

## Homework 1

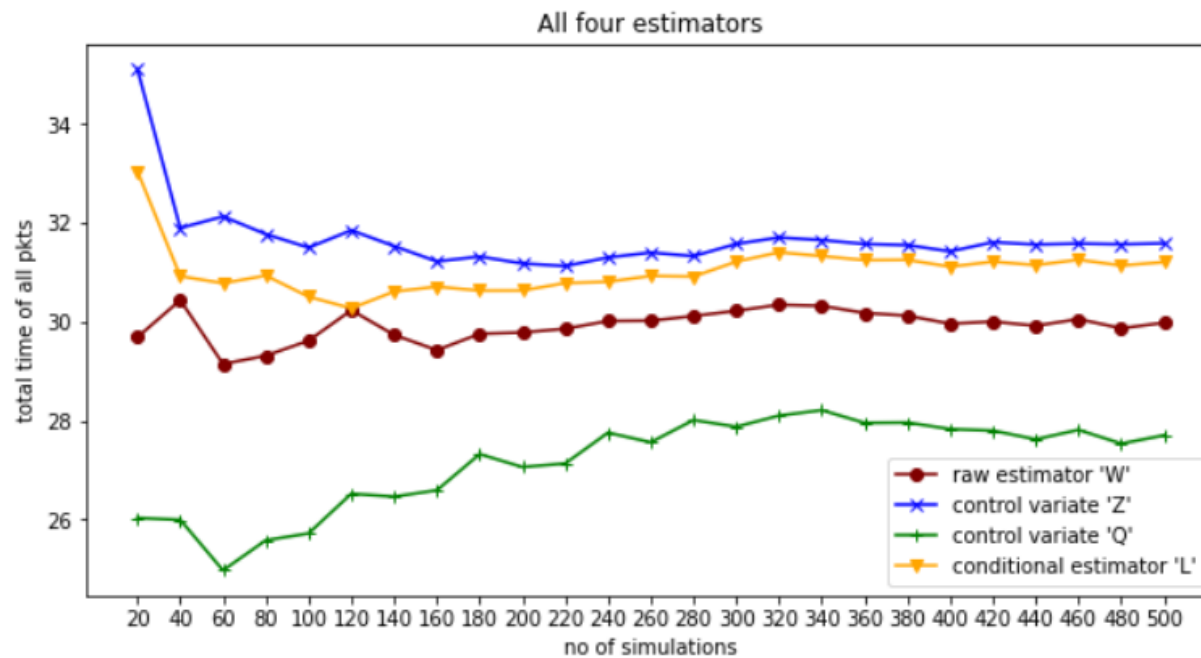
The summary statistics of the packets simulation in the router network is shown in the figure below.

	packets	arrival time	queue	serving time	departure time	time in queue	time in server	total time in system
0	1	0.27	1	0.27	0.27	0.00	0.00	0.00
1	2	0.91	1	0.91	1.07	0.00	0.16	0.16
2	3	1.09	1	1.09	1.29	0.00	0.21	0.21
3	4	1.14	2	1.29	1.80	0.16	0.51	0.66
4	5	1.35	2	1.80	2.95	0.45	1.16	1.61
5	6	1.73	3	2.95	2.98	1.22	0.03	1.25
6	7	2.01	3	2.98	4.09	0.98	1.11	2.09
7	8	2.12	4	4.09	7.54	1.97	3.45	5.42
8	9	3.17	3	7.54	7.63	4.37	0.09	4.46
9	10	3.44	4	7.63	7.67	4.19	0.04	4.23

