ALA 07 (HA) zum 06.06.2013

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1. (i)

$$\frac{x+1}{x^2-x-6} = \frac{A}{x+2} + \frac{B}{x-3} = \frac{(A+B)x+(2B-3A)}{(x+2)(x-3)}$$

$$\Rightarrow \begin{array}{rcl} A+B &=& 1 \\ 2B-3A &=& 1 \end{array} \qquad \Rightarrow \begin{array}{rcl} A &=& 1-B \\ 4 &=& 5B \end{array} \qquad \Rightarrow \begin{array}{rcl} A &=& 1/5 \\ B &=& 4/5 \end{array}$$

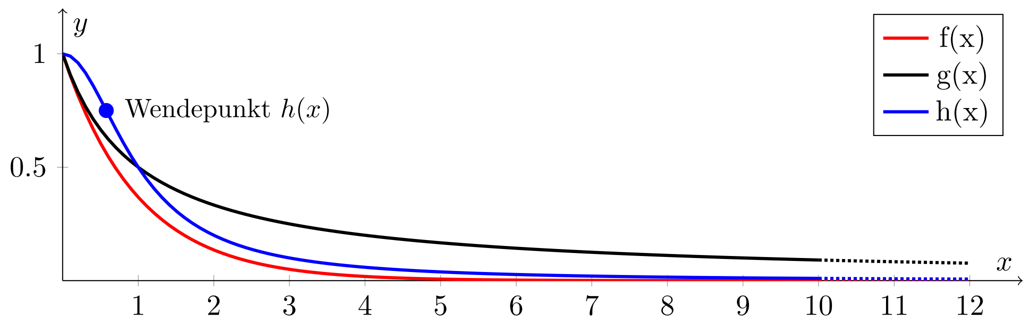
$$\int \frac{x+1}{x^2-x-6}\,\mathrm{d}x = \int \frac{1}{5(x+2)} + \frac{4}{5(x-3)}\,\mathrm{d}x = \frac{1}{5}\ln|x+2| + \frac{4}{5}\ln|x-3|$$

Probe:
$\frac{1}{5}\ln|x+2| + \frac{4}{5}\ln|x-3| = \frac{1}{5}\cdot\frac{1}{x+2} + \frac{4}{5}\cdot\frac{1}{x-3} = \frac{(x-3)+4(x+2)}{5(x+2)(x-3)} = \frac{x+1}{(x+2)(x-3)}$ $\square$

(ii) TODO

(iii) TODO

2. (a) Skizze:



Nur $h(x)$ hat einen Wendepunkt:
$h(x) = \frac{1}{1+x^2}$

$h'(x) = -\frac{2x}{(1+x^2)^2}$

$h''(x) = \frac{-2(1+x^2)^2+8x^2(1+x^2)}{(1+x^2)^4} = \frac{6x^2-2}{(1+x^2)^4}$

Wendestelle bei $6x^2 - 2 = 0 \Leftrightarrow x = \pm\sqrt{\frac{1}{3}}$.

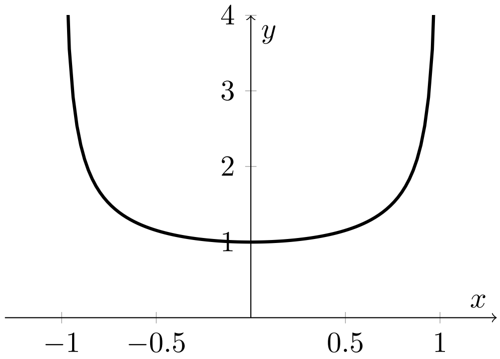
(b)

$$\lim_{z\to\infty}\left(\int_0^z e^{-x}\,\mathrm{d}x\right) = \lim_{z\to\infty}\left(\left[-e^{-x}\right]_0^z\right) = \lim_{z\to\infty}\left(e^0 - e^{-z}\right) = 1$$

$$\lim_{z\to\infty}\left(\int_0^z \frac{1}{1+x}\,\mathrm{d}x\right) = \lim_{z\to\infty}\left([\ln(x+1)]_0^z\right) = \lim_{z\to\infty}\left(\ln(z+1)\right) - \ln(1) = \infty$$

$$\lim_{z\to\infty}\left(\int_0^z \frac{1}{1+x^2}\,\mathrm{d}x\right) = \lim_{z\to\infty}\left([\arctan x]_0^z\right) = \lim_{z\to\infty}\left(\arctan(z)\right) - \arctan(0) = \frac{\pi}{2}$$

(c) Skizze:



Da die Funktion an der y-Achse spiegelsymmetrisch ist, gilt für die Fläche:

$$A = 2\cdot\int_0^1 \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = 2\left[\arcsin(x)\right]_0^1 = 2\left(\arcsin(1) - \arcsin(0)\right) = 2(\frac{\pi}{2} - 0) = \pi$$

3. (i) $\frac{1}{8}\left(\sin(0) + 2\sin(0.25) + 2\sin(0.5) + 2\sin(0.75) + \sin(1)\right) \approx 0.45720099376$

(ii) $\frac{1}{10}\left(\sin(0) + 2\sin(0.2) + \cdots + 2\sin(0.8) + \sin(1)\right) \approx 0.458164346$

