

Bayesian statistics with R

1. An introduction to Bayesian inference

Olivier Gimenez

November-December 2023

Credit where credit's due

- Ruth King, Byron Morgan, Steve Brooks (our workshops and [Bayesian analysis for population ecology book](#)).
- Richard McElreath ([Statistical rethinking](#) book and lecture videos).
- Jim Albert and Jingchen Hu ([Probability and Bayesian modelling](#) book).
- Materials shared by [Tristan Marh](#), [Jason Matthiopoulos](#), [Francisco Rodriguez Sanchez](#), [Kerrie Mengerson](#) and [Mark Lai](#).

Slides, code and data

- All material prepared with R.
- R Markdown used to write reproducible material.
- Dedicated website <https://oliviergimenez.github.io/bayesian-stats-with-R/>.

Objectives

- Try and demystify Bayesian statistics, and what we call MCMC.
- Make the difference between Bayesian and Frequentist analyses.
- Understand the Methods section of ecological papers doing Bayesian stuff.
- Run Bayesian analyses, safely hopefully.

BRACE YOURSELF



What is on our plate?

1. An introduction to Bayesian inference
2. The likelihood
3. Bayesian analyses by hand
4. A detour to explore priors
5. Markov chains Monte Carlo methods (MCMC)
6. Bayesian analyses in R with the Jags software
7. Contrast scientific hypotheses with model selection
8. Heterogeneity and multilevel models (aka mixed models)

I want mooooore

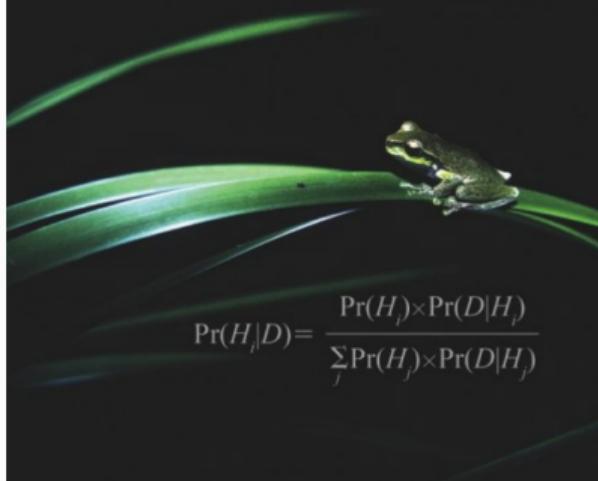
I ONLY LIKE TWO
THINGS:

THEY'RE BOTH BOOKS



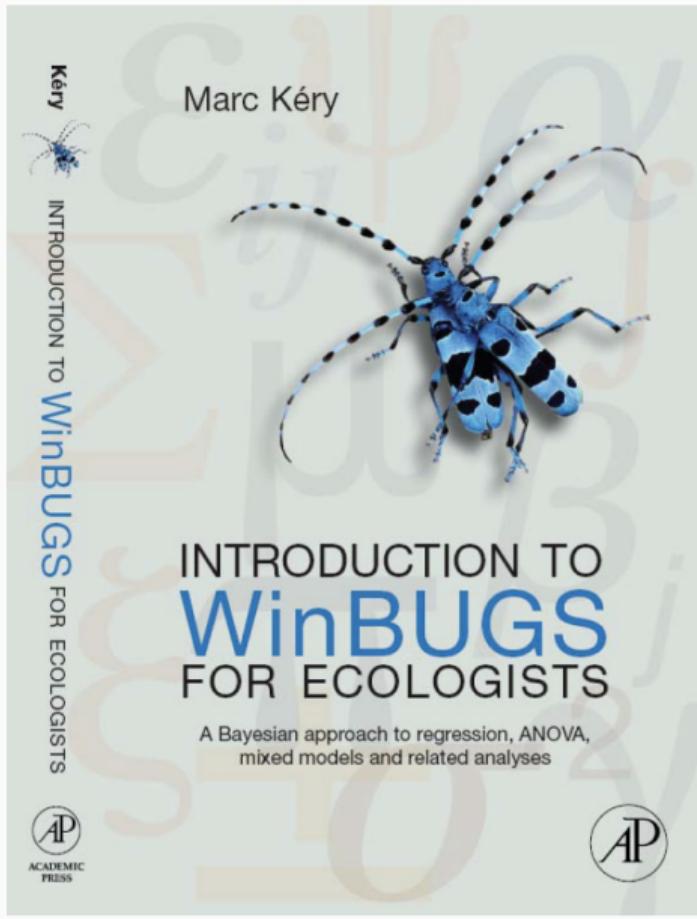
Bayesian Methods for Ecology

Michael A. McCarthy



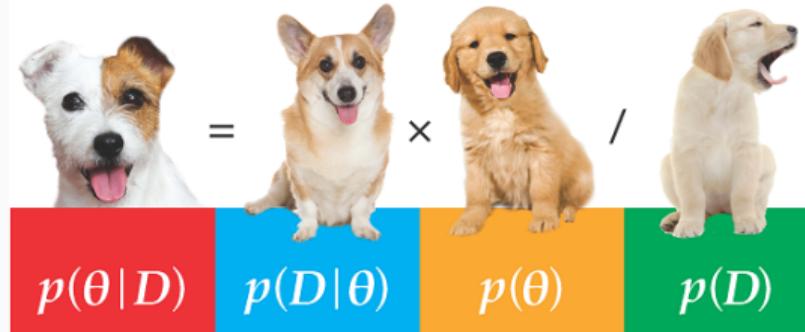
$$\Pr(H_i|D) = \frac{\Pr(H_i) \times \Pr(D|H_i)}{\sum_j \Pr(H_j) \times \Pr(D|H_j)}$$

