Optimierung Blatt 03 zum 04.11.2013

Paul Bienkowski, Arne Struck

4. November 2013

1. a) **Starttableau**:

$$\begin{array}{rcrcrcr} x_3 & = & 1 & + & 4x_1 & - & x_2 \\ x_4 & = & 2 & + & x_1 & - & x_2 \\ x_5 & = & 1 & - & \frac{1}{2}x_1 & + & x_2 \\ \hline z & = & & & x_1 & + & 2x_2 \end{array}$$

Eingangsvariable: x_2, Ausgangsvariable: x_3
$x_2 = 1 + 4x_1 - x_3$
$x_4 = 2 + x_1 - (1 + 4x_1 - x_3) = 1 - 3x_1 + x_3$
$x_5 = 1 - \frac{1}{2}x_1 + (1 + 4x_1 - x_3) = 2 + \frac{7}{2}x_1 - x_3$
$z = x_1 + 2(1 + 4x_1 - x_3) = 2 + 9x_1 - 2x_3$

Tableau nach 1. Iteration:

$$\begin{array}{rcrcrcr} x_2 & = & 1 & + & 4x_1 & - & x_3 \\ x_4 & = & 1 & - & 3x_1 & + & x_3 \\ x_5 & = & 2 & + & \frac{7}{2}x_1 & - & x_3 \\ \hline z & = & 2 & + & 9x_1 & - & 2x_3 \end{array}$$

Eingangsvariable: x_1, Ausgangsvariable: x_4
$x_1 = \frac{1}{3} + \frac{1}{3}x_3 - \frac{1}{3}x_4$
$x_2 = 1 + 4(\frac{1}{3} + \frac{1}{3}x_3 - \frac{1}{3}x_4) - x_3 = \frac{7}{3} + \frac{1}{3}x_3 - \frac{4}{3}x_4$
$x_5 = 2 + \frac{7}{2}(\frac{1}{3} + \frac{1}{3}x_3 - \frac{1}{3}x_4) - x_3 = \frac{19}{6} + \frac{1}{6}x_3 - \frac{7}{6}x_4$
$z = 2 + 9(\frac{1}{3} + \frac{1}{3}x_3 - \frac{1}{3}x_4) - 2x_3 = 5 + x_3 - 3x_4$

Tableau nach 2. Iteration:

$$\begin{array}{rcrcrcr} x_1 & = & \frac{1}{3} & + & \frac{1}{3}x_3 & - & \frac{1}{3}x_4 \\ x_2 & = & \frac{7}{3} & + & \frac{1}{3}x_3 & - & \frac{4}{3}x_4 \\ x_5 & = & \frac{19}{6} & + & \frac{1}{6}x_3 & - & \frac{7}{3}x_4 \\ \hline z & = & 5 & + & x_3 & - & 3x_4 \end{array}$$

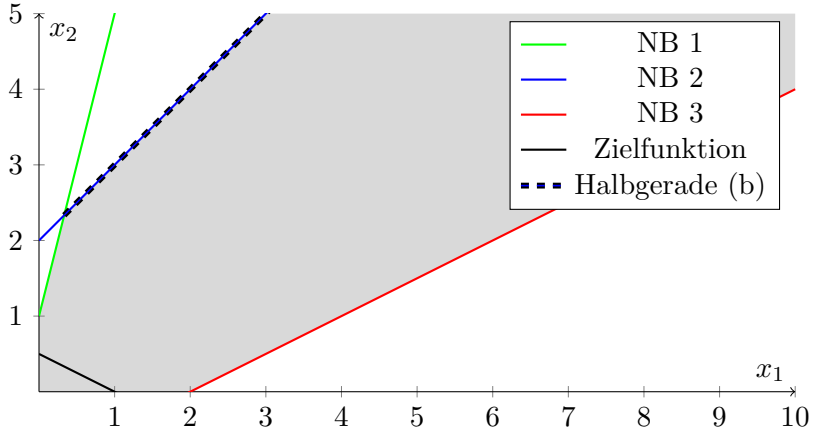
Eingangsvariable: x_3, Ausgangsvariable: keine möglich $\rightarrow$ unbeschränkt
$x_3 = t \Rightarrow x_1 = \frac{1}{3} + \frac{1}{3}t$, $x_2 = \frac{7}{3} + \frac{1}{3}t$