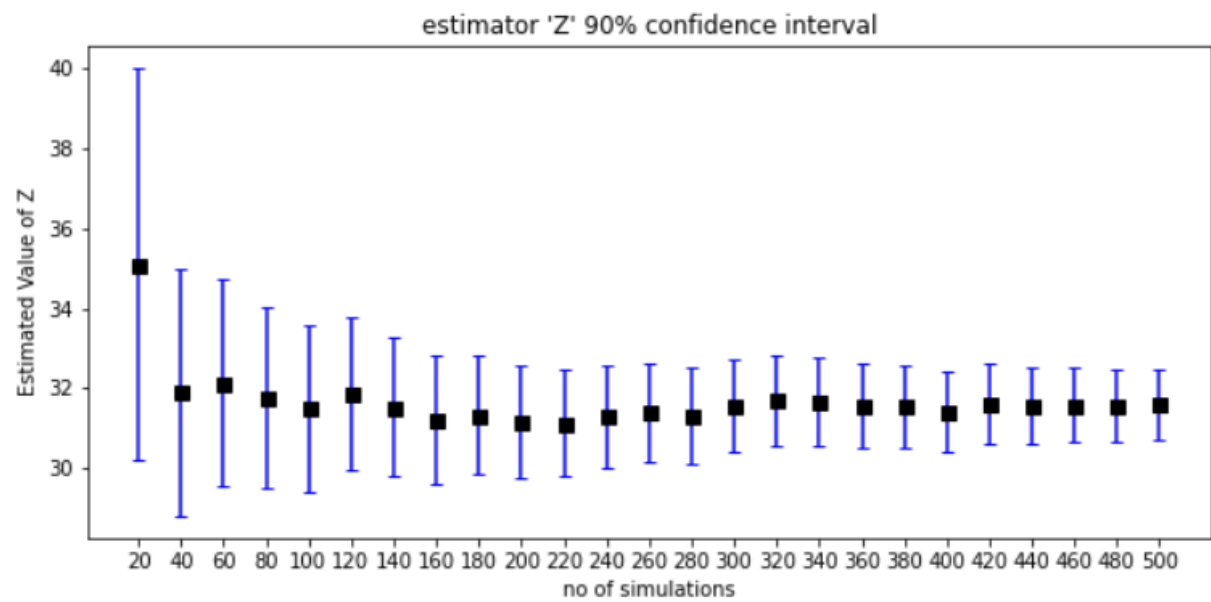
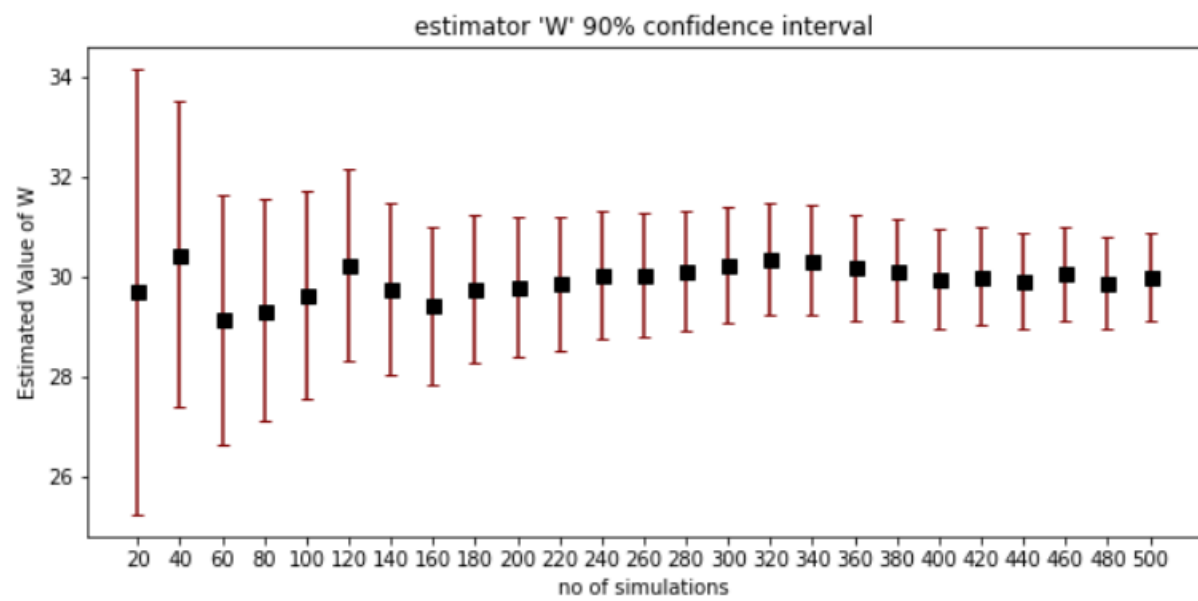
The average time for each simulation run was observed to be 0.04 seconds. We did 500 simulations to determine our parameter of interest (total time spent by the 10 packets in the Wi-Fi network) and it took about 255 seconds for consecutive simulation runs done in steps of 20. We also determine the average time a packet spent in each stage of the Wi-Fi system (queue, server, and the combined time in both) and the figure below shows the values obtained over 10 different simulations. This is understandable considering that packets arrival time and service times are not deterministic.