CAMBRIDGE



Doing Bayesian Data Analysis

A Tutorial with R, JAGS, and Stan



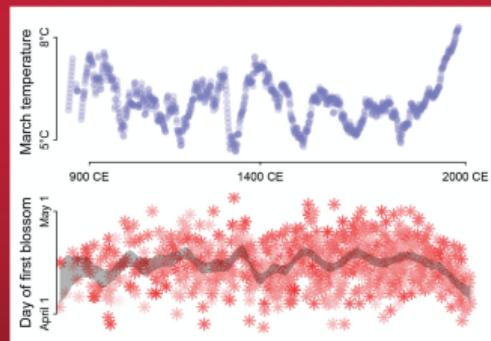
John K. Kruschke



Texts in Statistical Science

Statistical Rethinking

A Bayesian Course
with Examples in R and Stan
SECOND EDITION

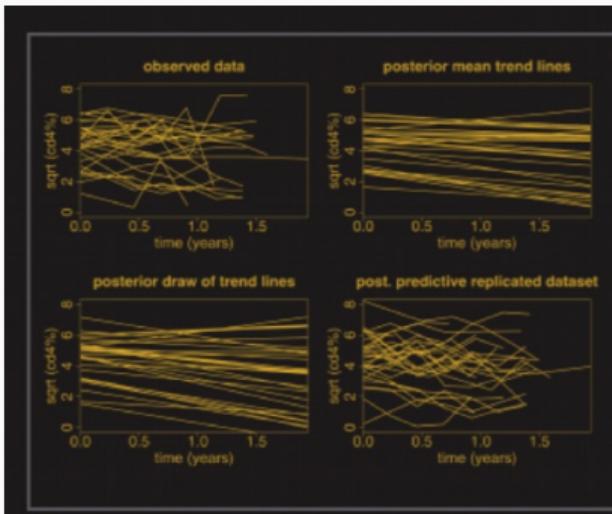


Richard McElreath



Taylor & Francis Group

A CHAPMAN & HALL BOOK

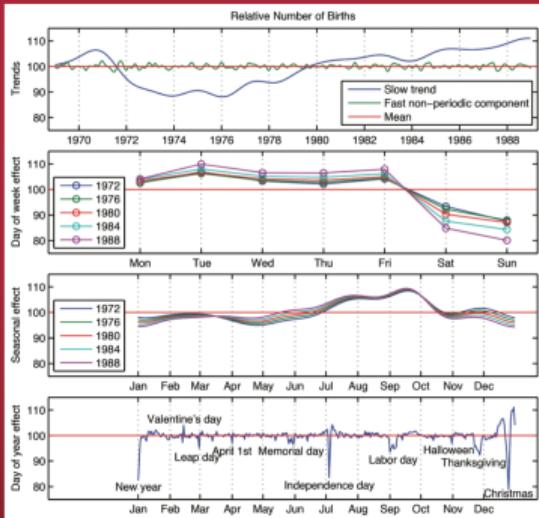


Data Analysis Using Regression and Multilevel/Hierarchical Models

ANDREW GELMAN
JENNIFER HILL

Bayesian Data Analysis

Third Edition

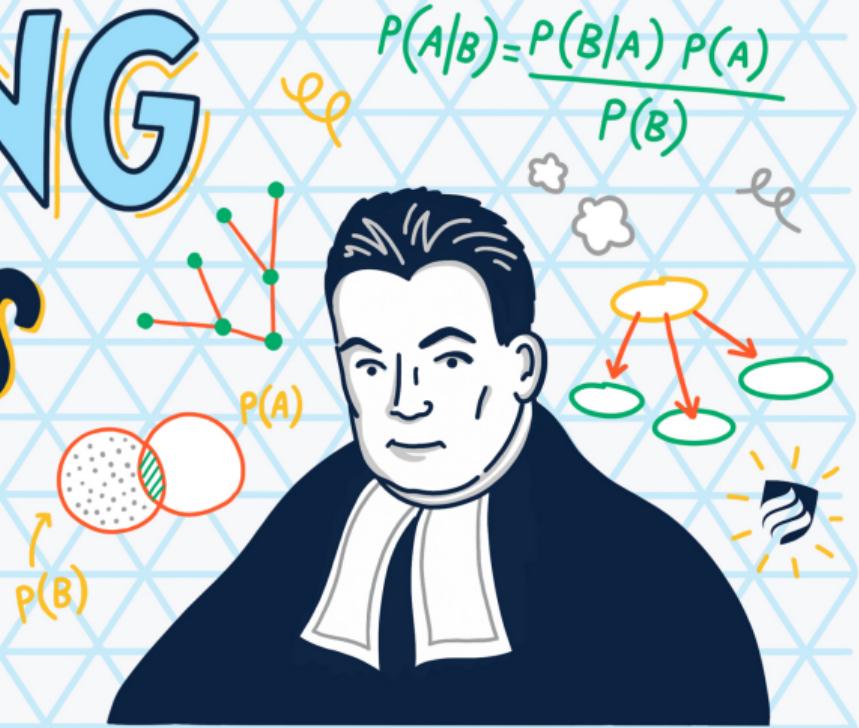


Andrew Gelman, John B. Carlin, Hal S. Stern,
David B. Dunson, Aki Vehtari, and Donald B. Rubin

Free at <http://www.stat.columbia.edu/~gelman/book/>

What is Bayesian inference?

THE AMAZING Thomas Bayes



A reminder on conditional probabilities

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HOW TO CURE VAMPIRES?

Screening for vampirism

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- The chance of a negative test given you are mortal is $\Pr(-|\text{mortal}) = 0.95$ (**specificity**).

What is the question?

- From the perspective of the test: Given a person is a vampire, what is the probability that the test is positive? $\Pr(+|\text{vampire}) = 0.90$.

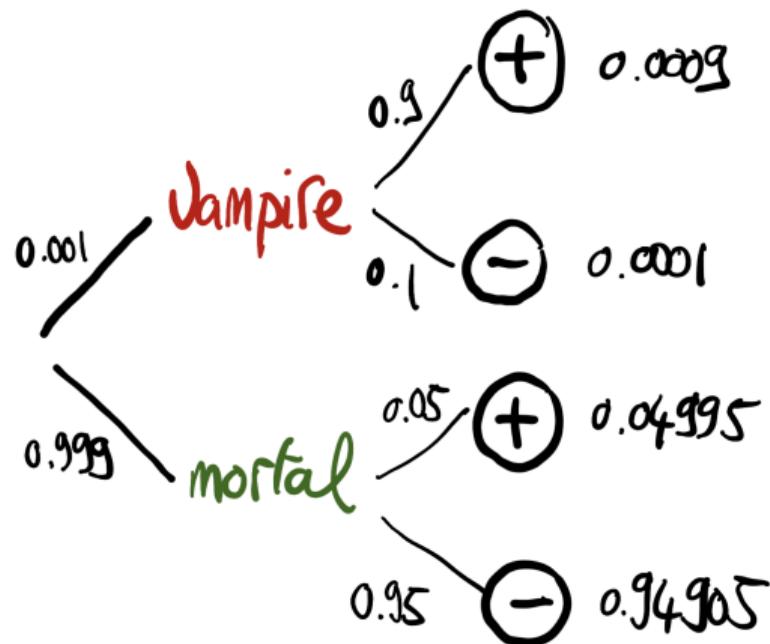
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- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$

What is the question?