(iii) $\frac{1}{20}\left(\sin(0) + 2\sin(0.1) + \cdots + 2\sin(0.9) + \sin(1)\right) \approx 0.4593145489$

4. (a) $f(1) = 10\cdot e^{-\frac{2}{5}} \approx 6.7032$

$f(2) = 20\cdot e^{-\frac{4}{5}} \approx 8.9866$

$f(6) = 60\cdot e^{-\frac{12}{5}} \approx 5.4431$

$f(12) = 120\cdot e^{-\frac{24}{5}} \approx 0.9876$

(b) $f'(x) = 10x \cdot e^{-\frac{2}{5}x} \cdot \left(-\frac{2}{5}\right) + 10e^{-\frac{2}{5}x} = 10e^{-\frac{2}{5}x}\left(1 - \frac{2}{5}\right) = 0 \Leftrightarrow x = \frac{5}{2}$

$f(\frac{5}{2}) = 25e \approx 9.1970$

Die maximale Konzentration von etwa $9.1970\frac{mg}{l}$ wird nach zweieinhalb Stunden erreicht.

(c)

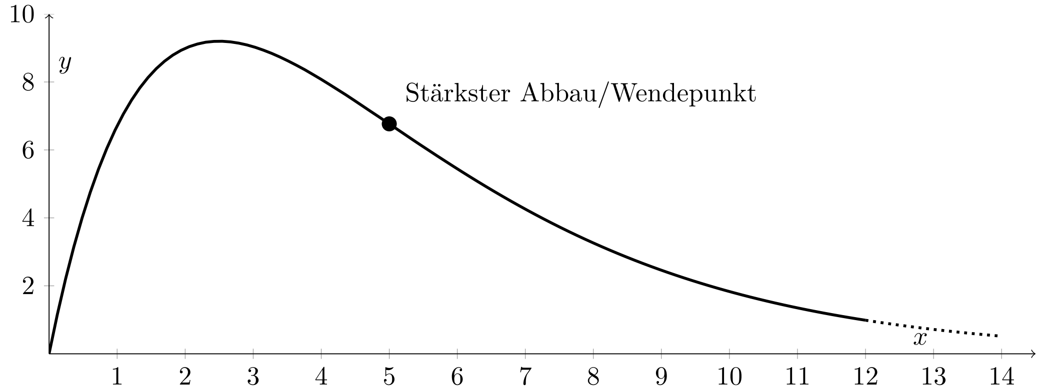
$$\begin{aligned}\int e^{-\frac{2}{5}x} \cdot 10x \,\mathrm{d}x &= -\frac{5}{2}e^{-\frac{2}{5}x} \cdot 10x - \int -\frac{5}{2}e^{-\frac{2}{5}x} \cdot 10 \,\mathrm{d}x \\ &= -25x \cdot e^{-\frac{2}{5}x} + 25 \int e^{-\frac{2}{5}x} \,\mathrm{d}x \\ &= -25x \cdot e^{-\frac{2}{5}x} + 25 \cdot \left(-\frac{5}{2}\right) \cdot e^{-\frac{2}{5}x} \\ &= -25x \cdot e^{-\frac{2}{5}x}\left(x + \frac{5}{2}\right)\end{aligned}$$

$$\frac{1}{6}\left[-25x \cdot e^{-\frac{2}{5}x}\left(x + \frac{5}{2}\right)\right]_0^6 \approx 7.2037$$

(d)

$$\frac{1}{6}\left[-25x \cdot e^{-\frac{2}{5}x}\left(x + \frac{5}{2}\right)\right]_6^{12} \approx 2.7157$$

(e) Skizze:



$f'(x) = 10e^{-\frac{2}{5}x}\left(1 - \frac{2}{5}x\right)$

$f''(x) = e^{-\frac{2}{5}x}\left(-8 + \frac{8}{5}x\right) = 0 \Rightarrow -8 + \frac{8}{5}x = 0 \Rightarrow x = 5$

5. (a)

$$h(x) = \left(x^2 + 1\right)^{\cos x} = e^{\ln\left(x^2+1\right)^{\cos x}} = e^{\cos x \cdot \ln\left(x^2+1\right)})$$

$$h'(x) = \left(x^2+1\right)^{\cos x} \cdot \left(-\sin x \cdot \ln\left(x^2+1\right) + \cos x \cdot \frac{2x}{x^2+1}\right)$$

(b) Es sei

$$t = \sqrt{\frac{x}{4}+3} \Leftrightarrow x = 4t^2 - 12 \Rightarrow \frac{dt}{dx} = 8t$$

Dann gilt

$$\begin{aligned}
\int \sin\left(\sqrt{\frac{x}{4}+3}\right) \mathrm{d}x &= \int \sin t \cdot 8t \, \mathrm{d}t \\
&= -\cos t \cdot 8t - \int -8cost \, \mathrm{d}t \\
&= -8t \cdot \cos t + 8 \sin t \\
&= -8\sqrt{\frac{x}{4}+3} \cdot \cos\left(\sqrt{\frac{x}{4}+3}\right) + 8 \sin\left(\sqrt{\frac{x}{4}+3}\right)
\end{aligned}$$

(c) TODO

(d) TODO