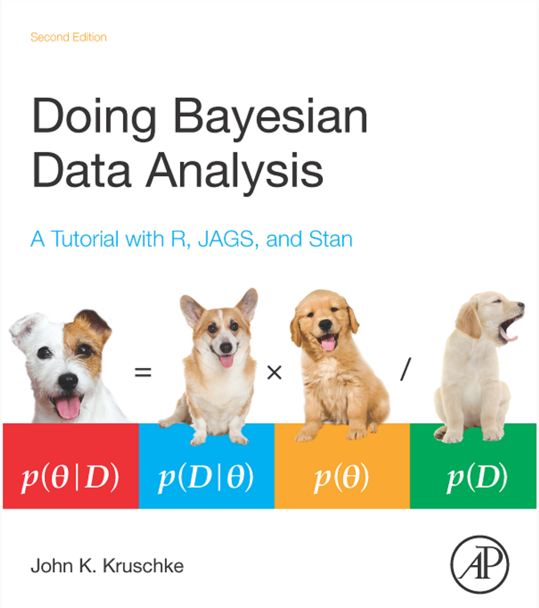
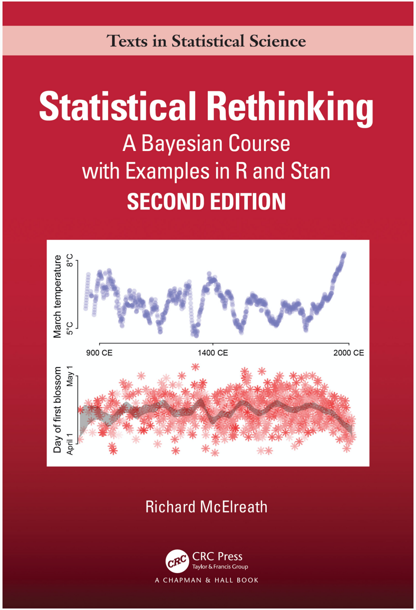
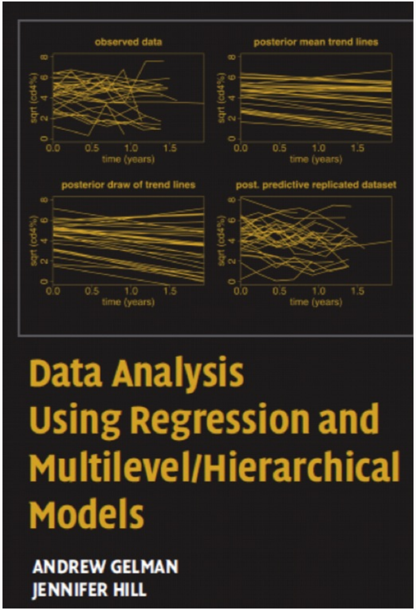

Kéry
INTRODUCTION TO WinBUGS FOR ECOLOGISTS
ACADEMIC PRESS
Marc Kéry
INTRODUCTION TO
WinBUGS
FOR ECOLOGISTS
A Bayesian approach to regression, ANOVA, mixed models and related analyses

