

Bayesian statistics with R

1. An introduction to Bayesian inference

Olivier Gimenez

March 2021

Credit where credit's due

- Ruth King, Byron Morgan, Steve Brooks (our workshops and [Bayesian analysis for population ecology book](#)).
- Richard McElreath ([Statistical rethinking](#) book and lecture videos).
- Jim Albert and Jingchen Hu ([Probability and Bayesian modelling](#) book).
- Materials shared by [Tristan Marh](#), [Jason Matthiopoulos](#), [Francisco Rodriguez Sanchez](#), [Kerrie Mengerson](#) and [Mark Lai](#).

Slides, code and data

- All material prepared with R.
- R Markdown used to write reproducible material.
- Dedicated website <https://oliviergimenez.github.io/bayesian-stats-with-R/>.

Objectives

- Try and demystify Bayesian statistics, and what we call MCMC.
- Make the difference between Bayesian and Frequentist analyses.
- Understand the Methods section of ecological papers doing Bayesian stuff.
- Run Bayesian analyses, safely hopefully.

BRACE YOURSELF



What is on our plate?

1. An introduction to Bayesian inference
2. The likelihood
3. Bayesian analyses by hand
4. A detour to explore priors
5. Markov chains Monte Carlo methods (MCMC)
6. Bayesian analyses in R with the Jags software
7. Contrast scientific hypotheses with model selection
8. Heterogeneity and multilevel models (aka mixed models)

I want mooooore

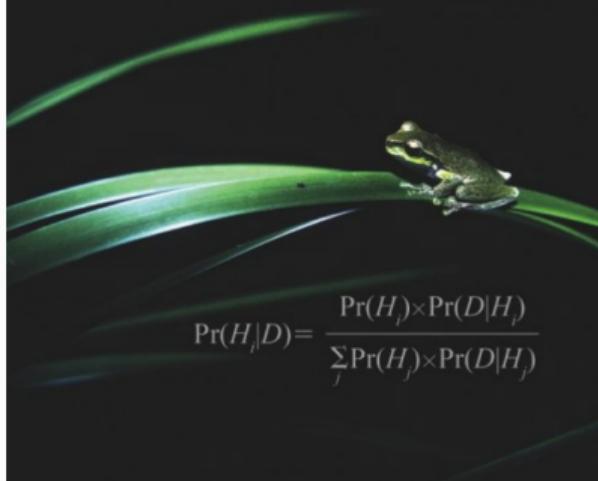
I ONLY LIKE TWO
THINGS:

THEY'RE BOTH BOOKS



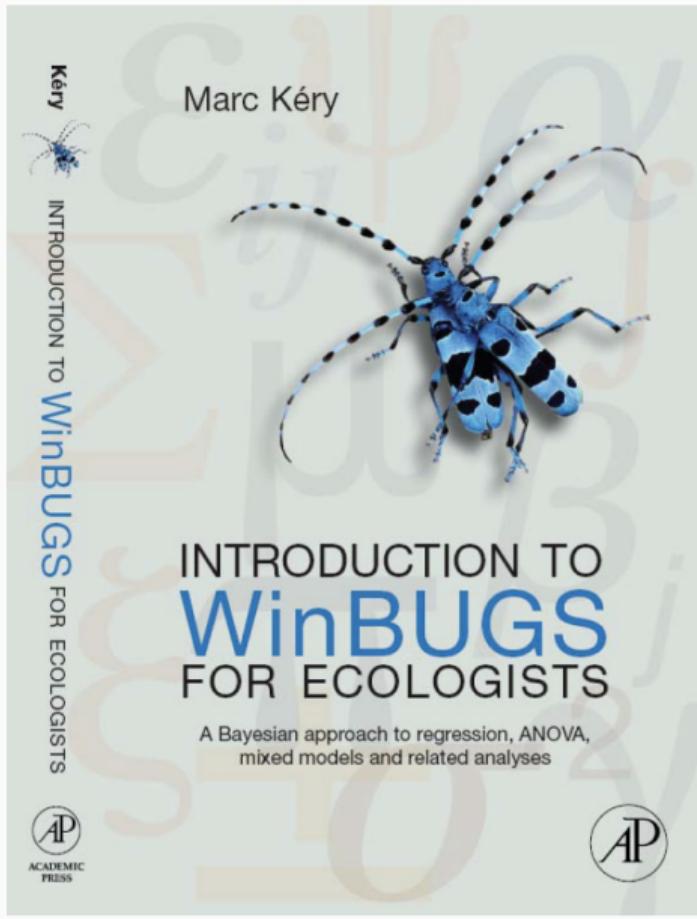
Bayesian Methods for Ecology

Michael A. McCarthy



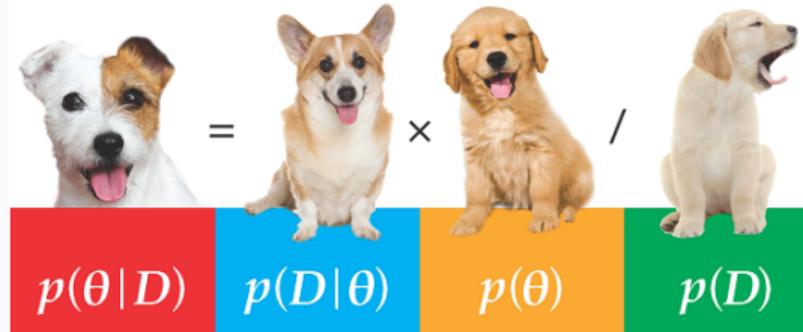
$$\Pr(H_i|D) = \frac{\Pr(H_i) \times \Pr(D|H_i)}{\sum_j \Pr(H_j) \times \Pr(D|H_j)}$$

