ALA 07 (HA) zum 06.06.2013

Paul Bienkowski, Hans Ole Hatzel

6. Juni 2013

1. (i)

$$\frac{x+1}{x^2-x-6} = \frac{A}{x+2} + \frac{B}{x-3} = \frac{(A+B)x+(2B-3A)}{(x+2)(x-3)}$$

$$\Rightarrow \begin{array}{rcl} A+B &=& 1 \\ 2B-3A &=& 1 \end{array} \qquad \Rightarrow \begin{array}{rcl} A &=& 1-B \\ 4 &=& 5B \end{array} \qquad \Rightarrow \begin{array}{rcl} A &=& 1/5 \\ B &=& 4/5 \end{array}$$

$$\int \frac{x+1}{x^2-x-6}\,\mathrm{d}x = \int \frac{1}{5(x+2)} + \frac{4}{5(x-3)}\,\mathrm{d}x = \frac{1}{5}\ln|x+2| + \frac{4}{5}\ln|x-3|$$

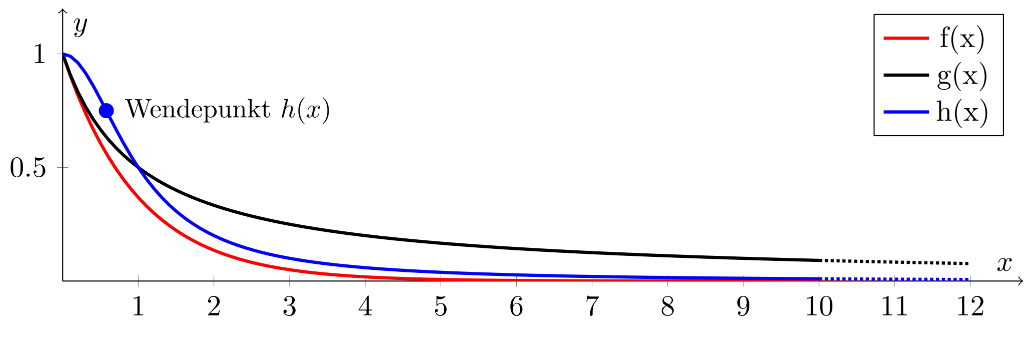
Probe:

$\frac{1}{5}\ln|x+2| + \frac{4}{5}\ln|x-3| = \frac{1}{5}\cdot\frac{1}{x+2} + \frac{4}{5}\cdot\frac{1}{x-3} = \frac{(x-3)+4(x+2)}{5(x+2)(x-3)} = \frac{x+1}{(x+2)(x-3)}$ $\square$

(ii)

(iii)

2. (a) Skizze:



Nur $h(x)$ hat einen Wendepunkt:

$h(x) = \frac{1}{1+x^2}$

$h'(x) = -\frac{2x}{(1+x^2)^2}$

$h''(x) = \frac{-2(1+x^2)^2+8x^2(1+x^2)}{(1+x^2)^4} = \frac{6x^2-2}{(1+x^2)^4}$

Wendestelle bei $6x^2 - 2 = 0 \Leftrightarrow x = \pm\sqrt{\frac{1}{3}}$.

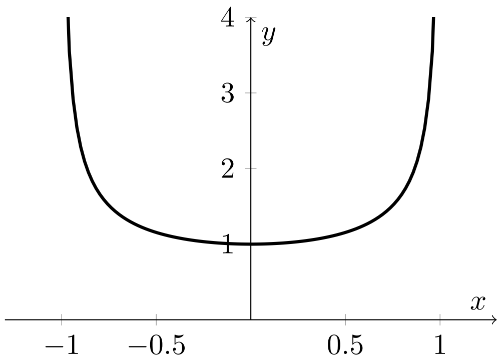
(b)

$$\lim_{z\to\infty}\left(\int_0^z e^{-x}\,\mathrm{d}x\right) = \lim_{z\to\infty}\left(\left[-e^{-x}\right]_0^z\right) = \lim_{z\to\infty}\left(e^0 - e^{-z}\right) = 1$$

$$\lim_{z\to\infty}\left(\int_0^z \frac{1}{1+x}\,\mathrm{d}x\right) = \lim_{z\to\infty}\left([\ln(x+1)]_0^z\right) = \lim_{z\to\infty}\left(\ln(z+1)\right) - \ln(1) = \infty$$

$$\lim_{z\to\infty}\left(\int_0^z \frac{1}{1+x^2}\,\mathrm{d}x\right) = \lim_{z\to\infty}\left([\arctan x]_0^z\right) = \lim_{z\to\infty}\left(\arctan(z)\right) - \arctan(0) = \frac{\pi}{2}$$

(c) Skizze:



Da die Funktion an der y-Achse spiegelsymmetrisch ist, gilt für die Fläche:

$$A = 2\cdot\int_0^1 \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = 2\left[\arcsin(x)\right]_0^1 = 2\left(\arcsin(1) - \arcsin(0)\right) = 2(\frac{\pi}{2} - 0) = \pi$$

3. (i)

(ii)

(iii)

4. (a)

(b)

(c)

(d)

(e)

5. (a)

(b)

(c)

(d)