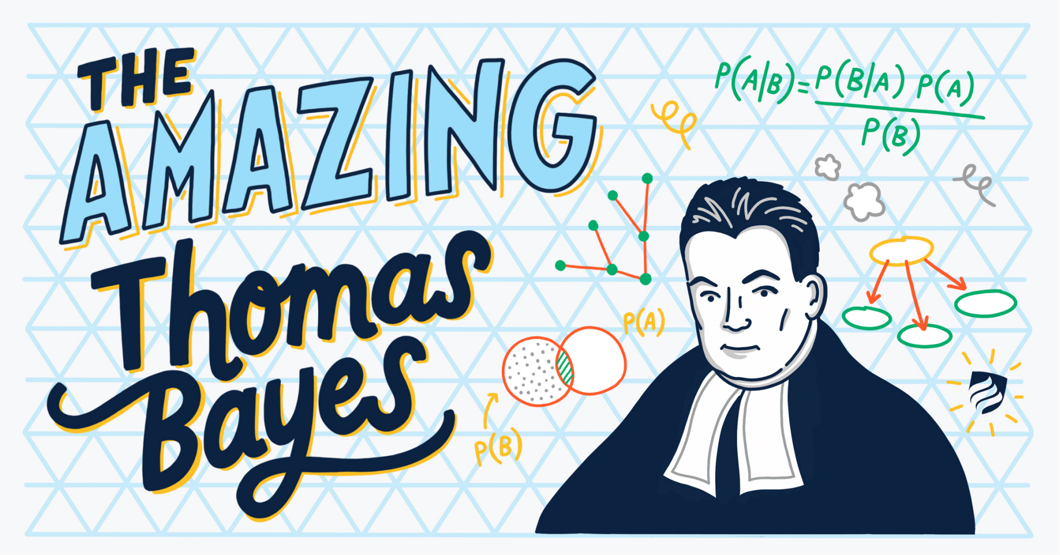

Second Edition
Doing Bayesian Data Analysis
A Tutorial with R, JAGS, and Stan
$p(\theta|D)$ = $p(D|\theta)$ × $p(\theta)$ / $p(D)$
John K. Kruschke


Texts in Statistical Science
Statistical Rethinking
A Bayesian Course
with Examples in R and Stan
SECOND EDITION
March temperature
8°C
5°C
900 CE
1400 CE
2000 CE
Day of first blossom
May 1
April 1
Richard McElreath
CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK



observed data
posterior mean trend lines
posterior draw of trend lines
post. predictive replicated dataset
sqrt (cd4%)
time (years)
Data Analysis Using Regression and Multilevel/Hierarchical Models
ANDREW GELMAN
JENNIFER HILL

What is Bayesian inference?


THE AMAZING
Thomas Bayes
P(A|B)= P(B|A) P(A) / P(B)
P(A)
P(B)

A reminder on conditional probabilities

- $\Pr(A \mid B)$: Probability of A given B

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HOW TO CURE VAMPIRES?
imgflip.com

Screening for vampirism

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- The chance of a negative test given you are mortal is $\Pr(-|\text{mortal}) = 0.95$ (**specificity**).

What is the question?

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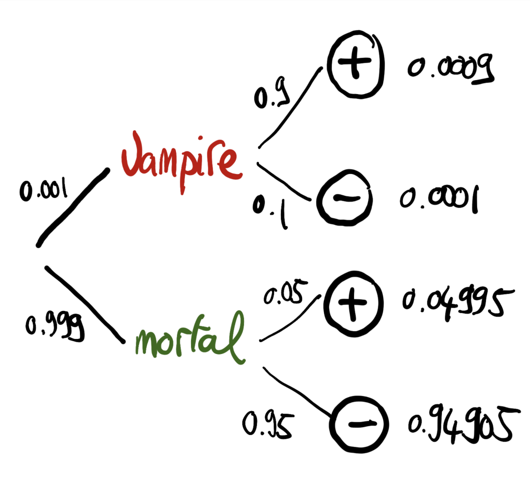
- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$
- Assume that vampires are rare, and represent only 0.1% of the population. This means that $\Pr(\text{vampire}) = 0.001$.

What is the answer? Bayes' theorem to the rescue!



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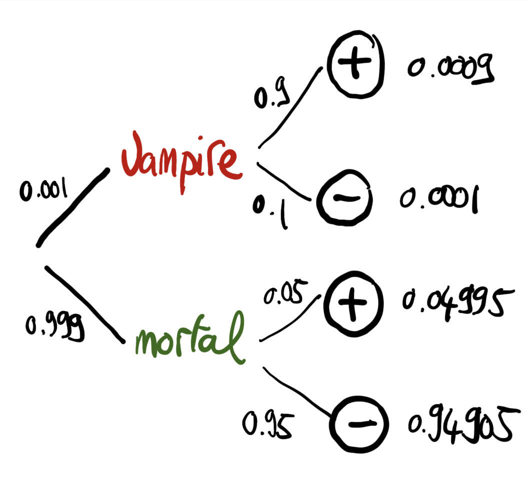