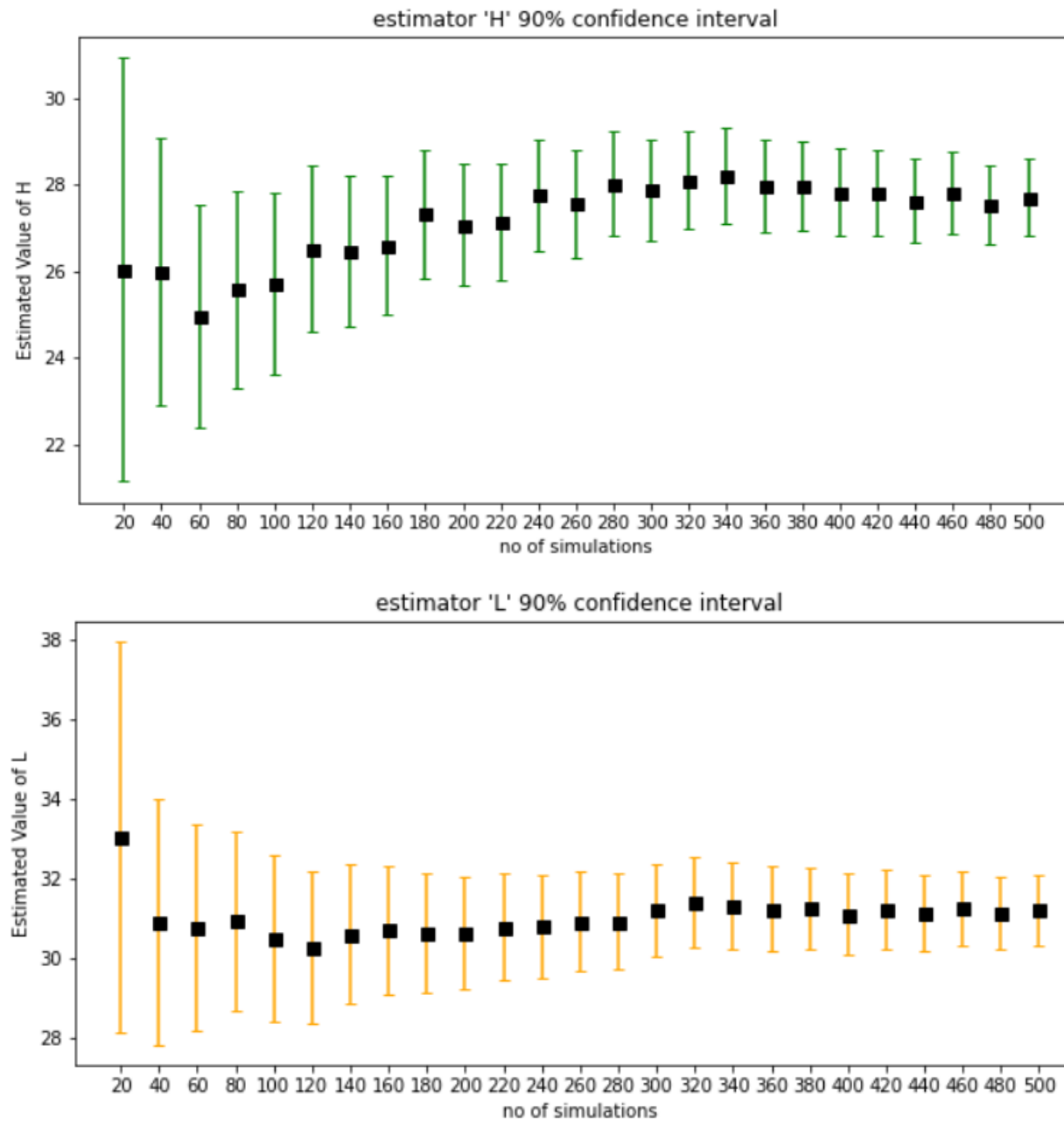
	run number	time in queue	time in server	time in system
0	1	1.812000	1.192000	3.006000
1	2	1.334000	0.676000	2.009000
2	3	2.262222	0.674444	2.935556
3	4	2.024444	0.994444	3.020000
4	5	2.079000	0.762000	2.841000
5	6	1.213000	0.545000	1.758000
6	7	4.270000	1.039000	5.310000
7	8	2.566667	1.720000	4.286667
8	9	1.171000	1.000000	2.171000
9	10	3.466000	1.006000	4.469000

**1a.** The plots of the four estimators are shown in the figure below. It is observed that the control variate estimators Z and the conditional estimator L seem to converge to a value faster than the raw estimator and the control variate estimator Q as the number of simulations increases. This clearly demonstrated that the estimators Z, Q, and L are better estimators of our parameter of interest. On the other hand, estimator Q appears to have a slightly different values to the other three estimators. Whether or not, estimator Q converge to the same value as the other three estimators might be explained by running the simulations for longer periods.



**1b.** The normalized 90% confidence interval width plots of the four estimators are shown in the figure below. As expected, the confidence interval shrinks for all four estimators as the number of simulations increases. Other observed behavior follows the preceding description.





**2a.** The minimum number of simulations required to be 90% confident that the true value of lies within  $\pm 10\%$  of our estimated values for all four estimators is:

Minimum number of simulations needed for estimator W is: 60  
 Minimum number of simulations needed for estimator Z is: 40  
 Minimum number of simulations needed for estimator H is: 40  
 Minimum number of simulations needed for estimator L is: 40

**2b.** Considering that we ran our simulations in incremental steps of 20, it is possible it could have taken fewer number of simulations than the values we obtained in the previous graphic for the two control variates estimators and the conditional estimator to better the raw estimator. But essentially, the results indicate that the other three estimators exceeds the raw estimator by a minimum of 20 simulation runs.