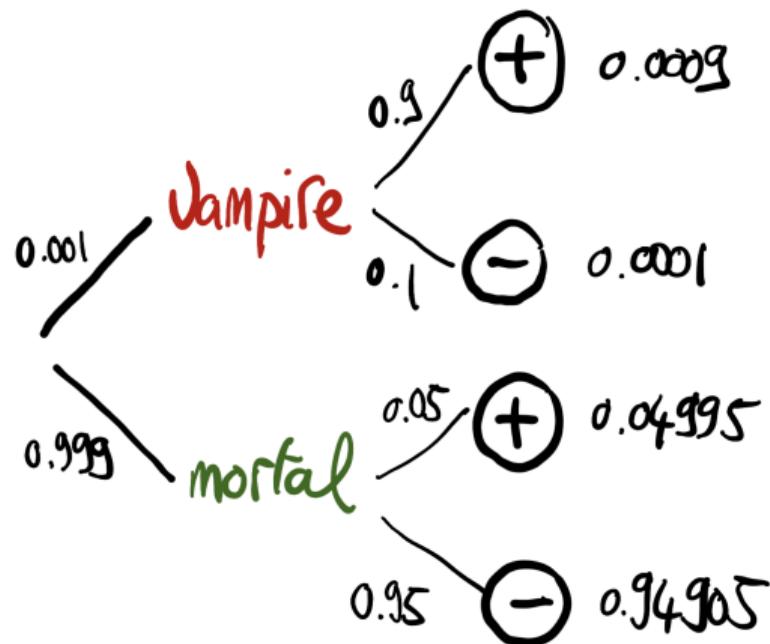
- From the perspective of the test: Given a person is a vampire, what is the probability that the test is positive? $\Pr(+|\text{vampire}) = 0.90$.
- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$
- Assume that vampires are rare, and represent only 0.1% of the population. This means that $\Pr(\text{vampire}) = 0.001$.

What is the answer? Bayes' theorem to the rescue!



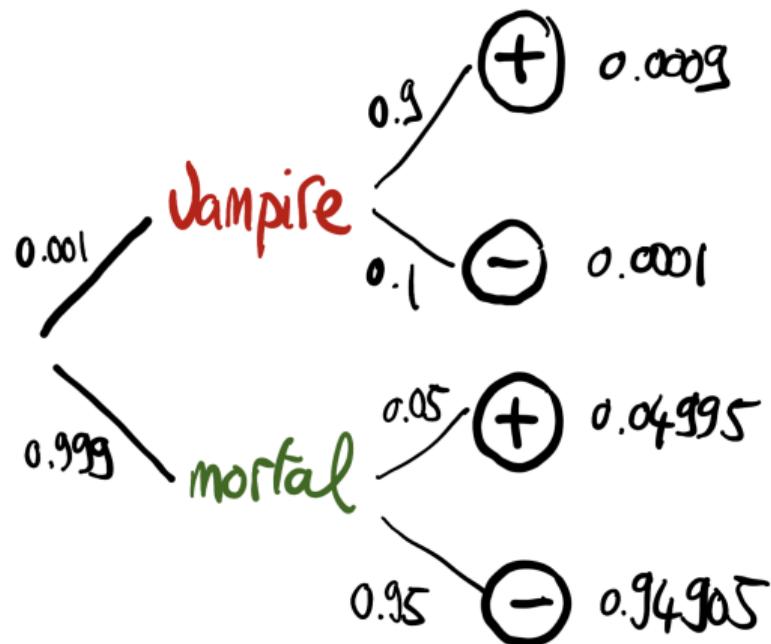
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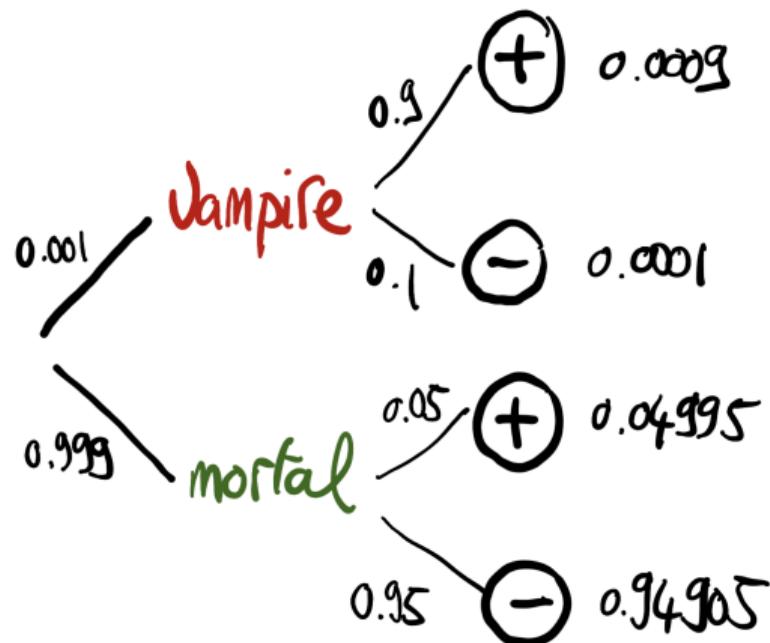
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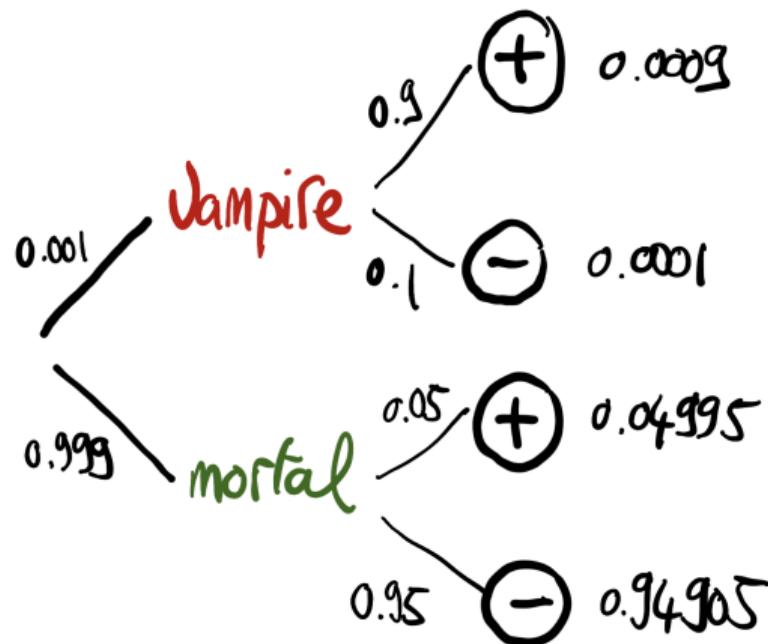
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$$\Pr(\text{vampire}|+) = \frac{\Pr(+|\text{vampire}) \Pr(\text{vampire})}{\Pr(+)}$$

