

ALA 09 (HA) zum 20.06.2013

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1. (a)

$$\begin{aligned} T_8(x) &= T_9(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \\ T_{10}(x) &= T_{11}(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} \\ T_{12}(x) &= T_{13}(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} \end{aligned}$$

$$\begin{aligned} T_9(1) &= \frac{1}{1} - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} = \frac{4357}{8064} \approx 0.540302579365 \\ T_{11}(1) &= \frac{1}{1} - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} - \frac{1}{10!} = \frac{1960649}{3628800} \approx 0.540302303791 \\ T_{13}(1) &= \dots = \dots \approx 0.540302305879 \end{aligned}$$

(b) Taylorpolynome für $f(x)$ und $g(x)$ an $x_0 = 0$:

$$\begin{aligned} f(x) &= \sqrt{1+x} & g(x) &= \frac{1}{\sqrt[3]{1+x}} \\ T_0(x) &= 1 & T_0(x) &= 1 \\ T_1(x) &= 1 + \frac{x}{2} & T_1(x) &= 1 - \frac{x}{3} \\ T_2(x) &= 1 + \frac{x}{2} - \frac{x^2}{4 \cdot 2!} & T_2(x) &= 1 - \frac{x}{3} + \frac{4x^2}{9 \cdot 2!} \\ T_3(x) &= 1 + \frac{x}{2} - \frac{x^2}{4 \cdot 2!} + \frac{x^3}{2 \cdot 3!} & T_3(x) &= 1 - \frac{x}{3} + \frac{4x^2}{9 \cdot 2!} - \frac{28x^3}{27 \cdot 3!} \\ T_4(x) &= 1 + \frac{x}{2} - \frac{x^2}{4 \cdot 2!} + \frac{x^3}{2 \cdot 3!} - \frac{5x^4}{4 \cdot 4!} & T_4(x) &= 1 - \frac{x}{3} + \frac{4x^2}{9 \cdot 2!} - \frac{28x^3}{27 \cdot 3!} + \frac{280x^4}{81 \cdot 4!} \end{aligned}$$

(c) Taylorpolynom für $f(x) = e^x \cdot \sin x$ an $x_0 = 0$:

$$T_5(x) = x + x^2 + \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{30}$$

2. (i)

(ii)

(iii)

(iv)

3. (a)

(b)

(c)

(d)

4. (a)

(b)

(c)