$$\vec{x} = \begin{pmatrix} \frac{1}{3} \\ \frac{7}{3} \end{pmatrix} + t \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

Zulässige Lösungen:

Startlösung :	$x_1 = 0$	$x_2 = 0$	$x_3 = 1$	$x_4 = 2$	$x_5 = 1$	$z = 0$
1. Iteration :	$x_1 = 0$	$x_2 = 1$	$x_3 = 0$	$x_4 = 1$	$x_5 = 2$	$z = 2$
2. Iteration :	$x_1 = \frac{1}{3}$	$x_2 = \frac{7}{3}$	$x_3 = 0$	$x_4 = 0$	$x_5 = \frac{19}{6}$	$z = 5$

b) **Grafisches Verfahren**:



2. a) **Hilfsproblem (als unzulässiges Tableau):**

$$\begin{array}{rcrcrcrcr} x_3 & = & -4 & + & x_1 & + & x_2 & + & x_0 \\ x_4 & = & 2 & - & x_1 & - & 2x_2 & + & x_0 \\ x_5 & = & -1 & + & x_1 & - & x_2 & + & x_0 \\ \hline w & = & & & & & & - & x_0 \end{array}$$

Eingangsvariable: x_0, Ausgangsvariable: x_3

$x_0 = 4 - x_1 - x_2 + x_3$

$x_4 = 2 - x_1 - 2x_2 + 4 - x_1 - x_2 + x_3 = 6 - 2x_1 - 3x_2 + x_3$

$x_5 = -1 + x_1 - x_2 + 4 - x_1 - x_2 + x_3 = 3 - 2x_2 + x_3$

$w = -4 + x_1 + x_2 - x_3$

Zulässiges Starttableau:

$$\begin{array}{rcrcrcrcr} x_0 & = & 4 & - & x_1 & - & x_2 & + & x_3 \\ x_4 & = & 6 & - & 2x_1 & - & 3x_2 & + & x_3 \\ x_5 & = & 3 & & & - & 2x_2 & + & x_3 \\ \hline w & = & -4 & + & x_1 & + & x_2 & - & x_3 \end{array}$$

Eingangsvariable: x_1, Ausgangsvariable: x_4

$x_1 = 3 - \frac{3}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_4$

$x_0 = 4 - \left(3 - \frac{3}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - x_2 + x_3 = 1 - \frac{5}{2}x_2 + \frac{3}{2}x_3 + \frac{1}{2}x_4$

$x_5 = 3 - 2x_2 + x_3$

$w = -4 + \left(3 - \frac{3}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + x_2 - x_3 = -1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4$

Tableau nach 1. Iteration:

$$\begin{array}{rcrcrcrcr} x_1 & = & 3 & - & \frac{3}{2}x_2 & + & \frac{1}{2}x_3 & - & \frac{1}{2}x_4 \\ x_0 & = & 1 & - & \frac{5}{2}x_2 & + & \frac{3}{2}x_3 & + & \frac{1}{2}x_4 \\ x_5 & = & 3 & & & - & 2x_2 & + & x_3 \\ \hline w & = & -1 & - & \frac{1}{2}x_2 & - & \frac{1}{2}x_3 & - & \frac{1}{2}x_4 \end{array}$$

Da $w, x_2, x_3, x_4 > 0$ gilt, ist dies unlösbar. Damit ist das Originalproblem unlösbar.