CAMBRIDGE



Doing Bayesian Data Analysis

A Tutorial with R, JAGS, and Stan



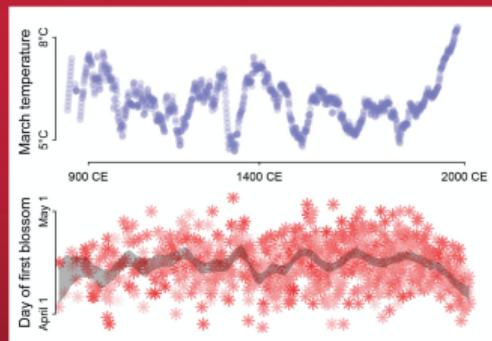
John K. Kruschke



Texts in Statistical Science

Statistical Rethinking

A Bayesian Course
with Examples in R and Stan
SECOND EDITION

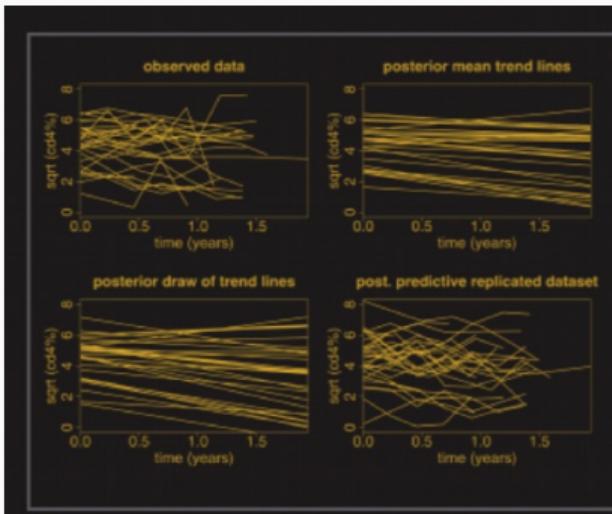


Richard McElreath



Taylor & Francis Group

A CHAPMAN & HALL BOOK

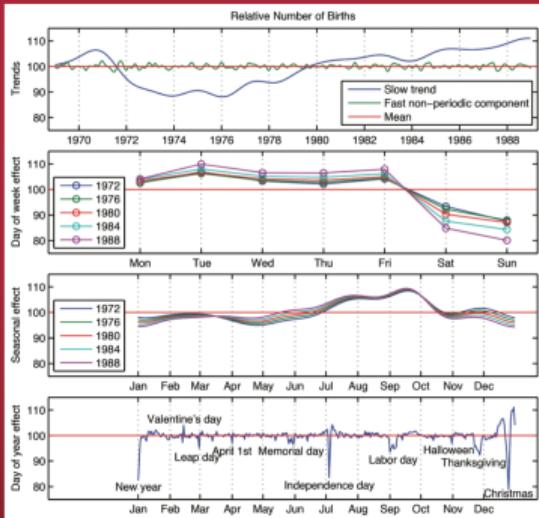


Data Analysis Using Regression and Multilevel/Hierarchical Models

ANDREW GELMAN
JENNIFER HILL

Bayesian Data Analysis

Third Edition

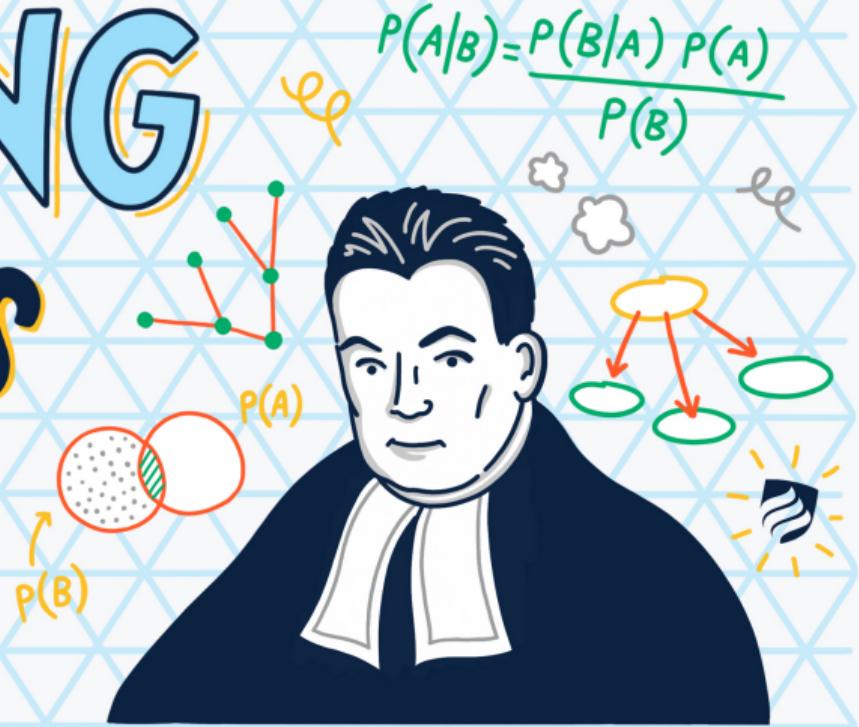


Andrew Gelman, John B. Carlin, Hal S. Stern,
David B. Dunson, Aki Vehtari, and Donald B. Rubin

Free at <http://www.stat.columbia.edu/~gelman/book/>

What is Bayesian inference?

THE AMAZING Thomas Bayes



A reminder on conditional probabilities

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HOW TO CURE VAMPIRES?

Screening for vampirism

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- The chance of a negative test given you are mortal is $\Pr(-|\text{mortal}) = 0.95$ (**specificity**).

What is the question?

- From the perspective of the test: Given a person is a vampire, what is the probability that the test is positive? $\Pr(+|\text{vampire}) = 0.90$.

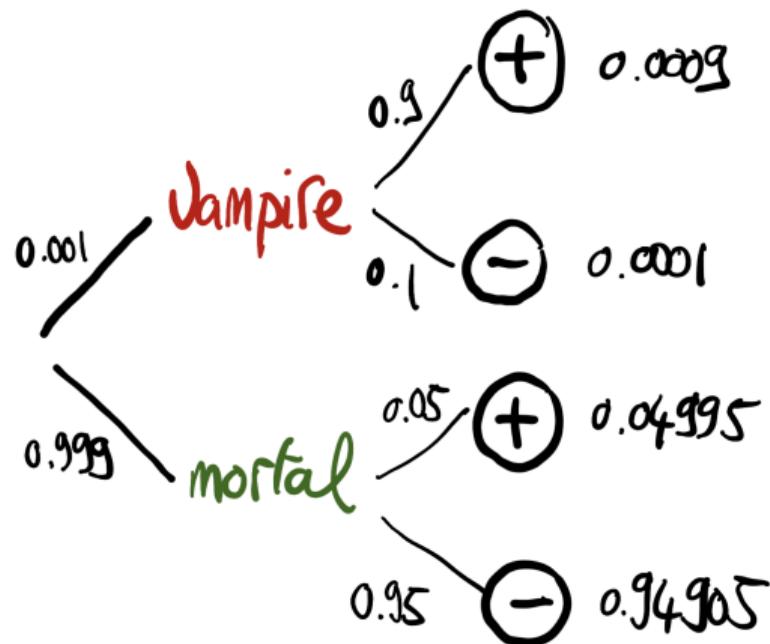
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- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$