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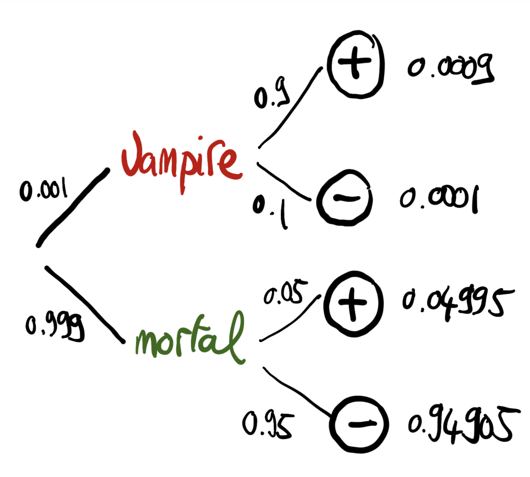

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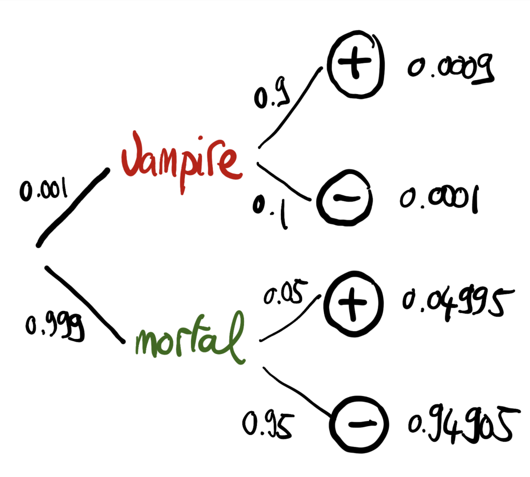

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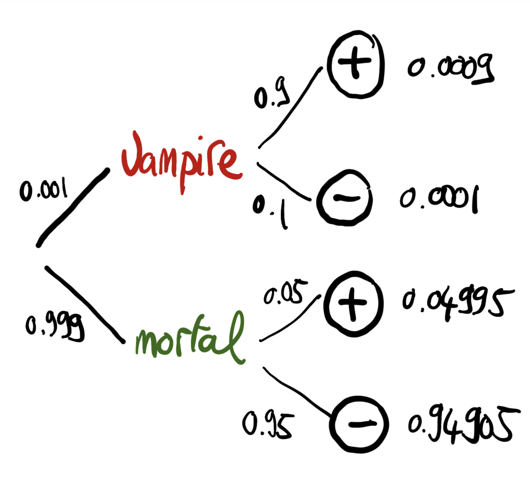

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$$\Pr(\text{vampire}|+) = \frac{\Pr(+|\text{vampire}) \; \Pr(\text{vampire})}{\Pr(+)}$$

Your turn

Screening for vampirism

- Suppose the diagnostic test has the same sensitivity and specificity but vampirism is more common: 10% of the population is vampire.
- What is the probability that a person is a vampire, given that the test is positive?

Solution

The probability that a person is a vampire, given that the test is positive

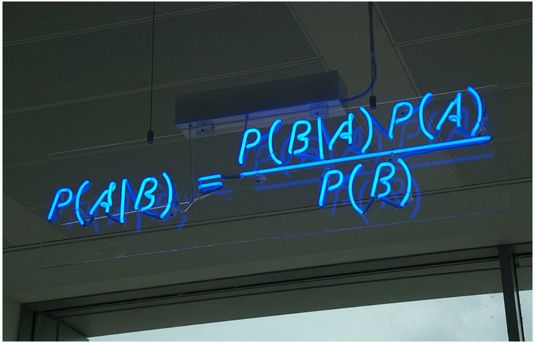
- $\Pr(+|\text{vampire}) = 0.9$
- $\Pr(-|\text{mortal}) = 0.95$
- $\Pr(\text{vampire}) = 0.1$

$$\begin{aligned}\Pr(+) &= \Pr(+|\text{vampire})\Pr(\text{vampire}) + \Pr(+|\text{mortal})\Pr(\text{mortal}) \\ &= 0.9 * 0.1 + 0.05 * 0.9 \\ &= 0.135\end{aligned}$$

$$\begin{aligned}\Pr(\text{vampire}|+) &= \Pr(+|\text{vampire})\Pr(\text{vampire})/\Pr(+) \\ &= 0.9 * 0.1/0.135\end{aligned}$$