Your turn: Practical 1

Bayes' theorem

- A theorem about conditional probabilities.
- $\Pr(B | A) = \frac{\Pr(A | B) \Pr(B)}{\Pr(A)}$

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Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

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- For regression models, the “hypothesis” is a parameter (intercept, slopes or error terms).
- Bayes theorem tells you the probability of the hypothesis given the data.

What is doing science after all?

How plausible is some hypothesis given the data?

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

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- Due to practical problems of implementing the Bayesian approach, and some wars of male statisticians's egos, little advance was made for over two centuries.
- Recent advances in computational power coupled with the development of new methodology have led to a great increase in the application of Bayesian methods within the last two decades.

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- Typical stats problems involve estimating parameter θ with available data.
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- Classical estimates generally provide a point estimate of the parameter of interest.
- The Bayesian approach assumes that the parameters are not fixed but have some fixed unknown distribution - a distribution for the parameter.

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- And then updates these beliefs on the basis of observed data.
- This updating procedure is based upon the Bayes' Theorem:

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$$\Pr(A | B) = \frac{\Pr(B | A) \Pr(A)}{\Pr(B)}$$

- Translates into:

$$\Pr(\theta | \text{data}) = \frac{\Pr(\text{data} | \theta) \Pr(\theta)}{\Pr(\text{data})}$$

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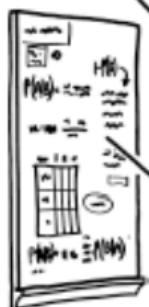
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- **$\Pr(\text{data}) = \int \Pr(\text{data} \mid \theta) \Pr(\theta) d\theta$:** Possibly high-dimensional integral, difficult if not impossible to calculate. This is one of the reasons why we need simulation (MCMC) methods - more soon.

GIVEN THESE PREVALENCES,
IS IT LIKELY THAT THE TEST
RESULT IS A FALSE POSITIVE?

WELL, THIS CHAPTER IS ON
BAYES' THEOREM, SO YES.



SOMETIMES, IF YOU UNDERSTAND
BAYES' THEOREM WELL ENOUGH,
YOU DON'T NEED IT.