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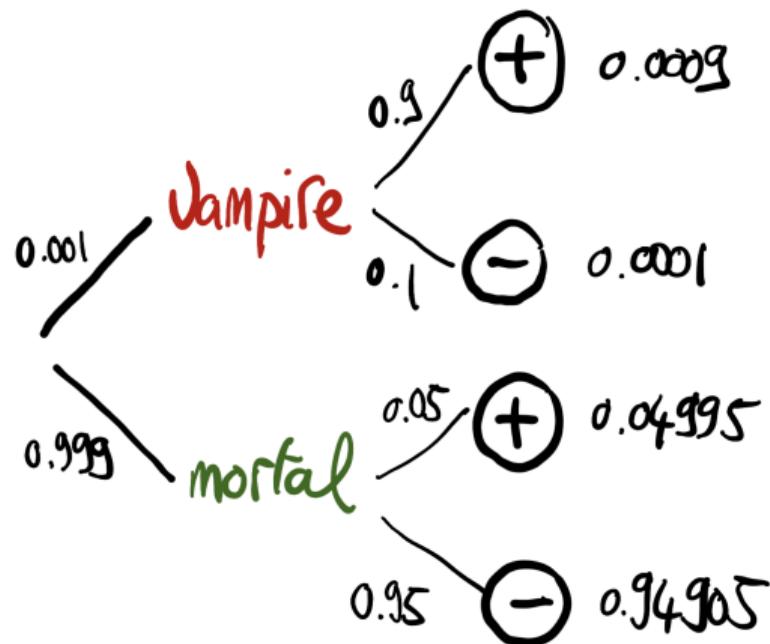
- From the perspective of the test: Given a person is a vampire, what is the probability that the test is positive? $\Pr(+|\text{vampire}) = 0.90$.
- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$
- Assume that vampires are rare, and represent only 0.1% of the population. This means that $\Pr(\text{vampire}) = 0.001$.

What is the answer? Bayes' theorem to the rescue!



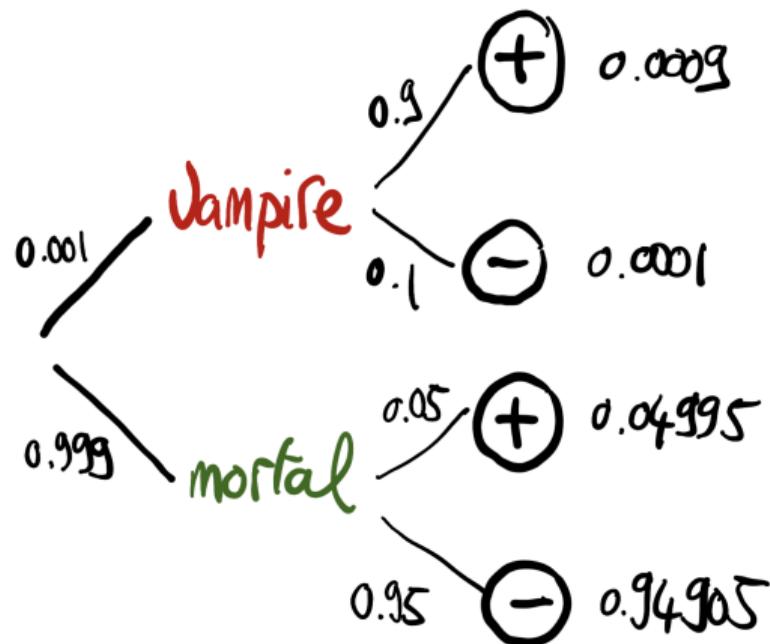
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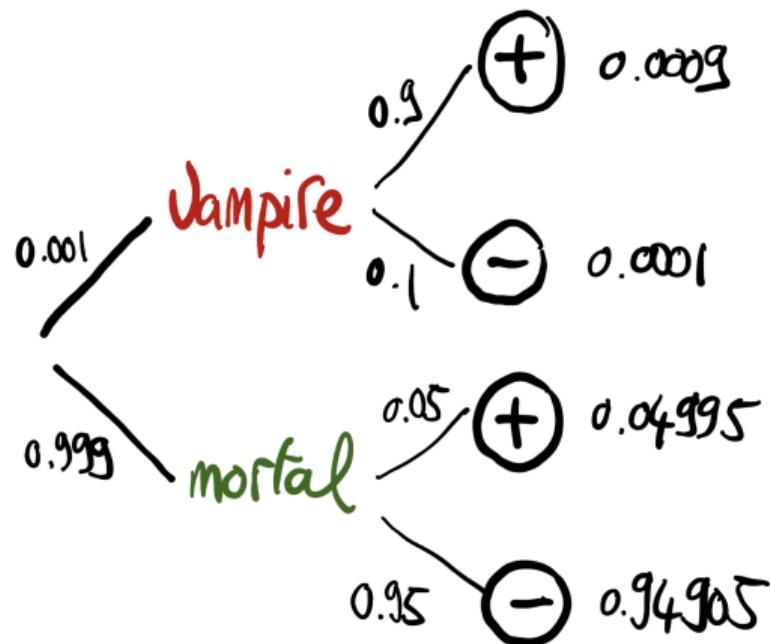
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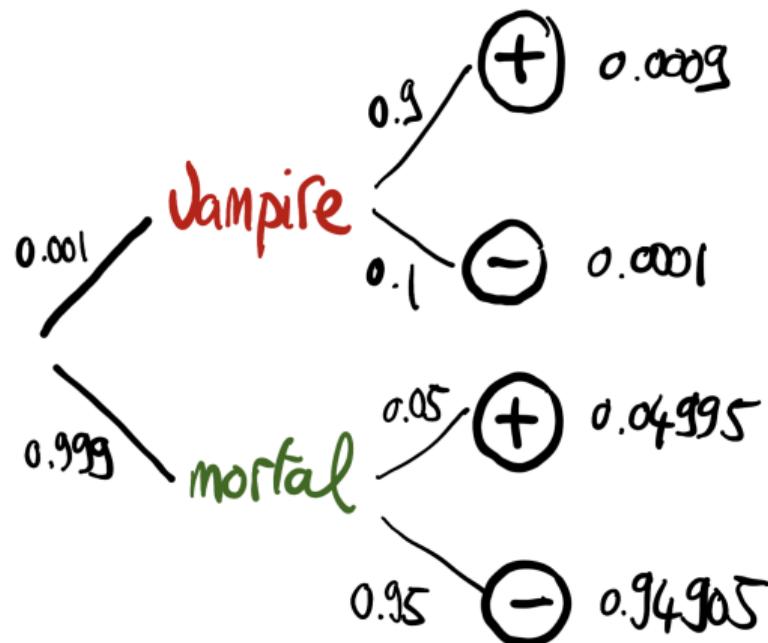
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$$\Pr(\text{vampire}|+) = \frac{\Pr(+|\text{vampire}) \Pr(\text{vampire})}{\Pr(+)}$$