b) **Hilfsproblem (als unzulässiges Tableau):**

$$\begin{array}{rcrcrcrcr} x_3 & = & 8 & - & x_1 & + & x_2 & + & x_0 \\ x_4 & = & -3 & + & x_1 & + & x_2 & + & x_0 \\ x_5 & = & 1 & + & x_1 & - & 4x_2 & + & x_0 \\ \hline w & = & & & & & & - & x_0 \end{array}$$

Eingangsvariable: x_0, Ausgangsvariable: x_4

$x_0 = 3 - x_1 - x_2 - x_4$

$x_3 = 8 - x_1 + x_2 + 3 - x_1 - x_2 - x_4 = 11 - 2x_1 - x_4$

$x_5 = 2 - 4x_2 + 3 - x_1 - x_2 - x_4 = 5 - 5x_2 - x_4$

$w = -3 + x_1 + x_2 + x_4$

Zulässiges Starttableau:

$$\begin{array}{rcrcrcrcr}
x_0 & = & 3 & - & x_1 & - & x_2 & - & x_4 \\
x_4 & = & 11 & - & 2x_1 & & & - & x_4 \\
x_5 & = & 5 & & & - & 5x_2 & - & x_4 \\
\hline
w & = & -3 & + & x_1 & + & x_2 & + & x_4
\end{array}$$

Eingangsvariable: x_2, Ausgangsvariable: x_5

$x_2 = 1 - \frac{1}{5}x_4 - \frac{1}{5}x_5$

$x_0 = 3 - x_1 - \left(1 - \frac{1}{5}x_4 - \frac{1}{5}x_5\right) - x_4 = 2 - x_1 - \frac{4}{5}x_4 + \frac{1}{5}x_5$

$x_3 = 11 - 2x_2 - x_4$

$w = -3 + x_1 + 1 - \frac{1}{5}x_4 - \frac{1}{5}x_5 + x_4 = -2 + x_1 + \frac{4}{5}x_4 - \frac{1}{5}x_5$

Tableau nach 1. Iteration:

$$\begin{array}{rcrcrcrcr}
x_2 & = & 1 & - & & & \frac{1}{5}x_4 & - & \frac{1}{5}x_5 \\
x_0 & = & 2 & - & x_1 & - & \frac{4}{5}x_4 & + & \frac{1}{5}x_5 \\
x_3 & = & 11 & & & - & 2x_2 & - & x_4 \\
\hline
w & = & -2 & + & x_1 & + & \frac{4}{5}x_4 & - & \frac{1}{5}x_5
\end{array}$$

Eingangsvariable: x_1, Ausgangsvariable: x_0

$x_1 = 2 - \frac{4}{5}x_4 + \frac{1}{5}x_5 - x_0$

$x_2 = 1 - \frac{1}{5}x_4 - \frac{1}{5}x_5$

$x_3 = 11 - 2\left(2 - \frac{4}{5}x_4 + \frac{1}{5}x_5 - x_0\right) - x_4 = 7 + \frac{3}{5}x_4 - \frac{2}{5}x_5 + 2x_0$

$w = -2 + 2 - \frac{4}{5}x_4 + \frac{1}{5}x_5 - x_0 + \frac{4}{5}x_4 - \frac{1}{5}x_5 = -x_0$

Tableau nach 2. Iteration:

$$\begin{array}{rcrcrcrcr}
x_1 & = & 2 & - & \frac{4}{5}x_4 & + & \frac{1}{5}x_5 & - & x_0 \\
x_2 & = & 1 & - & \frac{1}{5}x_4 & - & \frac{1}{5}x_5 & & \\
x_3 & = & 7 & + & \frac{3}{5}x_4 & - & \frac{2}{5}x_5 & + & 2x_0 \\
\hline
w & = & & & & & & & -x_0
\end{array}$$

Da $w, x_0 > 0$ gilt, ist dies unlösbar. Damit ist das Originalproblem unlösbar.