Bayes' theorem

- A theorem about conditional probabilities.
- $\Pr(B \mid A) = \dfrac{\Pr(A \mid B)\ \Pr(B)}{\Pr(A)}$



Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis})\ \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

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- The "hypothesis" is typically something unobserved or unknown. It's what you want to learn about using the data.
- For regression models, the "hypothesis" is a parameter (intercept, slopes or error terms).
- Bayes theorem tells you the probability of the hypothesis given the data.

What is doing science after all?

How plausible is some hypothesis given the data?

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \; \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

Why is Bayesian statistics not the default?

- Due to practical problems of implementing the Bayesian approach, and some wars of male statisticians's egos, little advance was made for over two centuries.

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- Due to practical problems of implementing the Bayesian approach, and some wars of male statisticians's egos, little advance was made for over two centuries.
- Recent advances in computational power coupled with the development of new methodology have led to a great increase in the application of Bayesian methods within the last two decades.

Frequentist versus Bayesian

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- Typical stats problems involve estimating parameter θ with available data.
- The frequentist approach (**maximum likelihood estimation** – MLE) assumes that the parameters are fixed, but have unknown values to be estimated.
- Classical estimates generally provide a point estimate of the parameter of interest.
- The Bayesian approach assumes that the parameters are not fixed but have some fixed unknown distribution - a distribution for the parameter.

What is the Bayesian approach?

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- The approach is based upon the idea that the experimenter begins with some prior beliefs about the system.
- And then updates these beliefs on the basis of observed data.
- This updating procedure is based upon the Bayes' Theorem:

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What is the Bayesian approach?

- Schematically if $A = \theta$ and $B = \text{data}$, then
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$$\Pr(A \mid B) = \frac{\Pr(B \mid A)\ \Pr(A)}{\Pr(B)}$$

- Translates into:

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta)\ \Pr(\theta)}{\Pr(\text{data})}$$

Bayes' theorem

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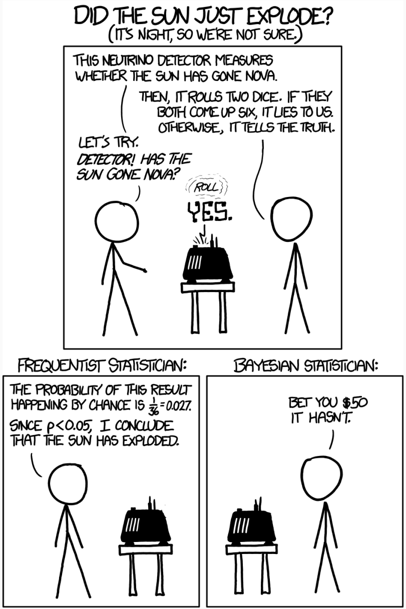
- Prior distribution: Represents what you know before seeing the data. The source of much discussion about the Bayesian approach.
- $\Pr(\text{data}) = \int L(\text{data} \mid \theta)\ \Pr(\theta) d\theta$: Possibly high-dimensional integral, difficult if not impossible to calculate. This is one of the reasons why we need simulation (MCMC) methods - more soon.


DID THE SUN JUST EXPLODE?
(IT'S NIGHT, SO WE'RE NOT SURE.)
THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.
THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.
LET'S TRY.
DETECTOR! HAS THE SUN GONE NOVA?
ROLL
YES.
FREQUENTIST STATISTICIAN:
THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS $\frac{1}{36}=0.027$.
SINCE $p<0.05$, I CONCLUDE THAT THE SUN HAS EXPLODED.
BAYESIAN STATISTICIAN:
BET YOU $50 IT HASN'T.