Your turn: Practical 1

Bayes' theorem

- A theorem about conditional probabilities.
- $\Pr(B | A) = \frac{\Pr(A | B) \Pr(B)}{\Pr(A)}$

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Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

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- For regression models, the “hypothesis” is a parameter (intercept, slopes or error terms).
- Bayes theorem tells you the probability of the hypothesis given the data.

What is doing science after all?

How plausible is some hypothesis given the data?

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

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- Due to practical problems of implementing the Bayesian approach, and some wars of male statisticians's egos, little advance was made for over two centuries.
- Recent advances in computational power coupled with the development of new methodology have led to a great increase in the application of Bayesian methods within the last two decades.

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- Typical stats problems involve estimating parameter θ with available data.
- The frequentist approach (**maximum likelihood estimation** – MLE) assumes that the parameters are fixed, but have unknown values to be estimated.
- Classical estimates generally provide a point estimate of the parameter of interest.
- The Bayesian approach assumes that the parameters are not fixed but have some fixed unknown distribution - a distribution for the parameter.

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- And then updates these beliefs on the basis of observed data.
- This updating procedure is based upon the Bayes' Theorem:

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- Schematically if $A = \theta$ and $B = \text{data}$, then
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$$\Pr(A | B) = \frac{\Pr(B | A) \Pr(A)}{\Pr(B)}$$

- Translates into:

$$\Pr(\theta | \text{data}) = \frac{\Pr(\text{data} | \theta) \Pr(\theta)}{\Pr(\text{data})}$$

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- **$\Pr(\text{data}) = \int \Pr(\text{data} \mid \theta) \Pr(\theta) d\theta$:** Possibly high-dimensional integral, difficult if not impossible to calculate. This is one of the reasons why we need simulation (MCMC) methods - more soon.

DID THE SUN JUST EXPLODE?

(IT'S NIGHT, SO WE'RE NOT SURE.)

THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.

THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.

LET'S TRY.

DETECTOR! HAS THE SUN GONE NOVA?

(ROLL)

YES.



FREQUENTIST STATISTICIAN:

THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS $\frac{1}{36} = 0.027$. SINCE $p < 0.05$, I CONCLUDE THAT THE SUN HAS EXPLODED.



Bayesian Statistician:

BET YOU \$50 IT HASN'T.

